Computer algebra independent integration tests

4-Trig-functions/4.5-Secant/4.5.1.3-d-sin-^n-a+b-sec-^m

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July 17, 2021 Compiled on July 17, 2021 at 5:06pm

Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [306]. This is test number [119].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.67 (305)	% 0.33 (1)
Mathematica	% 99.35 (304)	% 0.65 (2)
Maple	% 87.25 (267)	% 12.75 (39)
Maxima	% 57.19 (175)	% 42.81 (131)
Fricas	% 62.42 (191)	% 37.58 (115)
Sympy	% 2.29 (7)	% 97.71 (299)
Giac	% 62.42 (191)	% 37.58 (115)
Mupad	% 63.07 (193)	% 36.93 (113)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

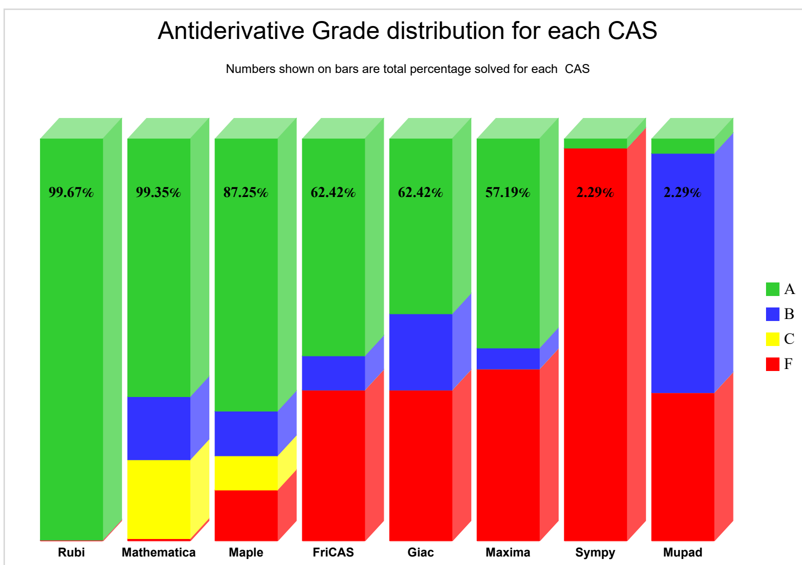
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

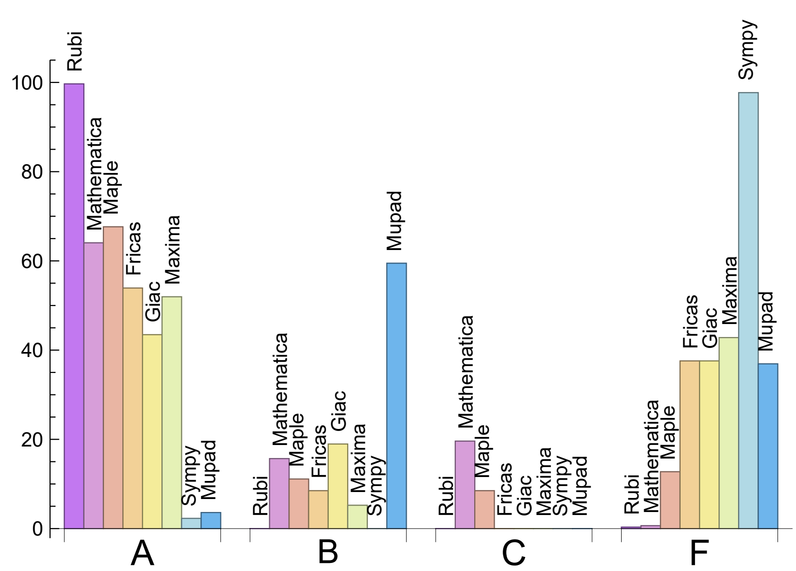
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.67	0.00	0.00	0.33
Mathematica	64.05	15.69	19.61	0.65
Maple	67.65	11.11	8.50	12.75
Maxima	51.96	5.23	0.00	42.81
Fricas	53.92	8.50	0.00	37.58
Sympy	2.29	0.00	0.00	97.71
Giac	43.46	18.95	0.00	37.58
Mupad	3.59	59.48	0.00	36.93

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	1	100.00 %	0.00 %	0.00 %
Mathematica	2	100.00 %	0.00 %	0.00 %
Maple	39	100.00 %	0.00 %	0.00 %
Maxima	131	61.07 %	26.72 %	12.21 %
Fricas	115	85.22 %	14.78 %	0.00 %
Sympy	299	55.52 %	44.48 %	0.00 %
Giac	115	96.52 %	1.74 %	1.74 %
Mupad	113	100.00 %	0.00 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

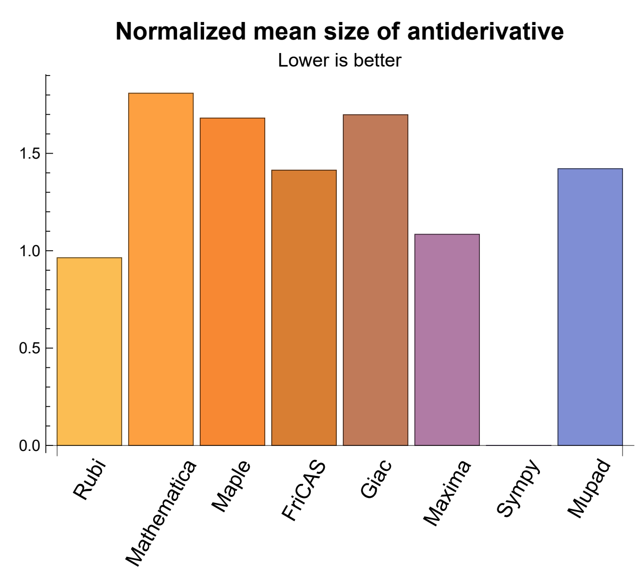
1.3 Performance

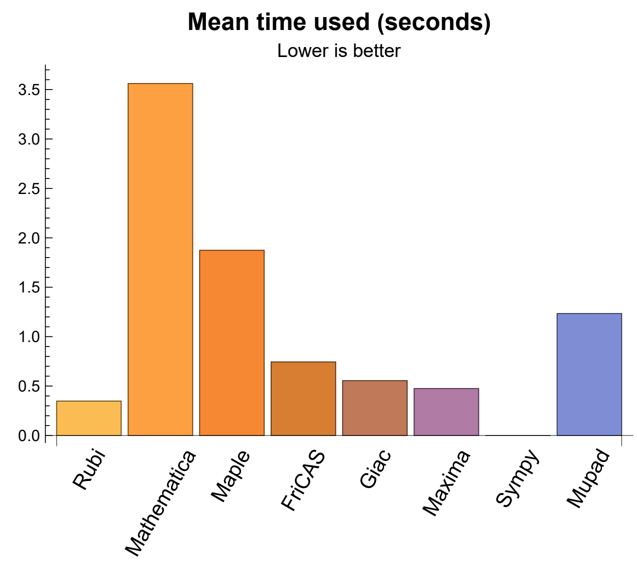
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.35	169.10	0.96	127.00	1.00
Mathematica	3.56	331.39	1.81	136.50	1.07
Maple	1.87	354.67	1.68	157.00	1.21
Maxima	0.47	127.21	1.08	106.00	0.97
Fricas	0.74	210.66	1.41	125.00	1.24
Sympy	0.00	0.00	0.00	0.00	0.00
Giac	0.55	248.10	1.70	149.00	1.42
Mupad	1.23	248.97	1.42	107.00	0.98

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{263, 264, 265, 266, 267, 273, 274, 277, 278, 279, 280}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {37, 56, 114, 115, 116, 117, 118, 119, 134, 135, 138, 140, 141, 142, 143, 144, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 254, 255, 260, 261, 262, 272, 275, 276, 287, 288, 289, 290, 291, 292}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

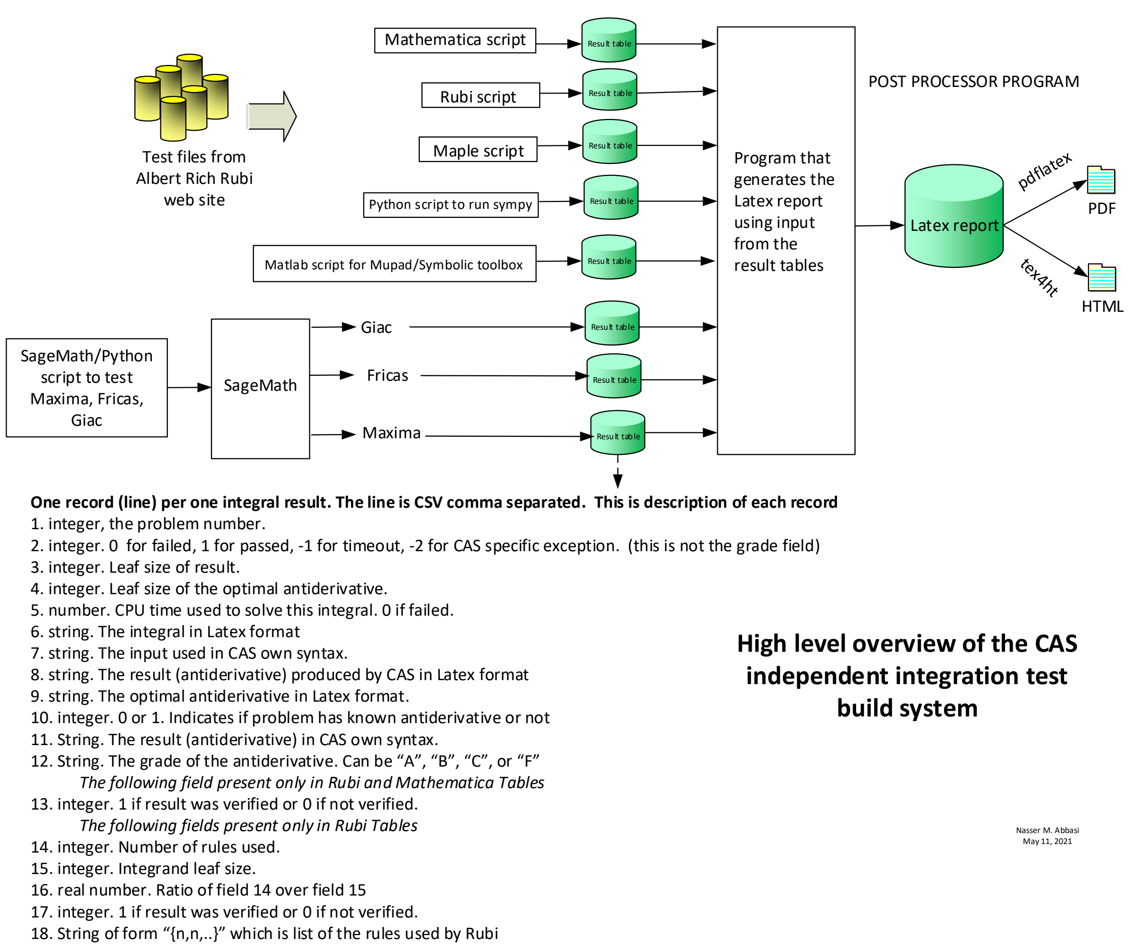
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

B grade: { }

C grade: { }

F grade: { 276 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 120, 122, 124, 126, 127, 129, 131, 133, 136, 145, 146, 147, 148, 150, 154, 155, 160, 161, 162, 163, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 179, 180, 181, 185, 186, 187, 188, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 228, 230, 231, 232, 249, 250, 252, 253, 254, 256, 257, 258, 259, 263, 264, 265, 266, 267, 269, 270, 271, 273, 274, 277, 278, 279, 280, 281, 283, 285, 293, 295, 297, 299, 300, 302, 304, 306 }

B grade: { 6, 32, 33, 34, 35, 36, 37, 51, 52, 53, 54, 55, 56, 70, 71, 72, 73, 140, 141, 142, 143, 144, 149, 151, 156, 157, 158, 159, 164, 178, 182, 183, 184, 189, 190, 191, 192, 193, 194, 195, 229, 260, 261, 262, 268, 272, 275, 276 }

C grade: { 14, 15, 16, 17, 18, 114, 115, 116, 117, 118, 119, 121, 123, 125, 128, 130, 132, 134, 135, 138, 152, 153, 171, 172, 173, 226, 227, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 251, 255, 282, 284, 286, 287, 288, 289, 290, 291, 292, 294, 296, 298, 301, 303, 305 }

F grade: { 137, 139 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 219, 220, 221, 222, 223, 224, 225, 226, 227, 231, 232, 239, 245, 246, 247, 248, 249, 253, 263, 264, 265, 266, 267, 273, 274, 277, 278, 279, 280 }

B grade: { 55, 56, 65, 66, 67, 68, 84, 100, 203, 204, 205, 216, 217, 218, 228, 229, 230, 233, 234, 235, 236, 237, 238, 240, 241, 242, 243, 244, 250, 251, 252, 254, 255, 256 }

C grade: { 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

F grade: { 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 257, 258, 259, 260, 261, 262, 268, 269, 270, 271, 272, 275, 276 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 69, 70, 74, 75, 76, 77, 78, 79, 80, 81, 82, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 209, 210, 211, 212, 213, 214, 221, 222, 223, 224, 225, 226, 263, 264, 265, 266, 267, 273, 274, 277, 278, 279, 280 }

B grade: { 65, 66, 67, 68, 71, 72, 73, 83, 84, 85, 86, 100, 101, 102, 215, 227 }

C grade: { }

F grade: { 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 203, 204, 205, 206, 207, 208, 216, 217, 218, 219, 220, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 270, 271, 272, 275, 276, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 160, 161, 162, 163, 164, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 216, 217, 218, 219, 220, 221, 222, 223, 224, 228, 229, 230, 231, 232, 263, 264, 265, 266, 267, 273, 274, 277, 278, 279, 280 }

B grade: { 9, 16, 17, 18, 28, 37, 46, 47, 64, 71, 72, 73, 82, 97, 98, 99, 165, 166, 167, 202, 208, 214, 215, 225, 226, 227 }

C grade: { }

F grade: { 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 270, 271, 272, 275, 276, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

2.1.6 Sympy

A grade: { 264, 265, 266, 274, 278, 279, 280 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 267, 268, 269, 270, 271, 272, 273, 275, 276, 277, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

2.1.7 Giac

A grade: { 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 41, 42, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 162, 163, 168, 175, 176, 177, 180, 186, 187, 191, 193, 194, 195, 198, 199, 200, 201, 206, 211, 212, 216, 217, 218, 219, 220, 223, 224, 229, 231, 232, 263, 264, 265, 266, 267, 273, 274, 277, 278, 279, 280 }

B grade: { 1, 2, 3, 19, 20, 21, 24, 38, 40, 43, 44, 76, 77, 81, 93, 160, 161, 164, 165, 166, 167, 169, 170, 171, 172, 173, 174, 178, 179, 181, 182, 183, 184, 185, 188, 189, 190, 192, 196, 197, 202, 203, 204, 205, 207, 208, 209, 210, 213, 214, 215, 221, 222, 225, 226, 227, 228, 230 }

C grade: { }

F grade: { 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 270, 271, 272, 275, 276, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

2.1.8 Mupad

A grade: { 263, 264, 265, 266, 267, 273, 274, 277, 278, 279, 280 }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 148, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 270 }

C grade: { }

F grade: { 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 271, 272, 275, 276, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	106	163	113	115	0	293	111
normalized size	1	1.00	0.70	1.07	0.74	0.76	0.00	1.93	0.73
time (sec)	N/A	0.107	0.219	0.668	0.337	1.705	0.000	0.252	0.131

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	86	129	91	93	0	247	89
normalized size	1	1.00	0.72	1.08	0.76	0.78	0.00	2.08	0.75
time (sec)	N/A	0.097	0.139	0.614	0.618	0.717	0.000	0.242	0.079

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	83	95	69	71	0	201	67
normalized size	1	1.00	0.95	1.09	0.79	0.82	0.00	2.31	0.77
time (sec)	N/A	0.087	0.087	0.595	0.388	0.510	0.000	0.583	0.056

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	61	47	49	0	66	45
normalized size	1	1.00	0.98	1.05	0.81	0.84	0.00	1.14	0.78
time (sec)	N/A	0.077	0.058	0.609	0.619	0.523	0.000	0.357	0.063

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	37	28	23	25	0	32	20
normalized size	1	1.00	1.42	1.08	0.88	0.96	0.00	1.23	0.77
time (sec)	N/A	0.031	0.020	0.204	0.349	0.512	0.000	1.452	0.038

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	63	15	26	31	0	58	17
normalized size	1	1.00	2.10	0.50	0.87	1.03	0.00	1.93	0.57
time (sec)	N/A	0.058	0.035	0.362	0.353	0.481	0.000	0.500	0.124

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	114	48	52	93	0	102	53
normalized size	1	1.00	1.56	0.66	0.71	1.27	0.00	1.40	0.73
time (sec)	N/A	0.095	0.843	0.590	0.355	0.487	0.000	0.425	0.963

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	164	80	95	193	0	149	99
normalized size	1	1.00	1.39	0.68	0.81	1.64	0.00	1.26	0.84
time (sec)	N/A	0.120	0.358	0.473	0.517	0.742	0.000	0.307	0.099

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	165	112	136	307	0	196	142
normalized size	1	1.00	1.01	0.69	0.83	1.88	0.00	1.20	0.87
time (sec)	N/A	0.150	0.460	0.460	0.450	0.775	0.000	0.315	0.994

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	106	164	127	123	0	174	150
normalized size	1	1.00	0.64	0.99	0.77	0.75	0.00	1.05	0.91
time (sec)	N/A	0.146	0.348	0.692	0.346	1.667	0.000	2.320	1.144

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	86	130	106	102	0	146	120
normalized size	1	1.00	0.68	1.02	0.83	0.80	0.00	1.15	0.94
time (sec)	N/A	0.128	0.195	0.684	0.474	0.768	0.000	0.843	1.047

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	86	96	81	79	0	118	90
normalized size	1	1.00	0.97	1.08	0.91	0.89	0.00	1.33	1.01
time (sec)	N/A	0.111	0.119	0.606	0.612	0.701	0.000	0.963	1.025

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	54	62	59	55	0	88	80
normalized size	1	1.00	1.06	1.22	1.16	1.08	0.00	1.73	1.57
time (sec)	N/A	0.082	0.057	0.285	0.325	0.652	0.000	0.267	1.069

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	41	47	50	63	0	50	29
normalized size	1	1.00	1.11	1.27	1.35	1.70	0.00	1.35	0.78
time (sec)	N/A	0.093	0.030	0.602	0.365	0.657	0.000	0.341	0.961

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	81	76	108	0	79	65
normalized size	1	1.00	1.00	1.17	1.10	1.57	0.00	1.14	0.94
time (sec)	N/A	0.103	0.029	0.726	0.421	0.723	0.000	0.238	0.983

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	91	115	96	190	0	107	97
normalized size	1	1.00	0.90	1.14	0.95	1.88	0.00	1.06	0.96
time (sec)	N/A	0.110	0.033	0.727	0.375	0.549	0.000	0.280	1.005

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	113	149	116	281	0	136	128
normalized size	1	1.00	0.86	1.14	0.89	2.15	0.00	1.04	0.98
time (sec)	N/A	0.117	0.051	0.813	0.451	0.628	0.000	0.620	1.166

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	135	183	136	366	0	164	159
normalized size	1	1.00	0.82	1.11	0.82	2.22	0.00	0.99	0.96
time (sec)	N/A	0.127	0.057	0.813	0.331	0.688	0.000	0.275	1.643

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	127	206	146	167	0	370	146
normalized size	1	1.00	0.69	1.13	0.80	0.91	0.00	2.02	0.80
time (sec)	N/A	0.188	0.937	0.744	0.333	0.568	0.000	1.927	0.993

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	107	168	107	128	0	320	105
normalized size	1	1.00	0.82	1.28	0.82	0.98	0.00	2.44	0.80
time (sec)	N/A	0.168	0.561	0.754	0.337	0.690	0.000	0.361	0.950

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	87	130	94	115	0	270	91
normalized size	1	1.00	0.78	1.16	0.84	1.03	0.00	2.41	0.81
time (sec)	N/A	0.158	0.309	0.740	0.349	0.737	0.000	0.378	0.886

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	65	92	56	76	0	74	54
normalized size	1	1.00	1.05	1.48	0.90	1.23	0.00	1.19	0.87
time (sec)	N/A	0.124	0.213	0.734	0.324	0.848	0.000	0.295	0.058

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	31	46	41	51	0	51	41
normalized size	1	1.00	0.72	1.07	0.95	1.19	0.00	1.19	0.95
time (sec)	N/A	0.077	0.119	0.250	0.345	0.586	0.000	0.494	0.059

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	36	32	43	61	0	115	35
normalized size	1	1.00	0.75	0.67	0.90	1.27	0.00	2.40	0.73
time (sec)	N/A	0.115	0.085	0.408	0.328	0.691	0.000	0.281	0.082

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	75	50	68	112	0	135	61
normalized size	1	1.00	1.09	0.72	0.99	1.62	0.00	1.96	0.88
time (sec)	N/A	0.144	0.558	0.587	0.324	0.571	0.000	0.672	0.901

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	103	85	104	209	0	191	109
normalized size	1	1.00	0.90	0.74	0.90	1.82	0.00	1.66	0.95
time (sec)	N/A	0.171	1.572	0.494	0.319	0.593	0.000	0.494	0.095

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	136	121	143	289	0	238	147
normalized size	1	1.00	0.85	0.76	0.89	1.81	0.00	1.49	0.92
time (sec)	N/A	0.199	1.338	0.503	0.327	0.625	0.000	0.397	0.110

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	164	157	197	461	0	291	203
normalized size	1	1.00	0.80	0.77	0.96	2.25	0.00	1.42	0.99
time (sec)	N/A	0.238	3.455	0.572	0.366	0.642	0.000	0.382	0.171

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	144	210	215	185	0	225	293
normalized size	1	1.00	0.72	1.06	1.08	0.93	0.00	1.13	1.47
time (sec)	N/A	0.361	0.963	0.704	0.438	0.711	0.000	0.786	2.540

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	124	172	174	158	0	193	235
normalized size	1	1.00	0.79	1.10	1.11	1.01	0.00	1.23	1.50
time (sec)	N/A	0.270	0.561	0.694	0.538	0.678	0.000	0.505	2.157

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	94	134	126	133	0	161	177
normalized size	1	1.00	0.82	1.17	1.10	1.16	0.00	1.40	1.54
time (sec)	N/A	0.268	0.243	0.696	0.474	0.946	0.000	1.213	1.825

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	243	86	81	104	0	128	117
normalized size	1	1.00	3.33	1.18	1.11	1.42	0.00	1.75	1.60
time (sec)	N/A	0.132	1.157	0.400	0.431	1.608	0.000	0.294	1.160

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	401	77	74	101	0	90	70
normalized size	1	1.00	7.04	1.35	1.30	1.77	0.00	1.58	1.23
time (sec)	N/A	0.246	6.158	0.687	0.333	0.643	0.000	0.535	1.126

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	228	140	113	159	0	104	91
normalized size	1	1.00	2.62	1.61	1.30	1.83	0.00	1.20	1.05
time (sec)	N/A	0.297	1.853	0.879	0.344	0.690	0.000	0.304	2.485

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	317	202	144	206	0	136	124
normalized size	1	1.00	2.46	1.57	1.12	1.60	0.00	1.05	0.96
time (sec)	N/A	0.226	0.995	0.900	0.352	0.571	0.000	0.695	1.284

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	428	264	175	272	0	168	159
normalized size	1	1.00	2.63	1.62	1.07	1.67	0.00	1.03	0.98
time (sec)	N/A	0.243	1.302	1.021	0.356	0.744	0.000	0.321	0.988

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	1050	326	204	406	0	200	194
normalized size	1	1.00	5.22	1.62	1.01	2.02	0.00	1.00	0.97
time (sec)	N/A	0.258	6.946	1.026	0.347	3.265	0.000	0.610	0.972

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	148	230	158	182	0	396	157
normalized size	1	1.00	0.73	1.13	0.78	0.90	0.00	1.95	0.77
time (sec)	N/A	0.196	1.790	0.772	0.329	0.702	0.000	0.546	0.965

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	106	130	107	121	0	239	107
normalized size	1	1.00	0.81	0.99	0.82	0.92	0.00	1.82	0.82
time (sec)	N/A	0.168	1.009	0.713	0.322	0.775	0.000	1.112	0.897

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	108	155	106	130	0	297	107
normalized size	1	1.00	0.81	1.16	0.79	0.97	0.00	2.22	0.80
time (sec)	N/A	0.167	0.655	0.778	0.326	0.687	0.000	1.276	0.887

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	86	109	80	104	0	102	80
normalized size	1	1.00	0.88	1.11	0.82	1.06	0.00	1.04	0.82
time (sec)	N/A	0.096	0.203	0.803	0.359	0.699	0.000	0.385	0.882

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	65	63	55	65	0	64	52
normalized size	1	1.00	1.05	1.02	0.89	1.05	0.00	1.03	0.84
time (sec)	N/A	0.091	0.250	0.250	0.408	0.861	0.000	0.316	0.055

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	81	49	56	76	0	142	49
normalized size	1	1.00	1.21	0.73	0.84	1.13	0.00	2.12	0.73
time (sec)	N/A	0.125	0.146	0.477	0.561	0.736	0.000	0.311	0.074

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	88	67	84	132	0	189	75
normalized size	1	1.00	1.00	0.76	0.95	1.50	0.00	2.15	0.85
time (sec)	N/A	0.156	0.904	0.727	0.501	0.742	0.000	0.401	0.095

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	100	85	103	177	0	186	96
normalized size	1	1.00	0.90	0.77	0.93	1.59	0.00	1.68	0.86
time (sec)	N/A	0.169	0.947	0.620	0.659	0.786	0.000	0.469	0.931

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	129	120	145	297	0	243	151
normalized size	1	1.00	0.82	0.76	0.92	1.89	0.00	1.55	0.96
time (sec)	N/A	0.195	1.056	0.609	0.425	0.671	0.000	0.765	0.965

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	159	156	189	419	0	292	195
normalized size	1	1.00	0.79	0.77	0.94	2.07	0.00	1.45	0.97
time (sec)	N/A	0.231	1.234	0.720	0.505	1.584	0.000	0.651	1.013

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	156	235	291	204	0	244	320
normalized size	1	1.00	0.74	1.12	1.39	0.97	0.00	1.16	1.52
time (sec)	N/A	0.389	2.104	0.929	0.469	0.736	0.000	0.507	2.454

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	136	197	240	177	0	212	261
normalized size	1	1.00	0.75	1.08	1.32	0.97	0.00	1.16	1.43
time (sec)	N/A	0.274	0.953	0.844	0.439	0.739	0.000	0.455	2.225

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	114	159	182	152	0	180	204
normalized size	1	1.00	0.83	1.15	1.32	1.10	0.00	1.30	1.48
time (sec)	N/A	0.228	0.412	0.755	0.784	0.736	0.000	0.401	1.966

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	300	111	127	125	0	102	90
normalized size	1	1.00	3.06	1.13	1.30	1.28	0.00	1.04	0.92
time (sec)	N/A	0.183	2.417	0.455	0.510	0.596	0.000	0.938	1.278

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	244	102	137	122	0	106	98
normalized size	1	1.00	3.05	1.28	1.71	1.52	0.00	1.32	1.22
time (sec)	N/A	0.194	1.160	0.746	0.596	0.500	0.000	0.395	2.460

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	678	188	188	178	0	123	116
normalized size	1	1.00	6.16	1.71	1.71	1.62	0.00	1.12	1.05
time (sec)	N/A	0.230	6.245	1.026	0.352	0.600	0.000	0.563	5.348

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	353	274	228	225	0	141	136
normalized size	1	1.00	2.14	1.66	1.38	1.36	0.00	0.85	0.82
time (sec)	N/A	0.436	1.224	1.054	0.487	0.744	0.000	0.402	4.913

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	430	360	268	278	0	169	169
normalized size	1	1.00	2.24	1.88	1.40	1.45	0.00	0.88	0.88
time (sec)	N/A	0.314	1.250	1.115	0.371	0.621	0.000	0.687	2.896

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	A	A	F(-1)	A	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	1000	446	308	375	0	202	204
normalized size	1	1.00	4.31	1.92	1.33	1.62	0.00	0.87	0.88
time (sec)	N/A	0.332	6.714	1.126	0.366	0.891	0.000	0.512	1.039

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	62	89	89	89	0	141	110
normalized size	1	1.00	0.68	0.98	0.98	0.98	0.00	1.55	1.21
time (sec)	N/A	0.161	4.869	0.542	0.331	0.821	0.000	0.275	0.087

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	52	70	69	69	0	119	84
normalized size	1	1.00	0.71	0.96	0.95	0.95	0.00	1.63	1.15
time (sec)	N/A	0.155	1.720	0.457	0.340	0.482	0.000	0.841	0.060

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	42	49	49	49	0	97	58
normalized size	1	1.00	0.76	0.89	0.89	0.89	0.00	1.76	1.05
time (sec)	N/A	0.148	0.398	0.445	0.323	0.530	0.000	0.221	0.074

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	32	30	29	29	0	32	26
normalized size	1	1.00	0.86	0.81	0.78	0.78	0.00	0.86	0.70
time (sec)	N/A	0.126	0.128	0.428	0.320	0.674	0.000	1.002	0.882

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	28	49	30	28	0	34	25
normalized size	1	1.00	0.90	1.58	0.97	0.90	0.00	1.10	0.81
time (sec)	N/A	0.071	0.087	0.056	0.323	0.833	0.000	0.220	0.054

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	67	54	47	60	0	56	33
normalized size	1	1.00	1.16	0.93	0.81	1.03	0.00	0.97	0.57
time (sec)	N/A	0.097	0.106	0.483	0.317	0.873	0.000	0.886	0.919

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	91	72	86	138	0	129	75
normalized size	1	1.00	1.11	0.88	1.05	1.68	0.00	1.57	0.91
time (sec)	N/A	0.158	0.394	0.652	0.364	0.731	0.000	0.236	0.935

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	122	108	130	217	0	182	115
normalized size	1	1.00	1.15	1.02	1.23	2.05	0.00	1.72	1.08
time (sec)	N/A	0.173	0.500	0.563	0.353	0.914	0.000	0.292	1.008

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	132	290	360	91	0	139	132
normalized size	1	1.00	1.06	2.32	2.88	0.73	0.00	1.11	1.06
time (sec)	N/A	0.210	1.307	0.444	0.485	0.855	0.000	0.297	3.896

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	112	222	278	70	0	113	106
normalized size	1	1.00	1.13	2.24	2.81	0.71	0.00	1.14	1.07
time (sec)	N/A	0.177	0.742	0.432	0.439	0.891	0.000	0.448	3.664

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	83	154	196	51	0	87	55
normalized size	1	1.00	1.14	2.11	2.68	0.70	0.00	1.19	0.75
time (sec)	N/A	0.150	0.649	0.416	0.427	1.012	0.000	0.199	0.984

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	68	85	112	27	0	58	30
normalized size	1	1.00	1.55	1.93	2.55	0.61	0.00	1.32	0.68
time (sec)	N/A	0.109	0.282	0.330	0.446	1.685	0.000	0.320	0.963

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	66	36	49	41	0	37	32
normalized size	1	1.00	1.78	0.97	1.32	1.11	0.00	1.00	0.86
time (sec)	N/A	0.126	0.225	0.468	0.336	0.710	0.000	0.543	0.927

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	116	62	96	89	0	74	60
normalized size	1	1.00	2.11	1.13	1.75	1.62	0.00	1.35	1.09
time (sec)	N/A	0.143	0.536	0.572	0.328	0.668	0.000	0.245	1.087

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	158	88	136	131	0	103	153
normalized size	1	1.00	2.16	1.21	1.86	1.79	0.00	1.41	2.10
time (sec)	N/A	0.147	0.655	0.546	0.337	0.881	0.000	0.253	1.161

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	200	114	176	177	0	132	201
normalized size	1	1.00	2.20	1.25	1.93	1.95	0.00	1.45	2.21
time (sec)	N/A	0.151	1.061	0.605	0.403	0.763	0.000	0.277	1.454

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	242	140	216	219	0	161	139
normalized size	1	1.00	2.22	1.28	1.98	2.01	0.00	1.48	1.28
time (sec)	N/A	0.155	1.576	0.588	0.340	0.604	0.000	0.294	3.108

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	72	88	89	89	0	185	109
normalized size	1	1.00	0.53	0.64	0.65	0.65	0.00	1.35	0.80
time (sec)	N/A	0.186	5.631	0.691	0.327	1.512	0.000	0.895	0.087

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	62	79	79	79	0	141	96
normalized size	1	1.00	0.54	0.69	0.69	0.69	0.00	1.24	0.84
time (sec)	N/A	0.180	3.953	0.628	0.325	0.599	0.000	0.295	0.910

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	53	50	49	49	0	141	58
normalized size	1	1.00	0.73	0.68	0.67	0.67	0.00	1.93	0.79
time (sec)	N/A	0.159	1.967	0.595	0.324	0.749	0.000	0.282	0.918

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	42	39	39	39	0	119	36
normalized size	1	1.00	0.76	0.71	0.71	0.71	0.00	2.16	0.65
time (sec)	N/A	0.154	0.598	0.611	0.325	1.458	0.000	0.255	0.061

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	51	82	51	48	0	75	56
normalized size	1	1.00	0.77	1.24	0.77	0.73	0.00	1.14	0.85
time (sec)	N/A	0.163	0.243	0.650	0.332	0.454	0.000	1.424	0.064

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	64	68	46	58	0	52	46
normalized size	1	1.00	1.23	1.31	0.88	1.12	0.00	1.00	0.88
time (sec)	N/A	0.102	0.244	0.174	0.375	0.674	0.000	0.199	0.923

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	83	72	74	106	0	87	60
normalized size	1	1.00	1.38	1.20	1.23	1.77	0.00	1.45	1.00
time (sec)	N/A	0.127	0.206	0.570	0.395	1.464	0.000	1.135	0.096

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	38	57	59	60	0	82	58
normalized size	1	1.00	0.90	1.36	1.40	1.43	0.00	1.95	1.38
time (sec)	N/A	0.127	0.103	0.723	0.575	0.657	0.000	0.296	0.942

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	152	144	167	283	0	207	152
normalized size	1	1.00	1.04	0.99	1.14	1.94	0.00	1.42	1.04
time (sec)	N/A	0.217	0.791	0.697	0.372	0.641	0.000	0.304	1.053

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	131	290	378	90	0	139	133
normalized size	1	1.00	0.78	1.74	2.26	0.54	0.00	0.83	0.80
time (sec)	N/A	0.440	3.137	0.660	0.444	0.698	0.000	0.257	3.934

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	111	222	292	71	0	113	107
normalized size	1	1.00	1.07	2.13	2.81	0.68	0.00	1.09	1.03
time (sec)	N/A	0.311	0.948	0.634	0.545	1.434	0.000	0.259	3.750

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	91	154	206	50	0	87	81
normalized size	1	1.00	1.05	1.77	2.37	0.57	0.00	1.00	0.93
time (sec)	N/A	0.233	0.589	0.671	0.560	0.994	0.000	0.247	4.505

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	121	103	140	61	0	75	91
normalized size	1	1.00	1.75	1.49	2.03	0.88	0.00	1.09	1.32
time (sec)	N/A	0.316	0.359	0.626	0.508	0.880	0.000	0.731	1.030

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	105	60	90	71	0	74	71
normalized size	1	1.00	1.44	0.82	1.23	0.97	0.00	1.01	0.97
time (sec)	N/A	0.200	0.476	0.624	0.522	0.591	0.000	0.261	0.982

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	149	86	134	108	0	105	121
normalized size	1	1.00	1.64	0.95	1.47	1.19	0.00	1.15	1.33
time (sec)	N/A	0.345	0.700	0.800	0.480	0.794	0.000	0.375	1.068

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	191	112	174	169	0	134	106
normalized size	1	1.00	1.75	1.03	1.60	1.55	0.00	1.23	0.97
time (sec)	N/A	0.351	1.024	0.719	0.475	1.302	0.000	0.349	2.185

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	233	112	174	204	0	134	201
normalized size	1	1.00	1.86	0.90	1.39	1.63	0.00	1.07	1.61
time (sec)	N/A	0.367	1.500	0.840	0.432	1.541	0.000	0.346	1.687

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	120	90	89	89	0	207	110
normalized size	1	1.00	0.86	0.65	0.64	0.64	0.00	1.49	0.79
time (sec)	N/A	0.195	4.643	0.890	0.534	0.741	0.000	0.406	0.089

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	100	69	69	69	0	185	84
normalized size	1	1.00	0.92	0.63	0.63	0.63	0.00	1.70	0.77
time (sec)	N/A	0.179	3.095	0.721	0.513	1.295	0.000	0.439	0.921

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	80	50	49	49	0	163	58
normalized size	1	1.00	1.10	0.68	0.67	0.67	0.00	2.23	0.79
time (sec)	N/A	0.165	1.756	0.639	0.322	0.670	0.000	1.274	0.070

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	73	114	73	70	0	172	82
normalized size	1	1.00	0.72	1.12	0.72	0.69	0.00	1.69	0.80
time (sec)	N/A	0.182	0.957	0.716	0.331	0.819	0.000	0.418	0.895

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	99	100	72	82	0	94	75
normalized size	1	1.00	1.11	1.12	0.81	0.92	0.00	1.06	0.84
time (sec)	N/A	0.184	0.439	0.726	0.339	0.796	0.000	0.643	0.888

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	103	86	71	96	0	63	59
normalized size	1	1.00	1.37	1.15	0.95	1.28	0.00	0.84	0.79
time (sec)	N/A	0.117	0.363	0.258	0.511	0.563	0.000	0.291	0.079

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	97	90	98	151	0	113	83
normalized size	1	1.00	1.18	1.10	1.20	1.84	0.00	1.38	1.01
time (sec)	N/A	0.151	0.346	0.687	0.324	0.670	0.000	0.297	0.111

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	138	126	146	240	0	182	130
normalized size	1	1.00	1.10	1.00	1.16	1.90	0.00	1.44	1.03
time (sec)	N/A	0.134	0.651	0.865	0.399	0.763	0.000	0.694	0.167

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	137	126	188	317	0	232	173
normalized size	1	1.00	1.07	0.98	1.47	2.48	0.00	1.81	1.35
time (sec)	N/A	0.206	5.290	0.801	0.380	0.855	0.000	1.065	1.090

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	131	290	378	91	0	139	132
normalized size	1	1.00	0.83	1.85	2.41	0.58	0.00	0.89	0.84
time (sec)	N/A	0.461	4.998	0.678	0.450	0.685	0.000	0.805	3.865

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	111	222	292	70	0	113	106
normalized size	1	1.00	0.86	1.72	2.26	0.54	0.00	0.88	0.82
time (sec)	N/A	0.291	1.964	0.733	0.500	0.721	0.000	0.350	3.669

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	173	171	227	83	0	101	98
normalized size	1	1.00	1.60	1.58	2.10	0.77	0.00	0.94	0.91
time (sec)	N/A	0.318	0.684	0.753	0.504	0.637	0.000	0.301	2.659

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	177	122	164	99	0	96	115
normalized size	1	1.00	1.82	1.26	1.69	1.02	0.00	0.99	1.19
time (sec)	N/A	0.310	0.467	0.565	0.587	0.715	0.000	0.320	1.057

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	137	60	90	95	0	73	84
normalized size	1	1.00	1.54	0.67	1.01	1.07	0.00	0.82	0.94
time (sec)	N/A	0.367	0.644	0.726	0.400	0.549	0.000	0.429	1.009

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	175	60	92	146	0	73	105
normalized size	1	1.00	1.70	0.58	0.89	1.42	0.00	0.71	1.02
time (sec)	N/A	0.379	0.871	0.806	0.396	0.699	0.000	0.952	1.112

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	223	112	174	191	0	134	201
normalized size	1	1.00	1.76	0.88	1.37	1.50	0.00	1.06	1.58
time (sec)	N/A	0.408	1.348	0.796	0.564	0.677	0.000	0.464	1.435

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	265	138	214	214	0	163	249
normalized size	1	1.00	1.83	0.95	1.48	1.48	0.00	1.12	1.72
time (sec)	N/A	0.419	1.932	0.821	0.590	0.645	0.000	2.019	2.128

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	106	290	0	0	0	0	-1
normalized size	1	1.00	0.68	1.85	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.307	3.714	0.000	0.970	0.000	0.000	0.000

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	170	210	0	0	0	0	-1
normalized size	1	1.00	1.10	1.36	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.200	0.595	3.633	0.000	1.888	0.000	0.000	0.000

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	69	198	0	0	0	0	-1
normalized size	1	1.00	0.66	1.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.115	3.640	0.000	1.059	0.000	0.000	0.000

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	193	122	0	0	0	0	-1
normalized size	1	1.00	1.87	1.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	3.176	2.544	0.000	0.735	0.000	0.000	0.000

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	143	247	0	0	0	0	-1
normalized size	1	1.00	0.92	1.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	0.397	3.246	0.000	1.128	0.000	0.000	0.000

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	120	212	0	0	0	0	-1
normalized size	1	1.00	0.75	1.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.201	0.369	3.474	0.000	0.802	0.000	0.000	0.000

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	205	265	0	0	0	0	-1
normalized size	1	1.00	1.06	1.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.382	17.789	6.135	0.000	1.908	0.000	0.000	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	204	201	0	0	0	0	-1
normalized size	1	1.00	1.06	1.05	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.380	15.386	5.495	0.000	0.983	0.000	0.000	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	168	220	0	0	0	0	-1
normalized size	1	1.00	1.22	1.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.307	2.177	6.158	0.000	1.941	0.000	0.000	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	164	163	0	0	0	0	-1
normalized size	1	1.00	1.18	1.17	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.307	71.889	5.248	0.000	0.743	0.000	0.000	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	135	238	0	0	0	0	-1
normalized size	1	1.00	0.60	1.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.424	11.056	5.682	0.000	0.740	0.000	0.000	0.000

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	169	301	0	0	0	0	-1
normalized size	1	1.00	0.72	1.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.419	48.876	6.248	0.000	0.847	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	122	128	0	0	0	0	-1
normalized size	1	1.00	0.88	0.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.280	0.691	3.276	0.000	1.125	0.000	0.000	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	232	173	0	0	0	0	-1
normalized size	1	1.00	2.23	1.66	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.220	4.897	3.488	0.000	0.776	0.000	0.000	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	69	112	0	0	0	0	-1
normalized size	1	1.00	0.68	1.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.222	20.651	3.281	0.000	0.668	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	249	149	0	0	0	0	-1
normalized size	1	1.00	2.62	1.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.612	3.894	0.000	0.706	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	77	121	0	0	0	0	-1
normalized size	1	1.00	0.76	1.20	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.211	0.580	3.339	0.000	0.645	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	124	187	0	0	0	0	-1
normalized size	1	1.00	0.92	1.39	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.248	1.096	3.703	0.000	0.541	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	91	136	0	0	0	0	-1
normalized size	1	1.00	0.67	1.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.250	1.275	3.284	0.000	1.465	0.000	0.000	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	94	145	0	0	0	0	-1
normalized size	1	1.00	0.58	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.551	1.551	4.121	0.000	0.621	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	249	173	0	0	0	0	-1
normalized size	1	1.00	1.33	0.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.596	2.988	4.034	0.000	0.456	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	119	153	0	0	0	0	-1
normalized size	1	1.00	0.63	0.81	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.594	1.913	4.299	0.000	0.513	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	222	205	0	0	0	0	-1
normalized size	1	1.00	1.18	1.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.590	1.363	4.831	0.000	0.520	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	82	148	0	0	0	0	-1
normalized size	1	1.00	0.43	0.78	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.591	1.404	4.179	0.000	0.783	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	163	213	0	0	0	0	-1
normalized size	1	1.00	0.73	0.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.664	1.443	4.425	0.000	0.572	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	113	160	0	0	0	0	-1
normalized size	1	1.00	0.50	0.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.670	0.994	4.840	0.000	0.588	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	287	0	0	0	0	0	-1
normalized size	1	1.00	1.16	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.353	7.348	3.495	0.000	0.651	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	230	0	0	0	0	0	-1
normalized size	1	1.00	1.18	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.285	4.611	3.005	0.000	0.640	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	97	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.143	0.147	2.495	0.000	0.624	0.000	0.000	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.199	31.643	2.110	0.000	0.859	0.000	0.000	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	2833	0	0	0	0	0	-1
normalized size	1	1.00	13.69	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.526	22.699	1.463	0.000	0.946	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.635	10.742	1.341	0.000	0.876	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	1243	0	0	0	0	0	-1
normalized size	1	1.00	11.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.375	9.746	1.171	0.000	1.499	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	433	0	0	0	0	0	-1
normalized size	1	1.00	4.05	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.314	2.900	1.238	0.000	0.557	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	277	0	0	0	0	0	-1
normalized size	1	1.00	2.41	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.325	2.066	1.163	0.000	0.676	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	484	0	0	0	0	0	-1
normalized size	1	1.00	4.03	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.374	3.009	1.128	0.000	1.432	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	276	0	0	0	0	0	-1
normalized size	1	1.00	2.12	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.277	1.924	2.791	0.000	0.559	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	113	0	0	0	0	0	-1
normalized size	1	1.00	0.63	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	1.513	3.007	0.000	0.625	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	84	0	0	0	0	0	-1
normalized size	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.108	0.520	2.788	0.000	0.868	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	67	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.155	3.020	0.000	0.603	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	0	0	0	64
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.52
time (sec)	N/A	0.037	0.042	1.208	0.000	1.255	0.000	0.000	1.182

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	92	0	0	0	0	0	-1
normalized size	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.046	0.684	1.307	0.000	0.773	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	179	0	0	0	0	0	-1
normalized size	1	1.00	1.60	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	2.173	1.396	0.000	1.887	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	492	0	0	0	0	0	-1
normalized size	1	1.00	2.05	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.224	6.529	1.300	0.000	0.501	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	7069	0	0	0	0	0	-1
normalized size	1	1.00	30.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.668	22.865	3.554	0.000	2.277	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	4297	0	0	0	0	0	-1
normalized size	1	1.00	45.23	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.353	17.143	2.789	0.000	1.265	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	142	0	0	0	0	0	-1
normalized size	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.132	1.164	1.321	0.000	0.622	0.000	0.000	0.000

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	350	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.541	7.039	1.391	0.000	0.688	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	382	0	0	0	0	0	-1
normalized size	1	1.00	3.64	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.262	3.357	0.820	0.000	0.651	0.000	0.000	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	214	0	0	0	0	0	-1
normalized size	1	1.00	2.04	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.272	1.444	0.794	0.000	0.494	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	212	0	0	0	0	0	-1
normalized size	1	1.00	2.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.250	1.042	0.843	0.000	0.519	0.000	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	212	0	0	0	0	0	-1
normalized size	1	1.00	2.02	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.267	1.236	0.808	0.000	0.637	0.000	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	115	129	91	93	0	317	89
normalized size	1	1.00	0.97	1.08	0.76	0.78	0.00	2.66	0.75
time (sec)	N/A	0.110	0.148	0.632	0.338	0.728	0.000	0.259	0.934

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	83	95	69	71	0	248	67
normalized size	1	1.00	0.95	1.09	0.79	0.82	0.00	2.85	0.77
time (sec)	N/A	0.098	0.084	0.589	0.451	0.780	0.000	0.238	0.906

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	57	61	47	49	0	66	45
normalized size	1	1.00	0.98	1.05	0.81	0.84	0.00	1.14	0.78
time (sec)	N/A	0.086	0.047	0.601	0.511	0.623	0.000	0.338	0.057

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	37	28	23	25	0	32	23
normalized size	1	1.00	1.42	1.08	0.88	0.96	0.00	1.23	0.88
time (sec)	N/A	0.033	0.026	0.192	0.438	0.699	0.000	0.235	0.041

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	63	35	45	51	0	61	63
normalized size	1	1.00	2.42	1.35	1.73	1.96	0.00	2.35	2.42
time (sec)	N/A	0.073	0.036	0.344	0.548	0.483	0.000	0.207	0.108

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	114	68	71	123	0	169	76
normalized size	1	1.00	1.78	1.06	1.11	1.92	0.00	2.64	1.19
time (sec)	N/A	0.104	0.511	0.778	0.397	0.494	0.000	0.253	0.097

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	164	102	110	201	0	266	117
normalized size	1	1.00	1.64	1.02	1.10	2.01	0.00	2.66	1.17
time (sec)	N/A	0.124	0.651	0.730	0.674	0.509	0.000	0.243	0.984

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	216	136	143	284	0	357	148
normalized size	1	1.00	1.54	0.97	1.02	2.03	0.00	2.55	1.06
time (sec)	N/A	0.144	0.621	0.749	0.346	0.462	0.000	0.455	1.021

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	118	130	106	102	0	228	332
normalized size	1	1.00	0.93	1.02	0.83	0.80	0.00	1.80	2.61
time (sec)	N/A	0.128	0.212	0.651	0.333	0.493	0.000	0.446	2.169

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	86	96	81	79	0	172	267
normalized size	1	1.00	0.97	1.08	0.91	0.89	0.00	1.93	3.00
time (sec)	N/A	0.111	0.155	0.649	0.475	0.476	0.000	0.240	1.884

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	54	62	59	55	0	114	83
normalized size	1	1.00	1.06	1.22	1.16	1.08	0.00	2.24	1.63
time (sec)	N/A	0.083	0.063	0.312	0.687	0.503	0.000	0.195	1.094

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	41	47	50	63	0	77	60
normalized size	1	1.00	1.11	1.27	1.35	1.70	0.00	2.08	1.62
time (sec)	N/A	0.096	0.027	0.729	0.536	0.603	0.000	1.283	1.022

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	81	76	125	0	133	101
normalized size	1	1.00	1.00	1.17	1.10	1.81	0.00	1.93	1.46
time (sec)	N/A	0.105	0.026	0.863	0.665	0.696	0.000	0.251	0.999

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	91	115	96	174	0	194	142
normalized size	1	1.00	0.90	1.14	0.95	1.72	0.00	1.92	1.41
time (sec)	N/A	0.111	0.027	0.866	0.324	0.695	0.000	1.722	1.094

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	112	184	105	125	0	418	104
normalized size	1	1.00	0.90	1.48	0.85	1.01	0.00	3.37	0.84
time (sec)	N/A	0.196	0.361	0.631	0.419	0.726	0.000	0.428	0.951

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	72	125	71	92	0	100	69
normalized size	1	1.00	0.90	1.56	0.89	1.15	0.00	1.25	0.86
time (sec)	N/A	0.145	0.182	0.626	0.349	0.589	0.000	0.408	0.915

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	37	45	40	50	0	50	40
normalized size	1	1.00	0.88	1.07	0.95	1.19	0.00	1.19	0.95
time (sec)	N/A	0.078	0.058	0.194	0.628	0.626	0.000	0.339	0.052

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	91	77	73	97	0	124	62
normalized size	1	1.00	1.23	1.04	0.99	1.31	0.00	1.68	0.84
time (sec)	N/A	0.180	0.161	0.381	0.740	0.532	0.000	0.510	0.966

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	329	139	119	205	0	314	120
normalized size	1	1.00	2.89	1.22	1.04	1.80	0.00	2.75	1.05
time (sec)	N/A	0.294	0.633	0.743	0.672	0.536	0.000	0.290	0.121

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	193	246	173	176	0	379	231
normalized size	1	1.00	1.10	1.41	0.99	1.01	0.00	2.17	1.32
time (sec)	N/A	0.461	1.681	0.699	0.808	0.489	0.000	0.740	3.102

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	157	187	125	142	0	285	207
normalized size	1	1.00	0.88	1.05	0.70	0.80	0.00	1.60	1.16
time (sec)	N/A	0.556	1.046	0.638	0.883	0.622	0.000	0.304	1.227

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	121	99	80	108	0	159	143
normalized size	1	1.00	1.57	1.29	1.04	1.40	0.00	2.06	1.86
time (sec)	N/A	0.131	0.604	0.388	0.910	0.496	0.000	0.278	1.168

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	138	89	73	104	0	167	108
normalized size	1	1.00	2.34	1.51	1.24	1.76	0.00	2.83	1.83
time (sec)	N/A	0.414	0.488	0.648	0.541	0.480	0.000	1.223	1.071

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	259	151	112	178	0	226	182
normalized size	1	1.00	2.59	1.51	1.12	1.78	0.00	2.26	1.82
time (sec)	N/A	0.322	0.662	0.921	0.746	0.547	0.000	1.127	1.102

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	368	212	143	241	0	326	248
normalized size	1	1.00	2.57	1.48	1.00	1.69	0.00	2.28	1.73
time (sec)	N/A	0.408	0.739	0.980	0.714	0.528	0.000	0.327	1.045

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	154	266	142	175	0	695	143
normalized size	1	1.00	0.91	1.56	0.84	1.03	0.00	4.09	0.84
time (sec)	N/A	0.255	0.691	0.657	0.336	0.531	0.000	0.585	0.965

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	102	164	98	123	0	128	99
normalized size	1	1.00	0.88	1.41	0.84	1.06	0.00	1.10	0.85
time (sec)	N/A	0.128	0.350	0.651	0.607	0.506	0.000	0.403	0.072

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	56	65	57	67	0	66	57
normalized size	1	1.00	0.88	1.02	0.89	1.05	0.00	1.03	0.89
time (sec)	N/A	0.101	0.119	0.192	1.185	0.539	0.000	1.563	0.925

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	89	113	112	139	0	250	85
normalized size	1	1.00	0.87	1.11	1.10	1.36	0.00	2.45	0.83
time (sec)	N/A	0.219	0.300	0.388	0.499	0.507	0.000	0.498	0.130

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	669	201	171	290	0	482	159
normalized size	1	1.00	4.13	1.24	1.06	1.79	0.00	2.98	0.98
time (sec)	N/A	0.349	6.199	0.750	0.708	0.510	0.000	1.261	1.008

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	818	354	242	241	0	563	373
normalized size	1	1.00	2.74	1.18	0.81	0.81	0.00	1.88	1.25
time (sec)	N/A	0.336	6.258	0.697	0.914	0.535	0.000	0.464	1.685

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	696	276	183	196	0	431	281
normalized size	1	1.00	2.95	1.17	0.78	0.83	0.00	1.83	1.19
time (sec)	N/A	0.748	6.192	0.622	0.727	0.565	0.000	0.404	1.472

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	327	167	129	151	0	346	202
normalized size	1	1.00	2.37	1.21	0.93	1.09	0.00	2.51	1.46
time (sec)	N/A	0.504	0.967	0.415	1.036	0.506	0.000	0.379	1.269

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	406	158	139	151	0	225	181
normalized size	1	1.00	3.05	1.19	1.05	1.14	0.00	1.69	1.36
time (sec)	N/A	0.273	0.659	0.639	0.865	0.693	0.000	0.356	1.466

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	610	246	190	260	0	361	260
normalized size	1	1.00	2.98	1.20	0.93	1.27	0.00	1.76	1.27
time (sec)	N/A	0.291	0.972	0.921	1.564	1.212	0.000	0.516	1.165

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	812	334	230	354	0	498	363
normalized size	1	1.00	2.91	1.20	0.82	1.27	0.00	1.78	1.30
time (sec)	N/A	0.316	1.500	0.971	0.802	0.619	0.000	0.377	1.222

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	282	363	224	222	0	1559	249
normalized size	1	1.00	1.26	1.63	1.00	1.00	0.00	6.99	1.12
time (sec)	N/A	0.251	1.562	0.541	0.585	0.612	0.000	1.617	0.155

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	172	216	141	140	0	867	151
normalized size	1	1.00	1.13	1.42	0.93	0.92	0.00	5.70	0.99
time (sec)	N/A	0.194	0.377	0.403	0.724	0.610	0.000	0.442	0.091

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	89	106	80	78	0	102	79
normalized size	1	1.00	1.00	1.19	0.90	0.88	0.00	1.15	0.89
time (sec)	N/A	0.156	0.210	0.397	0.584	0.558	0.000	0.234	1.023

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	30	53	33	31	0	38	30
normalized size	1	1.00	0.88	1.56	0.97	0.91	0.00	1.12	0.88
time (sec)	N/A	0.077	0.020	0.062	0.529	0.514	0.000	0.240	0.058

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	63	75	64	64	0	100	68
normalized size	1	1.00	0.85	1.01	0.86	0.86	0.00	1.35	0.92
time (sec)	N/A	0.105	0.101	0.416	0.749	0.522	0.000	0.301	1.147

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	123	121	132	216	0	202	133
normalized size	1	1.00	1.06	1.04	1.14	1.86	0.00	1.74	1.15
time (sec)	N/A	0.213	0.627	0.550	0.734	0.553	0.000	0.400	0.303

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	207	259	268	469	0	419	297
normalized size	1	1.00	1.16	1.45	1.50	2.62	0.00	2.34	1.66
time (sec)	N/A	0.301	5.218	0.546	0.818	0.598	0.000	0.748	1.614

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	268	1566	0	553	0	781	3341
normalized size	1	1.00	1.17	6.81	0.00	2.40	0.00	3.40	14.53
time (sec)	N/A	0.608	2.400	0.523	0.000	0.597	0.000	0.289	3.879

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	172	769	0	393	0	407	317
normalized size	1	1.00	1.07	4.78	0.00	2.44	0.00	2.53	1.97
time (sec)	N/A	0.381	0.822	0.448	0.000	0.657	0.000	0.304	2.293

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	96	269	0	258	0	185	147
normalized size	1	1.00	0.96	2.69	0.00	2.58	0.00	1.85	1.47
time (sec)	N/A	0.207	0.318	0.362	0.000	0.491	0.000	0.267	1.478

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	118	96	0	300	0	129	109
normalized size	1	1.00	1.40	1.14	0.00	3.57	0.00	1.54	1.30
time (sec)	N/A	0.149	0.200	0.469	0.000	0.559	0.000	0.253	1.304

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	162	165	0	558	0	269	219
normalized size	1	1.00	1.16	1.18	0.00	3.99	0.00	1.92	1.56
time (sec)	N/A	0.306	0.923	0.541	0.000	0.853	0.000	0.272	1.375

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	277	282	0	861	0	541	387
normalized size	1	1.00	1.38	1.40	0.00	4.28	0.00	2.69	1.93
time (sec)	N/A	0.520	1.292	0.604	0.000	0.891	0.000	0.351	1.687

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	417	456	271	344	0	1861	588
normalized size	1	1.00	1.56	1.71	1.01	1.29	0.00	6.97	2.20
time (sec)	N/A	0.372	3.612	0.588	0.786	0.822	0.000	0.770	0.188

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	280	285	184	240	0	1102	253
normalized size	1	1.00	1.44	1.47	0.95	1.24	0.00	5.68	1.30
time (sec)	N/A	0.299	1.637	0.515	0.419	0.582	0.000	0.368	0.122

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	167	153	112	150	0	139	113
normalized size	1	1.00	1.40	1.29	0.94	1.26	0.00	1.17	0.95
time (sec)	N/A	0.228	0.556	0.442	0.511	0.554	0.000	0.258	0.090

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	76	75	55	75	0	61	60
normalized size	1	1.00	1.33	1.32	0.96	1.32	0.00	1.07	1.05
time (sec)	N/A	0.112	0.177	0.068	0.715	0.610	0.000	0.248	1.023

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	165	106	123	210	0	213	103
normalized size	1	1.00	1.51	0.97	1.13	1.93	0.00	1.95	0.94
time (sec)	N/A	0.226	0.351	0.438	0.845	0.639	0.000	0.268	0.226

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	224	224	274	630	0	456	228
normalized size	1	1.00	1.33	1.33	1.63	3.75	0.00	2.71	1.36
time (sec)	N/A	0.433	1.307	0.653	0.799	0.763	0.000	0.323	1.469

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	320	368	511	1205	0	710	447
normalized size	1	1.00	1.24	1.42	1.97	4.65	0.00	2.74	1.73
time (sec)	N/A	0.741	1.468	0.663	0.729	0.874	0.000	0.446	1.911

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	473	473	402	1735	0	793	0	870	3692
normalized size	1	1.00	0.85	3.67	0.00	1.68	0.00	1.84	7.81
time (sec)	N/A	1.713	7.187	0.638	0.000	0.648	0.000	0.370	4.865

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	282	883	0	581	0	482	2804
normalized size	1	1.00	1.08	3.38	0.00	2.23	0.00	1.85	10.74
time (sec)	N/A	0.825	3.361	0.457	0.000	0.566	0.000	0.887	3.861

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	178	325	0	551	0	240	1655
normalized size	1	1.00	1.17	2.14	0.00	3.62	0.00	1.58	10.89
time (sec)	N/A	0.571	1.333	0.440	0.000	0.553	0.000	0.257	3.569

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	128	162	0	526	0	289	245
normalized size	1	1.00	0.63	0.80	0.00	2.59	0.00	1.42	1.21
time (sec)	N/A	0.436	0.907	0.505	0.000	0.602	0.000	0.508	1.788

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	281	242	0	1040	0	457	403
normalized size	1	1.00	0.82	0.71	0.00	3.03	0.00	1.33	1.17
time (sec)	N/A	0.547	1.131	0.560	0.000	0.705	0.000	0.360	1.618

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	550	549	326	447	0	2150	762
normalized size	1	1.00	1.67	1.67	0.99	1.36	0.00	6.53	2.32
time (sec)	N/A	0.502	5.088	0.588	0.920	0.807	0.000	2.602	0.236

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	388	355	234	331	0	1337	315
normalized size	1	1.00	1.62	1.49	0.98	1.38	0.00	5.59	1.32
time (sec)	N/A	0.365	3.001	0.523	0.660	0.708	0.000	0.445	1.111

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	208	200	154	226	0	170	167
normalized size	1	1.00	1.32	1.27	0.97	1.43	0.00	1.08	1.06
time (sec)	N/A	0.272	0.783	0.517	0.873	0.549	0.000	0.505	0.105

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	111	96	87	126	0	77	93
normalized size	1	1.00	1.34	1.16	1.05	1.52	0.00	0.93	1.12
time (sec)	N/A	0.134	0.459	0.145	0.840	0.593	0.000	0.347	1.073

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	203	206	241	474	0	452	182
normalized size	1	1.00	1.25	1.26	1.48	2.91	0.00	2.77	1.12
time (sec)	N/A	0.319	0.606	0.510	0.825	0.597	0.000	1.508	1.394

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	332	322	435	1071	0	800	378
normalized size	1	1.00	1.45	1.41	1.90	4.68	0.00	3.49	1.65
time (sec)	N/A	0.513	6.337	0.680	0.793	0.894	0.000	2.648	1.702

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	496	427	707	1803	0	1551	673
normalized size	1	1.00	1.58	1.36	2.26	5.76	0.00	4.96	2.15
time (sec)	N/A	1.007	4.889	0.640	0.628	1.361	0.000	0.543	2.554

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	539	539	599	2251	0	1057	0	1030	3975
normalized size	1	1.00	1.11	4.18	0.00	1.96	0.00	1.91	7.37
time (sec)	N/A	2.436	12.695	0.594	0.000	0.697	0.000	0.810	5.710

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	1178	1227	0	1041	0	584	3255
normalized size	1	1.00	3.54	3.68	0.00	3.13	0.00	1.75	9.77
time (sec)	N/A	1.138	9.046	0.532	0.000	0.688	0.000	3.101	5.761

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	282	729	0	984	0	1193	4026
normalized size	1	1.00	1.06	2.73	0.00	3.69	0.00	4.47	15.08
time (sec)	N/A	0.940	4.196	0.464	0.000	0.611	0.000	0.647	9.095

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	231	234	0	841	0	386	423
normalized size	1	1.00	0.61	0.62	0.00	2.24	0.00	1.03	1.12
time (sec)	N/A	0.658	1.107	0.579	0.000	0.737	0.000	3.771	2.858

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	515	515	388	328	0	1550	0	709	588
normalized size	1	1.00	0.75	0.64	0.00	3.01	0.00	1.38	1.14
time (sec)	N/A	0.775	1.082	0.620	0.000	0.832	0.000	0.501	1.781

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	516	516	2049	1776	0	0	0	0	-1
normalized size	1	1.00	3.97	3.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.703	17.574	12.588	0.000	0.000	0.000	0.000	0.000

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	430	430	853	1195	0	0	0	0	-1
normalized size	1	1.00	1.98	2.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.109	15.040	10.726	0.000	0.000	0.000	0.000	0.000

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	1959	1120	0	0	0	0	-1
normalized size	1	1.00	4.41	2.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.044	17.345	10.484	0.000	0.000	0.000	0.000	0.000

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	351	919	0	0	0	0	-1
normalized size	1	1.00	0.99	2.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.762	20.749	6.692	0.000	0.000	0.000	0.000	0.000

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	370	370	546	937	0	0	0	0	-1
normalized size	1	1.00	1.48	2.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.781	6.308	7.104	0.000	0.000	0.000	0.000	0.000

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	430	430	834	1083	0	0	0	0	-1
normalized size	1	1.00	1.94	2.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.037	14.475	7.710	0.000	175.618	0.000	0.000	0.000

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	1233	681	0	0	0	0	-1
normalized size	1	1.00	2.73	1.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.050	12.708	11.400	0.000	0.000	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	511	511	930	1672	0	0	0	0	-1
normalized size	1	1.00	1.82	3.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.377	6.890	8.486	0.000	0.000	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1070	1070	974	3808	0	0	0	0	-1
normalized size	1	1.00	0.91	3.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.795	15.593	21.443	0.000	0.000	0.000	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1101	1101	2095	3412	0	0	0	0	-1
normalized size	1	1.00	1.90	3.10	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.930	17.616	21.386	0.000	0.000	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	850	850	886	2540	0	0	0	0	-1
normalized size	1	1.00	1.04	2.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.126	15.150	16.079	0.000	0.000	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	882	882	2012	2282	0	0	0	0	-1
normalized size	1	1.00	2.28	2.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.168	16.966	16.851	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	809	809	854	1563	0	0	0	0	-1
normalized size	1	1.00	1.06	1.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.833	15.269	13.396	0.000	0.000	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	838	838	1246	1475	0	0	0	0	-1
normalized size	1	1.00	1.49	1.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.924	13.306	15.314	0.000	0.000	0.000	0.000	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1054	1054	922	2263	0	0	0	0	-1
normalized size	1	1.00	0.87	2.15	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.692	6.932	17.212	0.000	0.000	0.000	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1089	1089	1320	2159	0	0	0	0	-1
normalized size	1	1.00	1.21	1.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.781	15.770	18.902	0.000	0.000	0.000	0.000	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	151	215	0	0	0	0	-1
normalized size	1	1.00	1.21	1.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.032	0.273	1.335	0.000	0.000	0.000	0.000	0.000

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	120	264	0	0	0	0	-1
normalized size	1	1.00	0.99	2.18	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	1.274	1.341	0.000	0.984	0.000	0.000	0.000

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	882	1199	0	0	0	0	-1
normalized size	1	1.00	2.85	3.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.230	6.139	1.366	0.000	0.000	0.000	0.000	0.000

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	276	849	0	0	0	0	-1
normalized size	1	1.00	1.21	3.72	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.240	11.255	1.388	0.000	0.651	0.000	0.000	0.000

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	138	178	0	0	0	0	-1
normalized size	1	1.00	1.30	1.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.022	0.222	1.357	0.000	31.823	0.000	0.000	0.000

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	259	852	0	0	0	0	-1
normalized size	1	1.00	1.02	3.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.320	7.747	1.490	0.000	0.539	0.000	0.000	0.000

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	1249	1209	0	0	0	0	-1
normalized size	1	1.00	3.60	3.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.333	6.153	1.450	0.000	26.217	0.000	0.000	0.000

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	259	1065	0	0	0	0	-1
normalized size	1	1.00	0.81	3.35	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.525	8.489	1.318	0.000	0.500	0.000	0.000	0.000

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	182	0	0	0	0	0	-1
normalized size	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.387	0.336	4.120	0.000	0.487	0.000	0.000	0.000

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	134	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.839	0.296	4.091	0.000	0.540	0.000	0.000	0.000

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	98	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.158	0.111	2.644	0.000	0.469	0.000	0.000	0.000

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	687	0	0	0	0	0	-1
normalized size	1	1.00	2.96	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.259	5.967	1.976	0.000	0.498	0.000	0.000	0.000

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	1433	0	0	0	0	0	-1
normalized size	1	1.00	3.54	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.456	11.748	1.135	0.000	0.587	0.000	0.000	0.000

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	580	580	2700	0	0	0	0	0	-1
normalized size	1	1.00	4.66	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.592	17.973	1.220	0.000	0.844	0.000	0.000	0.000

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.068	7.695	1.134	0.000	0.841	0.000	0.000	0.000

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.062	0.521	1.083	0.000	0.635	0.000	0.000	0.000

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.070	2.633	1.022	0.000	0.684	0.000	0.000	0.000

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	28	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.071	2.798	0.991	0.000	0.928	0.000	0.000	0.000

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	3.378	2.441	0.000	0.803	0.000	0.000	0.000

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	562	0	0	0	0	0	-1
normalized size	1	1.00	3.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	8.934	2.723	0.000	0.695	0.000	0.000	0.000

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	155	0	0	0	0	0	-1
normalized size	1	1.00	1.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	1.862	2.710	0.000	0.547	0.000	0.000	0.000

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	72	0	0	0	0	0	73
normalized size	1	1.00	1.50	0.00	0.00	0.00	0.00	0.00	1.52
time (sec)	N/A	0.039	0.536	0.987	0.000	0.550	0.000	0.000	1.386

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	132	0	0	0	0	0	-1
normalized size	1	1.00	1.15	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	0.976	1.066	0.000	0.618	0.000	0.000	0.000

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	710	0	0	0	0	0	-1
normalized size	1	1.00	3.07	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.195	17.054	1.141	0.000	0.977	0.000	0.000	0.000

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	14.436	3.605	0.000	0.788	0.000	0.000	0.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	3.903	2.834	0.000	0.631	0.000	0.000	0.000

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	3614	0	0	0	0	0	-1
normalized size	1	1.00	26.57	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	18.459	1.135	0.000	0.761	0.000	0.000	0.000

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	F	F	F	F(-1)	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	424	0	6403	0	0	0	0	0	-1
normalized size	1	0.00	15.10	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.040	23.725	1.260	0.000	1.265	0.000	0.000	0.000

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	1.875	0.895	0.000	0.874	0.000	0.000	0.000

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	4.988	0.805	0.000	0.744	0.000	0.000	0.000

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	2.749	0.822	0.000	0.869	0.000	0.000	0.000

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	2.913	0.828	0.000	1.051	0.000	0.000	0.000

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	135	679	0	0	0	0	-1
normalized size	1	1.00	0.71	3.57	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	1.515	2.098	0.000	0.647	0.000	0.000	0.000

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	146	1522	0	0	0	0	-1
normalized size	1	1.00	0.86	9.01	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.162	1.280	1.664	0.000	1.525	0.000	0.000	0.000

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	111	291	0	0	0	0	-1
normalized size	1	1.00	0.92	2.40	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.137	0.854	1.629	0.000	0.848	0.000	0.000	0.000

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	130	1535	0	0	0	0	-1
normalized size	1	1.00	1.07	12.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.826	1.443	0.000	0.946	0.000	0.000	0.000

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	135	710	0	0	0	0	-1
normalized size	1	1.00	0.74	3.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	11.032	1.324	0.000	1.075	0.000	0.000	0.000

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	165	1529	0	0	0	0	-1
normalized size	1	1.00	0.84	7.76	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	1.507	1.316	0.000	1.402	0.000	0.000	0.000

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	195	745	0	0	0	0	-1
normalized size	1	1.00	0.72	2.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.334	3.862	1.814	0.000	0.786	0.000	0.000	0.000

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	195	1593	0	0	0	0	-1
normalized size	1	1.00	0.81	6.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.331	4.923	1.663	0.000	1.087	0.000	0.000	0.000

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	168	744	0	0	0	0	-1
normalized size	1	1.00	1.09	4.83	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.263	2.425	1.899	0.000	0.731	0.000	0.000	0.000

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	287	1605	0	0	0	0	-1
normalized size	1	1.00	1.88	10.49	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.272	8.523	1.435	0.000	1.471	0.000	0.000	0.000

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	164	763	0	0	0	0	-1
normalized size	1	1.00	0.74	3.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.313	7.816	1.334	0.000	1.333	0.000	0.000	0.000

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	152	1636	0	0	0	0	-1
normalized size	1	1.00	0.64	6.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.320	11.312	1.321	0.000	1.049	0.000	0.000	0.000

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	131	465	0	0	0	0	-1
normalized size	1	1.00	0.85	3.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.224	1.035	1.331	0.000	0.478	0.000	0.000	0.000

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	230	799	0	0	0	0	-1
normalized size	1	1.00	1.59	5.51	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.229	1.386	1.294	0.000	0.453	0.000	0.000	0.000

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	60	326	0	0	0	0	-1
normalized size	1	1.00	0.57	3.10	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.203	0.377	1.344	0.000	0.494	0.000	0.000	0.000

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	95	536	0	0	0	0	-1
normalized size	1	1.00	0.96	5.41	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.211	0.617	1.171	0.000	0.478	0.000	0.000	0.000

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	70	195	0	0	0	0	-1
normalized size	1	1.00	0.66	1.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.227	0.395	1.146	0.000	0.583	0.000	0.000	0.000

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	100	551	0	0	0	0	-1
normalized size	1	1.00	0.83	4.59	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.222	0.830	1.398	0.000	0.495	0.000	0.000	0.000

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	91	221	0	0	0	0	-1
normalized size	1	1.00	0.61	1.48	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.254	0.578	1.228	0.000	0.527	0.000	0.000	0.000

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	115	609	0	0	0	0	-1
normalized size	1	1.00	0.43	2.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.505	1.199	1.402	0.000	0.476	0.000	0.000	0.000

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	247	1044	0	0	0	0	-1
normalized size	1	1.00	0.99	4.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.493	1.832	1.343	0.000	0.613	0.000	0.000	0.000

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	82	474	0	0	0	0	-1
normalized size	1	1.00	0.41	2.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.448	0.747	1.555	0.000	0.540	0.000	0.000	0.000

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	252	811	0	0	0	0	-1
normalized size	1	1.00	1.27	4.08	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.469	1.599	1.303	0.000	0.501	0.000	0.000	0.000

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	101	332	0	0	0	0	-1
normalized size	1	1.00	0.47	1.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.475	0.557	1.230	0.000	0.506	0.000	0.000	0.000

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	125	563	0	0	0	0	-1
normalized size	1	1.00	0.58	2.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.471	2.145	1.304	0.000	0.703	0.000	0.000	0.000

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	94	224	0	0	0	0	-1
normalized size	1	1.00	0.55	1.30	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.463	2.202	1.236	0.000	0.737	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [241] had the largest ratio of [.6400]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.00	19	0.210
2	A	5	4	1.00	19	0.210
3	A	5	4	1.00	19	0.210
4	A	5	4	1.00	19	0.210
5	A	4	3	1.00	17	0.176
6	A	6	6	1.00	17	0.353
7	A	5	4	1.00	19	0.210
8	A	5	4	1.00	19	0.210
9	A	5	4	1.00	19	0.210
10	A	11	7	1.00	19	0.368
11	A	10	7	1.00	19	0.368
12	A	9	7	1.00	19	0.368
13	A	7	7	1.00	19	0.368
14	A	7	7	1.00	19	0.368
15	A	8	6	1.00	19	0.316
16	A	8	6	1.00	19	0.316
17	A	8	6	1.00	19	0.316
18	A	8	6	1.00	19	0.316
19	A	5	4	1.00	21	0.190
20	A	5	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	5	4	1.00	21	0.190
22	A	5	4	1.00	21	0.190
23	A	5	4	1.00	19	0.210
24	A	5	4	1.00	19	0.210
25	A	5	4	1.00	21	0.190
26	A	5	4	1.00	21	0.190
27	A	5	4	1.00	21	0.190
28	A	5	4	1.00	21	0.190
29	A	27	8	1.00	21	0.381
30	A	18	8	1.00	21	0.381
31	A	14	8	1.00	21	0.381
32	A	9	7	1.00	21	0.333
33	A	11	9	1.00	21	0.429
34	A	8	8	1.00	21	0.381
35	A	12	8	1.00	21	0.381
36	A	12	8	1.00	21	0.381
37	A	12	8	1.00	21	0.381
38	A	5	4	1.00	21	0.190
39	A	5	4	1.00	21	0.190
40	A	5	4	1.00	21	0.190
41	A	4	3	1.00	21	0.143
42	A	5	4	1.00	19	0.210
43	A	5	4	1.00	19	0.210
44	A	5	4	1.00	21	0.190
45	A	5	4	1.00	21	0.190
46	A	5	4	1.00	21	0.190
47	A	5	4	1.00	21	0.190
48	A	29	9	1.00	21	0.429
49	A	18	9	1.00	21	0.429
50	A	16	9	1.00	21	0.429
51	A	11	8	1.00	21	0.381
52	A	9	7	1.00	21	0.333
53	A	11	8	1.00	21	0.381
54	A	10	9	1.00	21	0.429
55	A	17	9	1.00	21	0.429
56	A	17	9	1.00	21	0.429

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	7	6	1.00	21	0.286
58	A	7	6	1.00	21	0.286
59	A	7	6	1.00	21	0.286
60	A	6	5	1.00	21	0.238
61	A	5	4	1.00	19	0.210
62	A	6	6	1.00	19	0.316
63	A	7	7	1.00	21	0.333
64	A	8	7	1.00	21	0.333
65	A	9	7	1.00	21	0.333
66	A	8	7	1.00	21	0.333
67	A	7	7	1.00	21	0.333
68	A	5	5	1.00	21	0.238
69	A	6	5	1.00	21	0.238
70	A	7	6	1.00	21	0.286
71	A	7	6	1.00	21	0.286
72	A	7	6	1.00	21	0.286
73	A	7	6	1.00	21	0.286
74	A	5	4	1.00	21	0.190
75	A	5	4	1.00	21	0.190
76	A	5	4	1.00	21	0.190
77	A	5	4	1.00	21	0.190
78	A	5	4	1.00	21	0.190
79	A	5	4	1.00	19	0.210
80	A	6	5	1.00	19	0.263
81	A	4	4	1.00	21	0.190
82	A	6	5	1.00	21	0.238
83	A	16	8	1.00	21	0.381
84	A	7	6	1.00	21	0.286
85	A	11	6	1.00	21	0.286
86	A	9	8	1.00	21	0.381
87	A	11	6	1.00	21	0.286
88	A	13	7	1.00	21	0.333
89	A	13	7	1.00	21	0.333
90	A	13	7	1.00	21	0.333
91	A	5	4	1.00	21	0.190
92	A	5	4	1.00	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
93	A	5	4	1.00	21	0.190
94	A	5	4	1.00	21	0.190
95	A	5	4	1.00	21	0.190
96	A	5	4	1.00	19	0.210
97	A	6	5	1.00	19	0.263
98	A	5	4	1.00	21	0.190
99	A	6	5	1.00	21	0.238
100	A	19	9	1.00	21	0.429
101	A	15	6	1.00	21	0.286
102	A	13	8	1.00	21	0.381
103	A	10	8	1.00	21	0.381
104	A	15	8	1.00	21	0.381
105	A	16	7	1.00	21	0.333
106	A	16	7	1.00	21	0.333
107	A	16	7	1.00	21	0.333
108	A	11	11	1.00	23	0.478
109	A	11	11	1.00	23	0.478
110	A	9	9	1.00	23	0.391
111	A	9	9	1.00	23	0.391
112	A	11	11	1.00	23	0.478
113	A	11	11	1.00	23	0.478
114	A	15	12	1.00	25	0.480
115	A	15	12	1.00	25	0.480
116	A	13	10	1.00	25	0.400
117	A	13	10	1.00	25	0.400
118	A	16	13	1.00	25	0.520
119	A	16	13	1.00	25	0.520
120	A	8	8	1.00	25	0.320
121	A	7	7	1.00	25	0.280
122	A	7	7	1.00	25	0.280
123	A	7	7	1.00	25	0.280
124	A	7	7	1.00	25	0.280
125	A	8	8	1.00	25	0.320
126	A	8	8	1.00	25	0.320
127	A	14	8	1.00	25	0.320
128	A	14	9	1.00	25	0.360

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
129	A	14	9	1.00	25	0.360
130	A	15	9	1.00	25	0.360
131	A	15	9	1.00	25	0.360
132	A	17	9	1.00	25	0.360
133	A	17	9	1.00	25	0.360
134	A	9	6	1.00	23	0.261
135	A	7	6	1.00	23	0.261
136	A	5	5	1.00	21	0.238
137	A	5	5	1.00	23	0.217
138	A	9	6	1.00	23	0.261
139	A	12	7	1.00	23	0.304
140	A	5	4	1.00	25	0.160
141	A	5	4	1.00	25	0.160
142	A	5	4	1.00	25	0.160
143	A	5	4	1.00	25	0.160
144	A	5	4	1.00	23	0.174
145	A	4	4	1.00	21	0.190
146	A	4	4	1.00	21	0.190
147	A	3	3	1.00	21	0.143
148	A	2	2	1.00	19	0.105
149	A	2	2	1.00	19	0.105
150	A	4	4	1.00	21	0.190
151	A	5	5	1.00	21	0.238
152	A	11	9	1.00	21	0.429
153	A	6	5	1.00	21	0.238
154	A	4	4	1.00	21	0.190
155	A	7	6	1.00	21	0.286
156	A	5	4	1.00	23	0.174
157	A	5	4	1.00	23	0.174
158	A	5	4	1.00	23	0.174
159	A	5	4	1.00	23	0.174
160	A	5	4	1.00	19	0.210
161	A	5	4	1.00	19	0.210
162	A	5	4	1.00	19	0.210
163	A	4	3	1.00	17	0.176
164	A	5	5	1.00	17	0.294

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	7	6	1.00	19	0.316
166	A	9	7	1.00	19	0.368
167	A	10	7	1.00	19	0.368
168	A	10	7	1.00	19	0.368
169	A	9	7	1.00	19	0.368
170	A	7	7	1.00	19	0.368
171	A	7	7	1.00	19	0.368
172	A	8	6	1.00	19	0.316
173	A	8	6	1.00	19	0.316
174	A	5	4	1.00	21	0.190
175	A	5	4	1.00	21	0.190
176	A	5	4	1.00	19	0.210
177	A	5	4	1.00	19	0.210
178	A	6	5	1.00	21	0.238
179	A	12	10	1.00	21	0.476
180	A	7	7	1.00	21	0.333
181	A	10	8	1.00	21	0.381
182	A	8	6	1.00	21	0.286
183	A	9	6	1.00	21	0.286
184	A	9	6	1.00	21	0.286
185	A	5	4	1.00	21	0.190
186	A	4	3	1.00	21	0.143
187	A	5	4	1.00	19	0.210
188	A	5	4	1.00	19	0.210
189	A	6	5	1.00	21	0.238
190	A	21	11	1.00	21	0.524
191	A	8	7	1.00	21	0.333
192	A	8	8	1.00	21	0.381
193	A	15	10	1.00	21	0.476
194	A	17	9	1.00	21	0.429
195	A	17	9	1.00	21	0.429
196	A	5	4	1.00	21	0.190
197	A	5	4	1.00	21	0.190
198	A	5	4	1.00	21	0.190
199	A	5	4	1.00	19	0.210
200	A	4	3	1.00	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	6	5	1.00	21	0.238
202	A	7	5	1.00	21	0.238
203	A	7	5	1.00	21	0.238
204	A	6	5	1.00	21	0.238
205	A	5	5	1.00	21	0.238
206	A	5	5	1.00	21	0.238
207	A	6	5	1.00	21	0.238
208	A	7	5	1.00	21	0.238
209	A	5	4	1.00	21	0.190
210	A	5	4	1.00	21	0.190
211	A	5	4	1.00	21	0.190
212	A	5	4	1.00	19	0.210
213	A	5	4	1.00	19	0.210
214	A	6	5	1.00	21	0.238
215	A	7	5	1.00	21	0.238
216	A	10	8	1.00	21	0.381
217	A	8	7	1.00	21	0.333
218	A	8	8	1.00	21	0.381
219	A	11	7	1.00	21	0.333
220	A	15	8	1.00	21	0.381
221	A	5	4	1.00	21	0.190
222	A	5	4	1.00	21	0.190
223	A	5	4	1.00	21	0.190
224	A	5	4	1.00	19	0.210
225	A	5	4	1.00	19	0.210
226	A	5	4	1.00	21	0.190
227	A	7	5	1.00	21	0.238
228	A	11	8	1.00	21	0.381
229	A	9	7	1.00	21	0.333
230	A	9	8	1.00	21	0.381
231	A	16	8	1.00	21	0.381
232	A	20	9	1.00	21	0.429
233	A	15	12	1.00	25	0.480
234	A	14	12	1.00	25	0.480
235	A	14	12	1.00	25	0.480
236	A	13	11	1.00	25	0.440

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
237	A	13	11	1.00	25	0.440
238	A	14	12	1.00	25	0.480
239	A	14	12	1.00	25	0.480
240	A	15	12	1.00	25	0.480
241	A	35	16	1.00	25	0.640
242	A	35	16	1.00	25	0.640
243	A	32	15	1.00	25	0.600
244	A	32	15	1.00	25	0.600
245	A	27	13	1.00	25	0.520
246	A	27	13	1.00	25	0.520
247	A	33	16	1.00	25	0.640
248	A	33	16	1.00	25	0.640
249	A	1	1	1.00	14	0.071
250	A	2	2	1.00	23	0.087
251	A	5	5	1.00	14	0.357
252	A	4	4	1.00	23	0.174
253	A	1	1	1.00	14	0.071
254	A	6	6	1.00	23	0.261
255	A	6	6	1.00	14	0.429
256	A	6	6	1.00	23	0.261
257	A	9	6	1.00	23	0.261
258	A	9	8	1.00	23	0.348
259	A	5	5	1.00	21	0.238
260	A	4	4	1.00	23	0.174
261	A	6	4	1.00	23	0.174
262	A	7	4	1.00	23	0.174
263	A	0	0	0.00	0	0.000
264	A	0	0	0.00	0	0.000
265	A	0	0	0.00	0	0.000
266	A	0	0	0.00	0	0.000
267	A	0	0	0.00	0	0.000
268	A	6	3	1.00	21	0.143
269	A	3	3	1.00	21	0.143
270	A	2	2	1.00	19	0.105
271	A	6	4	1.00	19	0.210
272	A	9	4	1.00	21	0.190

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
273	A	0	0	0.00	0	0.000
274	A	0	0	0.00	0	0.000
275	A	4	4	1.00	21	0.190
276	F	0	0	N/A	0	N/A
277	A	0	0	0.00	0	0.000
278	A	0	0	0.00	0	0.000
279	A	0	0	0.00	0	0.000
280	A	0	0	0.00	0	0.000
281	A	11	11	1.00	23	0.478
282	A	11	11	1.00	23	0.478
283	A	9	9	1.00	23	0.391
284	A	9	9	1.00	23	0.391
285	A	11	11	1.00	23	0.478
286	A	11	11	1.00	23	0.478
287	A	15	13	1.00	25	0.520
288	A	15	13	1.00	25	0.520
289	A	12	10	1.00	25	0.400
290	A	12	10	1.00	25	0.400
291	A	14	12	1.00	25	0.480
292	A	14	12	1.00	25	0.480
293	A	8	8	1.00	25	0.320
294	A	8	8	1.00	25	0.320
295	A	7	7	1.00	25	0.280
296	A	7	7	1.00	25	0.280
297	A	7	7	1.00	25	0.280
298	A	7	7	1.00	25	0.280
299	A	8	8	1.00	25	0.320
300	A	16	9	1.00	25	0.360
301	A	16	9	1.00	25	0.360
302	A	14	9	1.00	25	0.360
303	A	14	9	1.00	25	0.360
304	A	13	9	1.00	25	0.360
305	A	13	9	1.00	25	0.360
306	A	13	8	1.00	25	0.320

Chapter 3

Listing of integrals

3.1 $\int (a + a\sec(c + dx))\sin^9(c + dx)\,dx$

Optimal. Leaf size=152

$$-\frac{a\cos^9(c+dx)}{9d}-\frac{a\cos^8(c+dx)}{8d}+\frac{4a\cos^7(c+dx)}{7d}+\frac{2a\cos^6(c+dx)}{3d}-\frac{6a\cos^5(c+dx)}{5d}-\frac{3a\cos^4(c+dx)}{2d}+\frac{4a\cos}{}$$

```
[Out] -a*cos(d*x+c)/d+2*a*cos(d*x+c)^2/d+4/3*a*cos(d*x+c)^3/d-3/2*a*cos(d*x+c)^4/
d-6/5*a*cos(d*x+c)^5/d+2/3*a*cos(d*x+c)^6/d+4/7*a*cos(d*x+c)^7/d-1/8*a*cos(
d*x+c)^8/d-1/9*a*cos(d*x+c)^9/d-a*ln(cos(d*x+c))/d
```

Rubi [A] time = 0.11, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3872, 2836, 12, 88}

$$-\frac{a\cos^9(c+dx)}{9d}-\frac{a\cos^8(c+dx)}{8d}+\frac{4a\cos^7(c+dx)}{7d}+\frac{2a\cos^6(c+dx)}{3d}-\frac{6a\cos^5(c+dx)}{5d}-\frac{3a\cos^4(c+dx)}{2d}+\frac{4a\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^9,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (2*a*Cos[c + d*x]^2)/d + (4*a*Cos[c + d*x]^3)/(3*d)
 - (3*a*Cos[c + d*x]^4)/(2*d) - (6*a*Cos[c + d*x]^5)/(5*d) + (2*a*Cos[c + d
*x]^6)/(3*d) + (4*a*Cos[c + d*x]^7)/(7*d) - (a*Cos[c + d*x]^8)/(8*d) - (a*C
os[c + d*x]^9)/(9*d) - (a*Log[Cos[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
```

```
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))\sin^9(c+dx)\,dx &= -\int (-a-a\cos(c+dx))\sin^8(c+dx)\tan(c+dx)\,dx\\ &= \frac{\text{Subst}\left(\int \frac{a(-a-x)^4(-a+x)^5}{x}\,dx, x, -a\cos(c+dx)\right)}{a^9 d}\\ &= \frac{\text{Subst}\left(\int \frac{(-a-x)^4(-a+x)^5}{x}\,dx, x, -a\cos(c+dx)\right)}{a^8 d}\\ &= \frac{\text{Subst}\left(\int \left(a^8-\frac{a^9}{x}+4a^7x-4a^6x^2-6a^5x^3+6a^4x^4+4a^3x^5-4a^2x^6-ax^7\right.}{a^8 d}\\ &= -\frac{a\cos(c+dx)}{d}+\frac{2a\cos^2(c+dx)}{d}+\frac{4a\cos^3(c+dx)}{3d}-\frac{3a\cos^4(c+dx)}{2d}-\end{aligned}$$

Mathematica [A] time = 0.22, size = 106, normalized size = 0.70

$$-\frac{a\left(10080\cos^8(c+dx)-53760\cos^6(c+dx)+120960\cos^4(c+dx)-161280\cos^2(c+dx)+39690\cos(c+dx)\right.}{806}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^9,x]
```

```
[Out] -1/80640*(a*(39690*Cos[c + d*x] - 161280*Cos[c + d*x]^2 + 120960*Cos[c + d*
x]^4 - 53760*Cos[c + d*x]^6 + 10080*Cos[c + d*x]^8 - 8820*Cos[3*(c + d*x)]
+ 2268*Cos[5*(c + d*x)] - 405*Cos[7*(c + d*x)] + 35*Cos[9*(c + d*x)] + 8064
0*Log[Cos[c + d*x]]))/d
```

fricas [A] time = 1.71, size = 115, normalized size = 0.76

$$-\frac{280\,a\cos(dx+c)^9+315\,a\cos(dx+c)^8-1440\,a\cos(dx+c)^7-1680\,a\cos(dx+c)^6+3024\,a\cos(dx+c)^5+3}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/2520*(280*a*cos(d*x + c)^9 + 315*a*cos(d*x + c)^8 - 1440*a*cos(d*x + c)^
7 - 1680*a*cos(d*x + c)^6 + 3024*a*cos(d*x + c)^5 + 3780*a*cos(d*x + c)^4 -
 3360*a*cos(d*x + c)^3 - 5040*a*cos(d*x + c)^2 + 2520*a*cos(d*x + c) + 2520
*a*log(-cos(d*x + c)))/d
```

giac [B] time = 0.25, size = 293, normalized size = 1.93

$$\frac{2520\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2520\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{9177\,a-\frac{87633\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{375732\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{953988\,a(\cos(}{(\cos(dx+c}}{}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/2520*(2520*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2520*
a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (9177*a - 87633*a*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 375732*a*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 - 953988*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1594
782*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 1336734*a*(cos(d*x + c) -
 1)^5/(cos(d*x + c) + 1)^5 + 781956*a*(cos(d*x + c) - 1)^6/(cos(d*x + c) +
1)^6 - 302004*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 69201*a*(cos(d*
x + c) - 1)^8/(cos(d*x + c) + 1)^8 - 7129*a*(cos(d*x + c) - 1)^9/(cos(d*x +
 c) + 1)^9)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^9)/d
```

maple [A] time = 0.67, size = 163, normalized size = 1.07

$$-\frac{128a\cos(dx+c)}{315d}-\frac{\left(\sin^8(dx+c)\right)\cos(dx+c)\,a}{9d}-\frac{8a\cos(dx+c)\left(\sin^6(dx+c)\right)}{63d}-\frac{16a\cos(dx+c)\left(\sin^4(dx+\right.}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^9,x)
```

```
[Out] -128/315*a*cos(d*x+c)/d-1/9/d*sin(d*x+c)^8*cos(d*x+c)*a-8/63/d*a*cos(d*x+c)
*sin(d*x+c)^6-16/105/d*a*cos(d*x+c)*sin(d*x+c)^4-64/315/d*a*cos(d*x+c)*sin(
d*x+c)^2-1/8/d*a*sin(d*x+c)^8-1/6/d*a*sin(d*x+c)^6-1/4/d*a*sin(d*x+c)^4-1/2
/d*a*sin(d*x+c)^2-a*ln(cos(d*x+c))/d
```

maxima [A] time = 0.34, size = 113, normalized size = 0.74

$$-\frac{280\,a\cos(dx+c)^9+315\,a\cos(dx+c)^8-1440\,a\cos(dx+c)^7-1680\,a\cos(dx+c)^6+3024\,a\cos(dx+c)^5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/2520*(280*a*cos(d*x + c)^9 + 315*a*cos(d*x + c)^8 - 1440*a*cos(d*x + c)^
7 - 1680*a*cos(d*x + c)^6 + 3024*a*cos(d*x + c)^5 + 3780*a*cos(d*x + c)^4 -
 3360*a*cos(d*x + c)^3 - 5040*a*cos(d*x + c)^2 + 2520*a*cos(d*x + c) + 2520
*a*log(cos(d*x + c)))/d
```

mupad [B] time = 0.13, size = 111, normalized size = 0.73

$$-\frac{a\cos(c+d\,x)-2\,a\cos(c+d\,x)^2-\frac{4\,a\cos(c+d\,x)^3}{3}+\frac{3\,a\cos(c+d\,x)^4}{2}+\frac{6\,a\cos(c+d\,x)^5}{5}-\frac{2\,a\cos(c+d\,x)^6}{3}-\frac{4\,a\cos(c+d\,x)^7}{7}+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^9*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*cos(c + d*x) - 2*a*cos(c + d*x)^2 - (4*a*cos(c + d*x)^3)/3 + (3*a*cos(c
 + d*x)^4)/2 + (6*a*cos(c + d*x)^5)/5 - (2*a*cos(c + d*x)^6)/3 - (4*a*cos(c
 + d*x)^7)/7 + (a*cos(c + d*x)^8)/8 + (a*cos(c + d*x)^9)/9 + a*log(cos(c +
d*x)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**9,x)
```

```
[Out] Timed out
```

3.2 $\int (a+a\sec (c+dx))\sin^7(c+dx)\,dx$

Optimal. Leaf size=119

$$\frac{a\cos^7(c+dx)}{7d}+\frac{a\cos^6(c+dx)}{6d}-\frac{3a\cos^5(c+dx)}{5d}-\frac{3a\cos^4(c+dx)}{4d}+\frac{a\cos^3(c+dx)}{d}+\frac{3a\cos^2(c+dx)}{2d}-\frac{a\cos(c+dx)}{d}$$

```
[Out] -a*cos(d*x+c)/d+3/2*a*cos(d*x+c)^2/d+a*cos(d*x+c)^3/d-3/4*a*cos(d*x+c)^4/d-
3/5*a*cos(d*x+c)^5/d+1/6*a*cos(d*x+c)^6/d+1/7*a*cos(d*x+c)^7/d-a*ln(cos(d*x
+c))/d
```

Rubi [A] time = 0.10, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = $\{3872, 2836, 12, 88\}$

$$\frac{a\cos^7(c+dx)}{7d}+\frac{a\cos^6(c+dx)}{6d}-\frac{3a\cos^5(c+dx)}{5d}-\frac{3a\cos^4(c+dx)}{4d}+\frac{a\cos^3(c+dx)}{d}+\frac{3a\cos^2(c+dx)}{2d}-\frac{a\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^7,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (3*a*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]^3)/d -
 (3*a*Cos[c + d*x]^4)/(4*d) - (3*a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c + d*x]^
6)/(6*d) + (a*Cos[c + d*x]^7)/(7*d) - (a*Log[Cos[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))\sin^7(c+dx)\,dx = -\int (-a-a\cos(c+dx))\sin^6(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a(-a-x)^3(-a+x)^4}{x}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^3(-a+x)^4}{x}\,dx, x, -a\cos(c+dx)\right)}{a^6 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^6-\frac{a^7}{x}+3a^5x-3a^4x^2-3a^3x^3+3a^2x^4+ax^5-x^6\right)dx, x, -\right.}{a^6 d}$$
$$= -\frac{a\cos(c+dx)}{d}+\frac{3a\cos^2(c+dx)}{2d}+\frac{a\cos^3(c+dx)}{d}-\frac{3a\cos^4(c+dx)}{4d}$$

Mathematica [A] time = 0.14, size = 86, normalized size = 0.72

$$\frac{a\left(1120\cos^6(c+dx)-5040\cos^4(c+dx)+10080\cos^2(c+dx)-3675\cos(c+dx)+735\cos(3(c+dx))-147\right.}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^7,x]
```

```
[Out] (a*(-3675*Cos[c + d*x] + 10080*Cos[c + d*x]^2 - 5040*Cos[c + d*x]^4 + 1120*
Cos[c + d*x]^6 + 735*Cos[3*(c + d*x)] - 147*Cos[5*(c + d*x)] + 15*Cos[7*(c
+ d*x)] - 6720*Log[Cos[c + d*x]]))/(6720*d)
```

fricas [A] time = 0.72, size = 93, normalized size = 0.78

$$\frac{60\,a\cos(dx+c)^7+70\,a\cos(dx+c)^6-252\,a\cos(dx+c)^5-315\,a\cos(dx+c)^4+420\,a\cos(dx+c)^3+630\,a}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/420*(60*a*cos(d*x + c)^7 + 70*a*cos(d*x + c)^6 - 252*a*cos(d*x + c)^5 - 3
15*a*cos(d*x + c)^4 + 420*a*cos(d*x + c)^3 + 630*a*cos(d*x + c)^2 - 420*a*c
os(d*x + c) - 420*a*log(-cos(d*x + c)))/d
```

giac [B] time = 0.24, size = 247, normalized size = 2.08

$$\frac{420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{1473\,a-\frac{11151\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{36813\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{69475\,a(\cos(dx}{(\cos(dx+c)}}{}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/420*(420*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (1473*a - 11151*a*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 36813*a*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 - 69475*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*a*(
cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 28749*a*(cos(d*x + c) - 1)^5/(co
s(d*x + c) + 1)^5 + 8463*a*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 1089
*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) - 1)^7)/d
```

maple [A] time = 0.61, size = 129, normalized size = 1.08

$$-\frac{16a\cos(dx+c)}{35d}-\frac{a\cos(dx+c)\left(\sin^{6}(dx+c)\right)}{7d}-\frac{6a\cos(dx+c)\left(\sin^{4}(dx+c)\right)}{35d}-\frac{8a\cos(dx+c)\left(\sin^{2}(dx+c)\right)}{35d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^7,x)
```

```
[Out] -16/35*a*cos(d*x+c)/d-1/7/d*a*cos(d*x+c)*sin(d*x+c)^6-6/35/d*a*cos(d*x+c)*s
in(d*x+c)^4-8/35/d*a*cos(d*x+c)*sin(d*x+c)^2-1/6/d*a*sin(d*x+c)^6-1/4/d*a*s
in(d*x+c)^4-1/2/d*a*sin(d*x+c)^2-a*ln(cos(d*x+c))/d
```

maxima [A] time = 0.62, size = 91, normalized size = 0.76

$$\frac{60\,a\cos(dx+c)^{7}+70\,a\cos(dx+c)^{6}-252\,a\cos(dx+c)^{5}-315\,a\cos(dx+c)^{4}+420\,a\cos(dx+c)^{3}+630\,a\,\mathrm{co}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/420*(60*a*cos(d*x + c)^7 + 70*a*cos(d*x + c)^6 - 252*a*cos(d*x + c)^5 - 3
15*a*cos(d*x + c)^4 + 420*a*cos(d*x + c)^3 + 630*a*cos(d*x + c)^2 - 420*a*c
os(d*x + c) - 420*a*log(cos(d*x + c)))/d
```

mupad [B] time = 0.08, size = 89, normalized size = 0.75

$$-\frac{a\cos(c+d\,x)-\frac{3\,a\cos(c+d\,x)^{2}}{2}-a\cos(c+d\,x)^{3}+\frac{3\,a\cos(c+d\,x)^{4}}{4}+\frac{3\,a\cos(c+d\,x)^{5}}{5}-\frac{a\cos(c+d\,x)^{6}}{6}-\frac{a\cos(c+d\,x)^{7}}{7}+a\,\ln(\mathrm{co}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*cos(c + d*x) - (3*a*cos(c + d*x)^2)/2 - a*cos(c + d*x)^3 + (3*a*cos(c +
 d*x)^4)/4 + (3*a*cos(c + d*x)^5)/5 - (a*cos(c + d*x)^6)/6 - (a*cos(c + d*x
)^7)/7 + a*log(cos(c + d*x)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**7,x)
```

```
[Out] Timed out
```

3.3 $\int (a+a\sec(c+dx))\sin^5(c+dx)\,dx$

Optimal. Leaf size=87

$$-\frac{a\cos^5(c+dx)}{5d}-\frac{a\cos^4(c+dx)}{4d}+\frac{2a\cos^3(c+dx)}{3d}+\frac{a\cos^2(c+dx)}{d}-\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[Out] -a*cos(d*x+c)/d+a*cos(d*x+c)^2/d+2/3*a*cos(d*x+c)^3/d-1/4*a*cos(d*x+c)^4/d-
1/5*a*cos(d*x+c)^5/d-a*ln(cos(d*x+c))/d
```

Rubi [A] time = 0.09, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 2836, 12, 88}

$$-\frac{a\cos^5(c+dx)}{5d}-\frac{a\cos^4(c+dx)}{4d}+\frac{2a\cos^3(c+dx)}{3d}+\frac{a\cos^2(c+dx)}{d}-\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^5,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (a*Cos[c + d*x]^2)/d + (2*a*Cos[c + d*x]^3)/(3*d) -
 (a*Cos[c + d*x]^4)/(4*d) - (a*Cos[c + d*x]^5)/(5*d) - (a*Log[Cos[c + d*x]]
)/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))\sin^5(c+dx)\,dx = -\int (-a-a\cos(c+dx))\sin^4(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a(-a-x)^2(-a+x)^3}{x}\,dx, x, -a\cos(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^2(-a+x)^3}{x}\,dx, x, -a\cos(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^4-\frac{a^5}{x}+2a^3x-2a^2x^2-ax^3+x^4\right)dx, x, -a\cos(c+dx)\right)}{a^4 d}$$
$$= -\frac{a\cos(c+dx)}{d}+\frac{a\cos^2(c+dx)}{d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{a\cos^4(c+dx)}{4d}-\frac{a\,c}{}$$

Mathematica [A] time = 0.09, size = 83, normalized size = 0.95

$$-\frac{5a\cos(c+dx)}{8d}+\frac{5a\cos(3(c+dx))}{48d}-\frac{a\cos(5(c+dx))}{80d}-\frac{a\left(\frac{1}{4}\cos^4(c+dx)-\cos^2(c+dx)+\log(\cos(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^5,x]
```

```
[Out] (-5*a*Cos[c + d*x])/(8*d) + (5*a*Cos[3*(c + d*x)])/(48*d) - (a*Cos[5*(c + d
*x)])/(80*d) - (a*(-Cos[c + d*x]^2 + Cos[c + d*x]^4/4 + Log[Cos[c + d*x]]))
/d
```

fricas [A] time = 0.51, size = 71, normalized size = 0.82

$$-\frac{12\,a\cos(dx+c)^5+15\,a\cos(dx+c)^4-40\,a\cos(dx+c)^3-60\,a\cos(dx+c)^2+60\,a\cos(dx+c)+60\,a\log(-c}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/60*(12*a*cos(d*x + c)^5 + 15*a*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 60
*a*cos(d*x + c)^2 + 60*a*cos(d*x + c) + 60*a*log(-cos(d*x + c)))/d
```

giac [B] time = 0.58, size = 201, normalized size = 2.31

$$\frac{60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{201\,a-\frac{1125\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2610\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{1970\,a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+80}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (201*a - 1125*a*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 2610*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 - 1970*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805*a*(cos(d*x +
 c) - 1)^4/(cos(d*x + c) + 1)^4 - 137*a*(cos(d*x + c) - 1)^5/(cos(d*x + c)
+ 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)/d
```

maple [A] time = 0.60, size = 95, normalized size = 1.09

$$-\frac{8a\cos(dx+c)}{15d}-\frac{a\cos(dx+c)\left(\sin^4(dx+c)\right)}{5d}-\frac{4a\cos(dx+c)\left(\sin^2(dx+c)\right)}{15d}-\frac{a\left(\sin^4(dx+c)\right)}{4d}-\frac{a\left(\sin^2(dx+c\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^5,x)
```

```
[Out] -8/15*a*cos(d*x+c)/d-1/5/d*a*cos(d*x+c)*sin(d*x+c)^4-4/15/d*a*cos(d*x+c)*si
n(d*x+c)^2-1/4/d*a*sin(d*x+c)^4-1/2/d*a*sin(d*x+c)^2-a*ln(cos(d*x+c))/d
```

maxima [A] time = 0.39, size = 69, normalized size = 0.79

$$-\frac{12\,a\cos\left(dx+c\right)^{5}+15\,a\cos\left(dx+c\right)^{4}-40\,a\cos\left(dx+c\right)^{3}-60\,a\cos\left(dx+c\right)^{2}+60\,a\cos\left(dx+c\right)+60\,a\log(\cdots}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(12*a*cos(d*x + c)^5 + 15*a*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 60
*a*cos(d*x + c)^2 + 60*a*cos(d*x + c) + 60*a*log(cos(d*x + c)))/d
```

mupad [B] time = 0.06, size = 67, normalized size = 0.77

$$-\frac{a\,\cos\left(c+d\,x\right)-a\cos\left(c+d\,x\right)^2-\frac{2\,a\cos\left(c+d\,x\right)^3}{3}+\frac{a\cos\left(c+d\,x\right)^4}{4}+\frac{a\cos\left(c+d\,x\right)^5}{5}+a\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*cos(c + d*x) - a*cos(c + d*x)^2 - (2*a*cos(c + d*x)^3)/3 + (a*cos(c + d
*x)^4)/4 + (a*cos(c + d*x)^5)/5 + a*log(cos(c + d*x)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin^{5}\left(c+dx\right)\sec\left(c+dx\right)dx+\int\sin^{5}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**5,x)
```

```
[Out] a*(Integral(sin(c + d*x)**5*sec(c + d*x), x) + Integral(sin(c + d*x)**5, x)
)
```

3.4 $\int (a + a\sec(c + dx))\sin^3(c + dx)\,dx$

Optimal. Leaf size=58

$$\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos^2(c+dx)}{2d}-\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[Out] -a*cos(d*x+c)/d+1/2*a*cos(d*x+c)^2/d+1/3*a*cos(d*x+c)^3/d-a*ln(cos(d*x+c))/
d
```

Rubi [A] time = 0.08, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 2836, 12, 75}

$$\frac{a\cos^3(c+dx)}{3d}+\frac{a\cos^2(c+dx)}{2d}-\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^3,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (a*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]^3)/(3*d)
 - (a*Log[Cos[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))\sin^3(c+dx)\,dx = -\int (-a-a\cos(c+dx))\sin^2(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a(-a-x)(-a+x)^2}{x}\,dx, x, -a\cos(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)(-a+x)^2}{x}\,dx, x, -a\cos(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2-\frac{a^3}{x}+ax-x^2\right)dx, x, -a\cos(c+dx)\right)}{a^2 d}$$
$$= -\frac{a\cos(c+dx)}{d}+\frac{a\cos^2(c+dx)}{2d}+\frac{a\cos^3(c+dx)}{3d}-\frac{a\log(\cos(c+dx)}{d}$$

Mathematica [A] time = 0.06, size = 57, normalized size = 0.98

$$-\frac{3a\cos(c+dx)}{4d}+\frac{a\cos(3(c+dx))}{12d}-\frac{a\left(\log(\cos(c+dx))-\frac{1}{2}\cos^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^3,x]
```

```
[Out] (-3*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)])/(12*d) - (a*(-1/2*Cos[c +
d*x]^2 + Log[Cos[c + d*x]]))/d
```

fricas [A] time = 0.52, size = 49, normalized size = 0.84

$$\frac{2\,a\cos(dx+c)^3+3\,a\cos(dx+c)^2-6\,a\cos(dx+c)-6\,a\log(-\cos(dx+c))}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*cos(d*x + c)^3 + 3*a*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 6*a*log(-
cos(d*x + c)))/d
```

giac [A] time = 0.36, size = 66, normalized size = 1.14

$$-\frac{a\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{2\,ad^2\cos(dx+c)^3+3\,ad^2\cos(dx+c)^2-6\,ad^2\cos(dx+c)}{6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -a*log(abs(cos(d*x + c))/abs(d))/d + 1/6*(2*a*d^2*cos(d*x + c)^3 + 3*a*d^2*
cos(d*x + c)^2 - 6*a*d^2*cos(d*x + c))/d^3
```

maple [A] time = 0.61, size = 61, normalized size = 1.05

$$-\frac{a\cos(dx+c)\left(\sin^2(dx+c)\right)}{3d}-\frac{2a\cos(dx+c)}{3d}-\frac{a\left(\sin^2(dx+c)\right)}{2d}-\frac{a\ln(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^3,x)
```

```
[Out] -1/3/d*a*cos(d*x+c)*sin(d*x+c)^2-2/3*a*cos(d*x+c)/d-1/2/d*a*sin(d*x+c)^2-a*
ln(cos(d*x+c))/d
```

maxima [A] time = 0.62, size = 47, normalized size = 0.81

$$\frac{2\,a\cos\left(dx+c\right)^3+3\,a\cos\left(dx+c\right)^2-6\,a\cos\left(dx+c\right)-6\,a\log\left(\cos\left(dx+c\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a*cos(d*x + c)^3 + 3*a*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 6*a*log(c
os(d*x + c)))/d
```

mupad [B] time = 0.06, size = 45, normalized size = 0.78

$$-\frac{a\,\cos\left(c+d\,x\right)-\frac{a\cos(c+d\,x)^2}{2}-\frac{a\cos(c+d\,x)^3}{3}+a\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + a/cos(c + d*x)),x)
```

```
[Out] -(a*cos(c + d*x) - (a*cos(c + d*x)^2)/2 - (a*cos(c + d*x)^3)/3 + a*log(cos(
c + d*x)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin^3\left(c+dx\right)\sec\left(c+dx\right)dx+\int\sin^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**3,x)
```

```
[Out] a*(Integral(sin(c + d*x)**3*sec(c + d*x), x) + Integral(sin(c + d*x)**3, x)
)
```

3.5 $\int (a+a\sec(c+dx))\sin(c+dx)\,dx$

Optimal. Leaf size=26

$$-\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[Out] -a*cos(d*x+c)/d-a*ln(cos(d*x+c))/d
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3872, 2707, 43}

$$-\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x],x]
```

```
[Out] -((a*Cos[c + d*x])/d) - (a*Log[Cos[c + d*x]])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(c+dx))\sin(c+dx)\,dx &= -\int (-a-a\cos(c+dx))\tan(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{-a+x}{x}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(1-\frac{a}{x}\right)\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= -\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 1.42

$$\frac{a\sin(c)\sin(dx)}{d}-\frac{a\cos(c)\cos(dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x],x]
[Out] -((a*Cos[c]*Cos[d*x])/d) - (a*Log[Cos[c + d*x]])/d + (a*Sin[c]*Sin[d*x])/d
```

fricas [A] time = 0.51, size = 25, normalized size = 0.96

$$-\frac{a\cos\left(dx+c\right)+a\log\left(-\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c),x, algorithm="fricas")
[Out] -(a*cos(d*x + c) + a*log(-cos(d*x + c)))/d
```

giac [A] time = 1.45, size = 32, normalized size = 1.23

$$-\frac{a\cos\left(dx+c\right)}{d}-\frac{a\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c),x, algorithm="giac")
[Out] -a*cos(d*x + c)/d - a*log(abs(cos(d*x + c))/abs(d))/d
```

maple [A] time = 0.20, size = 28, normalized size = 1.08

$$\frac{a\ln\left(\sec\left(dx+c\right)\right)}{d}-\frac{a}{d\sec\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c),x)
[Out] a/d*ln(sec(d*x+c))-a/d/sec(d*x+c)
```

maxima [A] time = 0.35, size = 23, normalized size = 0.88

$$-\frac{a\cos\left(dx+c\right)+a\log\left(\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c),x, algorithm="maxima")
[Out] -(a*cos(d*x + c) + a*log(cos(d*x + c)))/d
```

mupad [B] time = 0.04, size = 20, normalized size = 0.77

$$-\frac{a\,\left(\cos\left(c+d\,x\right)+\ln\left(\cos\left(c+d\,x\right)\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + a/cos(c + d*x)),x)
[Out] -(a*(cos(c + d*x) + log(cos(c + d*x))))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin\left(c+dx\right)\sec\left(c+dx\right)dx+\int\sin\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c),x)
```

```
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x), x) + Integral(sin(c + d*x), x))
```

3.6 $\int \csc(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=30

$$\frac{a\log(1-\cos(c+dx))}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[Out] a*ln(1-cos(d*x+c))/d-a*ln(cos(d*x+c))/d
```

Rubi [A] time = 0.06, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3872, 2836, 12, 36, 31, 29}

$$\frac{a\log(1-\cos(c+dx))}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Log[1 - Cos[c + d*x]])/d - (a*Log[Cos[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc(c+dx)(a+a\sec(c+dx))\,dx = -\int (-a-a\cos(c+dx))\csc(c+dx)\sec(c+dx)\,dx$$
$$= \frac{a\,\text{Subst}\left(\int \frac{a}{(-a-x)x}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{1}{(-a-x)x}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{-a-x}\,dx, x, -a\cos(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, -a\cos(c-\right.}{d}$$
$$= \frac{a\log(1-\cos(c+dx))}{d} - \frac{a\log(\cos(c+dx))}{d}$$

Mathematica [B] time = 0.04, size = 63, normalized size = 2.10

$$\frac{a\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{a\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{a(\log(\cos(c+dx))-\log(\sin(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sec[c + d*x]),x]
```

```
[Out] -((a*Log[Cos[c/2 + (d*x)/2]])/d) + (a*Log[Sin[c/2 + (d*x)/2]])/d - (a*(Log[
Cos[c + d*x]] - Log[Sin[c + d*x]]))/d
```

fricas [A] time = 0.48, size = 31, normalized size = 1.03

$$-\frac{a\log(-\cos(dx+c)) - a\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(a*log(-cos(d*x + c)) - a*log(-1/2*cos(d*x + c) + 1/2))/d
```

giac [A] time = 0.50, size = 58, normalized size = 1.93

$$\frac{a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a*log(abs(-(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - 1)))/d
```

maple [A] time = 0.36, size = 15, normalized size = 0.50

$$\frac{a\ln(-1+\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d*a*ln(-1+sec(d*x+c))
```

maxima [A] time = 0.35, size = 26, normalized size = 0.87

$$\frac{a \log\left(\cos\left(dx+c\right)-1\right)-a \log\left(\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(cos(d*x + c) - 1) - a*log(cos(d*x + c)))/d
```

mupad [B] time = 0.12, size = 17, normalized size = 0.57

$$\frac{2\,a\,\mathrm{atanh}\left(1-2\,\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x),x)
```

```
[Out] (2*a*atanh(1 - 2*cos(c + d*x)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \csc\left(c+dx\right)\sec\left(c+dx\right)dx+\int \csc\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(csc(c + d*x)*sec(c + d*x), x) + Integral(csc(c + d*x), x))
```

3.7 $\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=73

$$-\frac{a^2}{2d(a-a\cos(c+dx))}+\frac{3a\log(1-\cos(c+dx))}{4d}-\frac{a\log(\cos(c+dx))}{d}+\frac{a\log(\cos(c+dx)+1)}{4d}$$

```
[Out] -1/2*a^2/d/(a-a*cos(d*x+c))+3/4*a*ln(1-cos(d*x+c))/d-a*ln(cos(d*x+c))/d+1/4
*a*ln(1+cos(d*x+c))/d
```

Rubi [A] time = 0.10, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 2836, 12, 72}

$$-\frac{a^2}{2d(a-a\cos(c+dx))}+\frac{3a\log(1-\cos(c+dx))}{4d}-\frac{a\log(\cos(c+dx))}{d}+\frac{a\log(\cos(c+dx)+1)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] -a^2/(2*d*(a - a*Cos[c + d*x])) + (3*a*Log[1 - Cos[c + d*x]])/(4*d) - (a*Lo
g[Cos[c + d*x]])/d + (a*Log[1 + Cos[c + d*x]])/(4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx = -\int (-a-a\cos(c+dx))\csc^3(c+dx)\sec(c+dx)\,dx$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{a}{(-a-x)^2x(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^4\,\text{Subst}\left(\int \frac{1}{(-a-x)^2x(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^4\,\text{Subst}\left(\int \left(-\frac{1}{4a^3(a-x)}-\frac{1}{a^3x}+\frac{1}{2a^2(a+x)^2}+\frac{3}{4a^3(a+x)}\right)dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^2}{2d(a-a\cos(c+dx))}+\frac{3a\log(1-\cos(c+dx))}{4d}-\frac{a\log(\cos(c+dx))}{d}+$$

Mathematica [A] time = 0.84, size = 114, normalized size = 1.56

$$-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\left(\csc^2(c+dx)-2\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/8*(a*Csc[(c + d*x)/2]^2)/d - (a*Log[Cos[(c + d*x)/2]])/(2*d) + (a*Log[Si
n[(c + d*x)/2]])/(2*d) - (a*(Csc[c + d*x]^2 + 2*Log[Cos[c + d*x]] - 2*Log[S
in[c + d*x]]))/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

fricas [A] time = 0.49, size = 93, normalized size = 1.27

$$-\frac{4\,(a\cos(dx+c)-a)\log(-\cos(dx+c))-(a\cos(dx+c)-a)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3\,(a\cos(dx+c)-a)\,\mathrm{l}}{4\,(d\cos(dx+c)-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*(a*cos(d*x + c) - a)*log(-cos(d*x + c)) - (a*cos(d*x + c) - a)*log(
1/2*cos(d*x + c) + 1/2) - 3*(a*cos(d*x + c) - a)*log(-1/2*cos(d*x + c) + 1/
2) - 2*a)/(d*cos(d*x + c) - d)
```

giac [A] time = 0.43, size = 102, normalized size = 1.40

$$\frac{3\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-4\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{\left(a-\frac{3\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(3*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 4*a*log(abs(-(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a - 3*a*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

maple [A] time = 0.59, size = 48, normalized size = 0.66

$$-\frac{a}{2d\,(-1+\sec(dx+c))}+\frac{3a\ln(-1+\sec(dx+c))}{4d}+\frac{a\ln(1+\sec(dx+c))}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sec(d*x+c)),x)
```

```
[Out] -1/2/d*a/(-1+sec(d*x+c))+3/4/d*a*ln(-1+sec(d*x+c))+1/4/d*a*ln(1+sec(d*x+c))
```

maxima [A] time = 0.35, size = 52, normalized size = 0.71

$$\frac{a\log\left(\cos\left(dx+c\right)+1\right)+3\,a\log\left(\cos\left(dx+c\right)-1\right)-4\,a\log\left(\cos\left(dx+c\right)\right)+\frac{2\,a}{\cos(dx+c)-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*log(cos(d*x + c) + 1) + 3*a*log(cos(d*x + c) - 1) - 4*a*log(cos(d*x
+ c)) + 2*a/(cos(d*x + c) - 1))/d
```

mupad [B] time = 0.96, size = 53, normalized size = 0.73

$$\frac{\frac{a}{2\,(\cos(c+d\,x)-1)}-a\,\ln\left(\cos\left(c+d\,x\right)\right)+\frac{3\,a\,\ln(\cos(c+d\,x)-1)}{4}+\frac{a\,\ln(\cos(c+d\,x)+1)}{4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^3,x)
```

```
[Out] (a/(2*(cos(c + d*x) - 1)) - a*log(cos(c + d*x)) + (3*a*log(cos(c + d*x) - 1
))/4 + (a*log(cos(c + d*x) + 1))/4)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\csc^{3}\left(c+dx\right)\sec\left(c+dx\right)dx+\int\csc^{3}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(csc(c + d*x)**3*sec(c + d*x), x) + Integral(csc(c + d*x)**3, x)
)
```

3.8 $\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=118

$$-\frac{a^3}{8d(a-a\cos(c+dx))^2}-\frac{a^2}{2d(a-a\cos(c+dx))}-\frac{a^2}{8d(a\cos(c+dx)+a)}+\frac{11a\log(1-\cos(c+dx))}{16d}-\frac{a\log(\cos(c+}{d}$$

```
[Out] -1/8*a^3/d/(a-a*cos(d*x+c))^2-1/2*a^2/d/(a-a*cos(d*x+c))-1/8*a^2/d/(a+a*cos
(d*x+c))+11/16*a*ln(1-cos(d*x+c))/d-a*ln(cos(d*x+c))/d+5/16*a*ln(1+cos(d*x+
c))/d
```

Rubi [A] time = 0.12, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3872, 2836, 12, 88}

$$-\frac{a^3}{8d(a-a\cos(c+dx))^2}-\frac{a^2}{2d(a-a\cos(c+dx))}-\frac{a^2}{8d(a\cos(c+dx)+a)}+\frac{11a\log(1-\cos(c+dx))}{16d}-\frac{a\log(\cos(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + a*Sec[c + d*x]),x]
```

```
[Out] -a^3/(8*d*(a - a*Cos[c + d*x])^2) - a^2/(2*d*(a - a*Cos[c + d*x])) - a^2/(8
*d*(a + a*Cos[c + d*x])) + (11*a*Log[1 - Cos[c + d*x]])/(16*d) - (a*Log[Cos
[c + d*x]])/d + (5*a*Log[1 + Cos[c + d*x]])/(16*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx = -\int(-a-a\cos(c+dx))\csc^5(c+dx)\sec(c+dx)\,dx$$
$$= \frac{a^5\,\text{Subst}\left(\int \frac{a}{(-a-x)^3x(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \frac{1}{(-a-x)^3x(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \left(-\frac{1}{8a^4(a-x)^2} - \frac{5}{16a^5(a-x)} - \frac{1}{a^5x} + \frac{1}{4a^3(a+x)^3} + \frac{1}{2a^4(a+x)^2} + \frac{11}{16a^5(a+\right.\right.}{d}$$
$$= -\frac{a^3}{8d(a-a\cos(c+dx))^2} - \frac{a^2}{2d(a-a\cos(c+dx))} - \frac{a^2}{8d(a+a\cos(c+dx}$$

Mathematica [A] time = 0.36, size = 164, normalized size = 1.39

$$-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{3a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-3*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (3*a*Log
[Cos[(c + d*x)/2]])/(8*d) + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) - (a*(2*Csc[c
 + d*x]^2 + Csc[c + d*x]^4 + 4*Log[Cos[c + d*x]] - 4*Log[Sin[c + d*x]]))/(4
*d) + (3*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

fricas [A] time = 0.74, size = 193, normalized size = 1.64

$$\frac{6\,a\cos\left(dx+c\right)^2+2\,a\cos\left(dx+c\right)-16\left(a\cos\left(dx+c\right)^3-a\cos\left(dx+c\right)^2-a\cos\left(dx+c\right)+a\right)\log\left(-\cos\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(6*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - 16*(a*cos(d*x + c)^3 - a*cos(
d*x + c)^2 - a*cos(d*x + c) + a)*log(-cos(d*x + c)) + 5*(a*cos(d*x + c)^3 -
 a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(1/2*cos(d*x + c) + 1/2) + 11*(a
*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(-1/2*cos(d*x +
 c) + 1/2) - 12*a)/(d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) +
d)
```

giac [A] time = 0.31, size = 149, normalized size = 1.26

$$\frac{22\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-32\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\frac{\left(a-\frac{10\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{33\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(\cos(dx+c)-1)^2}+\frac{2\,a(\cos(dx+c}{\cos(dx+c)}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(22*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 32*a*log(abs
(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a - 10*a*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 33*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos
```

```
(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1))/d
```

maple [A] time = 0.47, size = 80, normalized size = 0.68

$$-\frac{a}{8d\left(-1+\sec\left(dx+c\right)\right)^{2}}-\frac{3a}{4d\left(-1+\sec\left(dx+c\right)\right)}+\frac{11a\ln\left(-1+\sec\left(dx+c\right)\right)}{16d}+\frac{a}{8d\left(1+\sec\left(dx+c\right)\right)}+\frac{5a\ln\left(1+\sec\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+a*sec(d*x+c)),x)
```

```
[Out] -1/8/d*a/(-1+sec(d*x+c))^2-3/4/d*a/(-1+sec(d*x+c))+11/16/d*a*ln(-1+sec(d*x+
c))+1/8/d*a/(1+sec(d*x+c))+5/16/d*a*ln(1+sec(d*x+c))
```

maxima [A] time = 0.52, size = 95, normalized size = 0.81

$$\frac{5\,a\log\left(\cos\left(dx+c\right)+1\right)+11\,a\log\left(\cos\left(dx+c\right)-1\right)-16\,a\log\left(\cos\left(dx+c\right)\right)+\frac{2\left(3\,a\cos\left(dx+c\right)^{2}+a\cos\left(dx+c\right)-6\,a\right)}{\cos\left(dx+c\right)^{3}-\cos\left(dx+c\right)^{2}-\cos\left(dx+c\right)+1}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(5*a*log(cos(d*x + c) + 1) + 11*a*log(cos(d*x + c) - 1) - 16*a*log(cos
(d*x + c)) + 2*(3*a*cos(d*x + c)^2 + a*cos(d*x + c) - 6*a)/(cos(d*x + c)^3
- cos(d*x + c)^2 - cos(d*x + c) + 1))/d
```

mupad [B] time = 0.10, size = 99, normalized size = 0.84

$$\frac{11\,a\,\ln\left(\cos\left(c+d\,x\right)-1\right)}{16\,d}-\frac{a\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}+\frac{5\,a\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{16\,d}+\frac{\frac{3\,a\cos\left(c+d\,x\right)^{2}}{8}+\frac{a\,\cos\left(c+d\,x\right)}{8}-\frac{3\,a}{4}}{d\,\left(\cos\left(c+d\,x\right)^{3}-\cos\left(c+d\,x\right)+\sin\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^5,x)
```

```
[Out] (11*a*log(cos(c + d*x) - 1))/(16*d) - (a*log(cos(c + d*x)))/d + (5*a*log(co
s(c + d*x) + 1))/(16*d) + ((a*cos(c + d*x))/8 - (3*a)/4 + (3*a*cos(c + d*x)
^2)/8)/(d*(cos(c + d*x)^3 - cos(c + d*x) + sin(c + d*x)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\csc^{5}\left(c+dx\right)\sec\left(c+dx\right)dx+\int\csc^{5}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(csc(c + d*x)**5*sec(c + d*x), x) + Integral(csc(c + d*x)**5, x)
)
```

3.9 $\int \csc^7(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=163

$$-\frac{a^4}{24d(a-a\cos(c+dx))^3}-\frac{5a^3}{32d(a-a\cos(c+dx))^2}-\frac{a^3}{32d(a\cos(c+dx)+a)^2}-\frac{a^2}{2d(a-a\cos(c+dx))}-\frac{\cdots}{16d(a\cos\cdots}$$

```
[Out] -1/24*a^4/d/(a-a*cos(d*x+c))^3-5/32*a^3/d/(a-a*cos(d*x+c))^2-1/2*a^2/d/(a-a
*cos(d*x+c))-1/32*a^3/d/(a+a*cos(d*x+c))^2-3/16*a^2/d/(a+a*cos(d*x+c))+21/3
2*a*ln(1-cos(d*x+c))/d-a*ln(cos(d*x+c))/d+11/32*a*ln(1+cos(d*x+c))/d
```

Rubi [A] time = 0.15, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 2836, 12, 88}

$$-\frac{a^4}{24d(a-a\cos(c+dx))^3}-\frac{5a^3}{32d(a-a\cos(c+dx))^2}-\frac{a^3}{32d(a\cos(c+dx)+a)^2}-\frac{a^2}{2d(a-a\cos(c+dx))}-\frac{\cdots}{16d(a\cos\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^7*(a + a*Sec[c + d*x]),x]
```

```
[Out] -a^4/(24*d*(a - a*Cos[c + d*x])^3) - (5*a^3)/(32*d*(a - a*Cos[c + d*x])^2)
- a^2/(2*d*(a - a*Cos[c + d*x])) - a^3/(32*d*(a + a*Cos[c + d*x])^2) - (3*a
^2)/(16*d*(a + a*Cos[c + d*x])) + (21*a*Log[1 - Cos[c + d*x]])/(32*d) - (a*
Log[Cos[c + d*x]])/d + (11*a*Log[1 + Cos[c + d*x]])/(32*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^7(c+dx)(a+a\sec(c+dx))\,dx = -\int(-a-a\cos(c+dx))\csc^7(c+dx)\sec(c+dx)\,dx$$
$$= \frac{a^7\,\text{Subst}\left(\int \frac{a}{(-a-x)^4x(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^8\,\text{Subst}\left(\int \frac{1}{(-a-x)^4x(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^8\,\text{Subst}\left(\int\left(-\frac{1}{16a^5(a-x)^3}-\frac{3}{16a^6(a-x)^2}-\frac{11}{32a^7(a-x)}-\frac{1}{a^7x}+\frac{1}{8a^4(a+x)^4}+\frac{5}{16a^5(a+x}\right.\right.}{d}$$
$$= -\frac{a^4}{24d(a-a\cos(c+dx))^3}-\frac{5a^3}{32d(a-a\cos(c+dx))^2}-\frac{a^2}{2d(a-a\cos(c+d}$$

Mathematica [A] time = 0.46, size = 165, normalized size = 1.01

$$-\frac{a\left(\csc^6\left(\frac{1}{2}(c+dx)\right)+64\csc^6(c+dx)+6\csc^4\left(\frac{1}{2}(c+dx)\right)+96\csc^4(c+dx)+30\csc^2\left(\frac{1}{2}(c+dx)\right)+192\csc^2(c\cdot\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^7*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/384*(a*(30*Csc[(c + d*x)/2]^2 + 6*Csc[(c + d*x)/2]^4 + Csc[(c + d*x)/2]^
6 + 192*Csc[c + d*x]^2 + 96*Csc[c + d*x]^4 + 64*Csc[c + d*x]^6 + 120*Log[Co
s[(c + d*x)/2]] + 384*Log[Cos[c + d*x]] - 120*Log[Sin[(c + d*x)/2]] - 384*L
og[Sin[c + d*x]] - 30*Sec[(c + d*x)/2]^2 - 6*Sec[(c + d*x)/2]^4 - Sec[(c +
d*x)/2]^6))/d
```

fricas [B] time = 0.77, size = 307, normalized size = 1.88

$$\frac{30\,a\cos(dx+c)^4+18\,a\cos(dx+c)^3-98\,a\cos(dx+c)^2-22\,a\cos(dx+c)-96\left(a\cos(dx+c)^5-a\cos(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(30*a*cos(d*x + c)^4 + 18*a*cos(d*x + c)^3 - 98*a*cos(d*x + c)^2 - 22*
a*cos(d*x + c) - 96*(a*cos(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x + c)
^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a)*log(-cos(d*x + c)) + 33*(a*co
s(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*x + c)^2 +
 a*cos(d*x + c) - a)*log(1/2*cos(d*x + c) + 1/2) + 63*(a*cos(d*x + c)^5 - a
*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c)
- a)*log(-1/2*cos(d*x + c) + 1/2) + 88*a)/(d*cos(d*x + c)^5 - d*cos(d*x + c
)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 + d*cos(d*x + c) - d)
```

giac [A] time = 0.32, size = 196, normalized size = 1.20

$$\frac{252\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-384\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{\left(2\,a-\frac{21\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{132\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{462\,a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}\right)(\cos(dx+c)\cdot}{(\cos(dx+c)-1)^3}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/384*(252*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 384*a*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2*a - 21*a*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 132*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
 - 462*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^3/(c
os(d*x + c) - 1)^3 + 42*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/d
```

maple [A] time = 0.46, size = 112, normalized size = 0.69

$$-\frac{a}{24d\left(-1+\sec\left(dx+c\right)\right)^{3}}-\frac{9a}{32d\left(-1+\sec\left(dx+c\right)\right)^{2}}-\frac{15a}{16d\left(-1+\sec\left(dx+c\right)\right)}+\frac{21a\ln\left(-1+\sec\left(dx+c\right)\right)}{32d}-\frac{}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^7*(a+a*sec(d*x+c)),x)
```

```
[Out] -1/24/d*a/(-1+sec(d*x+c))^3-9/32/d*a/(-1+sec(d*x+c))^2-15/16/d*a/(-1+sec(d*
x+c))+21/32/d*a*ln(-1+sec(d*x+c))-1/32/d*a/(1+sec(d*x+c))^2+1/4/d*a/(1+sec(
d*x+c))+11/32/d*a*ln(1+sec(d*x+c))
```

maxima [A] time = 0.45, size = 136, normalized size = 0.83

$$\frac{33\,a\log\left(\cos\left(dx+c\right)+1\right)+63\,a\log\left(\cos\left(dx+c\right)-1\right)-96\,a\log\left(\cos\left(dx+c\right)\right)+\frac{2\left(15\,a\cos\left(dx+c\right)^{4}+9\,a\cos\left(dx+c\right)^{3}-4\right.}{\cos\left(dx+c\right)^{5}-\cos\left(dx+c\right)^{4}-2\,\cos\left(dx\right.}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(33*a*log(cos(d*x + c) + 1) + 63*a*log(cos(d*x + c) - 1) - 96*a*log(co
s(d*x + c)) + 2*(15*a*cos(d*x + c)^4 + 9*a*cos(d*x + c)^3 - 49*a*cos(d*x +
c)^2 - 11*a*cos(d*x + c) + 44*a)/(cos(d*x + c)^5 - cos(d*x + c)^4 - 2*cos(d
*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c) - 1))/d
```

mupad [B] time = 0.99, size = 142, normalized size = 0.87

$$\frac{\frac{5\,a\cos\left(c+d\,x\right)^4}{16}+\frac{3\,a\cos\left(c+d\,x\right)^3}{16}-\frac{49\,a\cos\left(c+d\,x\right)^2}{48}-\frac{11\,a\,\cos\left(c+d\,x\right)}{48}+\frac{11\,a}{12}}{d\,\left({\cos\left(c+d\,x\right)}^5-{\cos\left(c+d\,x\right)}^4-2\,{\cos\left(c+d\,x\right)}^3+2\,{\cos\left(c+d\,x\right)}^2+\cos\left(c+d\,x\right)-1\right)}-\frac{a\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^7,x)
```

```
[Out] ((11*a)/12 - (11*a*cos(c + d*x))/48 - (49*a*cos(c + d*x)^2)/48 + (3*a*cos(c
 + d*x)^3)/16 + (5*a*cos(c + d*x)^4)/16)/(d*(cos(c + d*x) + 2*cos(c + d*x)^
2 - 2*cos(c + d*x)^3 - cos(c + d*x)^4 + cos(c + d*x)^5 - 1)) - (a*log(cos(c
 + d*x)))/d + (21*a*log(cos(c + d*x) - 1))/(32*d) + (11*a*log(cos(c + d*x)
+ 1))/(32*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**7*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.10 $\int (a+a\sec(c+dx))\sin^8(c+dx)\,dx$

Optimal. Leaf size=165

$$-\frac{a\sin^7(c+dx)}{7d}-\frac{a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\sin^7(c+dx)\cos(c+dx)}{8d}$$

```
[Out] 35/128*a*x+a*arctanh(sin(d*x+c))/d-a*sin(d*x+c)/d-35/128*a*cos(d*x+c)*sin(d
*x+c)/d-1/3*a*sin(d*x+c)^3/d-35/192*a*cos(d*x+c)*sin(d*x+c)^3/d-1/5*a*sin(d
*x+c)^5/d-7/48*a*cos(d*x+c)*sin(d*x+c)^5/d-1/7*a*sin(d*x+c)^7/d-1/8*a*cos(d
*x+c)*sin(d*x+c)^7/d
```

Rubi [A] time = 0.15, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3872, 2838, 2592, 302, 206, 2635, 8}

$$-\frac{a\sin^7(c+dx)}{7d}-\frac{a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\sin^7(c+dx)\cos(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^8,x]
```

```
[Out] (35*a*x)/128 + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (35*a*Cos
[c + d*x]*Sin[c + d*x])/(128*d) - (a*Sin[c + d*x]^3)/(3*d) - (35*a*Cos[c +
d*x]*Sin[c + d*x]^3)/(192*d) - (a*Sin[c + d*x]^5)/(5*d) - (7*a*Cos[c + d*x]
*Sin[c + d*x]^5)/(48*d) - (a*Sin[c + d*x]^7)/(7*d) - (a*Cos[c + d*x]*Sin[c
+ d*x]^7)/(8*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a + a\sec(c+dx))\sin^8(c+dx)\,dx = -\int (-a - a\cos(c+dx))\sin^7(c+dx)\tan(c+dx)\,dx$$
$$= a\int \sin^8(c+dx)\,dx + a\int \sin^7(c+dx)\tan(c+dx)\,dx$$
$$= -\frac{a\cos(c+dx)\sin^7(c+dx)}{8d} + \frac{1}{8}(7a)\int \sin^6(c+dx)\,dx + \frac{a\,\text{Subst}\left(\int \frac{}{1}\right.}{}$$
$$= -\frac{7a\cos(c+dx)\sin^5(c+dx)}{48d} - \frac{a\cos(c+dx)\sin^7(c+dx)}{8d} + \frac{1}{48}(35a)$$
$$= -\frac{a\sin(c+dx)}{d} - \frac{a\sin^3(c+dx)}{3d} - \frac{35a\cos(c+dx)\sin^3(c+dx)}{192d} - \frac{a\sin}{}$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{35a\cos(c+dx)\sin(c+dx)}{128d} -$$
$$= \frac{35ax}{128} + \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{35a\cos(c+dx)\sin(c}{128d}$$

Mathematica [A] time = 0.35, size = 106, normalized size = 0.64

$$\frac{a\left(-15360\sin^7(c+dx) - 21504\sin^5(c+dx) - 35840\sin^3(c+dx) - 107520\sin(c+dx) + 35(-672\sin(2(c+\right.}{107}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^8,x]
```

```
[Out] (a*(107520*ArcTanh[Sin[c + d*x]] - 107520*Sin[c + d*x] - 35840*Sin[c + d*x]
^3 - 21504*Sin[c + d*x]^5 - 15360*Sin[c + d*x]^7 + 35*(840*c + 840*d*x - 67
2*Sin[2*(c + d*x)] + 168*Sin[4*(c + d*x)] - 32*Sin[6*(c + d*x)] + 3*Sin[8*(
c + d*x)])))/(107520*d)
```

fricas [A] time = 1.67, size = 123, normalized size = 0.75

$$\frac{3675\,adx + 6720\,a\log(\sin(dx+c)+1) - 6720\,a\log(-\sin(dx+c)+1) + \left(1680\,a\cos(dx+c)^7 + 1920\,a\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/13440*(3675*a*d*x + 6720*a*log(sin(d*x + c) + 1) - 6720*a*log(-sin(d*x +
c) + 1) + (1680*a*cos(d*x + c)^7 + 1920*a*cos(d*x + c)^6 - 7000*a*cos(d*x +
 c)^5 - 8448*a*cos(d*x + c)^4 + 11410*a*cos(d*x + c)^3 + 15616*a*cos(d*x +
c)^2 - 9765*a*cos(d*x + c) - 22528*a)*sin(d*x + c))/d
```

giac [A] time = 2.32, size = 174, normalized size = 1.05

$$3675\,(dx+c)a + 13440\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 13440\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(9765\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^8,x, algorithm="giac")
```

```
[Out] 1/13440*(3675*(d*x + c)*a + 13440*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 13
440*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(9765*a*tan(1/2*d*x + 1/2*c)^1
5 + 83825*a*tan(1/2*d*x + 1/2*c)^13 + 321013*a*tan(1/2*d*x + 1/2*c)^11 + 72
4649*a*tan(1/2*d*x + 1/2*c)^9 + 1078359*a*tan(1/2*d*x + 1/2*c)^7 + 508683*a
*tan(1/2*d*x + 1/2*c)^5 + 140175*a*tan(1/2*d*x + 1/2*c)^3 + 17115*a*tan(1/2
*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^8)/d
```

maple [A] time = 0.69, size = 164, normalized size = 0.99

$$-\frac{a\cos(dx+c)\left(\sin^7(dx+c)\right)}{8d} - \frac{7a\cos(dx+c)\left(\sin^5(dx+c)\right)}{48d} - \frac{35a\cos(dx+c)\left(\sin^3(dx+c)\right)}{192d} - \frac{35a\cos(dx+c)}{128d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^8,x)
```

```
[Out] -1/8*a*cos(d*x+c)*sin(d*x+c)^7/d-7/48*a*cos(d*x+c)*sin(d*x+c)^5/d-35/192*a*
cos(d*x+c)*sin(d*x+c)^3/d-35/128*a*cos(d*x+c)*sin(d*x+c)/d+35/128*a*x+35/12
8/d*c*a-1/7*a*sin(d*x+c)^7/d-1/5*a*sin(d*x+c)^5/d-1/3*a*sin(d*x+c)^3/d-a*si
n(d*x+c)/d+1/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.35, size = 127, normalized size = 0.77

$$-\frac{512\left(30\sin(dx+c)^7 + 42\sin(dx+c)^5 + 70\sin(dx+c)^3 - 105\log(\sin(dx+c)+1) + 105\log(\sin(dx+c)-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/107520*(512*(30*sin(d*x + c)^7 + 42*sin(d*x + c)^5 + 70*sin(d*x + c)^3 -
 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1) + 210*sin(d*x + c))*
a - 35*(128*sin(2*d*x + 2*c)^3 + 840*d*x + 840*c + 3*sin(8*d*x + 8*c) + 168
*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a)/d
```

mupad [B] time = 1.14, size = 150, normalized size = 0.91

$$\frac{35\,a\,x}{128} + \frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d} - \frac{7\,a\,\sin(2\,c+2\,d\,x)}{32\,d} + \frac{37\,a\,\sin(3\,c+3\,d\,x)}{192\,d} + \frac{7\,a\,\sin(4\,c+4\,d\,x)}{128\,d} - \frac{9\,a\,\sin(5\,c+5\,d}{320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8*(a + a/cos(c + d*x)),x)
```

```
[Out] (35*a*x)/128 + (2*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (7*a*
sin(2*c + 2*d*x))/(32*d) + (37*a*sin(3*c + 3*d*x))/(192*d) + (7*a*sin(4*c +
 4*d*x))/(128*d) - (9*a*sin(5*c + 5*d*x))/(320*d) - (a*sin(6*c + 6*d*x))/(9
6*d) + (a*sin(7*c + 7*d*x))/(448*d) + (a*sin(8*c + 8*d*x))/(1024*d) - (93*a
*sin(c + d*x))/(64*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**8,x)
```

```
[Out] Timed out
```

3.11 $\int (a + a\sec(c + dx))\sin^6(c + dx)\,dx$

Optimal. Leaf size=127

$$-\frac{a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\sin^5(c+dx)\cos(c+dx)}{6d}-\frac{5a\sin^3(c+d}{2}$$

```
[Out] 5/16*a*x+a*arctanh(sin(d*x+c))/d-a*sin(d*x+c)/d-5/16*a*cos(d*x+c)*sin(d*x+c
)/d-1/3*a*sin(d*x+c)^3/d-5/24*a*cos(d*x+c)*sin(d*x+c)^3/d-1/5*a*sin(d*x+c)^
5/d-1/6*a*cos(d*x+c)*sin(d*x+c)^5/d
```

Rubi [A] time = 0.13, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3872, 2838, 2592, 302, 206, 2635, 8}

$$-\frac{a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\sin^5(c+dx)\cos(c+dx)}{6d}-\frac{5a\sin^3(c+d}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^6,x]
```

```
[Out] (5*a*x)/16 + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (5*a*Cos[c
+ d*x]*Sin[c + d*x])/(16*d) - (a*Sin[c + d*x]^3)/(3*d) - (5*a*Cos[c + d*x]*
Sin[c + d*x]^3)/(24*d) - (a*Sin[c + d*x]^5)/(5*d) - (a*Cos[c + d*x]*Sin[c +
 d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
```

```
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a + a\sec(c+dx))\sin^6(c+dx)\,dx = -\int (-a - a\cos(c+dx))\sin^5(c+dx)\tan(c+dx)\,dx$$
$$= a\int \sin^6(c+dx)\,dx + a\int \sin^5(c+dx)\tan(c+dx)\,dx$$
$$= -\frac{a\cos(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{6}(5a)\int \sin^4(c+dx)\,dx + \frac{a\,\mathrm{Subst}\big(\int \frac{}{1}$$
$$= -\frac{5a\cos(c+dx)\sin^3(c+dx)}{24d} - \frac{a\cos(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{8}(5a)\int \mathrm{s}$$
$$= -\frac{a\sin(c+dx)}{d} - \frac{5a\cos(c+dx)\sin(c+dx)}{16d} - \frac{a\sin^3(c+dx)}{3d} - \frac{5a\cos}{}$$
$$= \frac{5ax}{16} + \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{5a\cos(c+dx)\sin(c+}{16d}$$

Mathematica [A] time = 0.19, size = 86, normalized size = 0.68

$$\frac{a\left(-192\sin^5(c+dx) - 320\sin^3(c+dx) - 960\sin(c+dx) + 5(-45\sin(2(c+dx)) + 9\sin(4(c+dx)) - \sin(6(c\right.}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^6,x]
```

```
[Out] (a*(960*ArcTanh[Sin[c + d*x]] - 960*Sin[c + d*x] - 320*Sin[c + d*x]^3 - 192
*Sin[c + d*x]^5 + 5*(60*c + 60*d*x - 45*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x
)] - Sin[6*(c + d*x)])))/(960*d)
```

fricas [A] time = 0.77, size = 102, normalized size = 0.80

$$\frac{75\,adx + 120\,a\log(\sin(dx+c)+1) - 120\,a\log(-\sin(dx+c)+1) - \left(40\,a\cos(dx+c)^5 + 48\,a\cos(dx+c)^4\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(75*a*d*x + 120*a*log(sin(d*x + c) + 1) - 120*a*log(-sin(d*x + c) + 1
) - (40*a*cos(d*x + c)^5 + 48*a*cos(d*x + c)^4 - 130*a*cos(d*x + c)^3 - 176
*a*cos(d*x + c)^2 + 165*a*cos(d*x + c) + 368*a)*sin(d*x + c))/d
```

giac [A] time = 0.84, size = 146, normalized size = 1.15

$$\frac{75(dx+c)a + 240\,a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - 240\,a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) - \frac{2\left(165\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+1095\right.}{}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/240*(75*(d*x + c)*a + 240*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 240*a*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(165*a*tan(1/2*d*x + 1/2*c)^11 + 1095*
a*tan(1/2*d*x + 1/2*c)^9 + 3138*a*tan(1/2*d*x + 1/2*c)^7 + 5118*a*tan(1/2*d
*x + 1/2*c)^5 + 1945*a*tan(1/2*d*x + 1/2*c)^3 + 315*a*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

maple [A] time = 0.68, size = 130, normalized size = 1.02

$$-\frac{a\cos\left(dx+c\right)\left(\sin^{5}\left(dx+c\right)\right)}{6d}-\frac{5a\cos\left(dx+c\right)\left(\sin^{3}\left(dx+c\right)\right)}{24d}-\frac{5a\cos\left(dx+c\right)\sin\left(dx+c\right)}{16d}+\frac{5ax}{16}+\frac{5ca}{16d}-\frac{a\left(\sin^{5}\left(\right.\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^6,x)
```

```
[Out] -1/6*a*cos(d*x+c)*sin(d*x+c)^5/d-5/24*a*cos(d*x+c)*sin(d*x+c)^3/d-5/16*a*co
s(d*x+c)*sin(d*x+c)/d+5/16*a*x+5/16/d*c*a-1/5*a*sin(d*x+c)^5/d-1/3*a*sin(d*
x+c)^3/d-a*sin(d*x+c)/d+1/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.47, size = 106, normalized size = 0.83

$$-\frac{32\left(6\sin\left(dx+c\right)^{5}+10\sin\left(dx+c\right)^{3}-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)+30\sin\left(dx+c\right)\right)a}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] -1/960*(32*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d*x + c) + 1)
 + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a - 5*(4*sin(2*d*x + 2*c)^3
+ 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a)/d
```

mupad [B] time = 1.05, size = 120, normalized size = 0.94

$$\frac{5\,a\,x}{16}+\frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{15\,a\,\sin\left(2\,c+2\,d\,x\right)}{64\,d}+\frac{7\,a\,\sin\left(3\,c+3\,d\,x\right)}{48\,d}+\frac{3\,a\,\sin\left(4\,c+4\,d\,x\right)}{64\,d}-\frac{a\,\sin\left(5\,c+5\,d\,x\right)}{80\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6*(a + a/cos(c + d*x)),x)
```

```
[Out] (5*a*x)/16 + (2*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (15*a*s
in(2*c + 2*d*x))/(64*d) + (7*a*sin(3*c + 3*d*x))/(48*d) + (3*a*sin(4*c + 4*
d*x))/(64*d) - (a*sin(5*c + 5*d*x))/(80*d) - (a*sin(6*c + 6*d*x))/(192*d) -
 (11*a*sin(c + d*x))/(8*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin^{6}\left(c+dx\right)\sec\left(c+dx\right)dx+\int\sin^{6}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**6,x)
```

```
[Out] a*(Integral(sin(c + d*x)**6*sec(c + d*x), x) + Integral(sin(c + d*x)**6, x)
)
```

3.12 $\int (a + a \sec(c + dx)) \sin^4(c + dx)\, dx$

Optimal. Leaf size=89

$$-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{3a\sin(c+dx)\cos(c+dx)}{8d}$$

```
[Out] 3/8*a*x+a*arctanh(sin(d*x+c))/d-a*sin(d*x+c)/d-3/8*a*cos(d*x+c)*sin(d*x+c)/
d-1/3*a*sin(d*x+c)^3/d-1/4*a*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.11, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3872, 2838, 2592, 302, 206, 2635, 8}

$$-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{3a\sin(c+dx)\cos(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^4,x]
```

```
[Out] (3*a*x)/8 + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (3*a*Cos[c +
 d*x]*Sin[c + d*x])/(8*d) - (a*Sin[c + d*x]^3)/(3*d) - (a*Cos[c + d*x]*Sin[
c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
```

```
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))\sin^4(c+dx)\,dx &= -\int (-a-a\cos(c+dx))\sin^3(c+dx)\tan(c+dx)\,dx\\ &= a\int \sin^4(c+dx)\,dx + a\int \sin^3(c+dx)\tan(c+dx)\,dx\\ &= -\frac{a\cos(c+dx)\sin^3(c+dx)}{4d} + \frac{1}{4}(3a)\int \sin^2(c+dx)\,dx + \frac{a\,\text{Subst}\left(\int \frac{x^4}{1-x^2}\right.}{}\\ &= -\frac{3a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\cos(c+dx)\sin^3(c+dx)}{4d} + \frac{1}{8}(3a)\int 1\,dx\\ &= \frac{3ax}{8} - \frac{a\sin(c+dx)}{d} - \frac{3a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\sin^3(c+dx)}{3d} - \frac{a\cos}{}\\ &= \frac{3ax}{8} + \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{3a\cos(c+dx)\sin(c+dx)}{8d}\end{aligned}$$

Mathematica [A] time = 0.12, size = 86, normalized size = 0.97

$$\frac{3a(c+dx)}{8d} - \frac{a\sin^3(c+dx)}{3d} - \frac{a\sin(c+dx)}{d} - \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d} + \frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^4,x]
```

```
[Out] (3*a*(c + d*x))/(8*d) + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d -
(a*Sin[c + d*x]^3)/(3*d) - (a*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)]
)/(32*d)
```

fricas [A] time = 0.70, size = 79, normalized size = 0.89

$$\frac{9\,adx + 12\,a\log(\sin(dx+c)+1) - 12\,a\log(-\sin(dx+c)+1) + \left(6\,a\cos(dx+c)^3 + 8\,a\cos(dx+c)^2 - 15\,a\cos\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(9*a*d*x + 12*a*log(sin(d*x + c) + 1) - 12*a*log(-sin(d*x + c) + 1) +
(6*a*cos(d*x + c)^3 + 8*a*cos(d*x + c)^2 - 15*a*cos(d*x + c) - 32*a)*sin(d*
x + c))/d
```

giac [A] time = 0.96, size = 118, normalized size = 1.33

$$\frac{9(dx+c)a + 24\,a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - 24\,a\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) - \frac{2\left(15\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 + 71\,a\tan\left(\frac{1}{2}dx+\right.\right.}{\left(\tan\left(\right.\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/24*(9*(d*x + c)*a + 24*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*a*log(ab
s(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*a*tan(1/2*d*x + 1/2*c)^7 + 71*a*tan(1/
2*d*x + 1/2*c)^5 + 137*a*tan(1/2*d*x + 1/2*c)^3 + 33*a*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.61, size = 96, normalized size = 1.08

$$-\frac{a\cos(dx+c)\left(\sin^3(dx+c)\right)}{4d}-\frac{3a\cos(dx+c)\sin(dx+c)}{8d}+\frac{3ax}{8}+\frac{3ca}{8d}-\frac{a\left(\sin^3(dx+c)\right)}{3d}-\frac{a\sin(dx+c)}{d}+\frac{a\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^4,x)
```

```
[Out] -1/4*a*cos(d*x+c)*sin(d*x+c)^3/d-3/8*a*cos(d*x+c)*sin(d*x+c)/d+3/8*a*x+3/8/
d*c*a-1/3*a*sin(d*x+c)^3/d-a*sin(d*x+c)/d+1/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.61, size = 81, normalized size = 0.91

$$-\frac{16\left(2\sin(dx+c)^3-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)+6\sin(dx+c)\right)a-3(12dx+12c+}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/96*(16*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
- 1) + 6*sin(d*x + c))*a - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*
x + 2*c))*a)/d
```

mupad [B] time = 1.03, size = 90, normalized size = 1.01

$$\frac{3\,a\,x}{8}+\frac{2\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\,\sin(2\,c+2\,d\,x)}{4\,d}+\frac{a\,\sin(3\,c+3\,d\,x)}{12\,d}+\frac{a\,\sin(4\,c+4\,d\,x)}{32\,d}-\frac{5\,a\,\sin(c+d\,x)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + a/cos(c + d*x)),x)
```

```
[Out] (3*a*x)/8 + (2*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*sin(2
*c + 2*d*x))/(4*d) + (a*sin(3*c + 3*d*x))/(12*d) + (a*sin(4*c + 4*d*x))/(32
*d) - (5*a*sin(c + d*x))/(4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin^4(c+dx)\sec(c+dx)\,dx+\int\sin^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**4,x)
```

```
[Out] a*(Integral(sin(c + d*x)**4*sec(c + d*x), x) + Integral(sin(c + d*x)**4, x)
)
```

3.13 $\int (a + a\sec(c + dx))\sin^2(c + dx)\,dx$

Optimal. Leaf size=51

$$-\frac{a\sin(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

```
[Out] 1/2*a*x+a*arctanh(sin(d*x+c))/d-a*sin(d*x+c)/d-1/2*a*cos(d*x+c)*sin(d*x+c)/
d
```

Rubi [A] time = 0.08, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = $\{3872, 2838, 2592, 321, 206, 2635, 8\}$

$$-\frac{a\sin(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^2,x]
```

```
[Out] (a*x)/2 + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (a*Cos[c + d*x
]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
```

```
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))\sin^2(c+dx)\,dx &= -\int (-a-a\cos(c+dx))\sin(c+dx)\tan(c+dx)\,dx\\ &= a\int \sin^2(c+dx)\,dx + a\int \sin(c+dx)\tan(c+dx)\,dx\\ &= -\frac{a\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}a\int 1\,dx + \frac{a\,\mathrm{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(c\right.}{d}\\ &= \frac{ax}{2} - \frac{a\sin(c+dx)}{d} - \frac{a\cos(c+dx)\sin(c+dx)}{2d} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx, x\right.}{d}\\ &= \frac{ax}{2} + \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{a\cos(c+dx)\sin(c+dx}{2d}\end{aligned}$$

Mathematica [A] time = 0.06, size = 54, normalized size = 1.06

$$\frac{a(c+dx)}{2d} - \frac{a\sin(c+dx)}{d} - \frac{a\sin(2(c+dx))}{4d} + \frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^2,x]
[Out] (a*(c + d*x))/(2*d) + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (a
*Sin[2*(c + d*x)])/(4*d)
```

fricas [A] time = 0.65, size = 55, normalized size = 1.08

$$\frac{adx + a\log\left(\sin\left(dx+c\right)+1\right) - a\log\left(-\sin\left(dx+c\right)+1\right) - \left(a\cos\left(dx+c\right)+2\,a\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="fricas")
[Out] 1/2*(a*d*x + a*log(sin(d*x + c) + 1) - a*log(-sin(d*x + c) + 1) - (a*cos(d*
x + c) + 2*a)*sin(d*x + c))/d
```

giac [A] time = 0.27, size = 88, normalized size = 1.73

$$\frac{(dx+c)a + 2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)*a + 2*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) - 2*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1
/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.28, size = 62, normalized size = 1.22

$$-\frac{a\cos(dx+c)\sin(dx+c)}{2d}+\frac{ax}{2}+\frac{ca}{2d}-\frac{a\sin(dx+c)}{d}+\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^2,x)
```

```
[Out] -1/2*a*cos(d*x+c)*sin(d*x+c)/d+1/2*a*x+1/2/d*c*a-a*sin(d*x+c)/d+1/d*a*ln(se
c(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.32, size = 59, normalized size = 1.16

$$\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))a+2\,a\Big(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\,\sin(dx+c)\Big)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a + 2*a*(log(sin(d*x + c) + 1) - log(
sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

mupad [B] time = 1.07, size = 80, normalized size = 1.57

$$\frac{a\,x}{2}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+3\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + a/cos(c + d*x)),x)
```

```
[Out] (a*x)/2 - (3*a*tan(c/2 + (d*x)/2) + a*tan(c/2 + (d*x)/2)^3)/(d*(2*tan(c/2 +
 (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4 + 1)) + (2*a*atanh(tan(c/2 + (d*x)/2)))/
d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sin^2(c+dx)\sec(c+dx)\,dx+\int\sin^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**2,x)
```

```
[Out] a*(Integral(sin(c + d*x)**2*sec(c + d*x), x) + Integral(sin(c + d*x)**2, x)
)
```

3.14 $\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=37

$$-\frac{a\cot(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] a*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-a*csc(d*x+c)/d
```

Rubi [A] time = 0.09, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = $\{3872, 2838, 2621, 321, 207, 3767, 8\}$

$$-\frac{a\cot(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (a*Csc[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx &= -\int(-a-a\cos(c+dx))\csc^2(c+dx)\sec(c+dx)\,dx\\
&= a\int \csc^2(c+dx)\,dx + a\int \csc^2(c+dx)\sec(c+dx)\,dx\\
&= -\frac{a\,\text{Subst}(\int 1\,dx, x, \cot(c+dx))}{d} - \frac{a\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d}\\
&= -\frac{a\cot(c+dx)}{d} - \frac{a\csc(c+dx)}{d} - \frac{a\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d}\\
&= \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\csc(c+dx)}{d}
\end{aligned}$$

Mathematica [C] time = 0.03, size = 41, normalized size = 1.11

$$-\frac{a\csc(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\sin^2(c+dx)\right)}{d} - \frac{a\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (a*Csc[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin
[c + d*x]^2])/d
```

fricas [A] time = 0.66, size = 63, normalized size = 1.70

$$\frac{a\log(\sin(dx+c)+1)\sin(dx+c) - a\log(-\sin(dx+c)+1)\sin(dx+c) - 2\,a\cos(dx+c) - 2\,a}{2\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*log(sin(d*x + c) + 1)*sin(d*x + c) - a*log(-sin(d*x + c) + 1)*sin(d*
x + c) - 2*a*cos(d*x + c) - 2*a)/(d*sin(d*x + c))
```

giac [A] time = 0.34, size = 50, normalized size = 1.35

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - a/tan(1/2*d*x + 1/2*c))/d
```

maple [A] time = 0.60, size = 47, normalized size = 1.27

$$-\frac{a\cot(dx+c)}{d}-\frac{a}{d\sin(dx+c)}+\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sec(d*x+c)),x)
```

```
[Out] -a*cot(d*x+c)/d-1/d*a/sin(d*x+c)+1/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.37, size = 50, normalized size = 1.35

$$-\frac{a\left(\frac{2}{\sin(dx+c)}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+\frac{2a}{\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(a*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) +
2*a/tan(d*x + c))/d
```

mupad [B] time = 0.96, size = 29, normalized size = 0.78

$$\frac{a\left(2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)-\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^2,x)
```

```
[Out] (a*(2*atanh(tan(c/2 + (d*x)/2)) - cot(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\csc^2(c+dx)\sec(c+dx)\,dx+\int\csc^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(csc(c + d*x)**2*sec(c + d*x), x) + Integral(csc(c + d*x)**2, x)
)
```

3.15 $\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=69

$$-\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] a*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-1/3*a*cot(d*x+c)^3/d-a*csc(d*x+c)/d-
1/3*a*csc(d*x+c)^3/d
```

Rubi [A] time = 0.10, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = $\{3872, 2838, 2621, 302, 207, 3767\}$

$$-\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (a*Cot[c + d*x]^3)/(3*d)
 - (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
```

```
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx = -\int (-a-a\cos(c+dx))\csc^4(c+dx)\sec(c+dx)\,dx$$
$$= a\int \csc^4(c+dx)\,dx + a\int \csc^4(c+dx)\sec(c+dx)\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, \cot(\right.}{d}$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \text{c}\right.}{d}$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{3d} - \frac{a\,\text{S}}{}$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\csc(c+dx}{d}$$

Mathematica [C] time = 0.03, size = 69, normalized size = 1.00

$$-\frac{a\csc^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\sin^2(c+dx)\right)}{3d} - \frac{2a\cot(c+dx)}{3d} - \frac{a\cot(c+dx)\csc^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*a*Cot[c + d*x])/(3*d) - (a*Cot[c + d*x]*Csc[c + d*x]^2)/(3*d) - (a*Csc[
c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[c + d*x]^2])/(3*d)
```

fricas [A] time = 0.72, size = 108, normalized size = 1.57

$$-\frac{4\,a\cos\left(dx+c\right)^2 - 3\left(a\cos\left(dx+c\right)-a\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right) + 3\left(a\cos\left(dx+c\right)-a\right)\log\left(-\sin\left(\right.\right.}{6\left(d\cos\left(dx+c\right)-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(4*a*cos(d*x + c)^2 - 3*(a*cos(d*x + c) - a)*log(sin(d*x + c) + 1)*sin
(d*x + c) + 3*(a*cos(d*x + c) - a)*log(-sin(d*x + c) + 1)*sin(d*x + c) + 2*
a*cos(d*x + c) - 8*a)/((d*cos(d*x + c) - d)*sin(d*x + c))
```

giac [A] time = 0.24, size = 79, normalized size = 1.14

$$\frac{12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - 3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - \frac{12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 12*a*log(abs(tan(1/2*d*x +
1/2*c) - 1)) - 3*a*tan(1/2*d*x + 1/2*c) - (12*a*tan(1/2*d*x + 1/2*c)^2 + a)
/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.73, size = 81, normalized size = 1.17

$$-\frac{2a\cot(dx+c)}{3d}-\frac{a\cot(dx+c)\left(\csc^2(dx+c)\right)}{3d}-\frac{a}{3d\sin(dx+c)^3}-\frac{a}{d\sin(dx+c)}+\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sec(d*x+c)),x)
```

```
[Out] -2/3*a*cot(d*x+c)/d-1/3/d*a*cot(d*x+c)*csc(d*x+c)^2-1/3/d*a/sin(d*x+c)^3-1/
d*a/sin(d*x+c)+1/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.42, size = 76, normalized size = 1.10

$$-\frac{a\left(\frac{2\left(3\sin(dx+c)^2+1\right)}{\sin(dx+c)^3}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)+\frac{2\left(3\tan(dx+c)^2+1\right)a}{\tan(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(a*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1)
+ 3*log(sin(d*x + c) - 1)) + 2*(3*tan(d*x + c)^2 + 1)*a/tan(d*x + c)^3)/d
```

mupad [B] time = 0.98, size = 65, normalized size = 0.94

$$\frac{2a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{a}{12}}{d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}-\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^4,x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (a/12 + a*tan(c/2 + (d*x)/2)^2)/(d*tan(
c/2 + (d*x)/2)^3) - (a*tan(c/2 + (d*x)/2))/(4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\csc^4(c+dx)\sec(c+dx)\,dx+\int\csc^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(csc(c + d*x)**4*sec(c + d*x), x) + Integral(csc(c + d*x)**4, x)
)
```

3.16 $\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=101

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(}{d}$$

```
[Out] a*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-2/3*a*cot(d*x+c)^3/d-1/5*a*cot(d*x+c
)^5/d-a*csc(d*x+c)/d-1/3*a*csc(d*x+c)^3/d-1/5*a*csc(d*x+c)^5/d
```

Rubi [A] time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3872, 2838, 2621, 302, 207, 3767}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (2*a*Cot[c + d*x]^3)/(3*
d) - (a*Cot[c + d*x]^5)/(5*d) - (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*
d) - (a*Csc[c + d*x]^5)/(5*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
```

```
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx &= -\int (-a-a\cos(c+dx))\csc^6(c+dx)\sec(c+dx)\,dx\\ &= a\int \csc^6(c+dx)\,dx + a\int \csc^6(c+dx)\sec(c+dx)\,dx\\ &= -\frac{a\,\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,c\right.}{d}\\ &= -\frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{a\,\text{Subst}\left(\int \left(1+x^2+\right.\right.}{}\\ &= -\frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{a\csc(c+dx)}{d} - \frac{a\csc}{}\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d}\end{aligned}$$

Mathematica [C] time = 0.03, size = 91, normalized size = 0.90

$$-\frac{a\csc^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\sin^2(c+dx)\right)}{5d}-\frac{8a\cot(c+dx)}{15d}-\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}-\frac{4a\cot(c+dx)\csc^2(c+}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-8*a*Cot[c + d*x])/(15*d) - (4*a*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) - (a*
Cot[c + d*x]*Csc[c + d*x]^4)/(5*d) - (a*Csc[c + d*x]^5*Hypergeometric2F1[-5
/2, 1, -3/2, Sin[c + d*x]^2])/(5*d)
```

fricas [B] time = 0.55, size = 190, normalized size = 1.88

$$-\frac{16\,a\cos(dx+c)^4+14\,a\cos(dx+c)^3-54\,a\cos(dx+c)^2-15\left(a\cos(dx+c)^3-a\cos(dx+c)^2-a\cos(dx+c\right.}{30\left(d\cos\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/30*(16*a*cos(d*x + c)^4 + 14*a*cos(d*x + c)^3 - 54*a*cos(d*x + c)^2 - 15
*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(sin(d*x + c
) + 1)*sin(d*x + c) + 15*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x +
 c) + a)*log(-sin(d*x + c) + 1)*sin(d*x + c) - 16*a*cos(d*x + c) + 46*a)/((
d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) + d)*sin(d*x + c))
```

giac [A] time = 0.28, size = 107, normalized size = 1.06

$$-\frac{5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-240\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+240\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+90\,a\tan\left(\frac{1}{2}\,dx+\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(5*a*tan(1/2*d*x + 1/2*c)^3 - 240*a*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) + 240*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 90*a*tan(1/2*d*x + 1/2*c) +
 3*(80*a*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(1/2*d*x + 1/2*c)^2 + a)/tan(1/2*
d*x + 1/2*c)^5)/d
```

maple [A] time = 0.73, size = 115, normalized size = 1.14

$$-\frac{8a\cot(dx+c)}{15d}-\frac{a\cot(dx+c)\left(\csc^4(dx+c)\right)}{5d}-\frac{4a\cot(dx+c)\left(\csc^2(dx+c)\right)}{15d}-\frac{a}{5d\sin(dx+c)^5}-\frac{a}{3d\sin(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+a*sec(d*x+c)),x)
```

```
[Out] -8/15*a*cot(d*x+c)/d-1/5/d*a*cot(d*x+c)*csc(d*x+c)^4-4/15/d*a*cot(d*x+c)*cs
c(d*x+c)^2-1/5/d*a/sin(d*x+c)^5-1/3/d*a/sin(d*x+c)^3-1/d*a/sin(d*x+c)+1/d*a
*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.37, size = 96, normalized size = 0.95

$$-\frac{a\left(\frac{2\left(15\sin(dx+c)^4+5\sin(dx+c)^2+3\right)}{\sin(dx+c)^5}-15\log(\sin(dx+c)+1)+15\log(\sin(dx+c)-1)\right)+\frac{2\left(15\tan(dx+c)^4+10\tan(dx}{\tan(dx+c)^5}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*(a*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 15*
log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 2*(15*tan(d*x + c)^4 +
10*tan(d*x + c)^2 + 3)*a/tan(d*x + c)^5)/d
```

mupad [B] time = 1.00, size = 97, normalized size = 0.96

$$\frac{2a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{3a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d}-\frac{a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{48d}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^5\left(16a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+2a\tan\left(\frac{c}{2}\right.\right.}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^6,x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (3*a*tan(c/2 + (d*x)/2))/(8*d) - (a*tan
(c/2 + (d*x)/2)^3)/(48*d) - (cot(c/2 + (d*x)/2)^5*(a/5 + 2*a*tan(c/2 + (d*x
)/2)^2 + 16*a*tan(c/2 + (d*x)/2)^4))/(16*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.17 $\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=131

$$-\frac{a\cot^7(c+dx)}{7d}-\frac{3a\cot^5(c+dx)}{5d}-\frac{a\cot^3(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\csc^7(c+dx)}{7d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}-$$

```
[Out] a*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-a*cot(d*x+c)^3/d-3/5*a*cot(d*x+c)^5/
d-1/7*a*cot(d*x+c)^7/d-a*csc(d*x+c)/d-1/3*a*csc(d*x+c)^3/d-1/5*a*csc(d*x+c)
^5/d-1/7*a*csc(d*x+c)^7/d
```

Rubi [A] time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3872, 2838, 2621, 302, 207, 3767}

$$-\frac{a\cot^7(c+dx)}{7d}-\frac{3a\cot^5(c+dx)}{5d}-\frac{a\cot^3(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\csc^7(c+dx)}{7d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (a*Cot[c + d*x]^3)/d - (
3*a*Cot[c + d*x]^5)/(5*d) - (a*Cot[c + d*x]^7)/(7*d) - (a*Csc[c + d*x])/d -
 (a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^7)/(
7*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx = -\int(-a-a\cos(c+dx))\csc^8(c+dx)\sec(c+dx)\,dx$$
$$= a\int \csc^8(c+dx)\,dx + a\int \csc^8(c+dx)\sec(c+dx)\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{x^8}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \left(1+3x^2+3x^4+x\right.\right.}{d}$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{d} - \frac{3a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx)}{7d} - \cdots$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{d} - \frac{3a\cot^5(c+dx)}{5d} - \frac{a\cot^7(c+dx)}{7d} - \cdots$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{d} - \frac{3a\cot^5(c+}{5d}$$

Mathematica [C] time = 0.05, size = 113, normalized size = 0.86

$$-\frac{a\csc^7(c+dx)\,{}_2F_1\left(-\frac{7}{2},1;-\frac{5}{2};\sin^2(c+dx)\right)}{7d} - \frac{16a\cot(c+dx)}{35d} - \frac{a\cot(c+dx)\csc^6(c+dx)}{7d} - \frac{6a\cot(c+dx)\csc^4}{35d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-16*a*Cot[c + d*x])/(35*d) - (8*a*Cot[c + d*x]*Csc[c + d*x]^2)/(35*d) - (6
*a*Cot[c + d*x]*Csc[c + d*x]^4)/(35*d) - (a*Cot[c + d*x]*Csc[c + d*x]^6)/(7
*d) - (a*Csc[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2, Sin[c + d*x]^2])/(
7*d)
```

fricas [B] time = 0.63, size = 281, normalized size = 2.15

$$-\frac{96\,a\cos(dx+c)^6 + 114\,a\cos(dx+c)^5 - 450\,a\cos(dx+c)^4 - 250\,a\cos(dx+c)^3 + 670\,a\cos(dx+c)^2 - 10}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/210*(96*a*cos(d*x + c)^6 + 114*a*cos(d*x + c)^5 - 450*a*cos(d*x + c)^4 -
 250*a*cos(d*x + c)^3 + 670*a*cos(d*x + c)^2 - 105*(a*cos(d*x + c)^5 - a*co
s(d*x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a
)*log(sin(d*x + c) + 1)*sin(d*x + c) + 105*(a*cos(d*x + c)^5 - a*cos(d*x +
c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a)*log(-s
in(d*x + c) + 1)*sin(d*x + c) + 142*a*cos(d*x + c) - 352*a)/((d*cos(d*x + c
)^5 - d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 + d*cos(d*
x + c) - d)*sin(d*x + c))
```

giac [A] time = 0.62, size = 136, normalized size = 1.04

$$-\frac{21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 280\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 6720\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 6720\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx\right.\right.\right.}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6720*(21*a*tan(1/2*d*x + 1/2*c)^5 + 280*a*tan(1/2*d*x + 1/2*c)^3 - 6720*
a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 6720*a*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + 3045*a*tan(1/2*d*x + 1/2*c) + (6720*a*tan(1/2*d*x + 1/2*c)^6 + 1015
*a*tan(1/2*d*x + 1/2*c)^4 + 168*a*tan(1/2*d*x + 1/2*c)^2 + 15*a)/tan(1/2*d*
x + 1/2*c)^7)/d
```

maple [A] time = 0.81, size = 149, normalized size = 1.14

$$-\frac{16a\cot(dx+c)}{35d}-\frac{a\cot(dx+c)\left(\csc^6(dx+c)\right)}{7d}-\frac{6a\cot(dx+c)\left(\csc^4(dx+c)\right)}{35d}-\frac{8a\cot(dx+c)\left(\csc^2(dx+c)\right)}{35d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8*(a+a*sec(d*x+c)),x)
```

```
[Out] -16/35*a*cot(d*x+c)/d-1/7/d*a*cot(d*x+c)*csc(d*x+c)^6-6/35/d*a*cot(d*x+c)*c
sc(d*x+c)^4-8/35/d*a*cot(d*x+c)*csc(d*x+c)^2-1/7/d*a/sin(d*x+c)^7-1/5/d*a/s
in(d*x+c)^5-1/3/d*a/sin(d*x+c)^3-1/d*a/sin(d*x+c)+1/d*a*ln(sec(d*x+c)+tan(d
*x+c))
```

maxima [A] time = 0.45, size = 116, normalized size = 0.89

$$-\frac{a\left(\frac{2\left(105\sin(dx+c)^6+35\sin(dx+c)^4+21\sin(dx+c)^2+15\right)}{\sin(dx+c)^7}-105\log(\sin(dx+c)+1)+105\log(\sin(dx+c)-1)\right)+\frac{6(35\tan}{}}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/210*(a*(2*(105*sin(d*x + c)^6 + 35*sin(d*x + c)^4 + 21*sin(d*x + c)^2 +
15)/sin(d*x + c)^7 - 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1))
 + 6*(35*tan(d*x + c)^6 + 35*tan(d*x + c)^4 + 21*tan(d*x + c)^2 + 5)*a/tan(
d*x + c)^7)/d
```

mupad [B] time = 1.17, size = 128, normalized size = 0.98

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{29\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{320\,d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\left(64\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^8,x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (29*a*tan(c/2 + (d*x)/2))/(64*d) - (a*t
an(c/2 + (d*x)/2)^3)/(24*d) - (a*tan(c/2 + (d*x)/2)^5)/(320*d) - (cot(c/2 +
 (d*x)/2)^7*(a/7 + (8*a*tan(c/2 + (d*x)/2)^2)/5 + (29*a*tan(c/2 + (d*x)/2)^
4)/3 + 64*a*tan(c/2 + (d*x)/2)^6))/(64*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.18 $\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=165

$$-\frac{a\cot^9(c+dx)}{9d}-\frac{4a\cot^7(c+dx)}{7d}-\frac{6a\cot^5(c+dx)}{5d}-\frac{4a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\csc^9(c+dx)}{9d}-\frac{a\csc^7(c+}{7d}$$

```
[Out] a*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-4/3*a*cot(d*x+c)^3/d-6/5*a*cot(d*x+c
)^5/d-4/7*a*cot(d*x+c)^7/d-1/9*a*cot(d*x+c)^9/d-a*csc(d*x+c)/d-1/3*a*csc(d*
x+c)^3/d-1/5*a*csc(d*x+c)^5/d-1/7*a*csc(d*x+c)^7/d-1/9*a*csc(d*x+c)^9/d
```

Rubi [A] time = 0.13, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3872, 2838, 2621, 302, 207, 3767}

$$-\frac{a\cot^9(c+dx)}{9d}-\frac{4a\cot^7(c+dx)}{7d}-\frac{6a\cot^5(c+dx)}{5d}-\frac{4a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\csc^9(c+dx)}{9d}-\frac{a\csc^7(c+}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^10*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (4*a*Cot[c + d*x]^3)/(3*
d) - (6*a*Cot[c + d*x]^5)/(5*d) - (4*a*Cot[c + d*x]^7)/(7*d) - (a*Cot[c + d
*x]^9)/(9*d) - (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d
*x]^5)/(5*d) - (a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx = -\int (-a-a\cos(c+dx))\csc^{10}(c+dx)\sec(c+dx)\,dx$$
$$= a\int \csc^{10}(c+dx)\,dx + a\int \csc^{10}(c+dx)\sec(c+dx)\,dx$$
$$= -\frac{a\,\mathrm{Subst}\left(\int \frac{x^{10}}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\,\mathrm{Subst}\left(\int \left(1+4x^2+6x^4+4x^6\right.\right.}{d}$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{4a\cot^3(c+dx)}{3d} - \frac{6a\cot^5(c+dx)}{5d} - \frac{4a\cot^7(c+dx)}{7d} - $$
$$= -\frac{a\cot(c+dx)}{d} - \frac{4a\cot^3(c+dx)}{3d} - \frac{6a\cot^5(c+dx)}{5d} - \frac{4a\cot^7(c+dx)}{7d} - $$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{4a\cot^3(c+dx)}{3d} - \frac{6a\cot^5(c+d}{5d}$$

Mathematica [C] time = 0.06, size = 135, normalized size = 0.82

$$-\frac{a\csc^9(c+dx)\,{}_2F_1\left(-\frac{9}{2},1;-\frac{7}{2};\sin^2(c+dx)\right)}{9d} - \frac{128a\cot(c+dx)}{315d} - \frac{a\cot(c+dx)\csc^8(c+dx)}{9d} - \frac{8a\cot(c+dx)\csc^6(c}{63d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^10*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-128*a*Cot[c + d*x])/(315*d) - (64*a*Cot[c + d*x]*Csc[c + d*x]^2)/(315*d)
- (16*a*Cot[c + d*x]*Csc[c + d*x]^4)/(105*d) - (8*a*Cot[c + d*x]*Csc[c + d*
x]^6)/(63*d) - (a*Cot[c + d*x]*Csc[c + d*x]^8)/(9*d) - (a*Csc[c + d*x]^9*Hy
pergeometric2F1[-9/2, 1, -7/2, Sin[c + d*x]^2])/(9*d)
```

fricas [B] time = 0.69, size = 366, normalized size = 2.22

$$-\frac{256\,a\cos(dx+c)^8 + 374\,a\cos(dx+c)^7 - 1526\,a\cos(dx+c)^6 - 1204\,a\cos(dx+c)^5 + 3220\,a\cos(dx+c)^4 + 1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/630*(256*a*cos(d*x + c)^8 + 374*a*cos(d*x + c)^7 - 1526*a*cos(d*x + c)^6
 - 1204*a*cos(d*x + c)^5 + 3220*a*cos(d*x + c)^4 + 1316*a*cos(d*x + c)^3 -
2996*a*cos(d*x + c)^2 - 315*(a*cos(d*x + c)^7 - a*cos(d*x + c)^6 - 3*a*cos(
d*x + c)^5 + 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^3 - 3*a*cos(d*x + c)^2 -
 a*cos(d*x + c) + a)*log(sin(d*x + c) + 1)*sin(d*x + c) + 315*(a*cos(d*x +
c)^7 - a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^5 + 3*a*cos(d*x + c)^4 + 3*a*cos
(d*x + c)^3 - 3*a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(-sin(d*x + c) +
1)*sin(d*x + c) - 496*a*cos(d*x + c) + 1126*a)/((d*cos(d*x + c)^7 - d*cos(d
*x + c)^6 - 3*d*cos(d*x + c)^5 + 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^3 -
3*d*cos(d*x + c)^2 - d*cos(d*x + c) + d)*sin(d*x + c))
```

giac [A] time = 0.27, size = 164, normalized size = 0.99

$$-\frac{45\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+630\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4830\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-80640\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/80640*(45*a*tan(1/2*d*x + 1/2*c)^7 + 630*a*tan(1/2*d*x + 1/2*c)^5 + 4830
*a*tan(1/2*d*x + 1/2*c)^3 - 80640*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 80
640*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 40950*a*tan(1/2*d*x + 1/2*c) + (
80640*a*tan(1/2*d*x + 1/2*c)^8 + 13650*a*tan(1/2*d*x + 1/2*c)^6 + 2898*a*ta
n(1/2*d*x + 1/2*c)^4 + 450*a*tan(1/2*d*x + 1/2*c)^2 + 35*a)/tan(1/2*d*x + 1
/2*c)^9)/d
```

maple [A] time = 0.81, size = 183, normalized size = 1.11

$$-\frac{128a\cot\left(dx+c\right)}{315d}-\frac{a\cot\left(dx+c\right)\left(\csc^{8}\left(dx+c\right)\right)}{9d}-\frac{8a\cot\left(dx+c\right)\left(\csc^{6}\left(dx+c\right)\right)}{63d}-\frac{16a\cot\left(dx+c\right)\left(\csc^{4}\left(dx+\right.\right.}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^10*(a+a*sec(d*x+c)),x)
```

```
[Out] -128/315*a*cot(d*x+c)/d-1/9/d*a*cot(d*x+c)*csc(d*x+c)^8-8/63/d*a*cot(d*x+c)
*csc(d*x+c)^6-16/105/d*a*cot(d*x+c)*csc(d*x+c)^4-64/315/d*a*cot(d*x+c)*csc(
d*x+c)^2-1/9/d*a/sin(d*x+c)^9-1/7/d*a/sin(d*x+c)^7-1/5/d*a/sin(d*x+c)^5-1/3
/d*a/sin(d*x+c)^3-1/d*a/sin(d*x+c)+1/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.33, size = 136, normalized size = 0.82

$$-\frac{a\left(\frac{2\left(315\sin(dx+c)^{8}+105\sin(dx+c)^{6}+63\sin(dx+c)^{4}+45\sin(dx+c)^{2}+35\right)}{\sin(dx+c)^{9}}-315\log\left(\sin\left(dx+c\right)+1\right)+315\log\left(\sin\left(dx+c\right)\right.\right.}{630\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/630*(a*(2*(315*sin(d*x + c)^8 + 105*sin(d*x + c)^6 + 63*sin(d*x + c)^4 +
 45*sin(d*x + c)^2 + 35)/sin(d*x + c)^9 - 315*log(sin(d*x + c) + 1) + 315*l
og(sin(d*x + c) - 1)) + 2*(315*tan(d*x + c)^8 + 420*tan(d*x + c)^6 + 378*ta
n(d*x + c)^4 + 180*tan(d*x + c)^2 + 35)*a/tan(d*x + c)^9)/d
```

mupad [B] time = 1.64, size = 159, normalized size = 0.96

$$\frac{2\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{65\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{128\,d}-\frac{23\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{384\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{128\,d}-\frac{a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{1792\,d}-\frac{\cot\left(\frac{c}{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/sin(c + d*x)^10,x)
```

```
[Out] (2*a*atanh(tan(c/2 + (d*x)/2)))/d - (65*a*tan(c/2 + (d*x)/2))/(128*d) - (23
*a*tan(c/2 + (d*x)/2)^3)/(384*d) - (a*tan(c/2 + (d*x)/2)^5)/(128*d) - (a*ta
n(c/2 + (d*x)/2)^7)/(1792*d) - (cot(c/2 + (d*x)/2)^9*(a/9 + (10*a*tan(c/2 +
```

```
(d*x)/2)^2)/7 + (46*a*tan(c/2 + (d*x)/2)^4)/5 + (130*a*tan(c/2 + (d*x)/2)^
6)/3 + 256*a*tan(c/2 + (d*x)/2)^8))/(256*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**10*(a+a*sec(d*x+c)),x)
[Out] Timed out
```

3.19 $\int (a + a\sec(c + dx))^2 \sin^9(c + dx)\, dx$

Optimal. Leaf size=183

$$-\frac{a^2\cos^9(c+dx)}{9d}-\frac{a^2\cos^8(c+dx)}{4d}+\frac{3a^2\cos^7(c+dx)}{7d}+\frac{4a^2\cos^6(c+dx)}{3d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{3a^2\cos^4(c+dx)}{d}-$$

```
[Out] 3*a^2*cos(d*x+c)/d+4*a^2*cos(d*x+c)^2/d-2/3*a^2*cos(d*x+c)^3/d-3*a^2*cos(d*
x+c)^4/d-2/5*a^2*cos(d*x+c)^5/d+4/3*a^2*cos(d*x+c)^6/d+3/7*a^2*cos(d*x+c)^7
/d-1/4*a^2*cos(d*x+c)^8/d-1/9*a^2*cos(d*x+c)^9/d-2*a^2*ln(cos(d*x+c))/d+a^2
*sec(d*x+c)/d
```

Rubi [A] time = 0.19, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = $\{3872, 2836, 12, 88\}$

$$-\frac{a^2\cos^9(c+dx)}{9d}-\frac{a^2\cos^8(c+dx)}{4d}+\frac{3a^2\cos^7(c+dx)}{7d}+\frac{4a^2\cos^6(c+dx)}{3d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{3a^2\cos^4(c+dx)}{d}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^9,x]
```

```
[Out] (3*a^2*Cos[c + d*x])/d + (4*a^2*Cos[c + d*x]^2)/d - (2*a^2*Cos[c + d*x]^3)/
(3*d) - (3*a^2*Cos[c + d*x]^4)/d - (2*a^2*Cos[c + d*x]^5)/(5*d) + (4*a^2*Co
s[c + d*x]^6)/(3*d) + (3*a^2*Cos[c + d*x]^7)/(7*d) - (a^2*Cos[c + d*x]^8)/(
4*d) - (a^2*Cos[c + d*x]^9)/(9*d) - (2*a^2*Log[Cos[c + d*x]])/d + (a^2*Sec[
c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a + a\sec(c + dx))^2 \sin^9(c + dx)\, dx = \int (-a - a\cos(c + dx))^2 \sin^7(c + dx)\tan^2(c + dx)\, dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^2(-a-x)^4(-a+x)^6}{x^2} dx, x, -a\cos(c + dx)\right)}{a^9 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^4(-a+x)^6}{x^2} dx, x, -a\cos(c + dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(-3a^8 + \frac{a^{10}}{x^2} - \frac{2a^9}{x} + 8a^7x + 2a^6x^2 - 12a^5x^3 + 2a^4x^4 + 8a^3x^5 - \right.\right.}{a^7 d}$$
$$= \frac{3a^2\cos(c + dx)}{d} + \frac{4a^2\cos^2(c + dx)}{d} - \frac{2a^2\cos^3(c + dx)}{3d} - \frac{3a^2\cos^4(c + d}{d}$$

Mathematica [A] time = 0.94, size = 127, normalized size = 0.69

$$-\frac{a^2\sec(c + dx)(-361620\cos(2(c + dx)) - 134820\cos(3(c + dx)) + 29232\cos(4(c + dx)) + 24780\cos(5(c + dx))}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^9,x]
```

```
[Out] -1/322560*(a^2*(-714420 - 361620*Cos[2*(c + d*x)] - 134820*Cos[3*(c + d*x)]
 + 29232*Cos[4*(c + d*x)] + 24780*Cos[5*(c + d*x)] - 1458*Cos[6*(c + d*x)]
- 3885*Cos[7*(c + d*x)] - 380*Cos[8*(c + d*x)] + 315*Cos[9*(c + d*x)] + 70*
Cos[10*(c + d*x)] + 210*Cos[c + d*x]*(205 + 3072*Log[Cos[c + d*x]]))*Sec[c
+ d*x])/d
```

fricas [A] time = 0.57, size = 167, normalized size = 0.91

$$-\frac{17920\,a^2\cos(dx + c)^{10} + 40320\,a^2\cos(dx + c)^9 - 69120\,a^2\cos(dx + c)^8 - 215040\,a^2\cos(dx + c)^7 + 64512\,a^2 c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/161280*(17920*a^2*cos(d*x + c)^10 + 40320*a^2*cos(d*x + c)^9 - 69120*a^2
*cos(d*x + c)^8 - 215040*a^2*cos(d*x + c)^7 + 64512*a^2*cos(d*x + c)^6 + 48
3840*a^2*cos(d*x + c)^5 + 107520*a^2*cos(d*x + c)^4 - 645120*a^2*cos(d*x +
c)^3 - 483840*a^2*cos(d*x + c)^2 + 322560*a^2*cos(d*x + c)*log(-cos(d*x + c
)) + 197295*a^2*cos(d*x + c) - 161280*a^2)/(d*cos(d*x + c))
```

giac [B] time = 1.93, size = 370, normalized size = 2.02

$$\frac{2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right) - 2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) + \frac{2520\left(2\,a^2 + \frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1} + \frac{1457\,a^2 - \frac{20673\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/1260*(2520*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 252
0*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 2520*(2*a^2 +
a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 1) + (1457*a^2 - 20673*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
```

```
123012*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 421428*a^2*(cos(d*x
 + c) - 1)^3/(cos(d*x + c) + 1)^3 + 949662*a^2*(cos(d*x + c) - 1)^4/(cos(d*
x + c) + 1)^4 - 1009134*a^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 666
036*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 276804*a^2*(cos(d*x + c
) - 1)^7/(cos(d*x + c) + 1)^7 + 66681*a^2*(cos(d*x + c) - 1)^8/(cos(d*x + c
) + 1)^8 - 7129*a^2*(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9)/((cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 1)^9)/d
```

maple [A] time = 0.74, size = 206, normalized size = 1.13

$$\frac{1024a^2\cos(dx+c)}{315d}+\frac{8a^2\left(\sin^8(dx+c)\right)\cos(dx+c)}{9d}+\frac{64a^2\cos(dx+c)\left(\sin^6(dx+c)\right)}{63d}+\frac{128a^2\cos(dx+c)\left(\mathrm{si}\right.}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^9,x)
```

```
[Out] 1024/315*a^2*cos(d*x+c)/d+8/9/d*a^2*sin(d*x+c)^8*cos(d*x+c)+64/63/d*a^2*cos
(d*x+c)*sin(d*x+c)^6+128/105/d*a^2*cos(d*x+c)*sin(d*x+c)^4+512/315/d*a^2*co
s(d*x+c)*sin(d*x+c)^2-1/4/d*a^2*sin(d*x+c)^8-1/3/d*a^2*sin(d*x+c)^6-1/2/d*a
^2*sin(d*x+c)^4-1/d*a^2*sin(d*x+c)^2-2*a^2*ln(cos(d*x+c))/d+1/d*a^2*sin(d*x
+c)^10/cos(d*x+c)
```

maxima [A] time = 0.33, size = 146, normalized size = 0.80

$$-\frac{140\,a^2\cos(dx+c)^9+315\,a^2\cos(dx+c)^8-540\,a^2\cos(dx+c)^7-1680\,a^2\cos(dx+c)^6+504\,a^2\cos(dx+c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/1260*(140*a^2*cos(d*x + c)^9 + 315*a^2*cos(d*x + c)^8 - 540*a^2*cos(d*x
+ c)^7 - 1680*a^2*cos(d*x + c)^6 + 504*a^2*cos(d*x + c)^5 + 3780*a^2*cos(d*
x + c)^4 + 840*a^2*cos(d*x + c)^3 - 5040*a^2*cos(d*x + c)^2 - 3780*a^2*cos(
d*x + c) + 2520*a^2*log(cos(d*x + c)) - 1260*a^2/cos(d*x + c))/d
```

mupad [B] time = 0.99, size = 146, normalized size = 0.80

$$-\frac{\frac{2\,a^2\cos(c+d\,x)^3}{3}-\frac{a^2}{\cos(c+d\,x)}-4\,a^2\cos(c+d\,x)^2-3\,a^2\,\cos(c+d\,x)+3\,a^2\cos(c+d\,x)^4+\frac{2\,a^2\cos(c+d\,x)^5}{5}-\frac{4\,a^2\cos(}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^9*(a + a/cos(c + d*x))^2,x)
```

```
[Out] -((2*a^2*cos(c + d*x)^3)/3 - a^2/cos(c + d*x) - 4*a^2*cos(c + d*x)^2 - 3*a^
2*cos(c + d*x) + 3*a^2*cos(c + d*x)^4 + (2*a^2*cos(c + d*x)^5)/5 - (4*a^2*c
os(c + d*x)^6)/3 - (3*a^2*cos(c + d*x)^7)/7 + (a^2*cos(c + d*x)^8)/4 + (a^2
*cos(c + d*x)^9)/9 + 2*a^2*log(cos(c + d*x)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**9,x)
```

```
[Out] Timed out
```

3.20 $\int (a + a\sec(c + dx))^2 \sin^7(c + dx)\,dx$

Optimal. Leaf size=131

$$\frac{a^2\cos^7(c+dx)}{7d}+\frac{a^2\cos^6(c+dx)}{3d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{3a^2\cos^4(c+dx)}{2d}+\frac{3a^2\cos^2(c+dx)}{d}+\frac{2a^2\cos(c+dx)}{d}+\frac{a^2\,\text{sec}}{}$$

```
[Out] 2*a^2*cos(d*x+c)/d+3*a^2*cos(d*x+c)^2/d-3/2*a^2*cos(d*x+c)^4/d-2/5*a^2*cos(
d*x+c)^5/d+1/3*a^2*cos(d*x+c)^6/d+1/7*a^2*cos(d*x+c)^7/d-2*a^2*ln(cos(d*x+c
))/d+a^2*sec(d*x+c)/d
```

Rubi [A] time = 0.17, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2836, 12, 88}

$$\frac{a^2\cos^7(c+dx)}{7d}+\frac{a^2\cos^6(c+dx)}{3d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{3a^2\cos^4(c+dx)}{2d}+\frac{3a^2\cos^2(c+dx)}{d}+\frac{2a^2\cos(c+dx)}{d}+\frac{a^2\,\text{sec}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^7,x]
```

```
[Out] (2*a^2*Cos[c + d*x])/d + (3*a^2*Cos[c + d*x]^2)/d - (3*a^2*Cos[c + d*x]^4)/
(2*d) - (2*a^2*Cos[c + d*x]^5)/(5*d) + (a^2*Cos[c + d*x]^6)/(3*d) + (a^2*Co
s[c + d*x]^7)/(7*d) - (2*a^2*Log[Cos[c + d*x]])/d + (a^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sin^7(c+dx)\,dx = \int (-a-a\cos(c+dx))^2\sin^5(c+dx)\tan^2(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^2(-a-x)^3(-a+x)^5}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^3(-a+x)^5}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a^6+\frac{a^8}{x^2}-\frac{2a^7}{x}+6a^5x-6a^3x^3+2a^2x^4+2ax^5-x^6\right)dx, x\right.}{a^5d}$$
$$= \frac{2a^2\cos(c+dx)}{d}+\frac{3a^2\cos^2(c+dx)}{d}-\frac{3a^2\cos^4(c+dx)}{2d}-\frac{2a^2\cos^5(c}{5d}$$

Mathematica [A] time = 0.56, size = 107, normalized size = 0.82

$$\frac{a^2\sec(c+dx)(11760\cos(2(c+dx))+5250\cos(3(c+dx))-588\cos(4(c+dx))-770\cos(5(c+dx))-48\cos}{13440}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^7,x]
```

```
[Out] (a^2*(25725 + 11760*Cos[2*(c + d*x)] + 5250*Cos[3*(c + d*x)] - 588*Cos[4*(c
 + d*x)] - 770*Cos[5*(c + d*x)] - 48*Cos[6*(c + d*x)] + 70*Cos[7*(c + d*x)]
 + 15*Cos[8*(c + d*x)] - 70*Cos[c + d*x]*(5 + 384*Log[Cos[c + d*x]]))*Sec[c
 + d*x])/(13440*d)
```

fricas [A] time = 0.69, size = 128, normalized size = 0.98

$$\frac{120a^2\cos(dx+c)^8+280a^2\cos(dx+c)^7-336a^2\cos(dx+c)^6-1260a^2\cos(dx+c)^5+2520a^2\cos(dx+c}{840d\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/840*(120*a^2*cos(d*x + c)^8 + 280*a^2*cos(d*x + c)^7 - 336*a^2*cos(d*x +
c)^6 - 1260*a^2*cos(d*x + c)^5 + 2520*a^2*cos(d*x + c)^3 + 1680*a^2*cos(d*x
 + c)^2 - 1680*a^2*cos(d*x + c)*log(-cos(d*x + c)) - 875*a^2*cos(d*x + c) +
 840*a^2)/(d*cos(d*x + c))
```

giac [B] time = 0.36, size = 320, normalized size = 2.44

$$\frac{420a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-420a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{420\left(2a^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}+\frac{357a^2-\frac{3759a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+}{}}{210d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/210*(420*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a
^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 420*(2*a^2 + a^2*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 1) + (357*a^2 - 3759*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 16737
*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 42595*a^2*(cos(d*x + c) -
1)^3/(cos(d*x + c) + 1)^3 + 43855*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) +
```

```
1)^4 - 25389*a^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 8043*a^2*(cos(
d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 1089*a^2*(cos(d*x + c) - 1)^7/(cos(d
*x + c) + 1)^7)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7)/d
```

maple [A] time = 0.75, size = 168, normalized size = 1.28

$$\frac{96a^2\cos(dx+c)}{35d}+\frac{6a^2\cos(dx+c)\left(\sin^6(dx+c)\right)}{7d}+\frac{36a^2\cos(dx+c)\left(\sin^4(dx+c)\right)}{35d}+\frac{48a^2\cos(dx+c)\left(\sin^2(dx\right.}{35d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^7,x)
```

```
[Out] 96/35*a^2*cos(d*x+c)/d+6/7/d*a^2*cos(d*x+c)*sin(d*x+c)^6+36/35/d*a^2*cos(d*
x+c)*sin(d*x+c)^4+48/35/d*a^2*cos(d*x+c)*sin(d*x+c)^2-1/3/d*a^2*sin(d*x+c)^
6-1/2/d*a^2*sin(d*x+c)^4-1/d*a^2*sin(d*x+c)^2-2*a^2*ln(cos(d*x+c))/d+1/d*a^
2*sin(d*x+c)^8/cos(d*x+c)
```

maxima [A] time = 0.34, size = 107, normalized size = 0.82

$$\frac{30\,a^2\cos(dx+c)^7+70\,a^2\cos(dx+c)^6-84\,a^2\cos(dx+c)^5-315\,a^2\cos(dx+c)^4+630\,a^2\cos(dx+c)^2+420\,a}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/210*(30*a^2*cos(d*x + c)^7 + 70*a^2*cos(d*x + c)^6 - 84*a^2*cos(d*x + c)^
5 - 315*a^2*cos(d*x + c)^4 + 630*a^2*cos(d*x + c)^2 + 420*a^2*cos(d*x + c)
- 420*a^2*log(cos(d*x + c)) + 210*a^2/cos(d*x + c))/d
```

mupad [B] time = 0.95, size = 105, normalized size = 0.80

$$\frac{2\,a^2\,\cos(c+d\,x)+\frac{a^2}{\cos(c+d\,x)}+3\,a^2\cos(c+d\,x)^2-\frac{3\,a^2\cos(c+d\,x)^4}{2}-\frac{2\,a^2\cos(c+d\,x)^5}{5}+\frac{a^2\cos(c+d\,x)^6}{3}+\frac{a^2\cos(c+d\,x)^7}{7}-2\,a^2}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*a^2*cos(c + d*x) + a^2/cos(c + d*x) + 3*a^2*cos(c + d*x)^2 - (3*a^2*cos(
c + d*x)^4)/2 - (2*a^2*cos(c + d*x)^5)/5 + (a^2*cos(c + d*x)^6)/3 + (a^2*co
s(c + d*x)^7)/7 - 2*a^2*log(cos(c + d*x)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**7,x)
```

```
[Out] Timed out
```

3.21 $\int (a+a\sec(c+dx))^2 \sin^5(c+dx)\,dx$

Optimal. Leaf size=112

$$-\frac{a^2\cos^5(c+dx)}{5d}-\frac{a^2\cos^4(c+dx)}{2d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{2a^2\cos^2(c+dx)}{d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sec(c+dx)}{d}-\frac{2a^2 \text{lo}}{}$$

```
[Out] a^2*cos(d*x+c)/d+2*a^2*cos(d*x+c)^2/d+1/3*a^2*cos(d*x+c)^3/d-1/2*a^2*cos(d*
x+c)^4/d-1/5*a^2*cos(d*x+c)^5/d-2*a^2*ln(cos(d*x+c))/d+a^2*sec(d*x+c)/d
```

Rubi [A] time = 0.16, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3872, 2836, 12, 88}

$$-\frac{a^2\cos^5(c+dx)}{5d}-\frac{a^2\cos^4(c+dx)}{2d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{2a^2\cos^2(c+dx)}{d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sec(c+dx)}{d}-\frac{2a^2 \text{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^5,x]
```

```
[Out] (a^2*Cos[c + d*x])/d + (2*a^2*Cos[c + d*x]^2)/d + (a^2*Cos[c + d*x]^3)/(3*d
) - (a^2*Cos[c + d*x]^4)/(2*d) - (a^2*Cos[c + d*x]^5)/(5*d) - (2*a^2*Log[Co
s[c + d*x]])/d + (a^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a + a\sec(c + dx))^2 \sin^5(c + dx)\,dx = \int (-a - a\cos(c + dx))^2 \sin^3(c + dx)\tan^2(c + dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^2(-a-x)^2(-a+x)^4}{x^2}\,dx, x, -a\cos(c + dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^2(-a+x)^4}{x^2}\,dx, x, -a\cos(c + dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^4 + \frac{a^6}{x^2} - \frac{2a^5}{x} + 4a^3x - a^2x^2 - 2ax^3 + x^4\right) dx, x, -a\cos(c + d$$
$$= \frac{a^2\cos(c + dx)}{d} + \frac{2a^2\cos^2(c + dx)}{d} + \frac{a^2\cos^3(c + dx)}{3d} - \frac{a^2\cos^4(c + dx)}{2d} -$$

Mathematica [**A**] time = 0.31, size = 87, normalized size = 0.78

$$-\frac{a^2\sec(c + dx)(-275\cos(2(c + dx)) - 165\cos(3(c + dx)) - 2\cos(4(c + dx)) + 15\cos(5(c + dx)) + 3\cos(6(c + dx}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^5,x]
```

```
[Out] -1/480*(a^2*(-750 - 275*Cos[2*(c + d*x)] - 165*Cos[3*(c + d*x)] - 2*Cos[4*(
c + d*x)] + 15*Cos[5*(c + d*x)] + 3*Cos[6*(c + d*x)] + 30*Cos[c + d*x]*(-3
+ 32*Log[Cos[c + d*x]]))*Sec[c + d*x])/d
```

fricas [**A**] time = 0.74, size = 115, normalized size = 1.03

$$-\frac{48\,a^2\cos(dx + c)^6 + 120\,a^2\cos(dx + c)^5 - 80\,a^2\cos(dx + c)^4 - 480\,a^2\cos(dx + c)^3 - 240\,a^2\cos(dx + c)^2 + 48}{240\,d\cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/240*(48*a^2*cos(d*x + c)^6 + 120*a^2*cos(d*x + c)^5 - 80*a^2*cos(d*x + c
)^4 - 480*a^2*cos(d*x + c)^3 - 240*a^2*cos(d*x + c)^2 + 480*a^2*cos(d*x + c
)*log(-cos(d*x + c)) + 195*a^2*cos(d*x + c) - 240*a^2)/(d*cos(d*x + c))
```

giac [**B**] time = 0.38, size = 270, normalized size = 2.41

$$\frac{60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1\right|\right) - 60\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) + \frac{60\left(2\,a^2 + \frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1} + 1} + \frac{69\,a^2 - \frac{525\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{1650\,a^2(\cos(d}{(\cos(dx+c}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/30*(60*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^2*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 60*(2*a^2 + a^2*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 1) + (69*a^2 - 525*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1650*a^2*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1610*a^2*(cos(d*x + c) - 1)^3/(co
s(d*x + c) + 1)^3 + 745*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 137
*a^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - 1)^5)/d
```

maple [A] time = 0.74, size = 130, normalized size = 1.16

$$\frac{32a^2\cos(dx+c)}{15d}+\frac{4a^2\cos(dx+c)\left(\sin^4(dx+c)\right)}{5d}+\frac{16a^2\cos(dx+c)\left(\sin^2(dx+c)\right)}{15d}-\frac{a^2\left(\sin^4(dx+c)\right)}{2d}-\frac{a^2\left(s\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^5,x)
```

```
[Out] 32/15*a^2*cos(d*x+c)/d+4/5/d*a^2*cos(d*x+c)*sin(d*x+c)^4+16/15/d*a^2*cos(d*
x+c)*sin(d*x+c)^2-1/2/d*a^2*sin(d*x+c)^4-1/d*a^2*sin(d*x+c)^2-2*a^2*ln(cos(
d*x+c))/d+1/d*a^2*sin(d*x+c)^6/cos(d*x+c)
```

maxima [A] time = 0.35, size = 94, normalized size = 0.84

$$-\frac{6\,a^2\cos(dx+c)^5+15\,a^2\cos(dx+c)^4-10\,a^2\cos(dx+c)^3-60\,a^2\cos(dx+c)^2-30\,a^2\cos(dx+c)+60\,a^2}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/30*(6*a^2*cos(d*x + c)^5 + 15*a^2*cos(d*x + c)^4 - 10*a^2*cos(d*x + c)^3
 - 60*a^2*cos(d*x + c)^2 - 30*a^2*cos(d*x + c) + 60*a^2*log(cos(d*x + c)) -
 30*a^2/cos(d*x + c))/d
```

mupad [B] time = 0.89, size = 91, normalized size = 0.81

$$\frac{a^2\,\cos(c+d\,x)+\frac{a^2}{\cos(c+d\,x)}+2\,a^2\cos(c+d\,x)^2+\frac{a^2\cos(c+d\,x)^3}{3}-\frac{a^2\cos(c+d\,x)^4}{2}-\frac{a^2\cos(c+d\,x)^5}{5}-2\,a^2\,\ln(\cos(c+d}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (a^2*cos(c + d*x) + a^2/cos(c + d*x) + 2*a^2*cos(c + d*x)^2 + (a^2*cos(c +
d*x)^3)/3 - (a^2*cos(c + d*x)^4)/2 - (a^2*cos(c + d*x)^5)/5 - 2*a^2*log(cos
(c + d*x)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.22 $\int (a+a\sec(c+dx))^2\sin^3(c+dx)\,dx$

Optimal. Leaf size=62

$$\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos^2(c+dx)}{d}+\frac{a^2\sec(c+dx)}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

```
[Out] a^2*cos(d*x+c)^2/d+1/3*a^2*cos(d*x+c)^3/d-2*a^2*ln(cos(d*x+c))/d+a^2*sec(d*
x+c)/d
```

Rubi [A] time = 0.12, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2836, 12, 75}

$$\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos^2(c+dx)}{d}+\frac{a^2\sec(c+dx)}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^3,x]
```

```
[Out] (a^2*Cos[c + d*x]^2)/d + (a^2*Cos[c + d*x]^3)/(3*d) - (2*a^2*Log[Cos[c + d*
x]])/d + (a^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sin^3(c+dx)\,dx = \int (-a-a\cos(c+dx))^2\sin(c+dx)\tan^2(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^2(-a-x)(-a+x)^3}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)(-a+x)^3}{x^2}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^4}{x^2}-\frac{2a^3}{x}+2ax-x^2\right)dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{a^2\cos^2(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec(c}{d}$$

Mathematica [A] time = 0.21, size = 65, normalized size = 1.05

$$\frac{a^2\sec(c+dx)(4\cos(2(c+dx))+6\cos(3(c+dx))+\cos(4(c+dx))-6\cos(c+dx)(8\log(\cos(c+dx))+1)+2}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^3,x]
```

```
[Out] (a^2*(27 + 4*Cos[2*(c + d*x)] + 6*Cos[3*(c + d*x)] + Cos[4*(c + d*x)] - 6*C
os[c + d*x]*(1 + 8*Log[Cos[c + d*x]]))*Sec[c + d*x])/(24*d)
```

fricas [A] time = 0.85, size = 76, normalized size = 1.23

$$\frac{2\,a^2\cos\left(dx+c\right)^4+6\,a^2\cos\left(dx+c\right)^3-12\,a^2\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)-3\,a^2\cos\left(dx+c\right)+6\,a^2}{6\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^2*cos(d*x + c)^4 + 6*a^2*cos(d*x + c)^3 - 12*a^2*cos(d*x + c)*log(
-cos(d*x + c)) - 3*a^2*cos(d*x + c) + 6*a^2)/(d*cos(d*x + c))
```

giac [A] time = 0.29, size = 74, normalized size = 1.19

$$-\frac{2\,a^2\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{a^2}{d\cos\left(dx+c\right)}+\frac{a^2d^5\cos\left(dx+c\right)^3+3\,a^2d^5\cos\left(dx+c\right)^2}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -2*a^2*log(abs(cos(d*x + c))/abs(d))/d + a^2/(d*cos(d*x + c)) + 1/3*(a^2*d^
5*cos(d*x + c)^3 + 3*a^2*d^5*cos(d*x + c)^2)/d^6
```

maple [A] time = 0.73, size = 92, normalized size = 1.48

$$\frac{2a^2\cos\left(dx+c\right)\left(\sin^2\left(dx+c\right)\right)}{3d}+\frac{4a^2\cos\left(dx+c\right)}{3d}-\frac{a^2\left(\sin^2\left(dx+c\right)\right)}{d}-\frac{2a^2\ln\left(\cos\left(dx+c\right)\right)}{d}+\frac{a^2\left(\sin^4\left(dx+c\right)\right)}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^3,x)
```

```
[Out] 2/3/d*a^2*cos(d*x+c)*sin(d*x+c)^2+4/3*a^2*cos(d*x+c)/d-1/d*a^2*sin(d*x+c)^2
-2*a^2*ln(cos(d*x+c))/d+1/d*a^2*sin(d*x+c)^4/cos(d*x+c)
```

maxima [A] time = 0.32, size = 56, normalized size = 0.90

$$\frac{a^2 \cos (dx + c)^3 + 3\,a^2 \cos (dx + c)^2 - 6\,a^2 \log (\cos (dx + c)) + \frac{3\,a^2}{\cos(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(a^2*cos(d*x + c)^3 + 3*a^2*cos(d*x + c)^2 - 6*a^2*log(cos(d*x + c)) +
3*a^2/cos(d*x + c))/d
```

mupad [B] time = 0.06, size = 54, normalized size = 0.87

$$\frac{\frac{a^2}{\cos(c+d\,x)} + a^2 \cos (c + d\,x)^2 + \frac{a^2 \cos(c+d\,x)^3}{3} - 2\,a^2\, \ln (\cos (c + d\,x))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (a^2/cos(c + d*x) + a^2*cos(c + d*x)^2 + (a^2*cos(c + d*x)^3)/3 - 2*a^2*log
(cos(c + d*x)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2 \left(\int 2 \sin^3 (c + dx) \sec (c + dx)\, dx + \int \sin^3 (c + dx) \sec^2 (c + dx)\, dx + \int \sin^3 (c + dx)\, dx \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**3,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)**3*sec(c + d*x), x) + Integral(sin(c + d*x)**
3*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**3, x))
```

3.23 $\int (a + a\sec(c + dx))^2 \sin(c + dx)\, dx$

Optimal. Leaf size=43

$$-\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sec(c+dx)}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

```
[Out] -a^2*cos(d*x+c)/d-2*a^2*ln(cos(d*x+c))/d+a^2*sec(d*x+c)/d
```

Rubi [A] time = 0.08, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 2833, 12, 43}

$$-\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sec(c+dx)}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x],x]
```

```
[Out] -((a^2*Cos[c + d*x])/d) - (2*a^2*Log[Cos[c + d*x]])/d + (a^2*Sec[c + d*x])/
d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sin(c+dx)\,dx=\int(-a-a\cos(c+dx))^2\sec(c+dx)\tan(c+dx)\,dx$$
$$=\frac{\text{Subst}\left(\int\frac{a^2(-a+x)^2}{x^2}\,dx,x,-a\cos(c+dx)\right)}{ad}$$
$$=\frac{a\,\text{Subst}\left(\int\frac{(-a+x)^2}{x^2}\,dx,x,-a\cos(c+dx)\right)}{d}$$
$$=\frac{a\,\text{Subst}\left(\int\left(1+\frac{a^2}{x^2}-\frac{2a}{x}\right)dx,x,-a\cos(c+dx)\right)}{d}$$
$$=-\frac{a^2\cos(c+dx)}{d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec(c+dx)}{d}$$

Mathematica [A] time = 0.12, size = 31, normalized size = 0.72

$$\frac{a^2(\sin(c+dx)\tan(c+dx)-2\log(\cos(c+dx))+1)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x],x]
```

```
[Out] (a^2*(1 - 2*Log[Cos[c + d*x]] + Sin[c + d*x]*Tan[c + d*x]))/d
```

fricas [A] time = 0.59, size = 51, normalized size = 1.19

$$-\frac{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)\log(-\cos(dx+c))-a^2}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c)*log(-cos(d*x + c)) - a^2)/(d*cos(
d*x + c))
```

giac [A] time = 0.49, size = 51, normalized size = 1.19

$$-\frac{a^2\cos(dx+c)}{d}-\frac{2\,a^2\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{a^2}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -a^2*cos(d*x + c)/d - 2*a^2*log(abs(cos(d*x + c))/abs(d))/d + a^2/(d*cos(d*
x + c))
```

maple [A] time = 0.25, size = 46, normalized size = 1.07

$$\frac{a^2\sec(dx+c)}{d}+\frac{2a^2\ln(\sec(dx+c))}{d}-\frac{a^2}{d\sec(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c),x)
```

```
[Out] a^2*sec(d*x+c)/d+2*a^2/d*ln(sec(d*x+c))-a^2/d/sec(d*x+c)
```

maxima [A] time = 0.34, size = 41, normalized size = 0.95

$$-\frac{a^2\cos\left(dx+c\right)+2\,a^2\log\left(\cos\left(dx+c\right)\right)-\frac{a^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a^2*cos(d*x + c) + 2*a^2*log(cos(d*x + c)) - a^2/cos(d*x + c))/d
```

mupad [B] time = 0.06, size = 41, normalized size = 0.95

$$-\frac{a^2\,\left(2\,\cos\left(c+d\,x\right)\,\ln\left(\cos\left(c+d\,x\right)\right)+{\cos\left(c+d\,x\right)}^2-1\right)}{d\,\cos\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] -(a^2*(2*cos(c + d*x)*log(cos(c + d*x)) + cos(c + d*x)^2 - 1))/(d*cos(c + d
*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin\left(c+dx\right)\sec\left(c+dx\right)dx+\int\sin\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\sin\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c),x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*sec(c + d*x), x) + Integral(sin(c + d*x)*sec(
c + d*x)**2, x) + Integral(sin(c + d*x), x))
```

3.24 $\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=48

$$\frac{a^2\sec(c+dx)}{d}+\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

```
[Out] 2*a^2*ln(1-cos(d*x+c))/d-2*a^2*ln(cos(d*x+c))/d+a^2*sec(d*x+c)/d
```

Rubi [A] time = 0.12, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 2836, 12, 77}

$$\frac{a^2\sec(c+dx)}{d}+\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*Log[1 - Cos[c + d*x]])/d - (2*a^2*Log[Cos[c + d*x]])/d + (a^2*Sec[c
+ d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx = \int (-a-a\cos(c+dx))^2\csc(c+dx)\sec^2(c+dx)\,dx$$
$$= \frac{a\,\text{Subst}\left(\int \frac{a^2(-a+x)}{(-a-x)x^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{-a+x}{(-a-x)x^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{2}{ax}+\frac{2}{a(a+x)}\right)dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec(c+dx)}{d}$$

Mathematica [A] time = 0.09, size = 36, normalized size = 0.75

$$\frac{a^2\left(\sec(c+dx)+4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-2\log(\cos(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(-2*Log[Cos[c + d*x]] + 4*Log[Sin[(c + d*x)/2]] + Sec[c + d*x]))/d
```

fricas [A] time = 0.69, size = 61, normalized size = 1.27

$$-\frac{2\,a^2\cos(dx+c)\log(-\cos(dx+c))-2\,a^2\cos(dx+c)\log\left(-\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)-a^2}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^2*cos(d*x + c)*log(-cos(d*x + c)) - 2*a^2*cos(d*x + c)*log(-1/2*cos(d
*x + c) + 1/2) - a^2)/(d*cos(d*x + c))
```

giac [B] time = 0.28, size = 115, normalized size = 2.40

$$\frac{2\left(a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{2\,a^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a^2*log(abs(-(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2*a^2 + a^2*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

maple [A] time = 0.41, size = 32, normalized size = 0.67

$$\frac{a^2\sec(dx+c)}{d}+\frac{2a^2\ln(-1+\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] a^2*sec(d*x+c)/d+2/d*a^2*ln(-1+sec(d*x+c))
```

maxima [A] time = 0.33, size = 43, normalized size = 0.90

$$\frac{2\,a^2\log\left(\cos\left(dx+c\right)-1\right)-2\,a^2\log\left(\cos\left(dx+c\right)\right)+\frac{a^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (2*a^2*log(cos(d*x + c) - 1) - 2*a^2*log(cos(d*x + c)) + a^2/cos(d*x + c))/
d
```

mupad [B] time = 0.08, size = 35, normalized size = 0.73

$$\frac{a^2}{d\,\cos\left(c+d\,x\right)}-\frac{4\,a^2\,\mathrm{atanh}\left(2\,\cos\left(c+d\,x\right)-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x),x)
```

```
[Out] a^2/(d*cos(c + d*x)) - (4*a^2*atanh(2*cos(c + d*x) - 1))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\csc\left(c+dx\right)\sec\left(c+dx\right)dx+\int\csc\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\csc\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*csc(c + d*x)*sec(c + d*x), x) + Integral(csc(c + d*x)*sec(
c + d*x)**2, x) + Integral(csc(c + d*x), x))
```

3.25 $\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=69

$$-\frac{a^3}{d(a-a\cos(c+dx))}+\frac{a^2\sec(c+dx)}{d}+\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

```
[Out] -a^3/d/(a-a*cos(d*x+c))+2*a^2*ln(1-cos(d*x+c))/d-2*a^2*ln(cos(d*x+c))/d+a^2
*sec(d*x+c)/d
```

Rubi [A] time = 0.14, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2836, 12, 44}

$$-\frac{a^3}{d(a-a\cos(c+dx))}+\frac{a^2\sec(c+dx)}{d}+\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^3/(d*(a - a*Cos[c + d*x]))) + (2*a^2*Log[1 - Cos[c + d*x]])/d - (2*a^2*
Log[Cos[c + d*x]])/d + (a^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx = \int (-a-a\cos(c+dx))^2\csc^3(c+dx)\sec^2(c+dx)\,dx$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{a^2}{(-a-x)^2x^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^5\,\text{Subst}\left(\int \frac{1}{(-a-x)^2x^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^5\,\text{Subst}\left(\int \left(\frac{1}{a^2x^2}-\frac{2}{a^3x}+\frac{1}{a^2(a+x)^2}+\frac{2}{a^3(a+x)}\right)dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^3}{d(a-a\cos(c+dx))}+\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{2a^2\log(\cos(c+dx)}{d}$$

Mathematica [**A**] time = 0.56, size = 75, normalized size = 1.09

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\csc^2\left(\frac{1}{2}(c+dx)\right)-2\sec(c+dx)-8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\log(\cos(c\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/8*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(Csc[(c + d*x)/2]^2 + 4*L
og[Cos[c + d*x]] - 8*Log[Sin[(c + d*x)/2]] - 2*Sec[c + d*x]))/d
```

fricas [**A**] time = 0.57, size = 112, normalized size = 1.62

$$\frac{2\,a^2\cos(dx+c)-a^2-2\left(a^2\cos(dx+c)^2-a^2\cos(dx+c)\right)\log(-\cos(dx+c))+2\left(a^2\cos(dx+c)^2-a^2\cos(d\right.}{d\cos(dx+c)^2-d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (2*a^2*cos(d*x + c) - a^2 - 2*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-
cos(d*x + c)) + 2*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-1/2*cos(d*x
+ c) + 1/2))/(d*cos(d*x + c)^2 - d*cos(d*x + c))
```

giac [**A**] time = 0.67, size = 135, normalized size = 1.96

$$\frac{4\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-4\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{a^2+\frac{5\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 4*a^2*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a^2 + 5*a^2*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + (cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2))/d
```

maple [**A**] time = 0.59, size = 50, normalized size = 0.72

$$\frac{a^2\sec(dx+c)}{d}-\frac{a^2}{d(-1+\sec(dx+c))}+\frac{2a^2\ln(-1+\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sec(d*x+c))^2,x)
```

```
[Out] a^2*sec(d*x+c)/d-1/d*a^2/(-1+sec(d*x+c))+2/d*a^2*ln(-1+sec(d*x+c))
```

maxima [A] time = 0.32, size = 68, normalized size = 0.99

$$\frac{2\,a^2\log\left(\cos\left(dx+c\right)-1\right)-2\,a^2\log\left(\cos\left(dx+c\right)\right)+\frac{2\,a^2\cos(dx+c)-a^2}{\cos(dx+c)^2-\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (2*a^2*log(cos(d*x + c) - 1) - 2*a^2*log(cos(d*x + c)) + (2*a^2*cos(d*x + c
) - a^2)/(cos(d*x + c)^2 - cos(d*x + c)))/d
```

mupad [B] time = 0.90, size = 61, normalized size = 0.88

$$-\frac{2\,a^2\,\cos\left(c+d\,x\right)-a^2}{d\,\left(\cos\left(c+d\,x\right)-{\cos\left(c+d\,x\right)}^2\right)}-\frac{4\,a^2\,\mathrm{atanh}\left(2\,\cos\left(c+d\,x\right)-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^3,x)
```

```
[Out] - (2*a^2*cos(c + d*x) - a^2)/(d*(cos(c + d*x) - cos(c + d*x)^2)) - (4*a^2*a
tanh(2*cos(c + d*x) - 1))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\csc^3\left(c+dx\right)\sec\left(c+dx\right)dx+\int\csc^3\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\csc^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*csc(c + d*x)**3*sec(c + d*x), x) + Integral(csc(c + d*x)**
3*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**3, x))
```

3.26 $\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=115

$$-\frac{a^4}{4d(a-a\cos(c+dx))^2}-\frac{5a^3}{4d(a-a\cos(c+dx))}+\frac{a^2\sec(c+dx)}{d}+\frac{17a^2\log(1-\cos(c+dx))}{8d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

```
[Out] -1/4*a^4/d/(a-a*cos(d*x+c))^2-5/4*a^3/d/(a-a*cos(d*x+c))+17/8*a^2*ln(1-cos(
d*x+c))/d-2*a^2*ln(cos(d*x+c))/d-1/8*a^2*ln(1+cos(d*x+c))/d+a^2*sec(d*x+c)/
d
```

Rubi [A] time = 0.17, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2836, 12, 88}

$$-\frac{a^4}{4d(a-a\cos(c+dx))^2}-\frac{5a^3}{4d(a-a\cos(c+dx))}+\frac{a^2\sec(c+dx)}{d}+\frac{17a^2\log(1-\cos(c+dx))}{8d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -a^4/(4*d*(a - a*Cos[c + d*x])^2) - (5*a^3)/(4*d*(a - a*Cos[c + d*x])) + (1
7*a^2*Log[1 - Cos[c + d*x]])/(8*d) - (2*a^2*Log[Cos[c + d*x]])/d - (a^2*Log
[1 + Cos[c + d*x]])/(8*d) + (a^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx = \int (-a-a\cos(c+dx))^2\csc^5(c+dx)\sec^2(c+dx)\,dx$$
$$= \frac{a^5\operatorname{Subst}\left(\int \frac{a^2}{(-a-x)^3x^2(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^7\operatorname{Subst}\left(\int \frac{1}{(-a-x)^3x^2(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^7\operatorname{Subst}\left(\int \left(\frac{1}{8a^5(a-x)}+\frac{1}{a^4x^2}-\frac{2}{a^5x}+\frac{1}{2a^3(a+x)^3}+\frac{5}{4a^4(a+x)^2}+\frac{17}{8a^5(a+x)}\right)dx\right.}{d}$$
$$= -\frac{a^4}{4d(a-a\cos(c+dx))^2}-\frac{5a^3}{4d(a-a\cos(c+dx))}+\frac{17a^2\log(1-\cos(c}{8d}$$

Mathematica [A] time = 1.57, size = 103, normalized size = 0.90

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\csc^4\left(\frac{1}{2}(c+dx)\right)+10\csc^2\left(\frac{1}{2}(c+dx)\right)+4\left(-4\sec(c+dx)-17\log\left(\sin\right.\right.\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/64*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(10*Csc[(c + d*x)/2]^2 +
 Csc[(c + d*x)/2]^4 + 4*(Log[Cos[(c + d*x)/2]] + 8*Log[Cos[c + d*x]] - 17*L
og[Sin[(c + d*x)/2]] - 4*Sec[c + d*x])))/d
```

fricas [A] time = 0.59, size = 209, normalized size = 1.82

$$\frac{18\,a^2\cos(dx+c)^2-28\,a^2\cos(dx+c)+8\,a^2-16\left(a^2\cos(dx+c)^3-2\,a^2\cos(dx+c)^2+a^2\cos(dx+c)\right)\log(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(18*a^2*cos(d*x + c)^2 - 28*a^2*cos(d*x + c) + 8*a^2 - 16*(a^2*cos(d*x
+ c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(-cos(d*x + c)) - (a^2
*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(1/2*cos(d*x
+ c) + 1/2) + 17*(a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x +
 c))*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^3 - 2*d*cos(d*x + c)^2 +
 d*cos(d*x + c))
```

giac [A] time = 0.49, size = 191, normalized size = 1.66

$$\frac{34\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-32\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\frac{\left(a^2-\frac{12\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{51\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(\cos(dx+c)-1)^2}+\frac{32\,(2\,a}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(34*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 32*a^2*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^2 - 12*a^2*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 51*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
```

```
1)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 + 32*(2*a^2 + a^2*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/
d
```

maple [A] time = 0.49, size = 85, normalized size = 0.74

$$\frac{a^2 \sec(dx+c)}{d} - \frac{a^2}{4d(-1+\sec(dx+c))^2} - \frac{7a^2}{4d(-1+\sec(dx+c))} + \frac{17a^2 \ln(-1+\sec(dx+c))}{8d} - \frac{a^2 \ln(1+\sec(dx+}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+a*sec(d*x+c))^2,x)
```

```
[Out] a^2*sec(d*x+c)/d-1/4/d*a^2/(-1+sec(d*x+c))^2-7/4/d*a^2/(-1+sec(d*x+c))+17/8
/d*a^2*ln(-1+sec(d*x+c))-1/8/d*a^2*ln(1+sec(d*x+c))
```

maxima [A] time = 0.32, size = 104, normalized size = 0.90

$$-\frac{a^2 \log(\cos(dx+c)+1) - 17\,a^2 \log(\cos(dx+c)-1) + 16\,a^2 \log(\cos(dx+c)) - \frac{2\left(9\,a^2 \cos(dx+c)^2 - 14\,a^2 \cos(dx+c) + 4\,a\right.}{\cos(dx+c)^3 - 2\,\cos(dx+c)^2 + \cos(dx+c)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(a^2*log(cos(d*x + c) + 1) - 17*a^2*log(cos(d*x + c) - 1) + 16*a^2*log
(cos(d*x + c)) - 2*(9*a^2*cos(d*x + c)^2 - 14*a^2*cos(d*x + c) + 4*a^2)/(co
s(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c)))/d
```

mupad [B] time = 0.10, size = 109, normalized size = 0.95

$$\frac{17\,a^2\,\ln(\cos(c+d\,x)-1)}{8\,d} - \frac{a^2\,\ln(\cos(c+d\,x)+1)}{8\,d} + \frac{\frac{9\,a^2\,\cos(c+d\,x)^2}{4} - \frac{7\,a^2\,\cos(c+d\,x)}{2} + a^2}{d\,\left(\cos(c+d\,x)^3 - 2\,\cos(c+d\,x)^2 + \cos(c+d\,x)\right)} - \frac{2\,a^2\,\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^5,x)
```

```
[Out] (17*a^2*log(cos(c + d*x) - 1))/(8*d) - (a^2*log(cos(c + d*x) + 1))/(8*d) +
(a^2 - (7*a^2*cos(c + d*x))/2 + (9*a^2*cos(c + d*x)^2)/4)/(d*(cos(c + d*x)
- 2*cos(c + d*x)^2 + cos(c + d*x)^3)) - (2*a^2*log(cos(c + d*x)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.27 $\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=160

$$-\frac{a^5}{12d(a-a\cos(c+dx))^3}-\frac{3a^4}{8d(a-a\cos(c+dx))^2}-\frac{23a^3}{16d(a-a\cos(c+dx))}+\frac{a^3}{16d(a\cos(c+dx)+a)}+\frac{a^2\sec(c+}{d}$$

```
[Out] -1/12*a^5/d/(a-a*cos(d*x+c))^3-3/8*a^4/d/(a-a*cos(d*x+c))^2-23/16*a^3/d/(a-
a*cos(d*x+c))+1/16*a^3/d/(a+a*cos(d*x+c))+9/4*a^2*ln(1-cos(d*x+c))/d-2*a^2*
ln(cos(d*x+c))/d-1/4*a^2*ln(1+cos(d*x+c))/d+a^2*sec(d*x+c)/d
```

Rubi [A] time = 0.20, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3872, 2836, 12, 88}

$$-\frac{a^5}{12d(a-a\cos(c+dx))^3}-\frac{3a^4}{8d(a-a\cos(c+dx))^2}-\frac{23a^3}{16d(a-a\cos(c+dx))}+\frac{a^3}{16d(a\cos(c+dx)+a)}+\frac{a^2\sec(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^7*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -a^5/(12*d*(a - a*Cos[c + d*x])^3) - (3*a^4)/(8*d*(a - a*Cos[c + d*x])^2) -
 (23*a^3)/(16*d*(a - a*Cos[c + d*x])) + a^3/(16*d*(a + a*Cos[c + d*x])) + (
9*a^2*Log[1 - Cos[c + d*x]])/(4*d) - (2*a^2*Log[Cos[c + d*x]])/d - (a^2*Log
[1 + Cos[c + d*x]])/(4*d) + (a^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx = \int (-a-a\cos(c+dx))^2\csc^7(c+dx)\sec^2(c+dx)\,dx$$
$$= \frac{a^7\,\text{Subst}\left(\int \frac{a^2}{(-a-x)^4x^2(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^9\,\text{Subst}\left(\int \frac{1}{(-a-x)^4x^2(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^9\,\text{Subst}\left(\int \left(\frac{1}{16a^6(a-x)^2} + \frac{1}{4a^7(a-x)} + \frac{1}{a^6x^2} - \frac{2}{a^7x} + \frac{1}{4a^4(a+x)^4} + \frac{3}{4a^5(a+x)^3} + \frac{}{16a}\right.}{d}$$
$$= -\frac{a^5}{12d(a-a\cos(c+dx))^3} - \frac{3a^4}{8d(a-a\cos(c+dx))^2} - \frac{23a^3}{16d(a-a\cos(c+d}$$

Mathematica [A] time = 1.34, size = 136, normalized size = 0.85

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(36\csc^4\left(\frac{1}{2}(c+dx)\right)+120\csc^2\left(\frac{1}{2}(c+dx)\right)+\csc^6\left(\frac{1}{2}(c+dx)\right)\left(16-3\sec\right.\right.}{38}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^7*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/384*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(120*Csc[(c + d*x)/2]^2
 + 36*Csc[(c + d*x)/2]^4 + 48*(Log[Cos[(c + d*x)/2]] + 4*Log[Cos[c + d*x]]
- 9*Log[Sin[(c + d*x)/2]]) + Csc[(c + d*x)/2]^6*(16 - 3*Sec[(c + d*x)/2]^2*
(3 + 2*Sec[c + d*x]))))/d
```

fricas [A] time = 0.62, size = 289, normalized size = 1.81

$$\frac{30\,a^2\cos\left(dx+c\right)^4-48\,a^2\cos\left(dx+c\right)^3-14\,a^2\cos\left(dx+c\right)^2+46\,a^2\cos\left(dx+c\right)-12\,a^2-24\left(a^2\cos\left(dx+c\right)^5-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(30*a^2*cos(d*x + c)^4 - 48*a^2*cos(d*x + c)^3 - 14*a^2*cos(d*x + c)^2
 + 46*a^2*cos(d*x + c) - 12*a^2 - 24*(a^2*cos(d*x + c)^5 - 2*a^2*cos(d*x +
c)^4 + 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-cos(d*x + c)) - 3*(a^2
*cos(d*x + c)^5 - 2*a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x
 + c))*log(1/2*cos(d*x + c) + 1/2) + 27*(a^2*cos(d*x + c)^5 - 2*a^2*cos(d*x
 + c)^4 + 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-1/2*cos(d*x + c) +
1/2))/(d*cos(d*x + c)^5 - 2*d*cos(d*x + c)^4 + 2*d*cos(d*x + c)^2 - d*cos(d
*x + c))
```

giac [A] time = 0.40, size = 238, normalized size = 1.49

$$\frac{216\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-192\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\frac{3\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{\left(a^2-\frac{12\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{90\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{396\,a^{}}{(c}\right.}{(\cos(dx+c)-1)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/96*(216*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 192*a^2*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - 3*a^2*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + (a^2 - 12*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) + 90*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 396*a^2*(cos(d*x + c
) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^3/(cos(d*x + c) - 1)^3 +
192*(2*a^2 + a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + 1))/d
```

maple [A] time = 0.50, size = 121, normalized size = 0.76

$$\frac{a^2 \sec(dx+c)}{d} - \frac{a^2}{12d\left(-1+\sec(dx+c)\right)^3} - \frac{5a^2}{8d\left(-1+\sec(dx+c)\right)^2} - \frac{39a^2}{16d\left(-1+\sec(dx+c)\right)} + \frac{9a^2 \ln(-1+\sec(\ldots}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^7*(a+a*sec(d*x+c))^2,x)
```

```
[Out] a^2*sec(d*x+c)/d-1/12/d*a^2/(-1+sec(d*x+c))^3-5/8/d*a^2/(-1+sec(d*x+c))^2-3
9/16/d*a^2/(-1+sec(d*x+c))+9/4/d*a^2*ln(-1+sec(d*x+c))-1/16/d*a^2/(1+sec(d*
x+c))-1/4/d*a^2*ln(1+sec(d*x+c))
```

maxima [A] time = 0.33, size = 143, normalized size = 0.89

$$-\frac{3\,a^2 \log\left(\cos\left(dx+c\right)+1\right) - 27\,a^2 \log\left(\cos\left(dx+c\right)-1\right) + 24\,a^2 \log\left(\cos\left(dx+c\right)\right) - \frac{2\left(15\,a^2 \cos(dx+c)^4 - 24\,a^2 \cos(dx\ldots\right.}{\cos(dx+c)^5 - 2\,\cos(\ldots}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^2*log(cos(d*x + c) + 1) - 27*a^2*log(cos(d*x + c) - 1) + 24*a^2*
log(cos(d*x + c)) - 2*(15*a^2*cos(d*x + c)^4 - 24*a^2*cos(d*x + c)^3 - 7*a^
2*cos(d*x + c)^2 + 23*a^2*cos(d*x + c) - 6*a^2)/(cos(d*x + c)^5 - 2*cos(d*x
 + c)^4 + 2*cos(d*x + c)^2 - cos(d*x + c)))/d
```

mupad [B] time = 0.11, size = 147, normalized size = 0.92

$$\frac{9\,a^2\,\ln\left(\cos\left(c+d\,x\right)-1\right)}{4\,d} - \frac{a^2\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{4\,d} - \frac{2\,a^2\,\ln\left(\cos\left(c+d\,x\right)\right)}{d} + \frac{-\frac{5\,a^2\,\cos(c+d\,x)^4}{2} + 4\,a^2\,\cos\left(c+d\,x\right)^{3}\ldots}{d\,\left(-\cos\left(c+d\,x\right)^5 + 2\,\cos\left(c+d\,x\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^7,x)
```

```
[Out] (9*a^2*log(cos(c + d*x) - 1))/(4*d) - (a^2*log(cos(c + d*x) + 1))/(4*d) - (
2*a^2*log(cos(c + d*x)))/d + (a^2 - (23*a^2*cos(c + d*x))/6 + (7*a^2*cos(c
+ d*x)^2)/6 + 4*a^2*cos(c + d*x)^3 - (5*a^2*cos(c + d*x)^4)/2)/(d*(cos(c +
d*x) - 2*cos(c + d*x)^2 + 2*cos(c + d*x)^4 - cos(c + d*x)^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**7*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.28 $\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=205

$$-\frac{a^6}{32d(a-a\cos(c+dx))^4}-\frac{7a^5}{48d(a-a\cos(c+dx))^3}-\frac{15a^4}{32d(a-a\cos(c+dx))^2}+\frac{a^4}{64d(a\cos(c+dx)+a)^2}-\frac{5}{32d(a-a}$$

```
[Out] -1/32*a^6/d/(a-a*cos(d*x+c))^4-7/48*a^5/d/(a-a*cos(d*x+c))^3-15/32*a^4/d/(a
-a*cos(d*x+c))^2-51/32*a^3/d/(a-a*cos(d*x+c))+1/64*a^4/d/(a+a*cos(d*x+c))^2
+9/64*a^3/d/(a+a*cos(d*x+c))+303/128*a^2*ln(1-cos(d*x+c))/d-2*a^2*ln(cos(d*
x+c))/d-47/128*a^2*ln(1+cos(d*x+c))/d+a^2*sec(d*x+c)/d
```

Rubi [A] time = 0.24, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3872, 2836, 12, 88}

$$-\frac{a^6}{32d(a-a\cos(c+dx))^4}-\frac{7a^5}{48d(a-a\cos(c+dx))^3}-\frac{15a^4}{32d(a-a\cos(c+dx))^2}+\frac{a^4}{64d(a\cos(c+dx)+a)^2}-\frac{5}{32d(a-a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^9*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -a^6/(32*d*(a - a*Cos[c + d*x])^4) - (7*a^5)/(48*d*(a - a*Cos[c + d*x])^3)
- (15*a^4)/(32*d*(a - a*Cos[c + d*x])^2) - (51*a^3)/(32*d*(a - a*Cos[c + d*
x])) + a^4/(64*d*(a + a*Cos[c + d*x])^2) + (9*a^3)/(64*d*(a + a*Cos[c + d*x
])) + (303*a^2*Log[1 - Cos[c + d*x]])/(128*d) - (2*a^2*Log[Cos[c + d*x]])/d
 - (47*a^2*Log[1 + Cos[c + d*x]])/(128*d) + (a^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx = \int (-a-a\cos(c+dx))^2\csc^9(c+dx)\sec^2(c+dx)\,dx$$
$$= \frac{a^9\,\mathrm{Subst}\left(\int \frac{a^2}{(-a-x)^5x^2(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^{11}\,\mathrm{Subst}\left(\int \frac{1}{(-a-x)^5x^2(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^{11}\,\mathrm{Subst}\left(\int \left(\frac{1}{32a^7(a-x)^3} + \frac{9}{64a^8(a-x)^2} + \frac{47}{128a^9(a-x)} + \frac{1}{a^8x^2} - \frac{2}{a^9x} + \frac{1}{8a^5(a+x)}\right.\right.}{}$$
$$= -\frac{a^6}{32d(a-a\cos(c+dx))^4} - \frac{7a^5}{48d(a-a\cos(c+dx))^3} - \frac{15a^4}{32d(a-a\cos(}$$

Mathematica [A] time = 3.46, size = 164, normalized size = 0.80

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(3\csc^8\left(\frac{1}{2}(c+dx)\right)+28\csc^6\left(\frac{1}{2}(c+dx)\right)+180\csc^4\left(\frac{1}{2}(c+dx)\right)+1224\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^9*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/6144*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(1224*Csc[(c + d*x)/2]
^2 + 180*Csc[(c + d*x)/2]^4 + 28*Csc[(c + d*x)/2]^6 + 3*Csc[(c + d*x)/2]^8
- 6*(18*Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^4 + 4*(-47*Log[Cos[(c + d*x)/
2]] - 128*Log[Cos[c + d*x]] + 303*Log[Sin[(c + d*x)/2]] + 64*Sec[c + d*x]))
))/d
```

fricas [B] time = 0.64, size = 461, normalized size = 2.25

$$\frac{1050\,a^2\cos(dx+c)^6 - 1716\,a^2\cos(dx+c)^5 - 1468\,a^2\cos(dx+c)^4 + 3308\,a^2\cos(dx+c)^3 - 38\,a^2\cos(dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/384*(1050*a^2*cos(d*x + c)^6 - 1716*a^2*cos(d*x + c)^5 - 1468*a^2*cos(d*x
 + c)^4 + 3308*a^2*cos(d*x + c)^3 - 38*a^2*cos(d*x + c)^2 - 1568*a^2*cos(d*
x + c) + 384*a^2 - 768*(a^2*cos(d*x + c)^7 - 2*a^2*cos(d*x + c)^6 - a^2*cos
(d*x + c)^5 + 4*a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c
)^2 + a^2*cos(d*x + c))*log(-cos(d*x + c)) - 141*(a^2*cos(d*x + c)^7 - 2*a^
2*cos(d*x + c)^6 - a^2*cos(d*x + c)^5 + 4*a^2*cos(d*x + c)^4 - a^2*cos(d*x
+ c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/
2) + 909*(a^2*cos(d*x + c)^7 - 2*a^2*cos(d*x + c)^6 - a^2*cos(d*x + c)^5 +
4*a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(
d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^7 - 2*d*cos(d*x + c
)^6 - d*cos(d*x + c)^5 + 4*d*cos(d*x + c)^4 - d*cos(d*x + c)^3 - 2*d*cos(d*
x + c)^2 + d*cos(d*x + c))
```

giac [A] time = 0.38, size = 291, normalized size = 1.42

$$\frac{3636\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 3072\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) - \frac{120\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{6\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{\left(3\,a^2 - \frac{40\,a^2(\cos}{\cos(d}\right.}{}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1536*(3636*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 3072*a
^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - 120*a^2*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 6*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2 - (3*a^2 - 40*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 282*a^2*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1680*a^2*(cos(d*x + c) - 1)^3/(cos(d
*x + c) + 1)^3 + 7575*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)*(cos(d
*x + c) + 1)^4/(cos(d*x + c) - 1)^4 + 3072*(2*a^2 + a^2*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

maple [A] time = 0.57, size = 157, normalized size = 0.77

$$\frac{a^2 \sec(dx+c)}{d} - \frac{a^2}{32d(-1+\sec(dx+c))^4} - \frac{13a^2}{48d(-1+\sec(dx+c))^3} - \frac{35a^2}{32d(-1+\sec(dx+c))^2} - \frac{99a^2}{32d(-1+\sec(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^9*(a+a*sec(d*x+c))^2,x)
```

```
[Out] a^2*sec(d*x+c)/d-1/32/d*a^2/(-1+sec(d*x+c))^4-13/48/d*a^2/(-1+sec(d*x+c))^3
-35/32/d*a^2/(-1+sec(d*x+c))^2-99/32/d*a^2/(-1+sec(d*x+c))+303/128/d*a^2*ln
(-1+sec(d*x+c))+1/64/d*a^2/(1+sec(d*x+c))^2-11/64/d*a^2/(1+sec(d*x+c))-47/1
28/d*a^2*ln(1+sec(d*x+c))
```

maxima [A] time = 0.37, size = 197, normalized size = 0.96

$$-\frac{141\,a^2\log(\cos(dx+c)+1) - 909\,a^2\log(\cos(dx+c)-1) + 768\,a^2\log(\cos(dx+c)) - \frac{2\left(525\,a^2\cos(dx+c)^6 - 858\,a^2\cos\right.}{\cos(dx+c)^7 - 2}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/384*(141*a^2*log(cos(d*x + c) + 1) - 909*a^2*log(cos(d*x + c) - 1) + 768
*a^2*log(cos(d*x + c)) - 2*(525*a^2*cos(d*x + c)^6 - 858*a^2*cos(d*x + c)^5
 - 734*a^2*cos(d*x + c)^4 + 1654*a^2*cos(d*x + c)^3 - 19*a^2*cos(d*x + c)^2
 - 784*a^2*cos(d*x + c) + 192*a^2)/(cos(d*x + c)^7 - 2*cos(d*x + c)^6 - cos
(d*x + c)^5 + 4*cos(d*x + c)^4 - cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*
x + c)))/d
```

mupad [B] time = 0.17, size = 203, normalized size = 0.99

$$\frac{-\frac{175\,a^2\cos(c+d\,x)^6}{64} + \frac{143\,a^2\cos(c+d\,x)^5}{32} + \frac{367\,a^2\cos(c+d\,x)^4}{96} - \frac{827\,a^2\cos(c+d\,x)^3}{96} + \frac{19\,a^2\cos(c+d\,x)^2}{192} + \frac{49\,a^2\cos(c+d\,x)}{12} - a^2}{d\left(-\cos(c+d\,x)^7 + 2\cos(c+d\,x)^6 + \cos(c+d\,x)^5 - 4\cos(c+d\,x)^4 + \cos(c+d\,x)^3 + 2\cos(c+d\,x)^2 - \cos(c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^9,x)
```

```
[Out] ((49*a^2*cos(c + d*x))/12 - a^2 + (19*a^2*cos(c + d*x)^2)/192 - (827*a^2*co
s(c + d*x)^3)/96 + (367*a^2*cos(c + d*x)^4)/96 + (143*a^2*cos(c + d*x)^5)/3
2 - (175*a^2*cos(c + d*x)^6)/64)/(d*(2*cos(c + d*x)^2 - cos(c + d*x) + cos(
c + d*x)^3 - 4*cos(c + d*x)^4 + cos(c + d*x)^5 + 2*cos(c + d*x)^6 - cos(c +
 d*x)^7)) + (303*a^2*log(cos(c + d*x) - 1))/(128*d) - (47*a^2*log(cos(c + d
*x) + 1))/(128*d) - (2*a^2*log(cos(c + d*x)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**9*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.29 $\int (a + a\sec(c + dx))^2 \sin^8(c + dx)\,dx$

Optimal. Leaf size=199

$$-\frac{2a^2\sin^7(c+dx)}{7d}-\frac{2a^2\sin^5(c+dx)}{5d}-\frac{2a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] -245/128*a^2*x+2*a^2*arctanh(sin(d*x+c))/d-2*a^2*sin(d*x+c)/d+139/128*a^2*c
os(d*x+c)*sin(d*x+c)/d+11/192*a^2*cos(d*x+c)^3*sin(d*x+c)/d-17/48*a^2*cos(d
*x+c)^5*sin(d*x+c)/d+1/8*a^2*cos(d*x+c)^7*sin(d*x+c)/d-2/3*a^2*sin(d*x+c)^3
/d-2/5*a^2*sin(d*x+c)^5/d-2/7*a^2*sin(d*x+c)^7/d+a^2*tan(d*x+c)/d
```

Rubi [A] time = 0.36, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3872, 2872, 2637, 2635, 8, 2633, 3770, 3767}

$$-\frac{2a^2\sin^7(c+dx)}{7d}-\frac{2a^2\sin^5(c+dx)}{5d}-\frac{2a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^8,x]
```

```
[Out] (-245*a^2*x)/128 + (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d
 + (139*a^2*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (11*a^2*Cos[c + d*x]^3*Sin
[c + d*x])/(192*d) - (17*a^2*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (a^2*Cos
[c + d*x]^7*Sin[c + d*x])/(8*d) - (2*a^2*Sin[c + d*x]^3)/(3*d) - (2*a^2*Sin
[c + d*x]^5)/(5*d) - (2*a^2*Sin[c + d*x]^7)/(7*d) + (a^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sin^8(c+dx)\,dx = \int (-a-a\cos(c+dx))^2\sin^6(c+dx)\tan^2(c+dx)\,dx$$

$$= \frac{\int\left(-3a^{10}-8a^{10}\cos(c+dx)+2a^{10}\cos^2(c+dx)+12a^{10}\cos^3(c+dx)\right.}{}$$

$$= -3a^2x+a^2\int\cos^8(c+dx)\,dx+a^2\int\sec^2(c+dx)\,dx+\left(2a^2\right)\int\cos^2$$

$$= -3a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{8a^2\sin(c+dx)}{d}+\frac{a^2\cos(c+dx)}{d}$$

$$= -2a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{7a^2\cos(c+dx}{4d}$$

$$= -\frac{5a^2x}{4}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{13a^2\cos(c+d}{16}$$

$$= -\frac{35a^2x}{16}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{139a^2\cos(c+}{1}$$

$$= -\frac{245a^2x}{128}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{139a^2\cos(c}{}$$

Mathematica [A] time = 0.96, size = 144, normalized size = 0.72

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(30720\sin^7(c+dx)+43008\sin^5(c+dx)+71680\sin^3(c+dx)+2150\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^8,x]
```

```
[Out] -1/430080*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(168000*c + 168000*d
*x + 37800*ArcTan[Tan[c + d*x]] - 215040*ArcTanh[Sin[c + d*x]] + 215040*Sin
[c + d*x] + 71680*Sin[c + d*x]^3 + 43008*Sin[c + d*x]^5 + 30720*Sin[c + d*x
]^7 - 55440*Sin[2*(c + d*x)] + 2520*Sin[4*(c + d*x)] + 560*Sin[6*(c + d*x)]
 - 105*Sin[8*(c + d*x)] - 107520*Tan[c + d*x]))/d
```

fricas [A] time = 0.71, size = 185, normalized size = 0.93

$$-\frac{25725\,a^2dx\cos(dx+c)-13440\,a^2\cos(dx+c)\log(\sin(dx+c)+1)+13440\,a^2\cos(dx+c)\log(-\sin(dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/13440*(25725*a^2*d*x*cos(d*x + c) - 13440*a^2*cos(d*x + c)*log(sin(d*x +
 c) + 1) + 13440*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) - (1680*a^2*cos(d*
x + c)^8 + 3840*a^2*cos(d*x + c)^7 - 4760*a^2*cos(d*x + c)^6 - 16896*a^2*co
s(d*x + c)^5 + 770*a^2*cos(d*x + c)^4 + 31232*a^2*cos(d*x + c)^3 + 14595*a^
2*cos(d*x + c)^2 - 45056*a^2*cos(d*x + c) + 13440*a^2)*sin(d*x + c))/(d*cos
(d*x + c))
```

giac [A] time = 0.79, size = 225, normalized size = 1.13

$$-\frac{25725\,(dx+c)a^2 - 26880\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 26880\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{26880\,a^2\tan\left(\frac{1}{2}\,d\right.}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/13440*(25725*(d*x + c)*a^2 - 26880*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)
) + 26880*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 26880*a^2*tan(1/2*d*x +
1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(39165*a^2*tan(1/2*d*x + 1/2*c)^15
+ 300265*a^2*tan(1/2*d*x + 1/2*c)^13 + 989261*a^2*tan(1/2*d*x + 1/2*c)^11 +
 1791073*a^2*tan(1/2*d*x + 1/2*c)^9 + 1814943*a^2*tan(1/2*d*x + 1/2*c)^7 +
670131*a^2*tan(1/2*d*x + 1/2*c)^5 + 147735*a^2*tan(1/2*d*x + 1/2*c)^3 + 145
95*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^8)/d
```

maple [A] time = 0.70, size = 210, normalized size = 1.06

$$\frac{7a^2\left(\sin^7(dx+c)\right)\cos(dx+c)}{8d} + \frac{49a^2\cos(dx+c)\left(\sin^5(dx+c)\right)}{48d} + \frac{245a^2\cos(dx+c)\left(\sin^3(dx+c)\right)}{192d} + \frac{245a^2\cos}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^8,x)
```

```
[Out] 7/8/d*a^2*sin(d*x+c)^7*cos(d*x+c)+49/48/d*a^2*cos(d*x+c)*sin(d*x+c)^5+245/1
92/d*a^2*cos(d*x+c)*sin(d*x+c)^3+245/128*a^2*cos(d*x+c)*sin(d*x+c)/d-245/12
8*a^2*x-245/128/d*a^2*c-2/7*a^2*sin(d*x+c)^7/d-2/5*a^2*sin(d*x+c)^5/d-2/3*a
^2*sin(d*x+c)^3/d-2*a^2*sin(d*x+c)/d+2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*
a^2*sin(d*x+c)^9/cos(d*x+c)
```

maxima [A] time = 0.44, size = 215, normalized size = 1.08

$$-\frac{1024\left(30\,\sin(dx+c)^7 + 42\,\sin(dx+c)^5 + 70\,\sin(dx+c)^3 - 105\,\log(\sin(dx+c)+1) + 105\,\log(\sin(dx+c)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/107520*(1024*(30*sin(d*x + c)^7 + 42*sin(d*x + c)^5 + 70*sin(d*x + c)^3
- 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1) + 210*sin(d*x + c))
*a^2 - 35*(128*sin(2*d*x + 2*c)^3 + 840*d*x + 840*c + 3*sin(8*d*x + 8*c) +
168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a^2 + 2240*(105*d*x + 105*c -
(87*tan(d*x + c)^5 + 136*tan(d*x + c)^3 + 57*tan(d*x + c))/(tan(d*x + c)^6
+ 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1) - 48*tan(d*x + c))*a^2)/d
```

mupad [B] time = 2.54, size = 293, normalized size = 1.47

$$\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{245\,a^2\,x}{128}+\frac{\frac{501\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{64}+\frac{2633\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{48}+\frac{38047\,a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{240}+\frac{388613\,}{}}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}-7\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}-20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-28\,\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (245*a^2*x)/128 + ((13781*a^2*tan(c/2
 + (d*x)/2)^9)/96 - (1739*a^2*tan(c/2 + (d*x)/2)^5)/80 - (32681*a^2*tan(c/2
 + (d*x)/2)^7)/560 - (61*a^2*tan(c/2 + (d*x)/2)^3)/16 + (388613*a^2*tan(c/2
 + (d*x)/2)^11)/1680 + (38047*a^2*tan(c/2 + (d*x)/2)^13)/240 + (2633*a^2*ta
n(c/2 + (d*x)/2)^15)/48 + (501*a^2*tan(c/2 + (d*x)/2)^17)/64 - (11*a^2*tan(
c/2 + (d*x)/2))/64)/(d*(7*tan(c/2 + (d*x)/2)^2 + 20*tan(c/2 + (d*x)/2)^4 +
28*tan(c/2 + (d*x)/2)^6 + 14*tan(c/2 + (d*x)/2)^8 - 14*tan(c/2 + (d*x)/2)^1
0 - 28*tan(c/2 + (d*x)/2)^12 - 20*tan(c/2 + (d*x)/2)^14 - 7*tan(c/2 + (d*x)
/2)^16 - tan(c/2 + (d*x)/2)^18 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**8,x)
```

```
[Out] Timed out
```

3.30 $\int (a + a\sec(c + dx))^2 \sin^6(c + dx)\, dx$

Optimal. Leaf size=157

$$-\frac{2a^2\sin^5(c+dx)}{5d}-\frac{2a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2\sin(c+dx)\,c}{6d}$$

```
[Out] -25/16*a^2*x+2*a^2*arctanh(sin(d*x+c))/d-2*a^2*sin(d*x+c)/d+7/16*a^2*cos(d*
x+c)*sin(d*x+c)/d+7/24*a^2*cos(d*x+c)^3*sin(d*x+c)/d-1/6*a^2*cos(d*x+c)^5*s
in(d*x+c)/d-2/3*a^2*sin(d*x+c)^3/d-2/5*a^2*sin(d*x+c)^5/d+a^2*tan(d*x+c)/d
```

Rubi [A] time = 0.27, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3872, 2872, 2637, 2633, 2635, 8, 3770, 3767}

$$-\frac{2a^2\sin^5(c+dx)}{5d}-\frac{2a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2\sin(c+dx)\,c}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^6,x]
```

```
[Out] (-25*a^2*x)/16 + (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d +
 (7*a^2*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (7*a^2*Cos[c + d*x]^3*Sin[c + d
*x])/(24*d) - (a^2*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (2*a^2*Sin[c + d*x]
^3)/(3*d) - (2*a^2*Sin[c + d*x]^5)/(5*d) + (a^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sin^6(c+dx)\,dx = \int (-a-a\cos(c+dx))^2\sin^4(c+dx)\tan^2(c+dx)\,dx$$

$$= \frac{\int \left(-2a^8-6a^8\cos(c+dx)+6a^8\cos^3(c+dx)+2a^8\cos^4(c+dx)-2a$$

$$= -2a^2x-a^2\int \cos^6(c+dx)\,dx+a^2\int \sec^2(c+dx)\,dx+\left(2a^2\right)\int \cos^4$$

$$= -2a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{6a^2\sin(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{2d}$$

$$= -2a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{3a^2\cos(c+dx)}{4d}$$

$$= -\frac{5a^2x}{4}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{7a^2\cos(c+dx)}{16}$$

$$= -\frac{25a^2x}{16}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{7a^2\cos(c+d}{1}$$

Mathematica [A] time = 0.56, size = 124, normalized size = 0.79

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(384\sin^5(c+dx)+640\sin^3(c+dx)+1920\sin(c+dx)-255\sin(2(c$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^6,x]
```

```
[Out] -1/3840*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(1080*c + 1080*d*x + 4
20*ArcTan[Tan[c + d*x]] - 1920*ArcTanh[Sin[c + d*x]] + 1920*Sin[c + d*x] +
640*Sin[c + d*x]^3 + 384*Sin[c + d*x]^5 - 255*Sin[2*(c + d*x)] - 15*Sin[4*(
c + d*x)] + 5*Sin[6*(c + d*x)] - 960*Tan[c + d*x]))/d
```

fricas [A] time = 0.68, size = 158, normalized size = 1.01

$$-\frac{375\,a^2dx\cos(dx+c)-240\,a^2\cos(dx+c)\log(\sin(dx+c)+1)+240\,a^2\cos(dx+c)\log(-\sin(dx+c)+1$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/240*(375*a^2*d*x*cos(d*x + c) - 240*a^2*cos(d*x + c)*log(sin(d*x + c) +
1) + 240*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + (40*a^2*cos(d*x + c)^6 +
 96*a^2*cos(d*x + c)^5 - 70*a^2*cos(d*x + c)^4 - 352*a^2*cos(d*x + c)^3 - 1
05*a^2*cos(d*x + c)^2 + 736*a^2*cos(d*x + c) - 240*a^2)*sin(d*x + c))/(d*co
s(d*x + c))
```

giac [A] time = 0.51, size = 193, normalized size = 1.23

$$-\frac{375\,(dx+c)a^2 - 480\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 480\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{480\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} + \cdots}{24\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/240*(375*(d*x + c)*a^2 - 480*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 48
0*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 480*a^2*tan(1/2*d*x + 1/2*c)/(ta
n(1/2*d*x + 1/2*c)^2 - 1) + 2*(615*a^2*tan(1/2*d*x + 1/2*c)^11 + 3485*a^2*t
an(1/2*d*x + 1/2*c)^9 + 7926*a^2*tan(1/2*d*x + 1/2*c)^7 + 8586*a^2*tan(1/2*
d*x + 1/2*c)^5 + 2595*a^2*tan(1/2*d*x + 1/2*c)^3 + 345*a^2*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

maple [A] time = 0.69, size = 172, normalized size = 1.10

$$\frac{5a^2\cos(dx+c)\left(\sin^5(dx+c)\right)}{6d}+\frac{25a^2\cos(dx+c)\left(\sin^3(dx+c)\right)}{24d}+\frac{25a^2\cos(dx+c)\sin(dx+c)}{16d}-\frac{25a^2x}{16}-\frac{25a^2c}{16d}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^6,x)
```

```
[Out] 5/6/d*a^2*cos(d*x+c)*sin(d*x+c)^5+25/24/d*a^2*cos(d*x+c)*sin(d*x+c)^3+25/16
*a^2*cos(d*x+c)*sin(d*x+c)/d-25/16*a^2*x-25/16/d*a^2*c-2/5*a^2*sin(d*x+c)^5
/d-2/3*a^2*sin(d*x+c)^3/d-2*a^2*sin(d*x+c)/d+2/d*a^2*ln(sec(d*x+c)+tan(d*x+
c))+1/d*a^2*sin(d*x+c)^7/cos(d*x+c)
```

maxima [A] time = 0.54, size = 174, normalized size = 1.11

$$-\frac{64\left(6\sin(dx+c)^5+10\sin(dx+c)^3-15\log(\sin(dx+c)+1)+15\log(\sin(dx+c)-1)+30\sin(dx+c)\right)a^2\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] -1/960*(64*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d*x + c) + 1)
 + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a^2 - 5*(4*sin(2*d*x + 2*c)^
3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^2 + 120*(15
*d*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d
*x + c)^2 + 1) - 8*tan(d*x + c))*a^2)/d
```

mupad [B] time = 2.16, size = 235, normalized size = 1.50

$$\frac{\frac{57\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{8}+\frac{431\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{12}+\frac{8041\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{120}+\frac{91\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{2}-\frac{797\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{40}-\frac{27\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{4}\cdots}{d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}+5\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}+9\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6*(a + a/cos(c + d*x))^2,x)
```

```
[Out] ((91*a^2*tan(c/2 + (d*x)/2)^7)/2 - (797*a^2*tan(c/2 + (d*x)/2)^5)/40 - (27*
a^2*tan(c/2 + (d*x)/2)^3)/4 + (8041*a^2*tan(c/2 + (d*x)/2)^9)/120 + (431*a^
2*tan(c/2 + (d*x)/2)^11)/12 + (57*a^2*tan(c/2 + (d*x)/2)^13)/8 - (7*a^2*tan
(c/2 + (d*x)/2))/8)/(d*(5*tan(c/2 + (d*x)/2)^2 + 9*tan(c/2 + (d*x)/2)^4 + 5
*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 - 9*tan(c/2 + (d*x)/2)^10 -
5*tan(c/2 + (d*x)/2)^12 - tan(c/2 + (d*x)/2)^14 + 1)) - (25*a^2*x)/16 + (4*
a^2*atanh(tan(c/2 + (d*x)/2)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**6,x)
```

```
[Out] Timed out
```

3.31 $\int (a + a\sec(c + dx))^2 \sin^4(c + dx)\, dx$

Optimal. Leaf size=115

$$-\frac{2a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{a^2\sin}{}$$

```
[Out] -9/8*a^2*x+2*a^2*arctanh(sin(d*x+c))/d-2*a^2*sin(d*x+c)/d-1/8*a^2*cos(d*x+c
)*sin(d*x+c)/d+1/4*a^2*cos(d*x+c)^3*sin(d*x+c)/d-2/3*a^2*sin(d*x+c)^3/d+a^2
*tan(d*x+c)/d
```

Rubi [A] time = 0.27, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3872, 2872, 2637, 2635, 8, 2633, 3770, 3767}

$$-\frac{2a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{a^2\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^4,x]
```

```
[Out] (-9*a^2*x)/8 + (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d - (
a^2*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(4
*d) - (2*a^2*Sin[c + d*x]^3)/(3*d) + (a^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a + a\sec(c+dx))^2 \sin^4(c+dx)\,dx = \int (-a - a\cos(c+dx))^2 \sin^2(c+dx)\tan^2(c+dx)\,dx$$
$$= \frac{\int \left(-a^6 - 4a^6\cos(c+dx) - a^6\cos^2(c+dx) + 2a^6\cos^3(c+dx) + a^6 \mathrm{cc}\right.}{a^4}$$
$$= -a^2x - a^2\int \cos^2(c+dx)\,dx + a^2\int \cos^4(c+dx)\,dx + a^2\int \sec^2(c+$$
$$= -a^2x + \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d} - \frac{4a^2\sin(c+dx)}{d} - \frac{a^2\cos(c+dx)\,\mathrm{si}}{2d}$$
$$= -\frac{3a^2x}{2} + \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d} - \frac{2a^2\sin(c+dx)}{d} - \frac{a^2\cos(c+dx)}{8d}$$
$$= -\frac{9a^2x}{8} + \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d} - \frac{2a^2\sin(c+dx)}{d} - \frac{a^2\cos(c+dx)}{8d}$$

Mathematica [A] time = 0.24, size = 94, normalized size = 0.82

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(64\sin^3(c+dx)+192\sin(c+dx)-3\sin(4(c+dx))+60\tan^{-1}(\tan(c\right.}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^4,x]
```

```
[Out] -1/384*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(48*c + 48*d*x + 60*Arc
Tan[Tan[c + d*x]] - 192*ArcTanh[Sin[c + d*x]] + 192*Sin[c + d*x] + 64*Sin[c
 + d*x]^3 - 3*Sin[4*(c + d*x)] - 96*Tan[c + d*x]))/d
```

fricas [A] time = 0.95, size = 133, normalized size = 1.16

$$-\frac{27\,a^2dx\cos(dx+c) - 24\,a^2\cos(dx+c)\log(\sin(dx+c)+1) + 24\,a^2\cos(dx+c)\log(-\sin(dx+c)+1) - (}{24\,d\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/24*(27*a^2*d*x*cos(d*x + c) - 24*a^2*cos(d*x + c)*log(sin(d*x + c) + 1)
+ 24*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) - (6*a^2*cos(d*x + c)^4 + 16*a
^2*cos(d*x + c)^3 - 3*a^2*cos(d*x + c)^2 - 64*a^2*cos(d*x + c) + 24*a^2)*si
n(d*x + c))/(d*cos(d*x + c))
```

giac [A] time = 1.21, size = 161, normalized size = 1.40

$$-\frac{27\,(dx+c)a^2 - 48\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 48\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{48\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} + \frac{2\,(51\,a^2}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/24*(27*(d*x + c)*a^2 - 48*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 48*a^
2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 48*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2
*d*x + 1/2*c)^2 - 1) + 2*(51*a^2*tan(1/2*d*x + 1/2*c)^7 + 187*a^2*tan(1/2*d
*x + 1/2*c)^5 + 229*a^2*tan(1/2*d*x + 1/2*c)^3 + 45*a^2*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.70, size = 134, normalized size = 1.17

$$\frac{3a^2\cos(dx+c)\left(\sin^3(dx+c)\right)}{4d}+\frac{9a^2\cos(dx+c)\sin(dx+c)}{8d}-\frac{9a^2x}{8}-\frac{9a^2c}{8d}-\frac{2a^2\left(\sin^3(dx+c)\right)}{3d}-\frac{2a^2\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^4,x)
```

```
[Out] 3/4/d*a^2*cos(d*x+c)*sin(d*x+c)^3+9/8*a^2*cos(d*x+c)*sin(d*x+c)/d-9/8*a^2*x
-9/8/d*a^2*c-2/3*a^2*sin(d*x+c)^3/d-2*a^2*sin(d*x+c)/d+2/d*a^2*ln(sec(d*x+c
)+tan(d*x+c))+1/d*a^2*sin(d*x+c)^5/cos(d*x+c)
```

maxima [A] time = 0.47, size = 126, normalized size = 1.10

$$-\frac{32\left(2\,\sin(dx+c)^3-3\,\log(\sin(dx+c)+1)+3\,\log(\sin(dx+c)-1)+6\,\sin(dx+c)\right)a^2-3\,(12\,dx+12\,c+\mathrm{si}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/96*(32*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
- 1) + 6*sin(d*x + c))*a^2 - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*
d*x + 2*c))*a^2 + 48*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*t
an(d*x + c))*a^2)/d
```

mupad [B] time = 1.83, size = 177, normalized size = 1.54

$$\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{9\,a^2\,x}{8}+\frac{\frac{25\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+\frac{58\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{3}+\frac{31\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2}-\frac{22\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3}-}{d\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (9*a^2*x)/8 + ((31*a^2*tan(c/2 + (d*x
)/2)^5)/2 - (22*a^2*tan(c/2 + (d*x)/2)^3)/3 + (58*a^2*tan(c/2 + (d*x)/2)^7)
/3 + (25*a^2*tan(c/2 + (d*x)/2)^9)/4 - (7*a^2*tan(c/2 + (d*x)/2))/4)/(d*(3*
tan(c/2 + (d*x)/2)^2 + 2*tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^6 - 3*
tan(c/2 + (d*x)/2)^8 - tan(c/2 + (d*x)/2)^10 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int 2\sin^{4}\left(c+dx\right)\sec\left(c+dx\right)dx+\int\sin^{4}\left(c+dx\right)\sec^{2}\left(c+dx\right)dx+\int\sin^{4}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**4,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)**4*sec(c + d*x), x) + Integral(sin(c + d*x)**
4*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**4, x))
```

3.32 $\int (a+a\sec(c+dx))^2\sin^2(c+dx)\,dx$

Optimal. Leaf size=73

$$-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{a^2x}{2}$$

```
[Out] -1/2*a^2*x+2*a^2*arctanh(sin(d*x+c))/d-2*a^2*sin(d*x+c)/d-1/2*a^2*cos(d*x+c
)*sin(d*x+c)/d+a^2*tan(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2709, 2637, 2635, 8, 3770, 3767}

$$-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^2,x]
```

```
[Out] -(a^2*x)/2 + (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d - (a^
2*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a + a\sec(c + dx))^2 \sin^2(c + dx)\,dx = \int (-a - a\cos(c + dx))^2 \tan^2(c + dx)\,dx$$
$$= \frac{\int \left(-2a^4\cos(c + dx) - a^4\cos^2(c + dx) + 2a^4\sec(c + dx) + a^4\sec^2(c +}{a^2}$$
$$= -\left(a^2 \int \cos^2(c + dx)\,dx\right) + a^2 \int \sec^2(c + dx)\,dx - \left(2a^2\right) \int \cos(c + d$$
$$= \frac{2a^2\tanh^{-1}(\sin(c + dx))}{d} - \frac{2a^2\sin(c + dx)}{d} - \frac{a^2\cos(c + dx)\sin(c + d}{2d}$$
$$= -\frac{a^2x}{2} + \frac{2a^2\tanh^{-1}(\sin(c + dx))}{d} - \frac{2a^2\sin(c + dx)}{d} - \frac{a^2\cos(c + dx)\,\mathrm{s}}{2d}$$

Mathematica [B] time = 1.16, size = 243, normalized size = 3.33

$$\frac{1}{16}a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c + dx)\right)\left(-\frac{8\sin(c)\cos(dx)}{d} - \frac{\sin(2c)\cos(2dx)}{d} - \frac{8\cos(c)\sin(dx)}{d} - \frac{\cos(2c)\sin(}{d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^2,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-2*x - (8*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]])/d + (8*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d -
 (8*Cos[d*x]*Sin[c])/d - (Cos[2*d*x]*Sin[2*c])/d - (8*Cos[c]*Sin[d*x])/d -
(Cos[2*c]*Sin[2*d*x])/d + (4*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2])) + (4*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/16
```

fricas [A] time = 1.61, size = 104, normalized size = 1.42

$$-\frac{a^2dx\cos(dx + c) - 2\,a^2\cos(dx + c)\log(\sin(dx + c) + 1) + 2\,a^2\cos(dx + c)\log(-\sin(dx + c) + 1) + \left(a^2\,\mathrm{co}\right.}{2\,d\cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*d*x*cos(d*x + c) - 2*a^2*cos(d*x + c)*log(sin(d*x + c) + 1) + 2*a
^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + (a^2*cos(d*x + c)^2 + 4*a^2*cos(d*
x + c) - 2*a^2)*sin(d*x + c))/(d*cos(d*x + c))
```

giac [A] time = 0.29, size = 128, normalized size = 1.75

$$-\frac{(dx + c)a^2 - 4\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) + 4\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} + \frac{2\left(3\,a^2\tan\left(\right.\right.}{\left(\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)*a^2 - 4*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 4*a^2*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x +
1/2*c)^2 - 1) + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^3 + 5*a^2*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.40, size = 86, normalized size = 1.18

$$-\frac{a^2\cos(dx+c)\sin(dx+c)}{2d}-\frac{a^2x}{2}-\frac{a^2c}{2d}-\frac{2a^2\sin(dx+c)}{d}+\frac{2a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^2\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^2,x)
```

```
[Out] -1/2*a^2*cos(d*x+c)*sin(d*x+c)/d-1/2*a^2*x-1/2/d*a^2*c-2*a^2*sin(d*x+c)/d+2
/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+a^2*tan(d*x+c)/d
```

maxima [A] time = 0.43, size = 81, normalized size = 1.11

$$\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))a^2-4\,(dx+c-\tan(dx+c))a^2+4\,a^2\bigl(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a^2 - 4*(d*x + c - tan(d*x + c))*a^2
+ 4*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

mupad [B] time = 1.16, size = 117, normalized size = 1.60

$$\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^2\,x}{2}+\frac{5\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+6\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - (a^2*x)/2 + (6*a^2*tan(c/2 + (d*x)/2)
^3 + 5*a^2*tan(c/2 + (d*x)/2)^5 - 3*a^2*tan(c/2 + (d*x)/2))/(d*(tan(c/2 + (
d*x)/2)^2 - tan(c/2 + (d*x)/2)^4 - tan(c/2 + (d*x)/2)^6 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\sin^2(c+dx)\sec(c+dx)\,dx+\int\sin^2(c+dx)\sec^2(c+dx)\,dx+\int\sin^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)**2*sec(c + d*x), x) + Integral(sin(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**2, x))
```

3.33 $\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=57

$$\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}-\frac{2a^2\csc(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] 2*a^2*arctanh(sin(d*x+c))/d-2*a^2*cot(d*x+c)/d-2*a^2*csc(d*x+c)/d+a^2*tan(d
*x+c)/d
```

Rubi [A] time = 0.25, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3872, 2873, 3767, 8, 2621, 321, 207, 2620, 14}

$$\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}-\frac{2a^2\csc(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Cot[c + d*x])/d - (2*a^2*Csc[c + d
*x])/d + (a^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx &= \int (-a-a\cos(c+dx))^2\csc^2(c+dx)\sec^2(c+dx)\,dx \\
&= \int \left(a^2\csc^2(c+dx)+2a^2\csc^2(c+dx)\sec(c+dx)+a^2\csc^2(c+dx)\sec^2(\right. \\
&= a^2\int \csc^2(c+dx)\,dx + a^2\int \csc^2(c+dx)\sec^2(c+dx)\,dx + \left(2a^2\right)\int \csc \\
&= -\frac{a^2\,\mathrm{Subst}(\int 1\,dx, x, \cot(c+dx))}{d} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{1+x^2}{x^2}\,dx, x, \tan(c+dx)\right)}{d} \\
&= -\frac{a^2\cot(c+dx)}{d} - \frac{2a^2\csc(c+dx)}{d} + \frac{a^2\,\mathrm{Subst}\left(\int \left(1+\frac{1}{x^2}\right)dx, x, \tan(c+\right.}{d} \\
&= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d} - \frac{2a^2\cot(c+dx)}{d} - \frac{2a^2\csc(c+dx)}{d} + \frac{a^2\tan(c}{d}
\end{aligned}$$

Mathematica [B] time = 6.16, size = 401, normalized size = 7.04

$$\frac{\sin\left(\frac{dx}{2}\right)\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^2}{4d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{\sin\left(\frac{dx}{2}\right)\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^2}{4d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/2*(Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 +
 (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/d + (Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)
/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(2*d
) + (Cos[c + d*x]^2*Csc[c/2]*Csc[c/2 + (d*x)/2]*Sec[c/2 + (d*x)/2]^4*(a + a
*Sec[c + d*x])^2*Sin[(d*x)/2])/(2*d) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4
*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(4*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 +
 (d*x)/2] - Sin[c/2 + (d*x)/2])) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a
+ a*Sec[c + d*x])^2*Sin[(d*x)/2])/(4*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*
x)/2] + Sin[c/2 + (d*x)/2]))
```

fricas [A] time = 0.64, size = 101, normalized size = 1.77

$$\frac{a^2\cos(dx+c)\log(\sin(dx+c)+1)\sin(dx+c) - a^2\cos(dx+c)\log(-\sin(dx+c)+1)\sin(dx+c) - 3\,a^2\cos(}{d\cos(dx+c)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a^2*cos(d*x + c)*log(sin(d*x + c) + 1)*sin(d*x + c) - a^2*cos(d*x + c)*log
(-sin(d*x + c) + 1)*sin(d*x + c) - 3*a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c
) + a^2)/(d*cos(d*x + c)*sin(d*x + c))
```

giac [A] time = 0.54, size = 90, normalized size = 1.58

$$\frac{2\left(a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a^2*log(abs(tan(1/2*d*x + 1/2*c
) - 1)) - (2*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2)/(tan(1/2*d*x + 1/2*c)^3 - ta
n(1/2*d*x + 1/2*c)))/d
```

maple [A] time = 0.69, size = 77, normalized size = 1.35

$$-\frac{3a^2\cot(dx+c)}{d}-\frac{2a^2}{d\sin(dx+c)}+\frac{2a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{a^2}{d\sin(dx+c)\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -3*a^2*cot(d*x+c)/d-2/d*a^2/sin(d*x+c)+2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/
d*a^2/sin(d*x+c)/cos(d*x+c)
```

maxima [A] time = 0.33, size = 74, normalized size = 1.30

$$-\frac{a^2\left(\frac{2}{\sin(dx+c)}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+a^2\left(\frac{1}{\tan(dx+c)}-\tan(dx+c)\right)+\frac{a^2}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(a^2*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + a^
2*(1/tan(d*x + c) - tan(d*x + c)) + a^2/tan(d*x + c))/d
```

mupad [B] time = 1.13, size = 70, normalized size = 1.23

$$\frac{4\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-2\,a^2}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}+\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^2,x)
```

```
[Out] (4*a^2*tan(c/2 + (d*x)/2)^2 - 2*a^2)/(d*(tan(c/2 + (d*x)/2) - tan(c/2 + (d*
x)/2)^3)) + (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\csc^2(c+dx)\sec(c+dx)\,dx+\int\csc^2(c+dx)\sec^2(c+dx)\,dx+\int\csc^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*csc(c + d*x)**2*sec(c + d*x), x) + Integral(csc(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**2, x))
```

3.34 $\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=87

$$-\frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2}+\frac{10a^2\tan(c+dx)}{3d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))}$$

```
[Out] 2*a^2*arctanh(sin(d*x+c))/d+10/3*a^2*tan(d*x+c)/d-2*a^2*tan(d*x+c)/d/(1-cos
(d*x+c))-1/3*a^4*tan(d*x+c)/d/(a-a*cos(d*x+c))^2
```

Rubi [A] time = 0.30, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2869, 2766, 2978, 2748, 3767, 8, 3770}

$$\frac{10a^2\tan(c+dx)}{3d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2}-\frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d + (10*a^2*Tan[c + d*x])/(3*d) - (2*a^2*Tan[
c + d*x])/(d*(1 - Cos[c + d*x])) - (a^4*Tan[c + d*x])/(3*d*(a - a*Cos[c + d
*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx = \int (-a-a\cos(c+dx))^2\csc^4(c+dx)\sec^2(c+dx)\,dx$$
$$= a^4\int \frac{\sec^2(c+dx)}{(-a+a\cos(c+dx))^2}\,dx$$
$$= -\frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2} + \frac{1}{3}a^2\int \frac{(-4a-2a\cos(c+dx))\sec^2(c+dx)}{-a+a\cos(c+dx)}\,dx$$
$$= -\frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))} - \frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2} + \frac{1}{3}\int \left(10a^2+6a^2\cos(c+\right.$$
$$= -\frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))} - \frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2} + \left(2a^2\right)\int \sec(c+dx)\,dx$$
$$= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d} - \frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))} - \frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2}$$
$$= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d} + \frac{10a^2\tan(c+dx)}{3d} - \frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))} - \frac{}{3d(a}$$

Mathematica [B] time = 1.85, size = 228, normalized size = 2.62

$$a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-\cot\left(\frac{c}{2}\right)\csc^2\left(\frac{1}{2}(c+dx)\right)+6\left(\frac{\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}\right.\right.}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-(Cot[c/2]*Csc[(c + d*x)/2]^2
) - (-8 + 7*Cos[c + d*x])*Csc[c/2]*Csc[(c + d*x)/2]^3*Sin[(d*x)/2] + 6*(-2*
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]] + Sin[d*x]/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))))/(24
*d)
```

fricas [A] time = 0.69, size = 159, normalized size = 1.83

$$-\frac{10\,a^2\cos\left(dx+c\right)^3-4\,a^2\cos\left(dx+c\right)^2-11\,a^2\cos\left(dx+c\right)-3\left(a^2\cos\left(dx+c\right)^2-a^2\cos\left(dx+c\right)\right)\log\left(\sin\left(dx\right.\right.}{3\left(d\cos\left(dx+c\right)^2-d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(10*a^2*cos(d*x + c)^3 - 4*a^2*cos(d*x + c)^2 - 11*a^2*cos(d*x + c) -
3*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c
) + 3*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*
x + c) + 3*a^2)/((d*cos(d*x + c)^2 - d*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 0.30, size = 104, normalized size = 1.20

$$\frac{12\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-12\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-\frac{15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(12*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 12*a^2*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) - 12*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1)
- (15*a^2*tan(1/2*d*x + 1/2*c)^2 + a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.88, size = 140, normalized size = 1.61

$$-\frac{10a^2\cot\left(dx+c\right)}{3d}-\frac{a^2\cot\left(dx+c\right)\left(\csc^2\left(dx+c\right)\right)}{3d}-\frac{2a^2}{3d\sin\left(dx+c\right)^3}-\frac{2a^2}{d\sin\left(dx+c\right)}+\frac{2a^2\ln\left(\sec\left(dx+c\right)+\tan\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -10/3*a^2*cot(d*x+c)/d-1/3/d*a^2*cot(d*x+c)*csc(d*x+c)^2-2/3/d*a^2/sin(d*x+
c)^3-2/d*a^2/sin(d*x+c)+2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))-1/3/d*a^2/sin(d*x
+c)^3/cos(d*x+c)+4/3/d*a^2/sin(d*x+c)/cos(d*x+c)
```

maxima [A] time = 0.34, size = 113, normalized size = 1.30

$$-\frac{a^2\left(\frac{2\left(3\sin\left(dx+c\right)^2+1\right)}{\sin\left(dx+c\right)^3}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)+a^2\left(\frac{6\tan\left(dx+c\right)^2+1}{\tan\left(dx+c\right)^3}-3\tan\left(dx+c\right)\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(a^2*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)) + a^2*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 -
 3*tan(d*x + c)) + (3*tan(d*x + c)^2 + 1)*a^2/tan(d*x + c)^3)/d
```

mupad [B] time = 2.48, size = 91, normalized size = 1.05

$$\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{-9\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\frac{14\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{a^2}{3}}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^4,x)
```

```
[Out] (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - ((14*a^2*tan(c/2 + (d*x)/2)^2)/3 - 9*
a^2*tan(c/2 + (d*x)/2)^4 + a^2/3)/(d*(2*tan(c/2 + (d*x)/2)^3 - 2*tan(c/2 +
(d*x)/2)^5))
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int 2\csc^4(c+dx)\sec(c+dx)\,dx + \int \csc^4(c+dx)\sec^2(c+dx)\,dx + \int \csc^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*csc(c + d*x)**4*sec(c + d*x), x) + Integral(csc(c + d*x)**
4*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**4, x))
```

3.35 $\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=129

$$\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{5a^2\cot^3(c+dx)}{3d}-\frac{4a^2\cot(c+dx)}{d}-\frac{2a^2\csc^5(c+dx)}{5d}-\frac{2a^2\csc^3(c+dx)}{3d}-2a^2$$

```
[Out] 2*a^2*arctanh(sin(d*x+c))/d-4*a^2*cot(d*x+c)/d-5/3*a^2*cot(d*x+c)^3/d-2/5*a
^2*cot(d*x+c)^5/d-2*a^2*csc(d*x+c)/d-2/3*a^2*csc(d*x+c)^3/d-2/5*a^2*csc(d*x
+c)^5/d+a^2*tan(d*x+c)/d
```

Rubi [A] time = 0.23, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2873, 3767, 2621, 302, 207, 2620, 270}

$$\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{5a^2\cot^3(c+dx)}{3d}-\frac{4a^2\cot(c+dx)}{d}-\frac{2a^2\csc^5(c+dx)}{5d}-\frac{2a^2\csc^3(c+dx)}{3d}-2a^2$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d - (4*a^2*Cot[c + d*x])/d - (5*a^2*Cot[c + d
*x]^3)/(3*d) - (2*a^2*Cot[c + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x])/d - (2*a
^2*Csc[c + d*x]^3)/(3*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) + (a^2*Tan[c + d*x]
)/d
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx = \int (-a-a\cos(c+dx))^2\csc^6(c+dx)\sec^2(c+dx)\,dx$$
$$= \int \left(a^2\csc^6(c+dx)+2a^2\csc^6(c+dx)\sec(c+dx)+a^2\csc^6(c+dx)\sec^2(\ldots\right.$$
$$= a^2\int \csc^6(c+dx)\,dx + a^2\int \csc^6(c+dx)\sec^2(c+dx)\,dx + \left(2a^2\right)\int \csc^6\ldots$$
$$= \frac{a^2\,\mathrm{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^6}\,dx, x, \tan(c+dx)\right)}{d} - \frac{a^2\,\mathrm{Subst}\left(\int \left(1+2x^2+x^4\right)dx, \ldots\right.}{d}$$
$$= -\frac{a^2\cot(c+dx)}{d} - \frac{2a^2\cot^3(c+dx)}{3d} - \frac{a^2\cot^5(c+dx)}{5d} + \frac{a^2\,\mathrm{Subst}\left(\int \left(1+\ldots\right.\right.}{}$$
$$= -\frac{4a^2\cot(c+dx)}{d} - \frac{5a^2\cot^3(c+dx)}{3d} - \frac{2a^2\cot^5(c+dx)}{5d} - \frac{2a^2\csc(c+d\ldots}{d}$$
$$= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d} - \frac{4a^2\cot(c+dx)}{d} - \frac{5a^2\cot^3(c+dx)}{3d} - \frac{2a^2\cot^5\ldots}{5\ldots}$$

Mathematica [B] time = 1.00, size = 317, normalized size = 2.46

$$\frac{a^2\cos(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2\left(\csc(2c)(216\sin(c-dx)-416\sin(c+dx)+624\sin(2(c+dx))\ldots\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Cos[c + d*x]*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(-3840*Cos[c + d*
x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3840*Cos[c + d*x]*Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]] + Csc[2*c]*Csc[(c + d*x)/2]^4*Csc[c + d*x]*(3
20*Sin[2*c] - 596*Sin[d*x] + 864*Sin[2*d*x] + 216*Sin[c - d*x] - 416*Sin[c
+ d*x] + 624*Sin[2*(c + d*x)] - 416*Sin[3*(c + d*x)] + 104*Sin[4*(c + d*x)]
 - 596*Sin[2*c + d*x] - 680*Sin[3*c + d*x] + 894*Sin[c + 2*d*x] + 224*Sin[2
*(c + 2*d*x)] + 894*Sin[3*c + 2*d*x] + 480*Sin[4*c + 2*d*x] - 776*Sin[c + 3
*d*x] - 596*Sin[2*c + 3*d*x] - 596*Sin[4*c + 3*d*x] - 120*Sin[5*c + 3*d*x]
+ 149*Sin[3*c + 4*d*x] + 149*Sin[5*c + 4*d*x])))/(7680*d)
```

fricas [A] time = 0.57, size = 206, normalized size = 1.60

$$-\frac{56\,a^2\cos\left(dx+c\right)^4-82\,a^2\cos\left(dx+c\right)^3-32\,a^2\cos\left(dx+c\right)^2+76\,a^2\cos\left(dx+c\right)-15\left(a^2\cos\left(dx+c\right)^3-2\,a\right.}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(56*a^2*cos(d*x + c)^4 - 82*a^2*cos(d*x + c)^3 - 32*a^2*cos(d*x + c)^
2 + 76*a^2*cos(d*x + c) - 15*(a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a
^2*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + 15*(a^2*cos(d*x + c)^
3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*x
 + c) - 15*a^2)/((d*cos(d*x + c)^3 - 2*d*cos(d*x + c)^2 + d*cos(d*x + c))*s
in(d*x + c))
```

giac [A] time = 0.70, size = 136, normalized size = 1.05

$$\frac{240\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-240\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{240\,a^2\tan\left(\frac{1}{2}\,dx\right.}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/120*(240*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 240*a^2*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) + 15*a^2*tan(1/2*d*x + 1/2*c) - 240*a^2*tan(1/2*d*x + 1
/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (345*a^2*tan(1/2*d*x + 1/2*c)^4 + 35*a
^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2)/tan(1/2*d*x + 1/2*c)^5)/d
```

maple [A] time = 0.90, size = 202, normalized size = 1.57

$$-\frac{56a^2\cot\left(dx+c\right)}{15d}-\frac{a^2\cot\left(dx+c\right)\left(\csc^4\left(dx+c\right)\right)}{5d}-\frac{4a^2\cot\left(dx+c\right)\left(\csc^2\left(dx+c\right)\right)}{15d}-\frac{2a^2}{5d\sin\left(dx+c\right)^5}-\frac{2a}{3d\sin\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -56/15*a^2*cot(d*x+c)/d-1/5/d*a^2*cot(d*x+c)*csc(d*x+c)^4-4/15/d*a^2*cot(d*
x+c)*csc(d*x+c)^2-2/5/d*a^2/sin(d*x+c)^5-2/3/d*a^2/sin(d*x+c)^3-2/d*a^2/sin
(d*x+c)+2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))-1/5/d*a^2/sin(d*x+c)^5/cos(d*x+c)
-2/5/d*a^2/sin(d*x+c)^3/cos(d*x+c)+8/5/d*a^2/sin(d*x+c)/cos(d*x+c)
```

maxima [A] time = 0.35, size = 144, normalized size = 1.12

$$-\frac{a^2\left(\frac{2\left(15\sin\left(dx+c\right)^4+5\sin\left(dx+c\right)^2+3\right)}{\sin\left(dx+c\right)^5}-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)\right)+3\,a^2\left(\frac{15\tan\left(dx+c\right)^4+5\tan\right.}{\tan\left(dx+c\right)}\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/15*(a^2*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 1
5*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 3*a^2*((15*tan(d*x +
c)^4 + 5*tan(d*x + c)^2 + 1)/tan(d*x + c)^5 - 5*tan(d*x + c)) + (15*tan(d*x
 + c)^4 + 10*tan(d*x + c)^2 + 3)*a^2/tan(d*x + c)^5)/d
```

mupad [B] time = 1.28, size = 124, normalized size = 0.96

$$\frac{4a^2\operatorname{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{-39a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+\frac{62a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4}{3}+\frac{32a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2}{15}+\frac{a^2}{5}}{d\left(8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5-8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7\right)}+\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^6,x)
```

```
[Out] (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - ((32*a^2*tan(c/2 + (d*x)/2)^2)/15 + (
62*a^2*tan(c/2 + (d*x)/2)^4)/3 - 39*a^2*tan(c/2 + (d*x)/2)^6 + a^2/5)/(d*(8
*tan(c/2 + (d*x)/2)^5 - 8*tan(c/2 + (d*x)/2)^7)) + (a^2*tan(c/2 + (d*x)/2))
/(8*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.36 $\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=163

$$\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{7a^2\cot^5(c+dx)}{5d}-\frac{3a^2\cot^3(c+dx)}{d}-\frac{5a^2\cot(c+dx)}{d}-\frac{2a^2\csc^7(c+dx)}{7d}-\frac{2a^2}{}$$

```
[Out] 2*a^2*arctanh(sin(d*x+c))/d-5*a^2*cot(d*x+c)/d-3*a^2*cot(d*x+c)^3/d-7/5*a^2
*cot(d*x+c)^5/d-2/7*a^2*cot(d*x+c)^7/d-2*a^2*csc(d*x+c)/d-2/3*a^2*csc(d*x+c
)^3/d-2/5*a^2*csc(d*x+c)^5/d-2/7*a^2*csc(d*x+c)^7/d+a^2*tan(d*x+c)/d
```

Rubi [A] time = 0.24, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2873, 3767, 2621, 302, 207, 2620, 270}

$$\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{7a^2\cot^5(c+dx)}{5d}-\frac{3a^2\cot^3(c+dx)}{d}-\frac{5a^2\cot(c+dx)}{d}-\frac{2a^2\csc^7(c+dx)}{7d}-\frac{2a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d - (5*a^2*Cot[c + d*x])/d - (3*a^2*Cot[c + d
*x]^3)/d - (7*a^2*Cot[c + d*x]^5)/(5*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (2
*a^2*Csc[c + d*x])/d - (2*a^2*Csc[c + d*x]^3)/(3*d) - (2*a^2*Csc[c + d*x]^5
)/(5*d) - (2*a^2*Csc[c + d*x]^7)/(7*d) + (a^2*Tan[c + d*x])/d
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx = \int (-a-a\cos(c+dx))^2\csc^8(c+dx)\sec^2(c+dx)\,dx$$

$$= \int \left(a^2\csc^8(c+dx) + 2a^2\csc^8(c+dx)\sec(c+dx) + a^2\csc^8(c+dx)\sec^2($$

$$= a^2\int \csc^8(c+dx)\,dx + a^2\int \csc^8(c+dx)\sec^2(c+dx)\,dx + \left(2a^2\right)\int \csc^8$$

$$= \frac{a^2\,\text{Subst}\left(\int \frac{\left(1+x^2\right)^4}{x^8}\,dx, x, \tan(c+dx)\right)}{d} - \frac{a^2\,\text{Subst}\left(\int \left(1+3x^2+3x^4+x^{\cdot}\right.\right.}{d}$$

$$= -\frac{a^2\cot(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{d} - \frac{3a^2\cot^5(c+dx)}{5d} - \frac{a^2\cot^7(c+dx)}{7d} -$$

$$= -\frac{5a^2\cot(c+dx)}{d} - \frac{3a^2\cot^3(c+dx)}{d} - \frac{7a^2\cot^5(c+dx)}{5d} - \frac{2a^2\cot^7(c+d}{7d}$$

$$= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d} - \frac{5a^2\cot(c+dx)}{d} - \frac{3a^2\cot^3(c+dx)}{d} - \frac{7a^2\cot^5}{5}$$

Mathematica [B] time = 1.30, size = 428, normalized size = 2.63

$$\frac{a^2\cos(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2\left(-32\csc(2c)(-7264\sin(c-dx)+14208\sin(c+dx)-19536\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Cos[c + d*x]*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(-6881280*Cos[c +
 d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6881280*Cos[c + d*x]*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] - 32*Csc[2*c]*Csc[(c + d*x)/2]^4*Csc[c
+ d*x]^3*(-9856*Sin[2*c] + 17288*Sin[d*x] - 29056*Sin[2*d*x] - 7264*Sin[c -
 d*x] + 14208*Sin[c + d*x] - 19536*Sin[2*(c + d*x)] + 7104*Sin[3*(c + d*x)]
 + 7104*Sin[4*(c + d*x)] - 7104*Sin[5*(c + d*x)] + 1776*Sin[6*(c + d*x)] +
17288*Sin[2*c + d*x] + 20384*Sin[3*c + d*x] - 23771*Sin[c + 2*d*x] + 7104*S
in[2*(c + 2*d*x)] - 23771*Sin[3*c + 2*d*x] - 8960*Sin[4*c + 2*d*x] + 19984*
Sin[c + 3*d*x] + 8644*Sin[2*c + 3*d*x] + 8644*Sin[4*c + 3*d*x] - 6160*Sin[5
*c + 3*d*x] + 8644*Sin[3*c + 4*d*x] + 8644*Sin[5*c + 4*d*x] + 6720*Sin[6*c
```

```
+ 4*d*x] - 12144*Sin[3*c + 5*d*x] - 8644*Sin[4*c + 5*d*x] - 8644*Sin[6*c +
5*d*x] - 1680*Sin[7*c + 5*d*x] + 3456*Sin[4*c + 6*d*x] + 2161*Sin[5*c + 6*d
*x] + 2161*Sin[7*c + 6*d*x]))))/(13762560*d)
```

fricas [A] time = 0.74, size = 272, normalized size = 1.67

$$-\frac{432\,a^2\cos(dx+c)^6-654\,a^2\cos(dx+c)^5-636\,a^2\cos(dx+c)^4+1226\,a^2\cos(dx+c)^3+74\,a^2\cos(dx+c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/105*(432*a^2*cos(d*x + c)^6 - 654*a^2*cos(d*x + c)^5 - 636*a^2*cos(d*x +
 c)^4 + 1226*a^2*cos(d*x + c)^3 + 74*a^2*cos(d*x + c)^2 - 562*a^2*cos(d*x +
 c) - 105*(a^2*cos(d*x + c)^5 - 2*a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^2
 - a^2*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + 105*(a^2*cos(d*x
+ c)^5 - 2*a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*lo
g(-sin(d*x + c) + 1)*sin(d*x + c) + 105*a^2)/((d*cos(d*x + c)^5 - 2*d*cos(d
*x + c)^4 + 2*d*cos(d*x + c)^2 - d*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 0.32, size = 168, normalized size = 1.03

$$\frac{35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6720\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6720\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+945\,a^2\,\mathrm{ta}}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3360*(35*a^2*tan(1/2*d*x + 1/2*c)^3 + 6720*a^2*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - 6720*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 945*a^2*tan(1/2*d*
x + 1/2*c) - 6720*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (
10710*a^2*tan(1/2*d*x + 1/2*c)^6 + 1330*a^2*tan(1/2*d*x + 1/2*c)^4 + 189*a^
2*tan(1/2*d*x + 1/2*c)^2 + 15*a^2)/tan(1/2*d*x + 1/2*c)^7)/d
```

maple [A] time = 1.02, size = 264, normalized size = 1.62

$$-\frac{144a^2\cot(dx+c)}{35d}-\frac{a^2\cot(dx+c)\left(\csc^6(dx+c)\right)}{7d}-\frac{6a^2\cot(dx+c)\left(\csc^4(dx+c)\right)}{35d}-\frac{8a^2\cot(dx+c)\left(\csc^2(dx\right.}{35d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -144/35*a^2*cot(d*x+c)/d-1/7/d*a^2*cot(d*x+c)*csc(d*x+c)^6-6/35/d*a^2*cot(d
*x+c)*csc(d*x+c)^4-8/35/d*a^2*cot(d*x+c)*csc(d*x+c)^2-2/7/d*a^2/sin(d*x+c)^
7-2/5/d*a^2/sin(d*x+c)^5-2/3/d*a^2/sin(d*x+c)^3-2/d*a^2/sin(d*x+c)+2/d*a^2*
ln(sec(d*x+c)+tan(d*x+c))-1/7/d*a^2/sin(d*x+c)^7/cos(d*x+c)-8/35/d*a^2/sin(
d*x+c)^5/cos(d*x+c)-16/35/d*a^2/sin(d*x+c)^3/cos(d*x+c)+64/35/d*a^2/sin(d*x
+c)/cos(d*x+c)
```

maxima [A] time = 0.36, size = 175, normalized size = 1.07

$$-\frac{a^2\left(\frac{2\left(105\,\sin(dx+c)^6+35\,\sin(dx+c)^4+21\,\sin(dx+c)^2+15\right)}{\sin(dx+c)^7}-105\,\log\left(\sin(dx+c)+1\right)+105\,\log\left(\sin(dx+c)-1\right)\right)+3\,a^2}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/105*(a^2*(2*(105*sin(d*x + c)^6 + 35*sin(d*x + c)^4 + 21*sin(d*x + c)^2
+ 15)/sin(d*x + c)^7 - 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1
)) + 3*a^2*((140*tan(d*x + c)^6 + 70*tan(d*x + c)^4 + 28*tan(d*x + c)^2 + 5
)/tan(d*x + c)^7 - 35*tan(d*x + c)) + 3*(35*tan(d*x + c)^6 + 35*tan(d*x + c
)^4 + 21*tan(d*x + c)^2 + 5)*a^2/tan(d*x + c)^7)/d
```

mupad [B] time = 0.99, size = 159, normalized size = 0.98

$$\frac{a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3}{96\,d}+\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{9\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{32\,d}-\frac{-166\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+\frac{268\,a^2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6}{3}+\frac{163\,a}{}}{d\left(32\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7-32\,\mathrm{ta}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^8,x)
```

```
[Out] (a^2*tan(c/2 + (d*x)/2)^3)/(96*d) + (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d + (
9*a^2*tan(c/2 + (d*x)/2))/(32*d) - ((58*a^2*tan(c/2 + (d*x)/2)^2)/35 + (163
*a^2*tan(c/2 + (d*x)/2)^4)/15 + (268*a^2*tan(c/2 + (d*x)/2)^6)/3 - 166*a^2*
tan(c/2 + (d*x)/2)^8 + a^2/7)/(d*(32*tan(c/2 + (d*x)/2)^7 - 32*tan(c/2 + (d
*x)/2)^9))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.37 $\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=201

$$\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot^9(c+dx)}{9d}-\frac{9a^2\cot^7(c+dx)}{7d}-\frac{16a^2\cot^5(c+dx)}{5d}-\frac{14a^2\cot^3(c+dx)}{3d}-\frac{6a^2\cot(c+dx)}{d}-2a$$

```
[Out] 2*a^2*arctanh(sin(d*x+c))/d-6*a^2*cot(d*x+c)/d-14/3*a^2*cot(d*x+c)^3/d-16/5
*a^2*cot(d*x+c)^5/d-9/7*a^2*cot(d*x+c)^7/d-2/9*a^2*cot(d*x+c)^9/d-2*a^2*csc
(d*x+c)/d-2/3*a^2*csc(d*x+c)^3/d-2/5*a^2*csc(d*x+c)^5/d-2/7*a^2*csc(d*x+c)^
7/d-2/9*a^2*csc(d*x+c)^9/d+a^2*tan(d*x+c)/d
```

Rubi [A] time = 0.26, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3872, 2873, 3767, 2621, 302, 207, 2620, 270}

$$\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot^9(c+dx)}{9d}-\frac{9a^2\cot^7(c+dx)}{7d}-\frac{16a^2\cot^5(c+dx)}{5d}-\frac{14a^2\cot^3(c+dx)}{3d}-\frac{6a^2\cot(c+dx)}{d}-2a$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^10*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d - (6*a^2*Cot[c + d*x])/d - (14*a^2*Cot[c +
d*x]^3)/(3*d) - (16*a^2*Cot[c + d*x]^5)/(5*d) - (9*a^2*Cot[c + d*x]^7)/(7*d
) - (2*a^2*Cot[c + d*x]^9)/(9*d) - (2*a^2*Csc[c + d*x])/d - (2*a^2*Csc[c +
d*x]^3)/(3*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x]^7)/(7*d)
 - (2*a^2*Csc[c + d*x]^9)/(9*d) + (a^2*Tan[c + d*x])/d
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (-a-a\cos(c+dx))^2\csc^{10}(c+dx)\sec^2(c+dx)\,dx$$
$$= \int \left(a^2\csc^{10}(c+dx)+2a^2\csc^{10}(c+dx)\sec(c+dx)+a^2\csc^{10}(c+dx)\,\mathrm{s}\right.$$
$$= a^2\int \csc^{10}(c+dx)\,dx + a^2\int \csc^{10}(c+dx)\sec^2(c+dx)\,dx + \left(2a^2\right)\int \mathrm{c}$$
$$= \frac{a^2\,\mathrm{Subst}\left(\int \frac{\left(1+x^2\right)^5}{x^{10}}\,dx, x, \tan(c+dx)\right)}{d} - \frac{a^2\,\mathrm{Subst}\left(\int \left(1+4x^2+6x^4+\right.\right.}{d}$$
$$= -\frac{a^2\cot(c+dx)}{d} - \frac{4a^2\cot^3(c+dx)}{3d} - \frac{6a^2\cot^5(c+dx)}{5d} - \frac{4a^2\cot^7(c+d}{7d}$$
$$= -\frac{6a^2\cot(c+dx)}{d} - \frac{14a^2\cot^3(c+dx)}{3d} - \frac{16a^2\cot^5(c+dx)}{5d} - \frac{9a^2\cot^7(c}{7d}$$
$$= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d} - \frac{6a^2\cot(c+dx)}{d} - \frac{14a^2\cot^3(c+dx)}{3d} - \frac{16a^2\,\mathrm{c}}{}$$

Mathematica [B] time = 6.95, size = 1050, normalized size = 5.22

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^10*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-6899*Cos[c + d*x]^2*Cot[c/2]*Csc[c/2 + (d*x)/2]^2*Sec[c/2 + (d*x)/2]^4*(a
 + a*Sec[c + d*x])^2)/(80640*d) - (193*Cos[c + d*x]^2*Cot[c/2]*Csc[c/2 + (d
*x)/2]^4*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(13440*d) - (71*Cos[c
 + d*x]^2*Cot[c/2]*Csc[c/2 + (d*x)/2]^6*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c +
 d*x])^2)/(32256*d) - (Cos[c + d*x]^2*Cot[c/2]*Csc[c/2 + (d*x)/2]^8*Sec[c/2
 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(4608*d) - (Cos[c + d*x]^2*Log[Cos[c/
2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x]
)^2)/(2*d) + (Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*S
ec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(2*d) + (123041*Cos[c + d*x]^2*
Csc[c/2]*Csc[c/2 + (d*x)/2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin
```

```
[(d*x)/2])/(161280*d) + (6899*Cos[c + d*x]^2*Csc[c/2]*Csc[c/2 + (d*x)/2]^3*
Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(80640*d) + (193*
Cos[c + d*x]^2*Csc[c/2]*Csc[c/2 + (d*x)/2]^5*Sec[c/2 + (d*x)/2]^4*(a + a*Se
c[c + d*x])^2*Sin[(d*x)/2])/(13440*d) + (71*Cos[c + d*x]^2*Csc[c/2]*Csc[c/2
 + (d*x)/2]^7*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(32
256*d) + (Cos[c + d*x]^2*Csc[c/2]*Csc[c/2 + (d*x)/2]^9*Sec[c/2 + (d*x)/2]^4
*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(4608*d) + (803*Cos[c + d*x]^2*Sec[c/
2]*Sec[c/2 + (d*x)/2]^5*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(7680*d) + (49
*Cos[c + d*x]^2*Sec[c/2]*Sec[c/2 + (d*x)/2]^7*(a + a*Sec[c + d*x])^2*Sin[(d
*x)/2])/(7680*d) + (Cos[c + d*x]^2*Sec[c/2]*Sec[c/2 + (d*x)/2]^9*(a + a*Sec
[c + d*x])^2*Sin[(d*x)/2])/(2560*d) + (Cos[c + d*x]*Sec[c]*Sec[c/2 + (d*x)/
2]^4*(a + a*Sec[c + d*x])^2*Sin[d*x])/(4*d) + (49*Cos[c + d*x]^2*Sec[c/2 +
(d*x)/2]^6*(a + a*Sec[c + d*x])^2*Tan[c/2])/(7680*d) + (Cos[c + d*x]^2*Sec[
c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^2*Tan[c/2])/(2560*d)
```

fricas [B] time = 3.26, size = 406, normalized size = 2.02

$$-\frac{1408\,a^2\cos\left(dx+c\right)^8 - 2186\,a^2\cos\left(dx+c\right)^7 - 3372\,a^2\cos\left(dx+c\right)^6 + 6200\,a^2\cos\left(dx+c\right)^5 + 2060\,a^2\cos\left(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/315*(1408*a^2*cos(d*x + c)^8 - 2186*a^2*cos(d*x + c)^7 - 3372*a^2*cos(d*
x + c)^6 + 6200*a^2*cos(d*x + c)^5 + 2060*a^2*cos(d*x + c)^4 - 5784*a^2*cos
(d*x + c)^3 + 268*a^2*cos(d*x + c)^2 + 1756*a^2*cos(d*x + c) - 315*(a^2*cos
(d*x + c)^7 - 2*a^2*cos(d*x + c)^6 - a^2*cos(d*x + c)^5 + 4*a^2*cos(d*x + c
)^4 - a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(sin
(d*x + c) + 1)*sin(d*x + c) + 315*(a^2*cos(d*x + c)^7 - 2*a^2*cos(d*x + c)^
6 - a^2*cos(d*x + c)^5 + 4*a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^3 - 2*a^2*
cos(d*x + c)^2 + a^2*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*x + c) - 31
5*a^2)/((d*cos(d*x + c)^7 - 2*d*cos(d*x + c)^6 - d*cos(d*x + c)^5 + 4*d*cos
(d*x + c)^4 - d*cos(d*x + c)^3 - 2*d*cos(d*x + c)^2 + d*cos(d*x + c))*sin(d
*x + c))
```

giac [A] time = 0.61, size = 200, normalized size = 1.00

$$\frac{63\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 1155\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 80640\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 80640\,a^2\log\left(\left|\tan\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/40320*(63*a^2*tan(1/2*d*x + 1/2*c)^5 + 1155*a^2*tan(1/2*d*x + 1/2*c)^3 +
80640*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 80640*a^2*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) + 17955*a^2*tan(1/2*d*x + 1/2*c) - 80640*a^2*tan(1/2*d*x +
 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (139545*a^2*tan(1/2*d*x + 1/2*c)^8 +
 19635*a^2*tan(1/2*d*x + 1/2*c)^6 + 3591*a^2*tan(1/2*d*x + 1/2*c)^4 + 495*a
^2*tan(1/2*d*x + 1/2*c)^2 + 35*a^2)/tan(1/2*d*x + 1/2*c)^9)/d
```

maple [A] time = 1.03, size = 326, normalized size = 1.62

$$-\frac{1408a^2\cot\left(dx+c\right)}{315d}-\frac{a^2\cot\left(dx+c\right)\left(\csc^8\left(dx+c\right)\right)}{9d}-\frac{8a^2\cot\left(dx+c\right)\left(\csc^6\left(dx+c\right)\right)}{63d}-\frac{16a^2\cot\left(dx+c\right)\left(\csc^4\left(\right.\right.}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^10*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1408/315*a^2*cot(d*x+c)/d-1/9/d*a^2*cot(d*x+c)*csc(d*x+c)^8-8/63/d*a^2*cot
(d*x+c)*csc(d*x+c)^6-16/105/d*a^2*cot(d*x+c)*csc(d*x+c)^4-64/315/d*a^2*cot(
d*x+c)*csc(d*x+c)^2-2/9/d*a^2/sin(d*x+c)^9-2/7/d*a^2/sin(d*x+c)^7-2/5/d*a^2
/sin(d*x+c)^5-2/3/d*a^2/sin(d*x+c)^3-2/d*a^2/sin(d*x+c)+2/d*a^2*ln(sec(d*x+
c)+tan(d*x+c))-1/9/d*a^2/sin(d*x+c)^9/cos(d*x+c)-10/63/d*a^2/sin(d*x+c)^7/c
os(d*x+c)-16/63/d*a^2/sin(d*x+c)^5/cos(d*x+c)-32/63/d*a^2/sin(d*x+c)^3/cos(
d*x+c)+128/63/d*a^2/sin(d*x+c)/cos(d*x+c)
```

maxima [A] time = 0.35, size = 204, normalized size = 1.01

$$-\frac{a^2\left(\frac{2\left(315\,\sin(dx+c)^8+105\,\sin(dx+c)^6+63\,\sin(dx+c)^4+45\,\sin(dx+c)^2+35\right)}{\sin(dx+c)^9} - 315\,\log\left(\sin\left(dx+c\right)+1\right)+315\,\log\left(\sin\left(dx+c\right)-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/315*(a^2*(2*(315*sin(d*x + c)^8 + 105*sin(d*x + c)^6 + 63*sin(d*x + c)^4
 + 45*sin(d*x + c)^2 + 35)/sin(d*x + c)^9 - 315*log(sin(d*x + c) + 1) + 315
*log(sin(d*x + c) - 1)) + 5*a^2*((315*tan(d*x + c)^8 + 210*tan(d*x + c)^6 +
 126*tan(d*x + c)^4 + 45*tan(d*x + c)^2 + 7)/tan(d*x + c)^9 - 63*tan(d*x +
c)) + (315*tan(d*x + c)^8 + 420*tan(d*x + c)^6 + 378*tan(d*x + c)^4 + 180*t
an(d*x + c)^2 + 35)*a^2/tan(d*x + c)^9)/d
```

mupad [B] time = 0.97, size = 194, normalized size = 0.97

$$\frac{11\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{384\,d}+\frac{a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{640\,d}+\frac{4\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{-699\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+\frac{1142\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8}{3}+}{d\,\left(128\tan\left(\frac{c}{2}+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/sin(c + d*x)^10,x)
```

```
[Out] (11*a^2*tan(c/2 + (d*x)/2)^3)/(384*d) + (a^2*tan(c/2 + (d*x)/2)^5)/(640*d)
+ (4*a^2*atanh(tan(c/2 + (d*x)/2)))/d - ((92*a^2*tan(c/2 + (d*x)/2)^2)/63 +
 (344*a^2*tan(c/2 + (d*x)/2)^4)/35 + (764*a^2*tan(c/2 + (d*x)/2)^6)/15 + (1
142*a^2*tan(c/2 + (d*x)/2)^8)/3 - 699*a^2*tan(c/2 + (d*x)/2)^10 + a^2/9)/(d
*(128*tan(c/2 + (d*x)/2)^9 - 128*tan(c/2 + (d*x)/2)^11)) + (57*a^2*tan(c/2
+ (d*x)/2))/(128*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**10*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.38 $\int (a + a\sec(c + dx))^3 \sin^9(c + dx)\,dx$

Optimal. Leaf size=203

$$-\frac{a^3\cos^9(c+dx)}{9d}-\frac{3a^3\cos^8(c+dx)}{8d}+\frac{a^3\cos^7(c+dx)}{7d}+\frac{11a^3\cos^6(c+dx)}{6d}+\frac{6a^3\cos^5(c+dx)}{5d}-\frac{7a^3\cos^4(c+dx)}{2d}-$$

```
[Out] 11*a^3*cos(d*x+c)/d+3*a^3*cos(d*x+c)^2/d-14/3*a^3*cos(d*x+c)^3/d-7/2*a^3*co
s(d*x+c)^4/d+6/5*a^3*cos(d*x+c)^5/d+11/6*a^3*cos(d*x+c)^6/d+1/7*a^3*cos(d*x
+c)^7/d-3/8*a^3*cos(d*x+c)^8/d-1/9*a^3*cos(d*x+c)^9/d+a^3*ln(cos(d*x+c))/d+
3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

Rubi [A] time = 0.20, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3872, 2836, 12, 88}

$$-\frac{a^3\cos^9(c+dx)}{9d}-\frac{3a^3\cos^8(c+dx)}{8d}+\frac{a^3\cos^7(c+dx)}{7d}+\frac{11a^3\cos^6(c+dx)}{6d}+\frac{6a^3\cos^5(c+dx)}{5d}-\frac{7a^3\cos^4(c+dx)}{2d}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^9,x]
```

```
[Out] (11*a^3*Cos[c + d*x])/d + (3*a^3*Cos[c + d*x]^2)/d - (14*a^3*Cos[c + d*x]^3
)/(3*d) - (7*a^3*Cos[c + d*x]^4)/(2*d) + (6*a^3*Cos[c + d*x]^5)/(5*d) + (11
*a^3*Cos[c + d*x]^6)/(6*d) + (a^3*Cos[c + d*x]^7)/(7*d) - (3*a^3*Cos[c + d*
x]^8)/(8*d) - (a^3*Cos[c + d*x]^9)/(9*d) + (a^3*Log[Cos[c + d*x]])/d + (3*a
^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\sin^9(c+dx)\,dx = -\int (-a-a\cos(c+dx))^3\sin^6(c+dx)\tan^3(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^3(-a-x)^4(-a+x)^7}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^9 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^4(-a+x)^7}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^6 d}$$
$$= \frac{\text{Subst}\left(\int \left(-11a^8 - \frac{a^{11}}{x^3} + \frac{3a^{10}}{x^2} + \frac{a^9}{x} + 6a^7x + 14a^6x^2 - 14a^5x^3 - 6a^4x^4 + 1\right.\right.}{a^6 d}$$
$$= \frac{11a^3\cos(c+dx)}{d} + \frac{3a^3\cos^2(c+dx)}{d} - \frac{14a^3\cos^3(c+dx)}{3d} - \frac{7a^3\cos^4(c+}{2d}$$

Mathematica [A] time = 1.79, size = 148, normalized size = 0.73

$$\frac{a^3\sec^2(c+dx)(11624760\cos(c+dx) + 2188872\cos(3(c+dx)) + 41160\cos(4(c+dx)) - 204156\cos(5(c+dx))}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^9,x]
```

```
[Out] (a^3*(471450 + 11624760*Cos[c + d*x] + 2188872*Cos[3*(c + d*x)] + 41160*Cos
[4*(c + d*x)] - 204156*Cos[5*(c + d*x)] - 35805*Cos[6*(c + d*x)] + 22972*Co
s[7*(c + d*x)] + 9030*Cos[8*(c + d*x)] - 820*Cos[9*(c + d*x)] - 945*Cos[10*
(c + d*x)] - 140*Cos[11*(c + d*x)] + 645120*Log[Cos[c + d*x]] + 210*Cos[2*(
c + d*x)]*(-413 + 3072*Log[Cos[c + d*x]]))*Sec[c + d*x]^2)/(1290240*d)
```

fricas [A] time = 0.70, size = 182, normalized size = 0.90

$$-\frac{35840\,a^3\cos(dx+c)^{11} + 120960\,a^3\cos(dx+c)^{10} - 46080\,a^3\cos(dx+c)^9 - 591360\,a^3\cos(dx+c)^8 - 387072}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/322560*(35840*a^3*cos(d*x + c)^11 + 120960*a^3*cos(d*x + c)^10 - 46080*a
^3*cos(d*x + c)^9 - 591360*a^3*cos(d*x + c)^8 - 387072*a^3*cos(d*x + c)^7 +
 1128960*a^3*cos(d*x + c)^6 + 1505280*a^3*cos(d*x + c)^5 - 967680*a^3*cos(d
*x + c)^4 - 3548160*a^3*cos(d*x + c)^3 - 322560*a^3*cos(d*x + c)^2*log(-cos
(d*x + c)) + 212205*a^3*cos(d*x + c)^2 - 967680*a^3*cos(d*x + c) - 161280*a
^3)/(d*cos(d*x + c)^2)
```

giac [B] time = 0.55, size = 396, normalized size = 1.95

$$-\frac{2520\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 2520\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) - \frac{1260\left(9\,a^3+\frac{2\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2} + \frac{45257}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^9,x, algorithm="giac")
```

```
[Out] -1/2520*(2520*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 25
20*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - 1260*(9*a^3 +
 2*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a^3*(cos(d*x + c) - 1)^2/(
```

```
cos(d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2 + (45257
*a^3 - 392193*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1467972*a^3*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3001908*a^3*(cos(d*x + c) - 1)^3/(co
s(d*x + c) + 1)^3 + 3232782*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 -
 2359854*a^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 1190196*a^3*(cos(d
*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 397764*a^3*(cos(d*x + c) - 1)^7/(cos(
d*x + c) + 1)^7 + 79281*a^3*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8 - 712
9*a^3*(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9)/((cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - 1)^9)/d
```

maple [A] time = 0.77, size = 230, normalized size = 1.13

$$\frac{3328a^3\cos(dx+c)}{315d}+\frac{26a^3\left(\sin^8(dx+c)\right)\cos(dx+c)}{9d}+\frac{208a^3\cos(dx+c)\left(\sin^6(dx+c)\right)}{63d}+\frac{416a^3\cos(dx+c)}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^9,x)
```

```
[Out] 3328/315*a^3*cos(d*x+c)/d+26/9/d*a^3*sin(d*x+c)^8*cos(d*x+c)+208/63/d*a^3*c
os(d*x+c)*sin(d*x+c)^6+416/105/d*a^3*cos(d*x+c)*sin(d*x+c)^4+1664/315/d*a^3
*cos(d*x+c)*sin(d*x+c)^2+1/8/d*a^3*sin(d*x+c)^8+1/6/d*a^3*sin(d*x+c)^6+1/4/
d*a^3*sin(d*x+c)^4+1/2/d*a^3*sin(d*x+c)^2+a^3*ln(cos(d*x+c))/d+3/d*a^3*sin(
d*x+c)^10/cos(d*x+c)+1/2/d*a^3*sin(d*x+c)^10/cos(d*x+c)^2
```

maxima [A] time = 0.33, size = 158, normalized size = 0.78

$$-\frac{280\,a^3\cos(dx+c)^9+945\,a^3\cos(dx+c)^8-360\,a^3\cos(dx+c)^7-4620\,a^3\cos(dx+c)^6-3024\,a^3\cos(dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/2520*(280*a^3*cos(d*x + c)^9 + 945*a^3*cos(d*x + c)^8 - 360*a^3*cos(d*x
+ c)^7 - 4620*a^3*cos(d*x + c)^6 - 3024*a^3*cos(d*x + c)^5 + 8820*a^3*cos(d
*x + c)^4 + 11760*a^3*cos(d*x + c)^3 - 7560*a^3*cos(d*x + c)^2 - 27720*a^3*
cos(d*x + c) - 2520*a^3*log(cos(d*x + c)) - 1260*(6*a^3*cos(d*x + c) + a^3)
/cos(d*x + c)^2)/d
```

mupad [B] time = 0.96, size = 157, normalized size = 0.77

$$\frac{\frac{3\,a^3\,\cos(c+d\,x)+\frac{a^3}{2}}{\cos(c+d\,x)^2}+11\,a^3\,\cos(c+d\,x)+3\,a^3\,\cos(c+d\,x)^2-\frac{14\,a^3\cos(c+d\,x)^3}{3}-\frac{7\,a^3\cos(c+d\,x)^4}{2}+\frac{6\,a^3\cos(c+d\,x)^5}{5}+\frac{11\,a^3}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^9*(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((3*a^3*cos(c + d*x) + a^3/2)/cos(c + d*x)^2 + 11*a^3*cos(c + d*x) + 3*a^3*
cos(c + d*x)^2 - (14*a^3*cos(c + d*x)^3)/3 - (7*a^3*cos(c + d*x)^4)/2 + (6*
a^3*cos(c + d*x)^5)/5 + (11*a^3*cos(c + d*x)^6)/6 + (a^3*cos(c + d*x)^7)/7
- (3*a^3*cos(c + d*x)^8)/8 - (a^3*cos(c + d*x)^9)/9 + a^3*log(cos(c + d*x))
)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**9,x)
```

```
[Out] Timed out
```

3.39 $\int (a+a\sec(c+dx))^3\sin^7(c+dx)\,dx$

Optimal. Leaf size=131

$$\frac{a^3\cos^7(c+dx)}{7d}+\frac{a^3\cos^6(c+dx)}{2d}-\frac{2a^3\cos^4(c+dx)}{d}-\frac{2a^3\cos^3(c+dx)}{d}+\frac{3a^3\cos^2(c+dx)}{d}+\frac{8a^3\cos(c+dx)}{d}+\frac{a^3\sec}{}$$

```
[Out] 8*a^3*cos(d*x+c)/d+3*a^3*cos(d*x+c)^2/d-2*a^3*cos(d*x+c)^3/d-2*a^3*cos(d*x+
c)^4/d+1/2*a^3*cos(d*x+c)^6/d+1/7*a^3*cos(d*x+c)^7/d+3*a^3*sec(d*x+c)/d+1/2
*a^3*sec(d*x+c)^2/d
```

Rubi [A] time = 0.17, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2836, 12, 88}

$$\frac{a^3\cos^7(c+dx)}{7d}+\frac{a^3\cos^6(c+dx)}{2d}-\frac{2a^3\cos^4(c+dx)}{d}-\frac{2a^3\cos^3(c+dx)}{d}+\frac{3a^3\cos^2(c+dx)}{d}+\frac{8a^3\cos(c+dx)}{d}+\frac{a^3\sec}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^7,x]
```

```
[Out] (8*a^3*Cos[c + d*x])/d + (3*a^3*Cos[c + d*x]^2)/d - (2*a^3*Cos[c + d*x]^3)/
d - (2*a^3*Cos[c + d*x]^4)/d + (a^3*Cos[c + d*x]^6)/(2*d) + (a^3*Cos[c + d*
x]^7)/(7*d) + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\sin^7(c+dx)\,dx = -\int (-a-a\cos(c+dx))^3\sin^4(c+dx)\tan^3(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^3(-a-x)^3(-a+x)^6}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^3(-a+x)^6}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(-8a^6 - \frac{a^9}{x^3} + \frac{3a^8}{x^2} + 6a^5x + 6a^4x^2 - 8a^3x^3 + 3ax^5 - x^6\right) dx, x\right.}{a^4 d}$$
$$= \frac{8a^3\cos(c+dx)}{d} + \frac{3a^3\cos^2(c+dx)}{d} - \frac{2a^3\cos^3(c+dx)}{d} - \frac{2a^3\cos^4(c}{d}$$

Mathematica [A] time = 1.01, size = 106, normalized size = 0.81

$$\frac{a^3(14014\cos(c+dx) - 210\cos(2(c+dx)) + 2548\cos(3(c+dx)) + 196\cos(4(c+dx)) - 188\cos(5(c+dx)) -}{1792d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^7,x]
```

```
[Out] (a^3*(427 + 14014*Cos[c + d*x] - 210*Cos[2*(c + d*x)] + 2548*Cos[3*(c + d*x
)] + 196*Cos[4*(c + d*x)] - 188*Cos[5*(c + d*x)] - 56*Cos[6*(c + d*x)] + 9*
Cos[7*(c + d*x)] + 7*Cos[8*(c + d*x)] + Cos[9*(c + d*x)])*Sec[c + d*x]^2)/(
1792*d)
```

fricas [A] time = 0.77, size = 121, normalized size = 0.92

$$\frac{32\,a^3\cos(dx+c)^9 + 112\,a^3\cos(dx+c)^8 - 448\,a^3\cos(dx+c)^6 - 448\,a^3\cos(dx+c)^5 + 672\,a^3\cos(dx+c)^4 +}{224\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/224*(32*a^3*cos(d*x + c)^9 + 112*a^3*cos(d*x + c)^8 - 448*a^3*cos(d*x + c
)^6 - 448*a^3*cos(d*x + c)^5 + 672*a^3*cos(d*x + c)^4 + 1792*a^3*cos(d*x +
c)^3 - 203*a^3*cos(d*x + c)^2 + 672*a^3*cos(d*x + c) + 112*a^3)/(d*cos(d*x
+ c)^2)
```

giac [A] time = 1.11, size = 239, normalized size = 1.82

$$\frac{2\left(\frac{7\left(3\,a^3+\frac{2\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2} - \frac{43\,a^3-\frac{273\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{672\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{630\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{343\,a^3(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{105\,a^3(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{14\,a^3}{(\cos}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^7}\right.}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 2/7*(7*(3*a^3 + 2*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 1)^2 - (43*a^3 - 273*a^3*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) + 672*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 630*a^
3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 343*a^3*(cos(d*x + c) - 1)^4/
(cos(d*x + c) + 1)^4 - 105*a^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 +
```

```
14*a^3*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6)/((cos(d*x + c) - 1)/(cos(
d*x + c) + 1) - 1)^7)/d
```

maple [A] time = 0.71, size = 130, normalized size = 0.99

$$\frac{64a^3\cos(dx+c)}{7d}+\frac{20a^3\cos(dx+c)\left(\sin^6(dx+c)\right)}{7d}+\frac{24a^3\cos(dx+c)\left(\sin^4(dx+c)\right)}{7d}+\frac{32a^3\cos(dx+c)\left(\sin^2(d\right.}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^7,x)
```

```
[Out] 64/7*a^3*cos(d*x+c)/d+20/7/d*a^3*cos(d*x+c)*sin(d*x+c)^6+24/7/d*a^3*cos(d*x
+c)*sin(d*x+c)^4+32/7/d*a^3*cos(d*x+c)*sin(d*x+c)^2+3/d*a^3*sin(d*x+c)^8/co
s(d*x+c)+1/2/d*a^3*sin(d*x+c)^8/cos(d*x+c)^2
```

maxima [A] time = 0.32, size = 107, normalized size = 0.82

$$\frac{2\,a^3\cos(dx+c)^7+7\,a^3\cos(dx+c)^6-28\,a^3\cos(dx+c)^4-28\,a^3\cos(dx+c)^3+42\,a^3\cos(dx+c)^2+112\,a^3\cos}{14\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/14*(2*a^3*cos(d*x + c)^7 + 7*a^3*cos(d*x + c)^6 - 28*a^3*cos(d*x + c)^4 -
 28*a^3*cos(d*x + c)^3 + 42*a^3*cos(d*x + c)^2 + 112*a^3*cos(d*x + c) + 7*(
6*a^3*cos(d*x + c) + a^3)/cos(d*x + c)^2)/d
```

mupad [B] time = 0.90, size = 107, normalized size = 0.82

$$\frac{\frac{3\,a^3\,\cos(c+d\,x)+\frac{a^3}{2}}{\cos(c+d\,x)^2}+8\,a^3\,\cos(c+d\,x)+3\,a^3\cos(c+d\,x)^2-2\,a^3\cos(c+d\,x)^3-2\,a^3\cos(c+d\,x)^4+\frac{a^3\cos(c+d\,x)^6}{2}+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7*(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((3*a^3*cos(c + d*x) + a^3/2)/cos(c + d*x)^2 + 8*a^3*cos(c + d*x) + 3*a^3*c
os(c + d*x)^2 - 2*a^3*cos(c + d*x)^3 - 2*a^3*cos(c + d*x)^4 + (a^3*cos(c +
d*x)^6)/2 + (a^3*cos(c + d*x)^7)/7)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**7,x)
```

```
[Out] Timed out
```

3.40 $\int (a + a\sec(c + dx))^3 \sin^5(c + dx)\,dx$

Optimal. Leaf size=134

$$-\frac{a^3\cos^5(c+dx)}{5d}-\frac{3a^3\cos^4(c+dx)}{4d}-\frac{a^3\cos^3(c+dx)}{3d}+\frac{5a^3\cos^2(c+dx)}{2d}+\frac{5a^3\cos(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a}{}$$

```
[Out] 5*a^3*cos(d*x+c)/d+5/2*a^3*cos(d*x+c)^2/d-1/3*a^3*cos(d*x+c)^3/d-3/4*a^3*co
s(d*x+c)^4/d-1/5*a^3*cos(d*x+c)^5/d-a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d
+1/2*a^3*sec(d*x+c)^2/d
```

Rubi [A] time = 0.17, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3872, 2836, 12, 88}

$$-\frac{a^3\cos^5(c+dx)}{5d}-\frac{3a^3\cos^4(c+dx)}{4d}-\frac{a^3\cos^3(c+dx)}{3d}+\frac{5a^3\cos^2(c+dx)}{2d}+\frac{5a^3\cos(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^5,x]
```

```
[Out] (5*a^3*Cos[c + d*x])/d + (5*a^3*Cos[c + d*x]^2)/(2*d) - (a^3*Cos[c + d*x]^3
)/(3*d) - (3*a^3*Cos[c + d*x]^4)/(4*d) - (a^3*Cos[c + d*x]^5)/(5*d) - (a^3*
Log[Cos[c + d*x]])/d + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a + a\sec(c+dx))^3 \sin^5(c+dx)\,dx = -\int (-a - a\cos(c+dx))^3 \sin^2(c+dx)\tan^3(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^3(-a-x)^2(-a+x)^5}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^2(-a+x)^5}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(-5a^4 - \frac{a^7}{x^3} + \frac{3a^6}{x^2} - \frac{a^5}{x} + 5a^3x + a^2x^2 - 3ax^3 + x^4\right) dx, x, -a\cos\right.}{a^2 d}$$
$$= \frac{5a^3\cos(c+dx)}{d} + \frac{5a^3\cos^2(c+dx)}{2d} - \frac{a^3\cos^3(c+dx)}{3d} - \frac{3a^3\cos^4(c+dx}{4d}$$

Mathematica [A] time = 0.66, size = 108, normalized size = 0.81

$$-\frac{a^3\sec^2(c+dx)(-12350\cos(c+dx) - 2074\cos(3(c+dx)) - 330\cos(4(c+dx)) + 82\cos(5(c+dx)) + 45\cos(6(}{1920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^5,x]
```

```
[Out] -1/1920*(a^3*(-120 - 12350*Cos[c + d*x] - 2074*Cos[3*(c + d*x)] - 330*Cos[4
*(c + d*x)] + 82*Cos[5*(c + d*x)] + 45*Cos[6*(c + d*x)] + 6*Cos[7*(c + d*x)
] + 960*Log[Cos[c + d*x]] + 15*Cos[2*(c + d*x)]*(31 + 64*Log[Cos[c + d*x]])
)*Sec[c + d*x]^2)/d
```

fricas [A] time = 0.69, size = 130, normalized size = 0.97

$$-\frac{96\,a^3\cos(dx+c)^7 + 360\,a^3\cos(dx+c)^6 + 160\,a^3\cos(dx+c)^5 - 1200\,a^3\cos(dx+c)^4 - 2400\,a^3\cos(dx+c)^3}{480\,d\cos(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/480*(96*a^3*cos(d*x + c)^7 + 360*a^3*cos(d*x + c)^6 + 160*a^3*cos(d*x +
c)^5 - 1200*a^3*cos(d*x + c)^4 - 2400*a^3*cos(d*x + c)^3 + 480*a^3*cos(d*x
+ c)^2*log(-cos(d*x + c)) + 465*a^3*cos(d*x + c)^2 - 1440*a^3*cos(d*x + c)
- 240*a^3)/(d*cos(d*x + c)^2)
```

giac [B] time = 1.28, size = 297, normalized size = 2.22

$$\frac{60\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 60\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{30\left(15\,a^3 + \frac{14\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{3\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2} - \frac{399\,a^3 - \frac{1395\,a^3(}{\cos(}}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^3*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 30*(15*a^3 + 14*a^3*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^3*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2 - (399*a^3 - 13
95*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 390*a^3*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 650*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 -
```

```
565*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a^3*(cos(d*x + c)
- 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)
/d
```

maple [A] time = 0.78, size = 155, normalized size = 1.16

$$\frac{112a^3\cos(dx+c)}{15d}+\frac{14a^3\cos(dx+c)\left(\sin^4(dx+c)\right)}{5d}+\frac{56a^3\cos(dx+c)\left(\sin^2(dx+c)\right)}{15d}-\frac{a^3\left(\sin^4(dx+c)\right)}{4d}-\frac{a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^5,x)
```

```
[Out] 112/15*a^3*cos(d*x+c)/d+14/5/d*a^3*cos(d*x+c)*sin(d*x+c)^4+56/15/d*a^3*cos(
d*x+c)*sin(d*x+c)^2-1/4/d*a^3*sin(d*x+c)^4-1/2/d*a^3*sin(d*x+c)^2-a^3*ln(co
s(d*x+c))/d+3/d*a^3*sin(d*x+c)^6/cos(d*x+c)+1/2/d*a^3*sin(d*x+c)^6/cos(d*x+
c)^2
```

maxima [A] time = 0.33, size = 106, normalized size = 0.79

$$-\frac{12\,a^3\cos(dx+c)^5+45\,a^3\cos(dx+c)^4+20\,a^3\cos(dx+c)^3-150\,a^3\cos(dx+c)^2-300\,a^3\cos(dx+c)+6}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(12*a^3*cos(d*x + c)^5 + 45*a^3*cos(d*x + c)^4 + 20*a^3*cos(d*x + c)^
3 - 150*a^3*cos(d*x + c)^2 - 300*a^3*cos(d*x + c) + 60*a^3*log(cos(d*x + c)
) - 30*(6*a^3*cos(d*x + c) + a^3)/cos(d*x + c)^2)/d
```

mupad [B] time = 0.89, size = 107, normalized size = 0.80

$$-\frac{\frac{a^3\cos(c+d\,x)^3}{3}-5\,a^3\,\cos(c+d\,x)-\frac{5\,a^3\cos(c+d\,x)^2}{2}-\frac{3\,a^3\,\cos(c+d\,x)+\frac{a^3}{2}}{\cos(c+d\,x)^2}+\frac{3\,a^3\cos(c+d\,x)^4}{4}+\frac{a^3\cos(c+d\,x)^5}{5}+a^3\,\ln(\cos(c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5*(a + a/cos(c + d*x))^3,x)
```

```
[Out] -((a^3*cos(c + d*x)^3)/3 - 5*a^3*cos(c + d*x) - (5*a^3*cos(c + d*x)^2)/2 -
(3*a^3*cos(c + d*x) + a^3/2)/cos(c + d*x)^2 + (3*a^3*cos(c + d*x)^4)/4 + (a
^3*cos(c + d*x)^5)/5 + a^3*log(cos(c + d*x)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.41 $\int (a+a\sec(c+dx))^3 \sin^3(c+dx)\,dx$

Optimal. Leaf size=98

$$\frac{a^3\cos^3(c+dx)}{3d}+\frac{3a^3\cos^2(c+dx)}{2d}+\frac{2a^3\cos(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}-\frac{2a^3\log(\cos(c+dx))}{d}$$

```
[Out] 2*a^3*cos(d*x+c)/d+3/2*a^3*cos(d*x+c)^2/d+1/3*a^3*cos(d*x+c)^3/d-2*a^3*ln(c
os(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

Rubi [A] time = 0.10, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3872, 2707, 75}

$$\frac{a^3\cos^3(c+dx)}{3d}+\frac{3a^3\cos^2(c+dx)}{2d}+\frac{2a^3\cos(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}-\frac{2a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^3,x]
```

```
[Out] (2*a^3*Cos[c + d*x])/d + (3*a^3*Cos[c + d*x]^2)/(2*d) + (a^3*Cos[c + d*x]^3
)/(3*d) - (2*a^3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c
 + d*x]^2)/(2*d)
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^3\sin^3(c+dx)\,dx &= -\int (-a-a\cos(c+dx))^3\tan^3(c+dx)\,dx\\ &= \frac{\text{Subst}\left(\int \frac{(-a-x)(-a+x)^4}{x^3}\,dx, x, -a\cos(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(-2a^2-\frac{a^5}{x^3}+\frac{3a^4}{x^2}-\frac{2a^3}{x}+3ax-x^2\right)dx, x, -a\cos(c+dx)\right)}{d}\\ &= \frac{2a^3\cos(c+dx)}{d}+\frac{3a^3\cos^2(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{2a^3\log(\cos(c+}{d}\end{aligned}$$

Mathematica [A] time = 0.20, size = 86, normalized size = 0.88

$$\frac{a^3 \sec^2(c+dx)(226\cos(c+dx)+29\cos(3(c+dx))+9\cos(4(c+dx))+\cos(5(c+dx))-48\log(\cos(c+dx))}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^3,x]
```

```
[Out] (a^3*(-41 + 226*Cos[c + d*x] + 29*Cos[3*(c + d*x)] + 9*Cos[4*(c + d*x)] + C
os[5*(c + d*x)] - 48*Log[Cos[c + d*x]] - 8*Cos[2*(c + d*x)]*(7 + 6*Log[Cos[
c + d*x]]))*Sec[c + d*x]^2)/(48*d)
```

fricas [A] time = 0.70, size = 104, normalized size = 1.06

$$\frac{4\,a^3\cos(dx+c)^5+18\,a^3\cos(dx+c)^4+24\,a^3\cos(dx+c)^3-24\,a^3\cos(dx+c)^2\log(-\cos(dx+c))-9\,a^3\cos}{12\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a^3*cos(d*x + c)^5 + 18*a^3*cos(d*x + c)^4 + 24*a^3*cos(d*x + c)^3
- 24*a^3*cos(d*x + c)^2*log(-cos(d*x + c)) - 9*a^3*cos(d*x + c)^2 + 36*a^3*
cos(d*x + c) + 6*a^3)/(d*cos(d*x + c)^2)
```

giac [A] time = 0.39, size = 102, normalized size = 1.04

$$-\frac{2\,a^3\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{6\,a^3\cos(dx+c)+a^3}{2\,d\cos(dx+c)^2}+\frac{2\,a^3d^8\cos(dx+c)^3+9\,a^3d^8\cos(dx+c)^2+12\,a^3d^8\cos(dx+c}{6\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -2*a^3*log(abs(cos(d*x + c))/abs(d))/d + 1/2*(6*a^3*cos(d*x + c) + a^3)/(d*
cos(d*x + c)^2) + 1/6*(2*a^3*d^8*cos(d*x + c)^3 + 9*a^3*d^8*cos(d*x + c)^2
+ 12*a^3*d^8*cos(d*x + c))/d^9
```

maple [A] time = 0.80, size = 109, normalized size = 1.11

$$\frac{8a^3\cos(dx+c)\left(\sin^2(dx+c)\right)}{3d}+\frac{16a^3\cos(dx+c)}{3d}-\frac{3a^3\left(\sin^2(dx+c)\right)}{2d}-\frac{2a^3\ln(\cos(dx+c))}{d}+\frac{3a^3\left(\sin^4(dx+\right.}{d\cos(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^3,x)
```

```
[Out] 8/3/d*a^3*cos(d*x+c)*sin(d*x+c)^2+16/3*a^3*cos(d*x+c)/d-3/2/d*a^3*sin(d*x+c
)^2-2*a^3*ln(cos(d*x+c))/d+3/d*a^3*sin(d*x+c)^4/cos(d*x+c)+1/2*a^3*tan(d*x+
c)^2/d
```

maxima [A] time = 0.36, size = 80, normalized size = 0.82

$$\frac{2\,a^3\cos(dx+c)^3+9\,a^3\cos(dx+c)^2+12\,a^3\cos(dx+c)-12\,a^3\log(\cos(dx+c))+\frac{3\left(6\,a^3\cos(dx+c)+a^3\right)}{\cos(dx+c)^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^3*cos(d*x + c)^3 + 9*a^3*cos(d*x + c)^2 + 12*a^3*cos(d*x + c) - 12
*a^3*log(cos(d*x + c)) + 3*(6*a^3*cos(d*x + c) + a^3)/cos(d*x + c)^2)/d
```

mupad [B] time = 0.88, size = 80, normalized size = 0.82

$$\frac{\frac{3\,a^3\,\cos(c+d\,x)+\frac{a^3}{2}}{\cos(c+d\,x)^2}+2\,a^3\,\cos\left(c+d\,x\right)+\frac{3\,a^3\,\cos(c+d\,x)^2}{2}+\frac{a^3\,\cos(c+d\,x)^3}{3}-2\,a^3\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((3*a^3*cos(c + d*x) + a^3/2)/cos(c + d*x)^2 + 2*a^3*cos(c + d*x) + (3*a^3*
cos(c + d*x)^2)/2 + (a^3*cos(c + d*x)^3)/3 - 2*a^3*log(cos(c + d*x)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sin^3\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3\sin^3\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int \sin^3\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int \mathrm{s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**3,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)**3*sec(c + d*x), x) + Integral(3*sin(c + d*x)
**3*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**3*sec(c + d*x)**3, x) + In
tegral(sin(c + d*x)**3, x))
```

3.42 $\int (a+a\sec(c+dx))^3 \sin(c+dx)\,dx$

Optimal. Leaf size=62

$$-\frac{a^3\cos(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}-\frac{3a^3\log(\cos(c+dx))}{d}$$

```
[Out] -a^3*cos(d*x+c)/d-3*a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x
+c)^2/d
```

Rubi [A] time = 0.09, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 2833, 12, 43}

$$-\frac{a^3\cos(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}-\frac{3a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x],x]
```

```
[Out] -((a^3*Cos[c + d*x])/d) - (3*a^3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[c + d*x]
)/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\sin(c+dx)\,dx = -\int (-a-a\cos(c+dx))^3\sec^2(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^3(-a+x)^3}{x^3}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{(-a+x)^3}{x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(1-\frac{a^3}{x^3}+\frac{3a^2}{x^2}-\frac{3a}{x}\right)dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^3\cos(c+dx)}{d}-\frac{3a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+}{2d}$$

Mathematica [A] time = 0.25, size = 65, normalized size = 1.05

$$-\frac{a^3\sec^2(c+dx)(-9\cos(c+dx)+\cos(3(c+dx))+6\log(\cos(c+dx))+\cos(2(c+dx))(6\log(\cos(c+dx))-2)-}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x],x]
```

```
[Out] -1/4*(a^3*(-4 - 9*Cos[c + d*x] + Cos[3*(c + d*x)] + 6*Log[Cos[c + d*x]] + C
os[2*(c + d*x)]*(-2 + 6*Log[Cos[c + d*x]]))*Sec[c + d*x]^2)/d
```

fricas [A] time = 0.86, size = 65, normalized size = 1.05

$$-\frac{2\,a^3\cos(dx+c)^3+6\,a^3\cos(dx+c)^2\log(-\cos(dx+c))-6\,a^3\cos(dx+c)-a^3}{2\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*cos(d*x + c)^3 + 6*a^3*cos(d*x + c)^2*log(-cos(d*x + c)) - 6*a^
3*cos(d*x + c) - a^3)/(d*cos(d*x + c)^2)
```

giac [A] time = 0.32, size = 64, normalized size = 1.03

$$-\frac{a^3\cos(dx+c)}{d}-\frac{3\,a^3\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{6\,a^3\cos(dx+c)+a^3}{2\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -a^3*cos(d*x + c)/d - 3*a^3*log(abs(cos(d*x + c))/abs(d))/d + 1/2*(6*a^3*co
s(d*x + c) + a^3)/(d*cos(d*x + c)^2)
```

maple [A] time = 0.25, size = 63, normalized size = 1.02

$$\frac{a^3\left(\sec^2(dx+c)\right)}{2d}+\frac{3a^3\sec(dx+c)}{d}+\frac{3a^3\ln(\sec(dx+c))}{d}-\frac{a^3}{d\sec(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c),x)
```

```
[Out] 1/2*a^3*sec(d*x+c)^2/d+3*a^3*sec(d*x+c)/d+3*a^3/d*ln(sec(d*x+c))-a^3/d/sec(
d*x+c)
```

maxima [A] time = 0.41, size = 55, normalized size = 0.89

$$-\frac{2\,a^3\cos\left(dx+c\right)+6\,a^3\log\left(\cos\left(dx+c\right)\right)-\frac{6\,a^3}{\cos(dx+c)}-\frac{a^3}{\cos(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^3*cos(d*x + c) + 6*a^3*log(cos(d*x + c)) - 6*a^3/cos(d*x + c) - a
^3/cos(d*x + c)^2)/d
```

mupad [B] time = 0.06, size = 52, normalized size = 0.84

$$\frac{a^3\left(3\,\cos\left(c+d\,x\right)-\cos\left(c+d\,x\right)^3-3\cos\left(c+d\,x\right)^2\,\ln\left(\cos\left(c+d\,x\right)\right)+\frac{1}{2}\right)}{d\cos\left(c+d\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (a^3*(3*cos(c + d*x) - cos(c + d*x)^3 - 3*cos(c + d*x)^2*log(cos(c + d*x))
+ 1/2))/(d*cos(c + d*x)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sin\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3\sin\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int \sin\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int \mathrm{s}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c),x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*sec(c + d*x), x) + Integral(3*sin(c + d*x)*se
c(c + d*x)**2, x) + Integral(sin(c + d*x)*sec(c + d*x)**3, x) + Integral(si
n(c + d*x), x))
```

3.43 $\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=67

$$\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{4a^3\log(1-\cos(c+dx))}{d}-\frac{4a^3\log(\cos(c+dx))}{d}$$

```
[Out] 4*a^3*ln(1-cos(d*x+c))/d-4*a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*
sec(d*x+c)^2/d
```

Rubi [A] time = 0.13, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 2836, 12, 88}

$$\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{4a^3\log(1-\cos(c+dx))}{d}-\frac{4a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (4*a^3*Log[1 - Cos[c + d*x]])/d - (4*a^3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[
c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx = -\int (-a-a\cos(c+dx))^3\csc(c+dx)\sec^3(c+dx)\,dx$$
$$= \frac{a\,\text{Subst}\left(\int \frac{a^3(-a+x)^2}{(-a-x)x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^4\,\text{Subst}\left(\int \frac{(-a+x)^2}{(-a-x)x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^4\,\text{Subst}\left(\int \left(-\frac{a}{x^3}+\frac{3}{x^2}-\frac{4}{ax}+\frac{4}{a(a+x)}\right)dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{4a^3\log(1-\cos(c+dx))}{d} - \frac{4a^3\log(\cos(c+dx))}{d} + \frac{3a^3\sec(c+dx)}{d} + \cdots$$

Mathematica [A] time = 0.15, size = 81, normalized size = 1.21

$$\frac{a^3\sec^2(c+dx)\left(6\cos(c+dx)+8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-4\log(\cos(c+dx))-4\cos(2(c+dx))\left(\log(\cos(c+\cdots\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + 6*Cos[c + d*x] - 4*Log[Cos[c + d*x]] - 4*Cos[2*(c + d*x)]*(Log[Co
s[c + d*x]] - 2*Log[Sin[(c + d*x)/2]]) + 8*Log[Sin[(c + d*x)/2]])*Sec[c + d
*x]^2)/(2*d)
```

fricas [A] time = 0.74, size = 76, normalized size = 1.13

$$-\frac{8\,a^3\cos(dx+c)^2\log(-\cos(dx+c)) - 8\,a^3\cos(dx+c)^2\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - 6\,a^3\cos(dx+c) - a^3}{2\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(8*a^3*cos(d*x + c)^2*log(-cos(d*x + c)) - 8*a^3*cos(d*x + c)^2*log(-1
/2*cos(d*x + c) + 1/2) - 6*a^3*cos(d*x + c) - a^3)/(d*cos(d*x + c)^2)
```

giac [B] time = 0.31, size = 142, normalized size = 2.12

$$\frac{2\left(2\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 2\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{6\,a^3+\frac{8\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2*(2*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 2*a^3*log(abs(
-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (6*a^3 + 8*a^3*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 3*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)
/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

maple [A] time = 0.48, size = 49, normalized size = 0.73

$$\frac{a^3\left(\sec^2(dx+c)\right)}{2d} + \frac{3a^3\sec(dx+c)}{d} + \frac{4a^3\ln(-1+\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*sec(d*x+c)^2/d+3*a^3*sec(d*x+c)/d+4/d*a^3*ln(-1+sec(d*x+c))
```

maxima [A] time = 0.56, size = 56, normalized size = 0.84

$$\frac{8\,a^3 \log\left(\cos\left(dx+c\right)-1\right) - 8\,a^3 \log\left(\cos\left(dx+c\right)\right) + \frac{6\,a^3 \cos(dx+c) + a^3}{\cos(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(8*a^3*log(cos(d*x + c) - 1) - 8*a^3*log(cos(d*x + c)) + (6*a^3*cos(d*x
 + c) + a^3)/cos(d*x + c)^2)/d
```

mupad [B] time = 0.07, size = 49, normalized size = 0.73

$$\frac{3\,a^3\,\cos\left(c+d\,x\right)+\frac{a^3}{2}}{d\cos\left(c+d\,x\right)^2}-\frac{8\,a^3\,\mathrm{atanh}\left(2\,\cos\left(c+d\,x\right)-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x),x)
```

```
[Out] (3*a^3*cos(c + d*x) + a^3/2)/(d*cos(c + d*x)^2) - (8*a^3*atanh(2*cos(c + d*
x) - 1))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\csc\left(c+dx\right)\sec\left(c+dx\right)dx + \int 3\csc\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int \csc\left(c+dx\right)\sec^3\left(c+dx\right)dx + \int \csc$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*csc(c + d*x)*sec(c + d*x), x) + Integral(3*csc(c + d*x)*se
c(c + d*x)**2, x) + Integral(csc(c + d*x)*sec(c + d*x)**3, x) + Integral(cs
c(c + d*x), x))
```

3.44 $\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=88

$$-\frac{2a^4}{d(a-a\cos(c+dx))}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{5a^3\log(1-\cos(c+dx))}{d}-\frac{5a^3\log(\cos(c+dx))}{d}$$

```
[Out] -2*a^4/d/(a-a*cos(d*x+c))+5*a^3*ln(1-cos(d*x+c))/d-5*a^3*ln(cos(d*x+c))/d+3
*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

Rubi [A] time = 0.16, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2836, 12, 77}

$$-\frac{2a^4}{d(a-a\cos(c+dx))}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{5a^3\log(1-\cos(c+dx))}{d}-\frac{5a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-2*a^4)/(d*(a - a*Cos[c + d*x])) + (5*a^3*Log[1 - Cos[c + d*x]])/d - (5*a^
3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx = -\int (-a-a\cos(c+dx))^3\csc^3(c+dx)\sec^3(c+dx)\,dx$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{a^3(-a+x)}{(-a-x)^2x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \frac{-a+x}{(-a-x)^2x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \left(-\frac{1}{ax^3}+\frac{3}{a^2x^2}-\frac{5}{a^3x}+\frac{2}{a^2(a+x)^2}+\frac{5}{a^3(a+x)}\right)dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{2a^4}{d(a-a\cos(c+dx))}+\frac{5a^3\log(1-\cos(c+dx))}{d}-\frac{5a^3\log(\cos(c+dx))}{d}$$

Mathematica [A] time = 0.90, size = 88, normalized size = 1.00

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(2\csc^2\left(\frac{1}{2}(c+dx)\right)-\sec^2(c+dx)-6\sec(c+dx)+10\left(\log(\cos(c+dx))-\right.\right.}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/16*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(2*Csc[(c + d*x)/2]^2 +
10*(Log[Cos[c + d*x]] - 2*Log[Sin[(c + d*x)/2]]) - 6*Sec[c + d*x] - Sec[c +
 d*x]^2))/d
```

fricas [A] time = 0.74, size = 132, normalized size = 1.50

$$\frac{10\,a^3\cos(dx+c)^2-5\,a^3\cos(dx+c)-a^3-10\left(a^3\cos(dx+c)^3-a^3\cos(dx+c)^2\right)\log(-\cos(dx+c))+10\left(a^3\cos\right.}{2\left(d\cos(dx+c)^3-d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(10*a^3*cos(d*x + c)^2 - 5*a^3*cos(d*x + c) - a^3 - 10*(a^3*cos(d*x + c
)^3 - a^3*cos(d*x + c)^2)*log(-cos(d*x + c)) + 10*(a^3*cos(d*x + c)^3 - a^3
*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^3 - d*cos(d*
x + c)^2)
```

giac [B] time = 0.40, size = 189, normalized size = 2.15

$$\frac{10\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-10\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{2\left(a^3-\frac{5a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}+\frac{27\,a^3+\frac{38\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{15\,a^3(\cdots}{(\cos\cdots}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(10*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 10*a^3*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 2*(a^3 - 5*a^3*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (27*a
^3 + 38*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 15*a^3*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

maple [A] time = 0.73, size = 67, normalized size = 0.76

$$\frac{a^3\left(\sec^2(dx+c)\right)}{2d}+\frac{3a^3\sec(dx+c)}{d}-\frac{2a^3}{d\left(-1+\sec(dx+c)\right)}+\frac{5a^3\ln\left(-1+\sec(dx+c)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*sec(d*x+c)^2/d+3*a^3*sec(d*x+c)/d-2/d*a^3/(-1+sec(d*x+c))+5/d*a^3*l
n(-1+sec(d*x+c))
```

maxima [A] time = 0.50, size = 84, normalized size = 0.95

$$\frac{10\,a^3\log\left(\cos(dx+c)-1\right)-10\,a^3\log\left(\cos(dx+c)\right)+\frac{10\,a^3\cos(dx+c)^2-5\,a^3\cos(dx+c)-a^3}{\cos(dx+c)^3-\cos(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(10*a^3*log(cos(d*x + c) - 1) - 10*a^3*log(cos(d*x + c)) + (10*a^3*cos(
d*x + c)^2 - 5*a^3*cos(d*x + c) - a^3)/(cos(d*x + c)^3 - cos(d*x + c)^2))/d
```

mupad [B] time = 0.10, size = 75, normalized size = 0.85

$$\frac{-5\,a^3\cos\left(c+d\,x\right)^2+\frac{5\,a^3\cos(c+d\,x)}{2}+\frac{a^3}{2}}{d\left(\cos\left(c+d\,x\right)^2-\cos\left(c+d\,x\right)^3\right)}-\frac{10\,a^3\,\mathrm{atanh}\left(2\,\cos\left(c+d\,x\right)-1\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^3,x)
```

```
[Out] ((5*a^3*cos(c + d*x))/2 + a^3/2 - 5*a^3*cos(c + d*x)^2)/(d*(cos(c + d*x)^2
- cos(c + d*x)^3)) - (10*a^3*atanh(2*cos(c + d*x) - 1))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\csc^3(c+dx)\sec(c+dx)\,dx+\int 3\csc^3(c+dx)\sec^2(c+dx)\,dx+\int \csc^3(c+dx)\sec^3(c+dx)\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*csc(c + d*x)**3*sec(c + d*x), x) + Integral(3*csc(c + d*x)
**3*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**3*sec(c + d*x)**3, x) + In
tegral(csc(c + d*x)**3, x))
```

3.45 $\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=111

$$-\frac{a^5}{2d(a-a\cos(c+dx))^2}-\frac{3a^4}{d(a-a\cos(c+dx))}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{6a^3\log(1-\cos(c+dx))}{d}-6a^3\,\mathrm{l}$$

```
[Out] -1/2*a^5/d/(a-a*cos(d*x+c))^2-3*a^4/d/(a-a*cos(d*x+c))+6*a^3*ln(1-cos(d*x+c
))/d-6*a^3*ln(cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

Rubi [A] time = 0.17, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3872, 2836, 12, 44}

$$-\frac{a^5}{2d(a-a\cos(c+dx))^2}-\frac{3a^4}{d(a-a\cos(c+dx))}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{6a^3\log(1-\cos(c+dx))}{d}-6a^3\,\mathrm{l}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -a^5/(2*d*(a - a*Cos[c + d*x])^2) - (3*a^4)/(d*(a - a*Cos[c + d*x])) + (6*a
^3*Log[1 - Cos[c + d*x]])/d - (6*a^3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[c +
d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] && !(IGtQ[n, 0] && LtQ[m
+ n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx = -\int (-a-a\cos(c+dx))^3\csc^5(c+dx)\sec^3(c+dx)\,dx$$
$$= \frac{a^5\,\text{Subst}\left(\int \frac{a^3}{(-a-x)^3x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^8\,\text{Subst}\left(\int \frac{1}{(-a-x)^3x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^8\,\text{Subst}\left(\int \left(-\frac{1}{a^3x^3}+\frac{3}{a^4x^2}-\frac{6}{a^5x}+\frac{1}{a^3(a+x)^3}+\frac{3}{a^4(a+x)^2}+\frac{6}{a^5(a+x)}\right)dx, x, -\right.}{d}$$
$$= -\frac{a^5}{2d(a-a\cos(c+dx))^2}-\frac{3a^4}{d(a-a\cos(c+dx))}+\frac{6a^3\log(1-\cos(c+}{d}$$

Mathematica [A] time = 0.95, size = 100, normalized size = 0.90

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\csc^4\left(\frac{1}{2}(c+dx)\right)+12\csc^2\left(\frac{1}{2}(c+dx)\right)-4\sec^2(c+dx)-24\sec(c+d\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/64*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(12*Csc[(c + d*x)/2]^2 +
 Csc[(c + d*x)/2]^4 + 48*(Log[Cos[c + d*x]] - 2*Log[Sin[(c + d*x)/2]]) - 24
*Sec[c + d*x] - 4*Sec[c + d*x]^2))/d
```

fricas [A] time = 0.79, size = 177, normalized size = 1.59

$$\frac{12\,a^3\cos(dx+c)^3-18\,a^3\cos(dx+c)^2+4\,a^3\cos(dx+c)+a^3-12\left(a^3\cos(dx+c)^4-2\,a^3\cos(dx+c)^3+a^3\right.}{2\left(d\cos(dx+c)^4-2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(12*a^3*cos(d*x + c)^3 - 18*a^3*cos(d*x + c)^2 + 4*a^3*cos(d*x + c) + a
^3 - 12*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2)*lo
g(-cos(d*x + c)) + 12*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 + a^3*cos(
d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x +
 c)^3 + d*cos(d*x + c)^2)
```

giac [A] time = 0.47, size = 186, normalized size = 1.68

$$\frac{48\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-48\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\frac{a^3-\frac{12\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{75\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{46\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(48*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 48*a^3*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^3 - 12*a^3*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) - 75*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
```

```
)^2 - 46*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/((cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + (cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)^2)/d
```

maple [A] time = 0.62, size = 85, normalized size = 0.77

$$\frac{a^3\left(\sec^2(dx+c)\right)}{2d}+\frac{3a^3\sec(dx+c)}{d}-\frac{4a^3}{d(-1+\sec(dx+c))}-\frac{a^3}{2d(-1+\sec(dx+c))^2}+\frac{6a^3\ln(-1+\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*sec(d*x+c)^2/d+3*a^3*sec(d*x+c)/d-4/d*a^3/(-1+sec(d*x+c))-1/2/d*a^3
/(-1+sec(d*x+c))^2+6/d*a^3*ln(-1+sec(d*x+c))
```

maxima [A] time = 0.66, size = 103, normalized size = 0.93

$$\frac{12\,a^3\log(\cos(dx+c)-1)-12\,a^3\log(\cos(dx+c))+\frac{12\,a^3\cos(dx+c)^3-18\,a^3\cos(dx+c)^2+4\,a^3\cos(dx+c)+a^3}{\cos(dx+c)^4-2\,\cos(dx+c)^3+\cos(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(12*a^3*log(cos(d*x + c) - 1) - 12*a^3*log(cos(d*x + c)) + (12*a^3*cos(
d*x + c)^3 - 18*a^3*cos(d*x + c)^2 + 4*a^3*cos(d*x + c) + a^3)/(cos(d*x + c
)^4 - 2*cos(d*x + c)^3 + cos(d*x + c)^2))/d
```

mupad [B] time = 0.93, size = 96, normalized size = 0.86

$$\frac{6\,a^3\cos(c+d\,x)^3-9\,a^3\cos(c+d\,x)^2+2\,a^3\,\cos(c+d\,x)+\frac{a^3}{2}}{d\,\left(\cos(c+d\,x)^4-2\cos(c+d\,x)^3+\cos(c+d\,x)^2\right)}-\frac{12\,a^3\,\mathrm{atanh}(2\,\cos(c+d\,x)-1)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^5,x)
```

```
[Out] (2*a^3*cos(c + d*x) + a^3/2 - 9*a^3*cos(c + d*x)^2 + 6*a^3*cos(c + d*x)^3)/
(d*(cos(c + d*x)^2 - 2*cos(c + d*x)^3 + cos(c + d*x)^4)) - (12*a^3*atanh(2*
cos(c + d*x) - 1))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.46 $\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=157

$$-\frac{a^6}{6d(a-a\cos(c+dx))^3}-\frac{7a^5}{8d(a-a\cos(c+dx))^2}-\frac{31a^4}{8d(a-a\cos(c+dx))}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{11}{}$$

```
[Out] -1/6*a^6/d/(a-a*cos(d*x+c))^3-7/8*a^5/d/(a-a*cos(d*x+c))^2-31/8*a^4/d/(a-a*
cos(d*x+c))+111/16*a^3*ln(1-cos(d*x+c))/d-7*a^3*ln(cos(d*x+c))/d+1/16*a^3*l
n(1+cos(d*x+c))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

Rubi [A] time = 0.20, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2836, 12, 88}

$$-\frac{a^6}{6d(a-a\cos(c+dx))^3}-\frac{7a^5}{8d(a-a\cos(c+dx))^2}-\frac{31a^4}{8d(a-a\cos(c+dx))}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{11}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^7*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -a^6/(6*d*(a - a*Cos[c + d*x])^3) - (7*a^5)/(8*d*(a - a*Cos[c + d*x])^2) -
(31*a^4)/(8*d*(a - a*Cos[c + d*x])) + (111*a^3*Log[1 - Cos[c + d*x]])/(16*d
) - (7*a^3*Log[Cos[c + d*x]])/d + (a^3*Log[1 + Cos[c + d*x]])/(16*d) + (3*a
^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx = -\int (-a-a\cos(c+dx))^3\csc^7(c+dx)\sec^3(c+dx)\,dx$$
$$= \frac{a^7\,\text{Subst}\left(\int \frac{a^3}{(-a-x)^4x^3(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^{10}\,\text{Subst}\left(\int \frac{1}{(-a-x)^4x^3(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^{10}\,\text{Subst}\left(\int \left(-\frac{1}{16a^7(a-x)} - \frac{1}{a^5x^3} + \frac{3}{a^6x^2} - \frac{7}{a^7x} + \frac{1}{2a^4(a+x)^4} + \frac{7}{4a^5(a+x)^3} + \frac{3}{8a^6(a}\right.\right.}{d}$$
$$= -\frac{a^6}{6d(a-a\cos(c+dx))^3} - \frac{7a^5}{8d(a-a\cos(c+dx))^2} - \frac{31a^4}{8d(a-a\cos(c+dx)}$$

Mathematica [A] time = 1.06, size = 129, normalized size = 0.82

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(2\csc^6\left(\frac{1}{2}(c+dx)\right)+21\csc^4\left(\frac{1}{2}(c+dx)\right)+186\csc^2\left(\frac{1}{2}(c+dx)\right)-12\left(4\,\text{se}\right.\right.}{768d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^7*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/768*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(186*Csc[(c + d*x)/2]^2
 + 21*Csc[(c + d*x)/2]^4 + 2*Csc[(c + d*x)/2]^6 - 12*(Log[Cos[(c + d*x)/2]]
 - 56*Log[Cos[c + d*x]] + 111*Log[Sin[(c + d*x)/2]] + 24*Sec[c + d*x] + 4*S
ec[c + d*x]^2)))/d
```

fricas [B] time = 0.67, size = 297, normalized size = 1.89

$$330\,a^3\cos(dx+c)^4 - 822\,a^3\cos(dx+c)^3 + 596\,a^3\cos(dx+c)^2 - 72\,a^3\cos(dx+c) - 24\,a^3 - 336\left(a^3\cos(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(330*a^3*cos(d*x + c)^4 - 822*a^3*cos(d*x + c)^3 + 596*a^3*cos(d*x + c
)^2 - 72*a^3*cos(d*x + c) - 24*a^3 - 336*(a^3*cos(d*x + c)^5 - 3*a^3*cos(d*
x + c)^4 + 3*a^3*cos(d*x + c)^3 - a^3*cos(d*x + c)^2)*log(-cos(d*x + c)) +
3*(a^3*cos(d*x + c)^5 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 - a^3*c
os(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + 333*(a^3*cos(d*x + c)^5 - 3*a^
3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 - a^3*cos(d*x + c)^2)*log(-1/2*cos(
d*x + c) + 1/2))/(d*cos(d*x + c)^5 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^
3 - d*cos(d*x + c)^2)
```

giac [A] time = 0.76, size = 243, normalized size = 1.55

$$\frac{666\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 672\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{\left(2\,a^3 - \frac{27\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{234\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{1221\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}\right)(\text{co}}{(\cos(dx+c)-1)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(666*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 672*a^3*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2*a^3 - 27*a^3*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + 234*a^3*(cos(d*x + c) - 1)^2/(cos(d*x +
c) + 1)^2 - 1221*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x +
c) + 1)^3/(cos(d*x + c) - 1)^3 + 48*(33*a^3 + 50*a^3*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 21*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

maple [A] time = 0.61, size = 120, normalized size = 0.76

$$\frac{a^3 \left(\sec^2 (dx+c)\right)}{2d}+\frac{3a^3 \sec (dx+c)}{d}-\frac{a^3}{6d\left(-1+\sec (dx+c)\right)^3}-\frac{11a^3}{8d\left(-1+\sec (dx+c)\right)^2}-\frac{49a^3}{8d\left(-1+\sec (dx+c)\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^7*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*sec(d*x+c)^2/d+3*a^3*sec(d*x+c)/d-1/6/d*a^3/(-1+sec(d*x+c))^3-11/8/
d*a^3/(-1+sec(d*x+c))^2-49/8/d*a^3/(-1+sec(d*x+c))+111/16/d*a^3*ln(-1+sec(d
*x+c))+1/16/d*a^3*ln(1+sec(d*x+c))
```

maxima [A] time = 0.43, size = 145, normalized size = 0.92

$$\frac{3\,a^3 \log\left(\cos\left(dx+c\right)+1\right)+333\,a^3 \log\left(\cos\left(dx+c\right)-1\right)-336\,a^3 \log\left(\cos\left(dx+c\right)\right)+\frac{2\left(165\,a^3 \cos(dx+c)^4-411\,a^3 \cos\right.}{\cos(dx+c)^5-3\,c}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/48*(3*a^3*log(cos(d*x + c) + 1) + 333*a^3*log(cos(d*x + c) - 1) - 336*a^3
*log(cos(d*x + c)) + 2*(165*a^3*cos(d*x + c)^4 - 411*a^3*cos(d*x + c)^3 + 2
98*a^3*cos(d*x + c)^2 - 36*a^3*cos(d*x + c) - 12*a^3)/(cos(d*x + c)^5 - 3*c
os(d*x + c)^4 + 3*cos(d*x + c)^3 - cos(d*x + c)^2))/d
```

mupad [B] time = 0.96, size = 151, normalized size = 0.96

$$\frac{111\,a^3\,\ln\left(\cos\left(c+d\,x\right)-1\right)}{16\,d}+\frac{a^3\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{16\,d}+\frac{-\frac{55\,a^3\cos(c+d\,x)^4}{8}+\frac{137\,a^3\cos(c+d\,x)^3}{8}-\frac{149\,a^3\cos(c+d\,x)^2}{12}+\frac{3\,a}{}}{d\,\left(-\cos\left(c+d\,x\right)^5+3\cos\left(c+d\,x\right)^4-3\cos\left(c+d\,x\right)^3+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^7,x)
```

```
[Out] (111*a^3*log(cos(c + d*x) - 1))/(16*d) + (a^3*log(cos(c + d*x) + 1))/(16*d)
 + ((3*a^3*cos(c + d*x))/2 + a^3/2 - (149*a^3*cos(c + d*x)^2)/12 + (137*a^3
*cos(c + d*x)^3)/8 - (55*a^3*cos(c + d*x)^4)/8)/(d*(cos(c + d*x)^2 - 3*cos(
c + d*x)^3 + 3*cos(c + d*x)^4 - cos(c + d*x)^5)) - (7*a^3*log(cos(c + d*x))
)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**7*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.47 $\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=202

$$-\frac{a^7}{16d(a-a\cos(c+dx))^4}-\frac{a^6}{3d(a-a\cos(c+dx))^3}-\frac{39a^5}{32d(a-a\cos(c+dx))^2}-\frac{75a^4}{16d(a-a\cos(c+dx))}-\frac{a^4}{32d(a\cos(c}$$

```
[Out] -1/16*a^7/d/(a-a*cos(d*x+c))^4-1/3*a^6/d/(a-a*cos(d*x+c))^3-39/32*a^5/d/(a-
a*cos(d*x+c))^2-75/16*a^4/d/(a-a*cos(d*x+c))-1/32*a^4/d/(a+a*cos(d*x+c))+50
1/64*a^3*ln(1-cos(d*x+c))/d-8*a^3*ln(cos(d*x+c))/d+11/64*a^3*ln(1+cos(d*x+c
))/d+3*a^3*sec(d*x+c)/d+1/2*a^3*sec(d*x+c)^2/d
```

Rubi [A] time = 0.23, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3872, 2836, 12, 88}

$$-\frac{a^7}{16d(a-a\cos(c+dx))^4}-\frac{a^6}{3d(a-a\cos(c+dx))^3}-\frac{39a^5}{32d(a-a\cos(c+dx))^2}-\frac{75a^4}{16d(a-a\cos(c+dx))}-\frac{a^4}{32d(a\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^9*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -a^7/(16*d*(a - a*Cos[c + d*x])^4) - a^6/(3*d*(a - a*Cos[c + d*x])^3) - (39
*a^5)/(32*d*(a - a*Cos[c + d*x])^2) - (75*a^4)/(16*d*(a - a*Cos[c + d*x]))
- a^4/(32*d*(a + a*Cos[c + d*x])) + (501*a^3*Log[1 - Cos[c + d*x]])/(64*d)
- (8*a^3*Log[Cos[c + d*x]])/d + (11*a^3*Log[1 + Cos[c + d*x]])/(64*d) + (3*
a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx = -\int (-a-a\cos(c+dx))^3\csc^9(c+dx)\sec^3(c+dx)\,dx$$
$$= \frac{a^9\,\text{Subst}\left(\int \frac{a^3}{(-a-x)^5x^3(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^{12}\,\text{Subst}\left(\int \frac{1}{(-a-x)^5x^3(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^{12}\,\text{Subst}\left(\int \left(-\frac{1}{32a^8(a-x)^2} - \frac{11}{64a^9(a-x)} - \frac{1}{a^7x^3} + \frac{3}{a^8x^2} - \frac{8}{a^9x} + \frac{1}{4a^5(a+x)^5} + \right.\right.}{d}$$
$$= -\frac{a^7}{16d(a-a\cos(c+dx))^4} - \frac{a^6}{3d(a-a\cos(c+dx))^3} - \frac{39a^5}{32d(a-a\cos(c}$$

Mathematica [A] time = 1.23, size = 159, normalized size = 0.79

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(3\csc^8\left(\frac{1}{2}(c+dx)\right)+32\csc^6\left(\frac{1}{2}(c+dx)\right)+234\csc^4\left(\frac{1}{2}(c+dx)\right)+180\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^9*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/6144*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(1800*Csc[(c + d*x)/2]
^2 + 234*Csc[(c + d*x)/2]^4 + 32*Csc[(c + d*x)/2]^6 + 3*Csc[(c + d*x)/2]^8
- 12*(22*Log[Cos[(c + d*x)/2]] - 512*Log[Cos[c + d*x]] + 1002*Log[Sin[(c +
d*x)/2]] - Sec[(c + d*x)/2]^2 + 192*Sec[c + d*x] + 32*Sec[c + d*x]^2)))/d
```

fricas [B] time = 1.58, size = 419, normalized size = 2.07

$$\frac{1470\,a^3\cos(dx+c)^6 - 3642\,a^3\cos(dx+c)^5 + 1126\,a^3\cos(dx+c)^4 + 3390\,a^3\cos(dx+c)^3 - 2752\,a^3\cos(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/192*(1470*a^3*cos(d*x + c)^6 - 3642*a^3*cos(d*x + c)^5 + 1126*a^3*cos(d*x
 + c)^4 + 3390*a^3*cos(d*x + c)^3 - 2752*a^3*cos(d*x + c)^2 + 288*a^3*cos(d
*x + c) + 96*a^3 - 1536*(a^3*cos(d*x + c)^7 - 3*a^3*cos(d*x + c)^6 + 2*a^3*
cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^3 + a^3*cos(d*x
+ c)^2)*log(-cos(d*x + c)) + 33*(a^3*cos(d*x + c)^7 - 3*a^3*cos(d*x + c)^6
+ 2*a^3*cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^3 + a^3*
cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + 1503*(a^3*cos(d*x + c)^7 - 3*
a^3*cos(d*x + c)^6 + 2*a^3*cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*co
s(d*x + c)^3 + a^3*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x
 + c)^7 - 3*d*cos(d*x + c)^6 + 2*d*cos(d*x + c)^5 + 2*d*cos(d*x + c)^4 - 3*
d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

giac [A] time = 0.65, size = 292, normalized size = 1.45

$$\frac{6012\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 6144\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{12\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{\left(3\,a^3 - \frac{44\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{348\,a^3(\cos(dx+c}{(\cos(dx+c)+1}\right.}{}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/768*(6012*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 6144*a^
3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 12*a^3*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - (3*a^3 - 44*a^3*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) + 348*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 2376*a^3*(cos
(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 12525*a^3*(cos(d*x + c) - 1)^4/(cos
(d*x + c) + 1)^4)*(cos(d*x + c) + 1)^4/(cos(d*x + c) - 1)^4 + 1536*(9*a^3 +
 14*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*a^3*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

maple [A] time = 0.72, size = 156, normalized size = 0.77

$$\frac{a^3 \left(\sec^2 (dx+c)\right)}{2d}+\frac{3a^3 \sec (dx+c)}{d}-\frac{a^3}{16d \left(-1+\sec (dx+c)\right)^4}-\frac{7a^3}{12d \left(-1+\sec (dx+c)\right)^3}-\frac{83a^3}{32d \left(-1+\sec (dx+c)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^9*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*sec(d*x+c)^2/d+3*a^3*sec(d*x+c)/d-1/16/d*a^3/(-1+sec(d*x+c))^4-7/12
/d*a^3/(-1+sec(d*x+c))^3-83/32/d*a^3/(-1+sec(d*x+c))^2-67/8/d*a^3/(-1+sec(d
*x+c))+501/64/d*a^3*ln(-1+sec(d*x+c))+1/32/d*a^3/(1+sec(d*x+c))+11/64/d*a^3
*ln(1+sec(d*x+c))
```

maxima [A] time = 0.51, size = 189, normalized size = 0.94

$$\frac{33\,a^3 \log\left(\cos\left(dx+c\right)+1\right)+1503\,a^3 \log\left(\cos\left(dx+c\right)-1\right)-1536\,a^3 \log\left(\cos\left(dx+c\right)\right)+\frac{2\left(735\,a^3 \cos(dx+c)^6-1821\,a^3 \cdots\right)}{\cos(dx+\cdots}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/192*(33*a^3*log(cos(d*x + c) + 1) + 1503*a^3*log(cos(d*x + c) - 1) - 1536
*a^3*log(cos(d*x + c)) + 2*(735*a^3*cos(d*x + c)^6 - 1821*a^3*cos(d*x + c)^
5 + 563*a^3*cos(d*x + c)^4 + 1695*a^3*cos(d*x + c)^3 - 1376*a^3*cos(d*x + c
)^2 + 144*a^3*cos(d*x + c) + 48*a^3)/(cos(d*x + c)^7 - 3*cos(d*x + c)^6 + 2
*cos(d*x + c)^5 + 2*cos(d*x + c)^4 - 3*cos(d*x + c)^3 + cos(d*x + c)^2))/d
```

mupad [B] time = 1.01, size = 195, normalized size = 0.97

$$\frac{501\,a^3\,\ln\left(\cos\left(c+d\,x\right)-1\right)}{64\,d}+\frac{11\,a^3\,\ln\left(\cos\left(c+d\,x\right)+1\right)}{64\,d}+\frac{\frac{245\,a^3\,\cos(c+d\,x)^6}{32}-\frac{607\,a^3\,\cos(c+d\,x)^5}{32}+\frac{563\,a^3\,\cos(c+d\,x)^4}{96}+\cdots}{d\,\left(\cos\left(c+d\,x\right)^7-3\cos\left(c+d\,x\right)^6+2\cos\left(c+d\,x\right)^5+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^9,x)
```

```
[Out] (501*a^3*log(cos(c + d*x) - 1))/(64*d) + (11*a^3*log(cos(c + d*x) + 1))/(64
*d) + ((3*a^3*cos(c + d*x))/2 + a^3/2 - (43*a^3*cos(c + d*x)^2)/3 + (565*a^
3*cos(c + d*x)^3)/32 + (563*a^3*cos(c + d*x)^4)/96 - (607*a^3*cos(c + d*x)^
5)/32 + (245*a^3*cos(c + d*x)^6)/32)/(d*(cos(c + d*x)^2 - 3*cos(c + d*x)^3
+ 2*cos(c + d*x)^4 + 2*cos(c + d*x)^5 - 3*cos(c + d*x)^6 + cos(c + d*x)^7))
 - (8*a^3*log(cos(c + d*x)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**9*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.48 $\int (a+a\sec(c+dx))^3 \sin^8(c+dx)\,dx$

Optimal. Leaf size=210

$$-\frac{3a^3\sin^7(c+dx)}{7d}-\frac{2a^3\sin^5(c+dx)}{5d}-\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\tan(c+dx)}{d}-\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^3\sin(c+dx)\,\mathrm{co}}{8d}$$

```
[Out] -805/128*a^3*x-1/2*a^3*arctanh(sin(d*x+c))/d+603/128*a^3*cos(d*x+c)*sin(d*x
+c)/d-293/192*a^3*cos(d*x+c)^3*sin(d*x+c)/d-1/48*a^3*cos(d*x+c)^5*sin(d*x+c
)/d+1/8*a^3*cos(d*x+c)^7*sin(d*x+c)/d-1/3*a^3*sin(d*x+c)^3/d-2/5*a^3*sin(d*
x+c)^5/d-3/7*a^3*sin(d*x+c)^7/d+3*a^3*tan(d*x+c)/d+1/2*a^3*sec(d*x+c)*tan(d
*x+c)/d
```

Rubi [A] time = 0.39, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 29, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3872, 2872, 2637, 2635, 8, 2633, 3770, 3767, 3768}

$$-\frac{3a^3\sin^7(c+dx)}{7d}-\frac{2a^3\sin^5(c+dx)}{5d}-\frac{a^3\sin^3(c+dx)}{3d}+\frac{3a^3\tan(c+dx)}{d}-\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^3\sin(c+dx)\,\mathrm{co}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^8,x]
```

```
[Out] (-805*a^3*x)/128 - (a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (603*a^3*Cos[c + d*x
]*Sin[c + d*x])/(128*d) - (293*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) - (
a^3*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (a^3*Cos[c + d*x]^7*Sin[c + d*x])
/(8*d) - (a^3*Sin[c + d*x]^3)/(3*d) - (2*a^3*Sin[c + d*x]^5)/(5*d) - (3*a^3
*Sin[c + d*x]^7)/(7*d) + (3*a^3*Tan[c + d*x])/d + (a^3*Sec[c + d*x]*Tan[c +
 d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
```

```
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\sec(c+dx))^3\sin^8(c+dx)\,dx &= -\int (-a-a\cos(c+dx))^3\sin^5(c+dx)\tan^3(c+dx)\,dx\\
&= -\frac{\int \left(11a^{11}+6a^{11}\cos(c+dx)-14a^{11}\cos^2(c+dx)-14a^{11}\cos^3(c+d\right.}{}\\
&= -11a^3x-a^3\int\cos^6(c+dx)\,dx+a^3\int\cos^8(c+dx)\,dx-a^3\int\sec(c+\\
&= -11a^3x-\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{6a^3\sin(c+dx)}{d}+\frac{7a^3\cos(c+dx)}{d}\\
&= -4a^3x-\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{19a^3\cos(c+dx)\sin(c+dx)}{4d}-\frac{41a}{}\\
&= -\frac{25a^3x}{4}-\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{71a^3\cos(c+dx)\sin(c+dx)}{16d}-\frac{2}{}\\
&= -\frac{105a^3x}{16}-\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{603a^3\cos(c+dx)\sin(c+dx)}{128d}-\\
&= -\frac{805a^3x}{128}-\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{603a^3\cos(c+dx)\sin(c+dx)}{128d}-
\end{aligned}
$$

Mathematica [A] time = 2.10, size = 156, normalized size = 0.74

$$\frac{a^3\sec^2(c+dx)\left(173600\sin(c+dx)+1052520\sin(2(c+dx))-11648\sin(3(c+dx))+175280\sin(4(c+dx))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^8,x]
```

```
[Out] (a^3*Sec[c + d*x]^2*(-1352400*c - 1352400*d*x - 215040*ArcTanh[Sin[c + d*x]
]*Cos[c + d*x]^2 - 1352400*(c + d*x)*Cos[2*(c + d*x)] + 173600*Sin[c + d*x]
```

```
 + 1052520*Sin[2*(c + d*x)] - 11648*Sin[3*(c + d*x)] + 175280*Sin[4*(c + d*
x)] + 22784*Sin[5*(c + d*x)] - 18095*Sin[6*(c + d*x)] - 6288*Sin[7*(c + d*x
)] + 770*Sin[8*(c + d*x)] + 720*Sin[9*(c + d*x)] + 105*Sin[10*(c + d*x)]))/
(430080*d)
```

fricas [A] time = 0.74, size = 204, normalized size = 0.97

$$-\frac{84525\,a^3dx\cos\left(dx+c\right)^2+3360\,a^3\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-3360\,a^3\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/13440*(84525*a^3*d*x*cos(d*x + c)^2 + 3360*a^3*cos(d*x + c)^2*log(sin(d*
x + c) + 1) - 3360*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - (1680*a^3*co
s(d*x + c)^9 + 5760*a^3*cos(d*x + c)^8 - 280*a^3*cos(d*x + c)^7 - 22656*a^3
*cos(d*x + c)^6 - 20510*a^3*cos(d*x + c)^5 + 32512*a^3*cos(d*x + c)^4 + 633
15*a^3*cos(d*x + c)^3 - 15616*a^3*cos(d*x + c)^2 + 40320*a^3*cos(d*x + c) +
 6720*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [A] time = 0.51, size = 244, normalized size = 1.16

$$-\frac{84525\,(dx+c)a^3+6720\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6720\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{13440\left(5\,a^3\tan\left(\frac{1}{2}\,dx\right.\right.}{\left(\tan\left(\frac{1}{2}\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/13440*(84525*(d*x + c)*a^3 + 6720*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1))
 - 6720*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 13440*(5*a^3*tan(1/2*d*x +
 1/2*c)^3 - 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + 2*
(44205*a^3*tan(1/2*d*x + 1/2*c)^15 + 303065*a^3*tan(1/2*d*x + 1/2*c)^13 + 8
41981*a^3*tan(1/2*d*x + 1/2*c)^11 + 1123793*a^3*tan(1/2*d*x + 1/2*c)^9 + 48
7983*a^3*tan(1/2*d*x + 1/2*c)^7 - 490749*a^3*tan(1/2*d*x + 1/2*c)^5 - 26722
5*a^3*tan(1/2*d*x + 1/2*c)^3 - 44205*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
 + 1/2*c)^2 + 1)^8)/d
```

maple [A] time = 0.93, size = 235, normalized size = 1.12

$$\frac{23a^3\left(\sin^7\left(dx+c\right)\right)\cos\left(dx+c\right)}{8d}+\frac{161a^3\cos\left(dx+c\right)\left(\sin^5\left(dx+c\right)\right)}{48d}+\frac{805a^3\cos\left(dx+c\right)\left(\sin^3\left(dx+c\right)\right)}{192d}+\frac{805a^3\cos\ldots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^8,x)
```

```
[Out] 23/8/d*a^3*sin(d*x+c)^7*cos(d*x+c)+161/48*a^3*cos(d*x+c)*sin(d*x+c)^5/d+805
/192*a^3*cos(d*x+c)*sin(d*x+c)^3/d+805/128*a^3*cos(d*x+c)*sin(d*x+c)/d-805/
128*a^3*x-805/128/d*a^3*c+1/14*a^3*sin(d*x+c)^7/d+1/10*a^3*sin(d*x+c)^5/d+1
/6*a^3*sin(d*x+c)^3/d+1/2*a^3*sin(d*x+c)/d-1/2/d*a^3*ln(sec(d*x+c)+tan(d*x+
c))+3/d*a^3*sin(d*x+c)^9/cos(d*x+c)+1/2/d*a^3*sin(d*x+c)^9/cos(d*x+c)^2
```

maxima [A] time = 0.47, size = 291, normalized size = 1.39

$$-\frac{1536\left(30\,\sin\left(dx+c\right)^7+42\,\sin\left(dx+c\right)^5+70\,\sin\left(dx+c\right)^3-105\,\log\left(\sin\left(dx+c\right)+1\right)+105\,\log\left(\sin\left(dx+c\right)\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/107520*(1536*(30*sin(d*x + c)^7 + 42*sin(d*x + c)^5 + 70*sin(d*x + c)^3
- 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1) + 210*sin(d*x + c))
*a^3 - 1792*(12*sin(d*x + c)^5 + 40*sin(d*x + c)^3 - 30*sin(d*x + c)/(sin(d
*x + c)^2 - 1) - 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1) + 18
0*sin(d*x + c))*a^3 - 35*(128*sin(2*d*x + 2*c)^3 + 840*d*x + 840*c + 3*sin(
8*d*x + 8*c) + 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a^3 + 6720*(105
*d*x + 105*c - (87*tan(d*x + c)^5 + 136*tan(d*x + c)^3 + 57*tan(d*x + c))/(
tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1) - 48*tan(d*x + c)
)*a^3)/d
```

mupad [B] time = 2.45, size = 320, normalized size = 1.52

$$\frac{-\frac{741\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{19}}{64}-\frac{12469\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{17}}{192}-\frac{5027\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{40}-\frac{19211\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{420}+\frac{199977\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{1120}+\frac{877061}{}}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{20}+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}+13\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}+8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}-14\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-28\tan\left(\frac{c}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8*(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((4967*a^3*tan(c/2 + (d*x)/2)^3)/64 + (6243*a^3*tan(c/2 + (d*x)/2)^5)/40 +
(10233*a^3*tan(c/2 + (d*x)/2)^7)/140 + (877061*a^3*tan(c/2 + (d*x)/2)^9)/33
60 + (199977*a^3*tan(c/2 + (d*x)/2)^11)/1120 - (19211*a^3*tan(c/2 + (d*x)/2
)^13)/420 - (5027*a^3*tan(c/2 + (d*x)/2)^15)/40 - (12469*a^3*tan(c/2 + (d*x
)/2)^17)/192 - (741*a^3*tan(c/2 + (d*x)/2)^19)/64 + (869*a^3*tan(c/2 + (d*x
)/2))/64)/(d*(6*tan(c/2 + (d*x)/2)^2 + 13*tan(c/2 + (d*x)/2)^4 + 8*tan(c/2
+ (d*x)/2)^6 - 14*tan(c/2 + (d*x)/2)^8 - 28*tan(c/2 + (d*x)/2)^10 - 14*tan(
c/2 + (d*x)/2)^12 + 8*tan(c/2 + (d*x)/2)^14 + 13*tan(c/2 + (d*x)/2)^16 + 6*
tan(c/2 + (d*x)/2)^18 + tan(c/2 + (d*x)/2)^20 + 1)) - (a^3*atanh(tan(c/2 +
(d*x)/2)))/d - (805*a^3*x)/128
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**8,x)
```

```
[Out] Timed out
```

3.49 $\int (a+a\sec(c+dx))^3\sin^6(c+dx)\,dx$

Optimal. Leaf size=182

$$-\frac{3a^3\sin^5(c+dx)}{5d}-\frac{2a^3\sin^3(c+dx)}{3d}-\frac{a^3\sin(c+dx)}{d}+\frac{3a^3\tan(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\sin(c+dx)\cos}{6d}$$

```
[Out] -85/16*a^3*x+1/2*a^3*arctanh(sin(d*x+c))/d-a^3*sin(d*x+c)/d+43/16*a^3*cos(d
*x+c)*sin(d*x+c)/d-5/24*a^3*cos(d*x+c)^3*sin(d*x+c)/d-1/6*a^3*cos(d*x+c)^5*
sin(d*x+c)/d-2/3*a^3*sin(d*x+c)^3/d-3/5*a^3*sin(d*x+c)^5/d+3*a^3*tan(d*x+c)
/d+1/2*a^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.27, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3872, 2872, 2637, 2635, 8, 2633, 3767, 3768, 3770}

$$-\frac{3a^3\sin^5(c+dx)}{5d}-\frac{2a^3\sin^3(c+dx)}{3d}-\frac{a^3\sin(c+dx)}{d}+\frac{3a^3\tan(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\sin(c+dx)\cos}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^6,x]
```

```
[Out] (-85*a^3*x)/16 + (a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (a^3*Sin[c + d*x])/d +
 (43*a^3*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (5*a^3*Cos[c + d*x]^3*Sin[c +
d*x])/(24*d) - (a^3*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (2*a^3*Sin[c + d*x
]^3)/(3*d) - (3*a^3*Sin[c + d*x]^5)/(5*d) + (3*a^3*Tan[c + d*x])/d + (a^3*S
ec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a + a\sec(c+dx))^3 \sin^6(c+dx)\,dx = -\int (-a - a\cos(c+dx))^3 \sin^3(c+dx)\tan^3(c+dx)\,dx$$
$$= -\frac{\int \left(8a^9 + 6a^9\cos(c+dx) - 6a^9\cos^2(c+dx) - 8a^9\cos^3(c+dx) + 3a \dots\right.}{\dots}$$
$$= -8a^3x - a^3\int \cos^6(c+dx)\,dx + a^3\int \sec^3(c+dx)\,dx - \left(3a^3\right)\int \cos^5 \dots$$
$$= -8a^3x - \frac{6a^3\sin(c+dx)}{d} + \frac{3a^3\cos(c+dx)\sin(c+dx)}{d} - \frac{a^3\cos^5(c+ \dots}{\dots}$$
$$= -5a^3x + \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d} - \frac{a^3\sin(c+dx)}{d} + \frac{3a^3\cos(c+dx)\sin \dots}{d}$$
$$= -5a^3x + \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d} - \frac{a^3\sin(c+dx)}{d} + \frac{43a^3\cos(c+dx) \dots}{16d}$$
$$= -\frac{85a^3x}{16} + \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d} - \frac{a^3\sin(c+dx)}{d} + \frac{43a^3\cos(c+dx \dots}{16d \dots}$$

Mathematica [A] time = 0.95, size = 136, normalized size = 0.75

$$-\frac{a^3\sec^2(c+dx)\left(-460\sin(c+dx) - 8145\sin(2(c+dx)) + 1156\sin(3(c+dx)) - 1120\sin(4(c+dx)) - 268\sin \dots\right.}{\dots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^6,x]
```

```
[Out] -1/3840*(a^3*Sec[c + d*x]^2*(10200*c + 10200*d*x - 1920*ArcTanh[Sin[c + d*x
]]*Cos[c + d*x]^2 + 10200*(c + d*x)*Cos[2*(c + d*x)] - 460*Sin[c + d*x] - 8
145*Sin[2*(c + d*x)] + 1156*Sin[3*(c + d*x)] - 1120*Sin[4*(c + d*x)] - 268*
Sin[5*(c + d*x)] + 55*Sin[6*(c + d*x)] + 36*Sin[7*(c + d*x)] + 5*Sin[8*(c +
 d*x)]))/d
```

fricas [A] time = 0.74, size = 177, normalized size = 0.97

$$-\frac{1275\,a^3dx\cos\left(dx+c\right)^2-60\,a^3\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)+60\,a^3\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/240*(1275*a^3*d*x*cos(d*x + c)^2 - 60*a^3*cos(d*x + c)^2*log(sin(d*x + c
) + 1) + 60*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + (40*a^3*cos(d*x + c
)^7 + 144*a^3*cos(d*x + c)^6 + 50*a^3*cos(d*x + c)^5 - 448*a^3*cos(d*x + c)
^4 - 645*a^3*cos(d*x + c)^3 + 544*a^3*cos(d*x + c)^2 - 720*a^3*cos(d*x + c)
 - 120*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [A] time = 0.45, size = 212, normalized size = 1.16

$$-\frac{1275\left(dx+c\right)a^3-120\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+120\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{240\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/240*(1275*(d*x + c)*a^3 - 120*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 1
20*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 240*(5*a^3*tan(1/2*d*x + 1/2*c)
^3 - 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + 2*(795*a^
3*tan(1/2*d*x + 1/2*c)^11 + 4025*a^3*tan(1/2*d*x + 1/2*c)^9 + 7614*a^3*tan(
1/2*d*x + 1/2*c)^7 + 5634*a^3*tan(1/2*d*x + 1/2*c)^5 - 345*a^3*tan(1/2*d*x
+ 1/2*c)^3 - 315*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/
d
```

maple [A] time = 0.84, size = 197, normalized size = 1.08

$$\frac{17a^3\cos\left(dx+c\right)\left(\sin^5\left(dx+c\right)\right)}{6d}+\frac{85a^3\cos\left(dx+c\right)\left(\sin^3\left(dx+c\right)\right)}{24d}+\frac{85a^3\cos\left(dx+c\right)\sin\left(dx+c\right)}{16d}-\frac{85a^3x}{16}-\frac{85a^3c}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^6,x)
```

```
[Out] 17/6*a^3*cos(d*x+c)*sin(d*x+c)^5/d+85/24*a^3*cos(d*x+c)*sin(d*x+c)^3/d+85/1
6*a^3*cos(d*x+c)*sin(d*x+c)/d-85/16*a^3*x-85/16/d*a^3*c-1/10*a^3*sin(d*x+c)
^5/d-1/6*a^3*sin(d*x+c)^3/d-1/2*a^3*sin(d*x+c)/d+1/2/d*a^3*ln(sec(d*x+c)+ta
n(d*x+c))+3/d*a^3*sin(d*x+c)^7/cos(d*x+c)+1/2/d*a^3*sin(d*x+c)^7/cos(d*x+c)
^2
```

maxima [A] time = 0.44, size = 240, normalized size = 1.32

$$-\frac{96\left(6\sin\left(dx+c\right)^5+10\sin\left(dx+c\right)^3-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)+30\sin\left(dx+c\right)\right)a^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] -1/960*(96*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d*x + c) + 1)
 + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a^3 - 5*(4*sin(2*d*x + 2*c)^
```

```
3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^3 - 80*(4*s
in(d*x + c)^3 - 6*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) +
 1) + 15*log(sin(d*x + c) - 1) + 24*sin(d*x + c))*a^3 + 360*(15*d*x + 15*c
- (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 +
1) - 8*tan(d*x + c))*a^3)/d
```

mupad [B]   time = 2.22, size = 261, normalized size = 1.43

$$\frac{a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{85\,a^3\,x}{16}+\frac{-\frac{93\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{8}-\frac{1039\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{24}-\frac{4319\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{120}+\frac{6169\,a^3\,\tan\left(\frac{c}{2}+\right.}{120}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}+4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (a^3*atanh(tan(c/2 + (d*x)/2)))/d - (85*a^3*x)/16 + ((277*a^3*tan(c/2 + (d*
x)/2)^3)/8 + (997*a^3*tan(c/2 + (d*x)/2)^5)/40 + (3933*a^3*tan(c/2 + (d*x)/
2)^7)/40 + (6169*a^3*tan(c/2 + (d*x)/2)^9)/120 - (4319*a^3*tan(c/2 + (d*x)/
2)^11)/120 - (1039*a^3*tan(c/2 + (d*x)/2)^13)/24 - (93*a^3*tan(c/2 + (d*x)/
2)^15)/8 + (77*a^3*tan(c/2 + (d*x)/2))/8)/(d*(4*tan(c/2 + (d*x)/2)^2 + 4*ta
n(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^6 - 10*tan(c/2 + (d*x)/2)^8 - 4*t
an(c/2 + (d*x)/2)^10 + 4*tan(c/2 + (d*x)/2)^12 + 4*tan(c/2 + (d*x)/2)^14 +
tan(c/2 + (d*x)/2)^16 + 1))
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**6,x)
```

```
[Out] Timed out
```

3.50 $\int (a + a\sec(c + dx))^3 \sin^4(c + dx)\, dx$

Optimal. Leaf size=138

$$-\frac{a^3\sin^3(c+dx)}{d}-\frac{2a^3\sin(c+dx)}{d}+\frac{3a^3\tan(c+dx)}{d}+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{7a^3\mathrm{s}}{}$$

```
[Out] -33/8*a^3*x+3/2*a^3*arctanh(sin(d*x+c))/d-2*a^3*sin(d*x+c)/d+7/8*a^3*cos(d*
x+c)*sin(d*x+c)/d+1/4*a^3*cos(d*x+c)^3*sin(d*x+c)/d-a^3*sin(d*x+c)^3/d+3*a^
3*tan(d*x+c)/d+1/2*a^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.23, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3872, 2872, 2637, 2635, 8, 2633, 3770, 3767, 3768}

$$-\frac{a^3\sin^3(c+dx)}{d}-\frac{2a^3\sin(c+dx)}{d}+\frac{3a^3\tan(c+dx)}{d}+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{7a^3\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^4,x]
```

```
[Out] (-33*a^3*x)/8 + (3*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (2*a^3*Sin[c + d*x])/
d + (7*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^3*Cos[c + d*x]^3*Sin[c + d
*x])/(4*d) - (a^3*Sin[c + d*x]^3)/d + (3*a^3*Tan[c + d*x])/d + (a^3*Sec[c +
 d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\sin^4(c+dx)\,dx = -\int (-a-a\cos(c+dx))^3\sin(c+dx)\tan^3(c+dx)\,dx$$
$$= -\frac{\int \left(5a^7+5a^7\cos(c+dx)-a^7\cos^2(c+dx)-3a^7\cos^3(c+dx)-a^7 c\right.}{}$$
$$= -5a^3x + a^3\int \cos^2(c+dx)\,dx + a^3\int \cos^4(c+dx)\,dx + a^3\int \sec(c+$$
$$= -5a^3x + \frac{a^3\tanh^{-1}(\sin(c+dx))}{d} - \frac{5a^3\sin(c+dx)}{d} + \frac{a^3\cos(c+dx)\,\mathrm{si}}{2d}$$
$$= -\frac{9a^3x}{2} + \frac{3a^3\tanh^{-1}(\sin(c+dx))}{2d} - \frac{2a^3\sin(c+dx)}{d} + \frac{7a^3\cos(c+dx}{8a}$$
$$= -\frac{33a^3x}{8} + \frac{3a^3\tanh^{-1}(\sin(c+dx))}{2d} - \frac{2a^3\sin(c+dx)}{d} + \frac{7a^3\cos(c+d}{8}$$

Mathematica [A] time = 0.41, size = 114, normalized size = 0.83

$$\frac{a^3\sec^2(c+dx)\left(-16\sin(c+dx)+225\sin(2(c+dx))-72\sin(3(c+dx))+18\sin(4(c+dx))+8\sin(5(c+dx)\right.}{128d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^4,x]
```

```
[Out] (a^3*Sec[c + d*x]^2*(-264*c - 264*d*x + 192*ArcTanh[Sin[c + d*x]]*Cos[c + d
*x]^2 - 264*(c + d*x)*Cos[2*(c + d*x)] - 16*Sin[c + d*x] + 225*Sin[2*(c + d
*x)] - 72*Sin[3*(c + d*x)] + 18*Sin[4*(c + d*x)] + 8*Sin[5*(c + d*x)] + Sin
[6*(c + d*x)]))/(128*d)
```

fricas [A] time = 0.74, size = 152, normalized size = 1.10

$$-\frac{33\,a^3dx\cos(dx+c)^2-6\,a^3\cos(dx+c)^2\log(\sin(dx+c)+1)+6\,a^3\cos(dx+c)^2\log(-\sin(dx+c)+1)-}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/8*(33*a^3*d*x*cos(d*x + c)^2 - 6*a^3*cos(d*x + c)^2*log(sin(d*x + c) + 1
) + 6*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - (2*a^3*cos(d*x + c)^5 + 8
*a^3*cos(d*x + c)^4 + 7*a^3*cos(d*x + c)^3 - 24*a^3*cos(d*x + c)^2 + 24*a^3
*cos(d*x + c) + 4*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [A] time = 0.40, size = 180, normalized size = 1.30

$$-\frac{33\,(dx+c)a^3 - 12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{8\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^3\tan\left(\cdots\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}\cdots}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/8*(33*(d*x + c)*a^3 - 12*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 12*a^3
*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 8*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a
^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + 2*(25*a^3*tan(1/2
*d*x + 1/2*c)^7 + 81*a^3*tan(1/2*d*x + 1/2*c)^5 + 79*a^3*tan(1/2*d*x + 1/2*
c)^3 + 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.76, size = 159, normalized size = 1.15

$$\frac{11a^3\cos(dx+c)\left(\sin^3(dx+c)\right)}{4d}+\frac{33a^3\cos(dx+c)\sin(dx+c)}{8d}-\frac{33a^3x}{8}-\frac{33a^3c}{8d}-\frac{a^3\left(\sin^3(dx+c)\right)}{2d}-\frac{3a^3\sin(dx+\cdots}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^4,x)
```

```
[Out] 11/4*a^3*cos(d*x+c)*sin(d*x+c)^3/d+33/8*a^3*cos(d*x+c)*sin(d*x+c)/d-33/8*a^
3*x-33/8/d*a^3*c-1/2*a^3*sin(d*x+c)^3/d-3/2*a^3*sin(d*x+c)/d+3/2/d*a^3*ln(s
ec(d*x+c)+tan(d*x+c))+3/d*a^3*sin(d*x+c)^5/cos(d*x+c)+1/2/d*a^3*sin(d*x+c)^
5/cos(d*x+c)^2
```

maxima [A] time = 0.78, size = 182, normalized size = 1.32

$$-\frac{16\left(2\,\sin(dx+c)^3-3\,\log(\sin(dx+c)+1)+3\,\log(\sin(dx+c)-1)+6\,\sin(dx+c)\right)a^3-(12\,dx+12\,c+\sin\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/32*(16*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
- 1) + 6*sin(d*x + c))*a^3 - (12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*
x + 2*c))*a^3 + 48*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan
(d*x + c))*a^3 + 8*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x
 + c) + 1) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)))/d
```

mupad [B] time = 1.97, size = 204, normalized size = 1.48

$$\frac{3\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{33\,a^3\,x}{8}+\frac{-\frac{45\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{4}-\frac{83\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{4}+\frac{25\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{2}+\frac{79\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{2}+\cdots}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (3*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (33*a^3*x)/8 + ((27*a^3*tan(c/2 + (d*
x)/2)^3)/4 + (79*a^3*tan(c/2 + (d*x)/2)^5)/2 + (25*a^3*tan(c/2 + (d*x)/2)^7
)/2 - (83*a^3*tan(c/2 + (d*x)/2)^9)/4 - (45*a^3*tan(c/2 + (d*x)/2)^11)/4 +
(21*a^3*tan(c/2 + (d*x)/2))/4)/(d*(2*tan(c/2 + (d*x)/2)^2 - tan(c/2 + (d*x)
/2)^4 - 4*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^8 + 2*tan(c/2 + (d*x)/2
)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.51 $\int (a+a\sec(c+dx))^3\sin^2(c+dx)\,dx$

Optimal. Leaf size=98

$$-\frac{3a^3\sin(c+dx)}{d}+\frac{3a^3\tan(c+dx)}{d}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^3\tan(c+dx)\sec(c+}{2d}$$

```
[Out] -5/2*a^3*x+5/2*a^3*arctanh(sin(d*x+c))/d-3*a^3*sin(d*x+c)/d-1/2*a^3*cos(d*x
+c)*sin(d*x+c)/d+3*a^3*tan(d*x+c)/d+1/2*a^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.18, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2872, 2637, 2635, 8, 3770, 3767, 3768}

$$-\frac{3a^3\sin(c+dx)}{d}+\frac{3a^3\tan(c+dx)}{d}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^3\tan(c+dx)\sec(c+}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^2,x]
```

```
[Out] (-5*a^3*x)/2 + (5*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^3*Sin[c + d*x])/d
 - (a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (3*a^3*Tan[c + d*x])/d + (a^3*Se
c[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\sin^2(c+dx)\,dx = -\int (-a-a\cos(c+dx))^3\sec(c+dx)\tan^2(c+dx)\,dx$$
$$= -\frac{\int \left(2a^5+3a^5\cos(c+dx)+a^5\cos^2(c+dx)-2a^5\sec(c+dx)-3a^5 s\right.}{a^2}$$
$$= -2a^3x-a^3\int\cos^2(c+dx)\,dx+a^3\int\sec^3(c+dx)\,dx+\left(2a^3\right)\int \sec($$
$$= -2a^3x+\frac{2a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\cos(c+dx)\,s}{2d}$$
$$= -\frac{5a^3x}{2}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\cos(c+dx)}{2d}$$

Mathematica [B] time = 2.42, size = 300, normalized size = 3.06

$$\frac{1}{32}a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{12\sin(c)\cos(dx)}{d}-\frac{\sin(2c)\cos(2dx)}{d}-\frac{12\cos(c)\sin(dx)}{d}-\frac{\cos(2c)\,s}{d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^2,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(-10*x - (10*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]])/d + (10*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/
d - (12*Cos[d*x]*Sin[c])/d - (Cos[2*d*x]*Sin[2*c])/d - (12*Cos[c]*Sin[d*x])
/d - (Cos[2*c]*Sin[2*d*x])/d + 1/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2
) + (12*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2])) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (12*Sin[(d*x)/
2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/32
```

fricas [A] time = 0.60, size = 125, normalized size = 1.28

$$-\frac{10\,a^3dx\cos(dx+c)^2-5\,a^3\cos(dx+c)^2\log(\sin(dx+c)+1)+5\,a^3\cos(dx+c)^2\log(-\sin(dx+c)+1)+}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(10*a^3*d*x*cos(d*x + c)^2 - 5*a^3*cos(d*x + c)^2*log(sin(d*x + c) + 1
) + 5*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x + c)^3 + 6
```

```
*a^3*cos(d*x + c)^2 - 6*a^3*cos(d*x + c) - a^3)*sin(d*x + c))/(d*cos(d*x +
c)^2)
```

giac [A] time = 0.94, size = 102, normalized size = 1.04

$$-\frac{5(dx+c)a^3-5a^3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)+5a^3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{4\left(5a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-9a^3\tan\left(\frac{1}{2}dx\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*(d*x + c)*a^3 - 5*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 5*a^3*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*(5*a^3*tan(1/2*d*x + 1/2*c)^7 - 9*a^3*
tan(1/2*d*x + 1/2*c)^3)/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/d
```

maple [A] time = 0.46, size = 111, normalized size = 1.13

$$-\frac{a^3\cos(dx+c)\sin(dx+c)}{2d}-\frac{5a^3x}{2}-\frac{5a^3c}{2d}-\frac{5a^3\sin(dx+c)}{2d}+\frac{5a^3\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{3a^3\tan(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^2,x)
```

```
[Out] -1/2*a^3*cos(d*x+c)*sin(d*x+c)/d-5/2*a^3*x-5/2/d*a^3*c-5/2*a^3*sin(d*x+c)/d
+5/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*tan(d*x+c)/d+1/2/d*a^3*sin(d*x+c
)^3/cos(d*x+c)^2
```

maxima [A] time = 0.51, size = 127, normalized size = 1.30

$$\frac{(2dx+2c-\sin(2dx+2c))a^3-12(dx+c-\tan(dx+c))a^3-a^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a^3 - 12*(d*x + c - tan(d*x + c))*a^3
 - a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) + 6*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) -
2*sin(d*x + c)))/d
```

mupad [B] time = 1.28, size = 90, normalized size = 0.92

$$\frac{5a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{5\,a^3\,x}{2}+\frac{18\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-10\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (5*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (5*a^3*x)/2 + (18*a^3*tan(c/2 + (d*x)
/2)^3 - 10*a^3*tan(c/2 + (d*x)/2)^7)/(d*(tan(c/2 + (d*x)/2)^8 - 2*tan(c/2 +
 (d*x)/2)^4 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\sin^2(c+dx)\sec(c+dx)\,dx+\int 3\sin^2(c+dx)\sec^2(c+dx)\,dx+\int \sin^2(c+dx)\sec^3(c+dx)\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**2,x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)**2*sec(c + d*x), x) + Integral(3*sin(c + d*x)
**2*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**2*sec(c + d*x)**3, x) + In
tegral(sin(c + d*x)**2, x))
```

3.52 $\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=80

$$\frac{3a^3\tan(c+dx)}{d}+\frac{9a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{4a^3\sin(c+dx)}{d(1-\cos(c+dx))}+\frac{a^3\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] 9/2*a^3*arctanh(sin(d*x+c))/d-4*a^3*sin(d*x+c)/d/(1-cos(d*x+c))+3*a^3*tan(d
*x+c)/d+1/2*a^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.19, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3872, 2872, 2648, 3770, 3767, 8, 3768}

$$\frac{3a^3\tan(c+dx)}{d}+\frac{9a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{4a^3\sin(c+dx)}{d(1-\cos(c+dx))}+\frac{a^3\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (9*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (4*a^3*Sin[c + d*x])/(d*(1 - Cos[c +
d*x])) + (3*a^3*Tan[c + d*x])/d + (a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx = -\int (-a-a\cos(c+dx))^3\csc^2(c+dx)\sec^3(c+dx)\,dx$$
$$= a^2\int\left(\frac{4a}{1-\cos(c+dx)}+4a\sec(c+dx)+3a\sec^2(c+dx)+a\sec^3(c-$$
$$= a^3\int\sec^3(c+dx)\,dx+\left(3a^3\right)\int\sec^2(c+dx)\,dx+\left(4a^3\right)\int\frac{1}{1-\cos(c}$$
$$= \frac{4a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{4a^3\sin(c+dx)}{d(1-\cos(c+dx))}+\frac{a^3\sec(c+dx)\tan(c}{2d}$$
$$= \frac{9a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{4a^3\sin(c+dx)}{d(1-\cos(c+dx))}+\frac{3a^3\tan(c+dx)}{d}+\frac{a^3}{}$$

Mathematica [B] time = 1.16, size = 244, normalized size = 3.05

$$a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{12\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(-18*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] + 18*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 16*Csc[c/2]
*Csc[(c + d*x)/2]*Sin[(d*x)/2] + (Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^(-2)
 - (Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^(-2) + (12*Sin[d*x])/((Cos[c/2] -
Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]))))/(32*d)
```

fricas [A] time = 0.50, size = 122, normalized size = 1.52

$$\frac{9\,a^3\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-9\,a^3\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-2}{4\,d\cos\left(dx+c\right)^2\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(9*a^3*cos(d*x + c)^2*log(sin(d*x + c) + 1)*sin(d*x + c) - 9*a^3*cos(d*
x + c)^2*log(-sin(d*x + c) + 1)*sin(d*x + c) - 28*a^3*cos(d*x + c)^3 - 18*a
^3*cos(d*x + c)^2 + 12*a^3*cos(d*x + c) + 2*a^3)/(d*cos(d*x + c)^2*sin(d*x
+ c))
```

giac [A] time = 0.40, size = 106, normalized size = 1.32

$$\frac{9\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-9\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{8\,a^3}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{2\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^3\tan\left(\frac{1}{2}\,d\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(9*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a^3*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 8*a^3/tan(1/2*d*x + 1/2*c) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^
3 - 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.75, size = 102, normalized size = 1.28

$$-\frac{7a^3\cot(dx+c)}{d}-\frac{9a^3}{2d\sin(dx+c)}+\frac{9a^3\ln(\sec(dx+c)+\tan(dx+c))}{2d}+\frac{3a^3}{d\sin(dx+c)\cos(dx+c)}+\frac{}{2d\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -7*a^3*cot(d*x+c)/d-9/2/d*a^3/sin(d*x+c)+9/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c)
)+3/d*a^3/sin(d*x+c)/cos(d*x+c)+1/2/d*a^3/sin(d*x+c)/cos(d*x+c)^2
```

maxima [A] time = 0.60, size = 137, normalized size = 1.71

$$-\frac{a^3\left(\frac{2\left(3\sin(dx+c)^2-2\right)}{\sin(dx+c)^3-\sin(dx+c)}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)+6a^3\left(\frac{2}{\sin(dx+c)}-\log(\sin(dx+c)+1\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(a^3*(2*(3*sin(d*x + c)^2 - 2)/(sin(d*x + c)^3 - sin(d*x + c)) - 3*log
(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 6*a^3*(2/sin(d*x + c) - log
(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^3*(1/tan(d*x + c) - tan(
d*x + c)) + 4*a^3/tan(d*x + c))/d
```

mupad [B] time = 2.46, size = 98, normalized size = 1.22

$$\frac{9\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{9\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-15\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+4\,a^3}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^2,x)
```

```
[Out] (9*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (9*a^3*tan(c/2 + (d*x)/2)^4 - 15*a^3*
tan(c/2 + (d*x)/2)^2 + 4*a^3)/(d*(tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2)
^3 + tan(c/2 + (d*x)/2)^5))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int 3\csc^2(c+dx)\sec(c+dx)\,dx+\int 3\csc^2(c+dx)\sec^2(c+dx)\,dx+\int \csc^2(c+dx)\sec^3(c+dx)\,dx+\int c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*csc(c + d*x)**2*sec(c + d*x), x) + Integral(3*csc(c + d*x)
**2*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**2*sec(c + d*x)**3, x) + In
tegral(csc(c + d*x)**2, x))
```

3.53 $\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=110

$$\frac{3a^3\tan(c+dx)}{d}+\frac{11a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{17a^3\sin(c+dx)}{3d(1-\cos(c+dx))}-\frac{2a^3\sin(c+dx)}{3d(1-\cos(c+dx))^2}+\frac{a^3\tan(c+dx)\sec(c+}{2d}$$

```
[Out] 11/2*a^3*arctanh(sin(d*x+c))/d-2/3*a^3*sin(d*x+c)/d/(1-cos(d*x+c))^2-17/3*a
^3*sin(d*x+c)/d/(1-cos(d*x+c))+3*a^3*tan(d*x+c)/d+1/2*a^3*sec(d*x+c)*tan(d*
x+c)/d
```

Rubi [A] time = 0.23, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2872, 2650, 2648, 3770, 3767, 8, 3768}

$$\frac{3a^3\tan(c+dx)}{d}+\frac{11a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{17a^3\sin(c+dx)}{3d(1-\cos(c+dx))}-\frac{2a^3\sin(c+dx)}{3d(1-\cos(c+dx))^2}+\frac{a^3\tan(c+dx)\sec(c+}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (11*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (2*a^3*Sin[c + d*x])/(3*d*(1 - Cos[c
 + d*x])^2) - (17*a^3*Sin[c + d*x])/(3*d*(1 - Cos[c + d*x])) + (3*a^3*Tan[c
 + d*x])/d + (a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx &= -\int (-a-a\cos(c+dx))^3\csc^4(c+dx)\sec^3(c+dx)\,dx\\ &= a^4\int\left(\frac{2}{a(1-\cos(c+dx))^2}+\frac{5}{a(1-\cos(c+dx))}+\frac{5\sec(c+dx)}{a}+\frac{3\sec\right.\\ &= a^3\int \sec^3(c+dx)\,dx+\left(2a^3\right)\int\frac{1}{(1-\cos(c+dx))^2}\,dx+\left(3a^3\right)\int\sec^2(c\\ &= \frac{5a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^3\sin(c+dx)}{3d(1-\cos(c+dx))^2}-\frac{5a^3\sin(c+dx)}{d(1-\cos(c+dx))}+\\ &= \frac{11a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{2a^3\sin(c+dx)}{3d(1-\cos(c+dx))^2}-\frac{17a^3\sin(c+dx)}{3d(1-\cos(c+dx))}\end{aligned}$$

Mathematica [B] time = 6.25, size = 678, normalized size = 6.16

$$\frac{3\sin\left(\frac{dx}{2}\right)\cos^3(c+dx)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^3}{8d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{3\sin\left(\frac{dx}{2}\right)\cos^3(c+dx)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a}{8d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/24*(Cos[c + d*x]^3*Cot[c/2]*Csc[c/2 + (d*x)/2]^2*Sec[c/2 + (d*x)/2]^6*(a
 + a*Sec[c + d*x])^3)/d - (11*Cos[c + d*x]^3*Log[Cos[c/2 + (d*x)/2] - Sin[c
/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(16*d) + (11*Co
s[c + d*x]^3*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2
]^6*(a + a*Sec[c + d*x])^3)/(16*d) + (17*Cos[c + d*x]^3*Csc[c/2]*Csc[c/2 +
(d*x)/2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(24*d) +
 (Cos[c + d*x]^3*Csc[c/2]*Csc[c/2 + (d*x)/2]^3*Sec[c/2 + (d*x)/2]^6*(a + a*
Sec[c + d*x])^3*Sin[(d*x)/2])/(24*d) + (Cos[c + d*x]^3*Sec[c/2 + (d*x)/2]^6
*(a + a*Sec[c + d*x])^3)/(32*d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2)
 + (3*Cos[c + d*x]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/
2])/(8*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) -
 (Cos[c + d*x]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(32*d*(Cos[c/
2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (3*Cos[c + d*x]^3*Sec[c/2 + (d*x)/2
]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(8*d*(Cos[c/2] + Sin[c/2])*(Cos[c/
2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

fricas [A] time = 0.60, size = 178, normalized size = 1.62

$$-\frac{104\,a^3\cos(dx+c)^4 - 38\,a^3\cos(dx+c)^3 - 118\,a^3\cos(dx+c)^2 + 30\,a^3\cos(dx+c) + 6\,a^3 - 33\left(a^3\cos(dx+\right.}{12\left(d\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/12*(104*a^3*cos(d*x + c)^4 - 38*a^3*cos(d*x + c)^3 - 118*a^3*cos(d*x + c
)^2 + 30*a^3*cos(d*x + c) + 6*a^3 - 33*(a^3*cos(d*x + c)^3 - a^3*cos(d*x +
c)^2)*log(sin(d*x + c) + 1)*sin(d*x + c) + 33*(a^3*cos(d*x + c)^3 - a^3*cos
(d*x + c)^2)*log(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^3 - d*co
s(d*x + c)^2)*sin(d*x + c))
```

giac [A] time = 0.56, size = 123, normalized size = 1.12

$$\frac{33\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 33\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{6\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2} - \frac{2(}{}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(33*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 33*a^3*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) - 6*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - 2*(18*a^3*tan(1/2*d*x + 1/2*c)^2 + a^
3)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 1.03, size = 188, normalized size = 1.71

$$-\frac{26a^3\cot(dx+c)}{3d} - \frac{a^3\cot(dx+c)\left(\csc^2(dx+c)\right)}{3d} - \frac{a^3}{d\sin(dx+c)^3} - \frac{11a^3}{2d\sin(dx+c)} + \frac{11a^3\ln(\sec(dx+c)+\tan}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -26/3*a^3*cot(d*x+c)/d-1/3/d*a^3*cot(d*x+c)*csc(d*x+c)^2-1/d*a^3/sin(d*x+c)
^3-11/2/d*a^3/sin(d*x+c)+11/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c))-1/d*a^3/sin(d
*x+c)^3/cos(d*x+c)+4/d*a^3/sin(d*x+c)/cos(d*x+c)-1/3/d*a^3/sin(d*x+c)^3/cos
(d*x+c)^2+5/6/d*a^3/sin(d*x+c)/cos(d*x+c)^2
```

maxima [A] time = 0.35, size = 188, normalized size = 1.71

$$-\frac{a^3\left(\frac{2\left(15\sin(dx+c)^4-10\sin(dx+c)^2-2\right)}{\sin(dx+c)^5-\sin(dx+c)^3} - 15\log(\sin(dx+c)+1) + 15\log(\sin(dx+c)-1)\right) + 6\,a^3\left(\frac{2\left(3\sin(dx+c)^2+1\right)}{\sin(dx+c)^3}\right.}{1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(a^3*(2*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 - 2)/(sin(d*x + c)^5 -
 sin(d*x + c)^3) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 6
*a^3*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3
*log(sin(d*x + c) - 1)) + 12*a^3*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 - 3
*tan(d*x + c)) + 4*(3*tan(d*x + c)^2 + 1)*a^3/tan(d*x + c)^3)/d
```

mupad [B] time = 5.35, size = 116, normalized size = 1.05

$$\frac{11\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{11\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-\frac{56\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{3}+\frac{16\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{3}+\frac{a^3}{3}}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7-2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^4,x)
```

```
[Out] (11*a^3*atanh(tan(c/2 + (d*x)/2)))/d - ((16*a^3*tan(c/2 + (d*x)/2)^2)/3 - (
56*a^3*tan(c/2 + (d*x)/2)^4)/3 + 11*a^3*tan(c/2 + (d*x)/2)^6 + a^3/3)/(d*(t
an(c/2 + (d*x)/2)^3 - 2*tan(c/2 + (d*x)/2)^5 + tan(c/2 + (d*x)/2)^7))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.54 $\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=165

$$-\frac{a^6\tan(c+dx)\sec(c+dx)}{5d(a-a\cos(c+dx))^3}-\frac{11a^5\tan(c+dx)\sec(c+dx)}{15d(a-a\cos(c+dx))^2}+\frac{152a^3\tan(c+dx)}{15d}+\frac{13a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{13a}{}$$

```
[Out] 13/2*a^3*arctanh(sin(d*x+c))/d+152/15*a^3*tan(d*x+c)/d+13/2*a^3*sec(d*x+c)*
tan(d*x+c)/d-1/5*a^6*sec(d*x+c)*tan(d*x+c)/d/(a-a*cos(d*x+c))^3-11/15*a^5*s
ec(d*x+c)*tan(d*x+c)/d/(a-a*cos(d*x+c))^2-76/15*a^6*sec(d*x+c)*tan(d*x+c)/d
/(a^3-a^3*cos(d*x+c))
```

Rubi [A] time = 0.44, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3872, 2869, 2766, 2978, 2748, 3768, 3770, 3767, 8}

$$\frac{152a^3\tan(c+dx)}{15d}+\frac{13a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{13a^3\tan(c+dx)\sec(c+dx)}{2d}-\frac{76a^6\tan(c+dx)\sec(c+dx)}{15d\left(a^3-a^3\cos(c+dx)\right)}-\frac{a^6}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (13*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (152*a^3*Tan[c + d*x])/(15*d) + (13*
a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (a^6*Sec[c + d*x]*Tan[c + d*x])/(5*d
*(a - a*Cos[c + d*x])^3) - (11*a^5*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a - a*
Cos[c + d*x])^2) - (76*a^6*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a^3 - a^3*Cos[
c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx = -\int (-a-a\cos(c+dx))^3\csc^6(c+dx)\sec^3(c+dx)\,dx$$
$$= -\left(a^6\int \frac{\sec^3(c+dx)}{(-a+a\cos(c+dx))^3}\,dx\right)$$
$$= -\frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3} - \frac{1}{5}a^4\int \frac{(-7a-4a\cos(c+dx))\sec^3(c+dx}{(-a+a\cos(c+dx))^2}$$
$$= -\frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3} - \frac{11a^5\sec(c+dx)\tan(c+dx)}{15d(a-a\cos(c+dx))^2} - \frac{1}{15}a^2\int \frac{(4}{}$$
$$= -\frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3} - \frac{11a^5\sec(c+dx)\tan(c+dx)}{15d(a-a\cos(c+dx))^2} - \frac{76a^4\sec(c}{15d(a-}$$
$$= -\frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3} - \frac{11a^5\sec(c+dx)\tan(c+dx)}{15d(a-a\cos(c+dx))^2} - \frac{76a^4\sec(c}{15d(a-}$$
$$= \frac{13a^3\sec(c+dx)\tan(c+dx)}{2d} - \frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3} - \frac{11a^5\sec(c+}{15d(a-a}$$
$$= \frac{13a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{152a^3\tan(c+dx)}{15d} + \frac{13a^3\sec(c+dx)\tan(c}{2d}$$

Mathematica [B] time = 1.22, size = 353, normalized size = 2.14

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(24960\cos^2(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/30720*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^2*(24960
*Cos[c + d*x]^2*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]]) + Csc[c/2]*Csc[(c + d*x)/2]^5*Sec[c]*(-1235*Sin[
(d*x)/2] + 3805*Sin[(3*d*x)/2] + 4329*Sin[c - (d*x)/2] - 1989*Sin[c + (d*x)
/2] - 3575*Sin[2*c + (d*x)/2] + 475*Sin[c + (3*d*x)/2] + 2005*Sin[2*c + (3*
d*x)/2] + 2275*Sin[3*c + (3*d*x)/2] - 2673*Sin[c + (5*d*x)/2] + 105*Sin[2*c
 + (5*d*x)/2] - 1593*Sin[3*c + (5*d*x)/2] - 975*Sin[4*c + (5*d*x)/2] + 1325
*Sin[2*c + (7*d*x)/2] - 255*Sin[3*c + (7*d*x)/2] + 875*Sin[4*c + (7*d*x)/2]
 + 195*Sin[5*c + (7*d*x)/2] - 304*Sin[3*c + (9*d*x)/2] + 90*Sin[4*c + (9*d*
x)/2] - 214*Sin[5*c + (9*d*x)/2])))/d
```

fricas [A] time = 0.74, size = 225, normalized size = 1.36

$$-\frac{608\,a^3\cos(dx+c)^5-826\,a^3\cos(dx+c)^4-476\,a^3\cos(dx+c)^3+868\,a^3\cos(dx+c)^2-120\,a^3\cos(dx+c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(608*a^3*cos(d*x + c)^5 - 826*a^3*cos(d*x + c)^4 - 476*a^3*cos(d*x +
c)^3 + 868*a^3*cos(d*x + c)^2 - 120*a^3*cos(d*x + c) - 30*a^3 - 195*(a^3*co
s(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2)*log(sin(d*x + c)
+ 1)*sin(d*x + c) + 195*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 + a^3*co
s(d*x + c)^2)*log(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d
*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sin(d*x + c))
```

giac [A] time = 0.40, size = 141, normalized size = 0.85

$$\frac{390\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-390\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{60\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}-}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(390*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 390*a^3*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 60*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*tan(1/2*d*x +
 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - (465*a^3*tan(1/2*d*x + 1/2*c)^4 +
 40*a^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

maple [A] time = 1.05, size = 274, normalized size = 1.66

$$-\frac{152a^3\cot(dx+c)}{15d}-\frac{a^3\cot(dx+c)\left(\csc^4(dx+c)\right)}{5d}-\frac{4a^3\cot(dx+c)\left(\csc^2(dx+c)\right)}{15d}-\frac{3a^3}{5d\sin(dx+c)^5}-\frac{a}{d\sin(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -152/15*a^3*cot(d*x+c)/d-1/5/d*a^3*cot(d*x+c)*csc(d*x+c)^4-4/15/d*a^3*cot(d
*x+c)*csc(d*x+c)^2-3/5/d*a^3/sin(d*x+c)^5-1/d*a^3/sin(d*x+c)^3-13/2/d*a^3/s
in(d*x+c)+13/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c))-3/5/d*a^3/sin(d*x+c)^5/cos(d
*x+c)-6/5/d*a^3/sin(d*x+c)^3/cos(d*x+c)+24/5/d*a^3/sin(d*x+c)/cos(d*x+c)-1/
5/d*a^3/sin(d*x+c)^5/cos(d*x+c)^2-7/15/d*a^3/sin(d*x+c)^3/cos(d*x+c)^2+7/6/
d*a^3/sin(d*x+c)/cos(d*x+c)^2
```

maxima [A] time = 0.49, size = 228, normalized size = 1.38

$$-\frac{a^3\left(\frac{2\left(105\sin(dx+c)^6-70\sin(dx+c)^4-14\sin(dx+c)^2-6\right)}{\sin(dx+c)^7-\sin(dx+c)^5}-105\log\left(\sin\left(dx+c\right)+1\right)+105\log\left(\sin\left(dx+c\right)-1\right)\right)+6a^3\left(\frac{2(1\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(a^3*(2*(105*sin(d*x + c)^6 - 70*sin(d*x + c)^4 - 14*sin(d*x + c)^2 -
 6)/(sin(d*x + c)^7 - sin(d*x + c)^5) - 105*log(sin(d*x + c) + 1) + 105*log
(sin(d*x + c) - 1)) + 6*a^3*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/s
in(d*x + c)^5 - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 36*a
^3*((15*tan(d*x + c)^4 + 5*tan(d*x + c)^2 + 1)/tan(d*x + c)^5 - 5*tan(d*x +
 c)) + 4*(15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 + 3)*a^3/tan(d*x + c)^5)/d
```

mupad [B] time = 4.91, size = 136, normalized size = 0.82

$$\frac{13\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{51\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-\frac{262\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6}{3}+\frac{388\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4}{15}+\frac{34\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2}{15}+\frac{a^3}{5}}{d\left(4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9-8\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^6,x)
```

```
[Out] (13*a^3*atanh(tan(c/2 + (d*x)/2)))/d - ((34*a^3*tan(c/2 + (d*x)/2)^2)/15 +
(388*a^3*tan(c/2 + (d*x)/2)^4)/15 - (262*a^3*tan(c/2 + (d*x)/2)^6)/3 + 51*a
^3*tan(c/2 + (d*x)/2)^8 + a^3/5)/(d*(4*tan(c/2 + (d*x)/2)^5 - 8*tan(c/2 + (
d*x)/2)^7 + 4*tan(c/2 + (d*x)/2)^9))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.55 $\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=192

$$\frac{3a^3\tan(c+dx)}{d}-\frac{4a^3\cot^7(c+dx)}{7d}-\frac{3a^3\cot^5(c+dx)}{d}-\frac{7a^3\cot^3(c+dx)}{d}-\frac{13a^3\cot(c+dx)}{d}-\frac{15a^3\csc^7(c+dx)}{14d}-3$$

```
[Out] 15/2*a^3*arctanh(sin(d*x+c))/d-13*a^3*cot(d*x+c)/d-7*a^3*cot(d*x+c)^3/d-3*a
^3*cot(d*x+c)^5/d-4/7*a^3*cot(d*x+c)^7/d-15/2*a^3*csc(d*x+c)/d-5/2*a^3*csc(
d*x+c)^3/d-3/2*a^3*csc(d*x+c)^5/d-15/14*a^3*csc(d*x+c)^7/d+1/2*a^3*csc(d*x+
c)^7*sec(d*x+c)^2/d+3*a^3*tan(d*x+c)/d
```

Rubi [A] time = 0.31, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3872, 2873, 3767, 2621, 302, 207, 2620, 270, 288}

$$\frac{3a^3\tan(c+dx)}{d}-\frac{4a^3\cot^7(c+dx)}{7d}-\frac{3a^3\cot^5(c+dx)}{d}-\frac{7a^3\cot^3(c+dx)}{d}-\frac{13a^3\cot(c+dx)}{d}-\frac{15a^3\csc^7(c+dx)}{14d}-3$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (15*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (13*a^3*Cot[c + d*x])/d - (7*a^3*Cot
[c + d*x]^3)/d - (3*a^3*Cot[c + d*x]^5)/d - (4*a^3*Cot[c + d*x]^7)/(7*d) -
(15*a^3*Csc[c + d*x])/(2*d) - (5*a^3*Csc[c + d*x]^3)/(2*d) - (3*a^3*Csc[c +
 d*x]^5)/(2*d) - (15*a^3*Csc[c + d*x]^7)/(14*d) + (a^3*Csc[c + d*x]^7*Sec[c
 + d*x]^2)/(2*d) + (3*a^3*Tan[c + d*x])/d
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx = -\int (-a-a\cos(c+dx))^3\csc^8(c+dx)\sec^3(c+dx)\,dx$$
$$= \int \left(a^3\csc^8(c+dx) + 3a^3\csc^8(c+dx)\sec(c+dx) + 3a^3\csc^8(c+dx)\text{sec}$$
$$= a^3\int \csc^8(c+dx)\,dx + a^3\int \csc^8(c+dx)\sec^3(c+dx)\,dx + \left(3a^3\right)\int \text{csc}$$
$$= -\frac{a^3\,\text{Subst}\left(\int \frac{x^{10}}{\left(-1+x^2\right)^2}\,dx, x, \csc(c+dx)\right)}{d} - \frac{a^3\,\text{Subst}\left(\int \left(1+3x^2+3x^4+\right.\right.}{d}$$
$$= -\frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{d} - \frac{3a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+dx)}{7d} -$$
$$= -\frac{13a^3\cot(c+dx)}{d} - \frac{7a^3\cot^3(c+dx)}{d} - \frac{3a^3\cot^5(c+dx)}{d} - \frac{4a^3\cot^7(c+}{7d}$$
$$= \frac{3a^3\tanh^{-1}(\sin(c+dx))}{d} - \frac{13a^3\cot(c+dx)}{d} - \frac{7a^3\cot^3(c+dx)}{d} - \frac{3a^3\text{co}}{}$$
$$= \frac{15a^3\tanh^{-1}(\sin(c+dx))}{2d} - \frac{13a^3\cot(c+dx)}{d} - \frac{7a^3\cot^3(c+dx)}{d} - \frac{3a^3\text{c}}{}$$

Mathematica [B] time = 1.25, size = 430, normalized size = 2.24

$$\frac{a^3\cos(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^3\left(-8\csc(2c)(2776\sin(c-dx)-6080\sin(c+dx)+8816\sin(2(c\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*Cos[c + d*x]*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(-860160*Cos[c +
d*x]^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 860160*Cos[c + d*x]^2*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 8*Csc[2*c]*Csc[(c + d*x)/2]^6*Csc[c
 + d*x]*(5264*Sin[2*c] - 9580*Sin[d*x] + 8480*Sin[2*d*x] + 2776*Sin[c - d*x
] - 6080*Sin[c + d*x] + 8816*Sin[2*(c + d*x)] - 7904*Sin[3*(c + d*x)] + 486
4*Sin[4*(c + d*x)] - 1824*Sin[5*(c + d*x)] + 304*Sin[6*(c + d*x)] - 9580*Si
n[2*c + d*x] - 10024*Sin[3*c + d*x] + 13891*Sin[c + 2*d*x] + 7720*Sin[2*(c
+ 2*d*x)] + 13891*Sin[3*c + 2*d*x] + 10080*Sin[4*c + 2*d*x] - 10060*Sin[c +
 3*d*x] - 12454*Sin[2*c + 3*d*x] - 12454*Sin[4*c + 3*d*x] - 6580*Sin[5*c +
3*d*x] + 7664*Sin[3*c + 4*d*x] + 7664*Sin[5*c + 4*d*x] + 2520*Sin[6*c + 4*d
*x] - 3420*Sin[3*c + 5*d*x] - 2874*Sin[4*c + 5*d*x] - 2874*Sin[6*c + 5*d*x]
 - 420*Sin[7*c + 5*d*x] + 640*Sin[4*c + 6*d*x] + 479*Sin[5*c + 6*d*x] + 479
*Sin[7*c + 6*d*x])))/(917504*d)
```

fricas [A] time = 0.62, size = 278, normalized size = 1.45

$$-\frac{320\,a^3\cos\left(dx+c\right)^6-750\,a^3\cos\left(dx+c\right)^5+170\,a^3\cos\left(dx+c\right)^4+720\,a^3\cos\left(dx+c\right)^3-520\,a^3\cos\left(dx+c\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/28*(320*a^3*cos(d*x + c)^6 - 750*a^3*cos(d*x + c)^5 + 170*a^3*cos(d*x +
c)^4 + 720*a^3*cos(d*x + c)^3 - 520*a^3*cos(d*x + c)^2 + 42*a^3*cos(d*x + c
) + 14*a^3 - 105*(a^3*cos(d*x + c)^5 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x
 + c)^3 - a^3*cos(d*x + c)^2)*log(sin(d*x + c) + 1)*sin(d*x + c) + 105*(a^3
*cos(d*x + c)^5 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 - a^3*cos(d*x
 + c)^2)*log(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^5 - 3*d*cos(
d*x + c)^4 + 3*d*cos(d*x + c)^3 - d*cos(d*x + c)^2)*sin(d*x + c))
```

giac [A] time = 0.69, size = 169, normalized size = 0.88

$$\frac{840\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-840\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{112\left(5\,a^3\tan\left(\frac{1}{2}\,d\right.\right.}{\left(\tan\right.}}{112\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/112*(840*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 840*a^3*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 7*a^3*tan(1/2*d*x + 1/2*c) - 112*(5*a^3*tan(1/2*d*x +
 1/2*c)^3 - 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - (1
050*a^3*tan(1/2*d*x + 1/2*c)^6 + 112*a^3*tan(1/2*d*x + 1/2*c)^4 + 14*a^3*ta
n(1/2*d*x + 1/2*c)^2 + a^3)/tan(1/2*d*x + 1/2*c)^7)/d
```

maple [B] time = 1.12, size = 360, normalized size = 1.88

$$-\frac{80a^3\cot\left(dx+c\right)}{7d}-\frac{a^3\cot\left(dx+c\right)\left(\csc^6\left(dx+c\right)\right)}{7d}-\frac{6a^3\cot\left(dx+c\right)\left(\csc^4\left(dx+c\right)\right)}{35d}-\frac{8a^3\cot\left(dx+c\right)\left(\csc^2\left(dx\right.\right.}{35d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -80/7*a^3*cot(d*x+c)/d-1/7/d*a^3*cot(d*x+c)*csc(d*x+c)^6-6/35/d*a^3*cot(d*x
+c)*csc(d*x+c)^4-8/35/d*a^3*cot(d*x+c)*csc(d*x+c)^2-3/7/d*a^3/sin(d*x+c)^7-
3/5/d*a^3/sin(d*x+c)^5-1/d*a^3/sin(d*x+c)^3-15/2/d*a^3/sin(d*x+c)+15/2/d*a^
```

```
3*ln(sec(d*x+c)+tan(d*x+c))-3/7/d*a^3/sin(d*x+c)^7/cos(d*x+c)-24/35/d*a^3/s
in(d*x+c)^5/cos(d*x+c)-48/35/d*a^3/sin(d*x+c)^3/cos(d*x+c)+192/35/d*a^3/sin
(d*x+c)/cos(d*x+c)-1/7/d*a^3/sin(d*x+c)^7/cos(d*x+c)^2-9/35/d*a^3/sin(d*x+c
)^5/cos(d*x+c)^2-3/5/d*a^3/sin(d*x+c)^3/cos(d*x+c)^2+3/2/d*a^3/sin(d*x+c)/c
os(d*x+c)^2
```

maxima [A] time = 0.37, size = 268, normalized size = 1.40

$$-\frac{a^3\left(\frac{2\left(315\,\sin(dx+c)^8-210\,\sin(dx+c)^6-42\,\sin(dx+c)^4-18\,\sin(dx+c)^2-10\right)}{\sin(dx+c)^9-\sin(dx+c)^7}-315\,\log\left(\sin\left(dx+c\right)+1\right)+315\,\log\left(\sin\left(dx+c\right)-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/140*(a^3*(2*(315*sin(d*x + c)^8 - 210*sin(d*x + c)^6 - 42*sin(d*x + c)^4
 - 18*sin(d*x + c)^2 - 10)/(sin(d*x + c)^9 - sin(d*x + c)^7) - 315*log(sin(
d*x + c) + 1) + 315*log(sin(d*x + c) - 1)) + 2*a^3*(2*(105*sin(d*x + c)^6 +
 35*sin(d*x + c)^4 + 21*sin(d*x + c)^2 + 15)/sin(d*x + c)^7 - 105*log(sin(d
*x + c) + 1) + 105*log(sin(d*x + c) - 1)) + 12*a^3*((140*tan(d*x + c)^6 + 7
0*tan(d*x + c)^4 + 28*tan(d*x + c)^2 + 5)/tan(d*x + c)^7 - 35*tan(d*x + c))
 + 4*(35*tan(d*x + c)^6 + 35*tan(d*x + c)^4 + 21*tan(d*x + c)^2 + 5)*a^3/ta
n(d*x + c)^7)/d
```

mupad [B] time = 2.90, size = 169, normalized size = 0.88

$$\frac{15\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{230\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-396\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+120\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\frac{85\,a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{7}}{d\,\left(16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}-32\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+16\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^8,x)
```

```
[Out] (15*a^3*atanh(tan(c/2 + (d*x)/2)))/d - ((12*a^3*tan(c/2 + (d*x)/2)^2)/7 + (
85*a^3*tan(c/2 + (d*x)/2)^4)/7 + 120*a^3*tan(c/2 + (d*x)/2)^6 - 396*a^3*tan
(c/2 + (d*x)/2)^8 + 230*a^3*tan(c/2 + (d*x)/2)^10 + a^3/7)/(d*(16*tan(c/2 +
 (d*x)/2)^7 - 32*tan(c/2 + (d*x)/2)^9 + 16*tan(c/2 + (d*x)/2)^11)) - (a^3*t
an(c/2 + (d*x)/2))/(16*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.56 $\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=232

$$\frac{3a^3\tan(c+dx)}{d}-\frac{4a^3\cot^9(c+dx)}{9d}-\frac{19a^3\cot^7(c+dx)}{7d}-\frac{36a^3\cot^5(c+dx)}{5d}-\frac{34a^3\cot^3(c+dx)}{3d}-\frac{16a^3\cot(c+dx)}{d}$$

```
[Out] 17/2*a^3*arctanh(sin(d*x+c))/d-16*a^3*cot(d*x+c)/d-34/3*a^3*cot(d*x+c)^3/d-
36/5*a^3*cot(d*x+c)^5/d-19/7*a^3*cot(d*x+c)^7/d-4/9*a^3*cot(d*x+c)^9/d-17/2
*a^3*csc(d*x+c)/d-17/6*a^3*csc(d*x+c)^3/d-17/10*a^3*csc(d*x+c)^5/d-17/14*a^
3*csc(d*x+c)^7/d-17/18*a^3*csc(d*x+c)^9/d+1/2*a^3*csc(d*x+c)^9*sec(d*x+c)^2
/d+3*a^3*tan(d*x+c)/d
```

Rubi [A] time = 0.33, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3872, 2873, 3767, 2621, 302, 207, 2620, 270, 288}

$$\frac{3a^3\tan(c+dx)}{d}-\frac{4a^3\cot^9(c+dx)}{9d}-\frac{19a^3\cot^7(c+dx)}{7d}-\frac{36a^3\cot^5(c+dx)}{5d}-\frac{34a^3\cot^3(c+dx)}{3d}-\frac{16a^3\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^10*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (17*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (16*a^3*Cot[c + d*x])/d - (34*a^3*Co
t[c + d*x]^3)/(3*d) - (36*a^3*Cot[c + d*x]^5)/(5*d) - (19*a^3*Cot[c + d*x]^
7)/(7*d) - (4*a^3*Cot[c + d*x]^9)/(9*d) - (17*a^3*Csc[c + d*x])/(2*d) - (17
*a^3*Csc[c + d*x]^3)/(6*d) - (17*a^3*Csc[c + d*x]^5)/(10*d) - (17*a^3*Csc[c
 + d*x]^7)/(14*d) - (17*a^3*Csc[c + d*x]^9)/(18*d) + (a^3*Csc[c + d*x]^9*Se
c[c + d*x]^2)/(2*d) + (3*a^3*Tan[c + d*x])/d
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
```

```
x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx = -\int (-a-a\cos(c+dx))^3\csc^{10}(c+dx)\sec^3(c+dx)\,dx$$

$$= \int \left(a^3\csc^{10}(c+dx)+3a^3\csc^{10}(c+dx)\sec(c+dx)+3a^3\csc^{10}(c+dx)\right.$$

$$= a^3\int \csc^{10}(c+dx)\,dx + a^3\int \csc^{10}(c+dx)\sec^3(c+dx)\,dx + \left(3a^3\right)\int \mathrm{c}$$

$$= -\frac{a^3\,\mathrm{Subst}\left(\int \frac{x^{12}}{\left(-1+x^2\right)^2}\,dx, x, \csc(c+dx)\right)}{d} - \frac{a^3\,\mathrm{Subst}\left(\int \left(1+4x^2+6x^4\right.\right.}{}$$

$$= -\frac{a^3\cot(c+dx)}{d} - \frac{4a^3\cot^3(c+dx)}{3d} - \frac{6a^3\cot^5(c+dx)}{5d} - \frac{4a^3\cot^7(c+d}{7d}$$

$$= -\frac{16a^3\cot(c+dx)}{d} - \frac{34a^3\cot^3(c+dx)}{3d} - \frac{36a^3\cot^5(c+dx)}{5d} - \frac{19a^3\cot^7}{7d}$$

$$= \frac{3a^3\tanh^{-1}(\sin(c+dx))}{d} - \frac{16a^3\cot(c+dx)}{d} - \frac{34a^3\cot^3(c+dx)}{3d} - \frac{36a^3}{}$$

$$= \frac{17a^3\tanh^{-1}(\sin(c+dx))}{2d} - \frac{16a^3\cot(c+dx)}{d} - \frac{34a^3\cot^3(c+dx)}{3d} - \frac{36a}{}$$

Mathematica [B] time = 6.71, size = 1000, normalized size = 4.31

$$\frac{\cos^3(c+dx)\csc\left(\frac{c}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(\sec(c+dx)a+a)^3\sin\left(\frac{dx}{2}\right)\csc^9\left(\frac{c}{2}+\frac{dx}{2}\right)}{4608d} - \frac{\cos^3(c+dx)\cot\left(\frac{c}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(}{4608}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^10*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-9833*Cos[c + d*x]^3*Cot[c/2]*Csc[c/2 + (d*x)/2]^2*Sec[c/2 + (d*x)/2]^6*(a
 + a*Sec[c + d*x])^3)/(80640*d) - (979*Cos[c + d*x]^3*Cot[c/2]*Csc[c/2 + (d
*x)/2]^4*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(53760*d) - (5*Cos[c
+ d*x]^3*Cot[c/2]*Csc[c/2 + (d*x)/2]^6*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c +
d*x])^3)/(2016*d) - (Cos[c + d*x]^3*Cot[c/2]*Csc[c/2 + (d*x)/2]^8*Sec[c/2 +
 (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(4608*d) - (17*Cos[c + d*x]^3*Log[Cos[c
/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x
])^3)/(16*d) + (17*Cos[c + d*x]^3*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/
2]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(16*d) + (197147*Cos[c + d
*x]^3*Csc[c/2]*Csc[c/2 + (d*x)/2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])
^3*Sin[(d*x)/2])/(161280*d) + (9833*Cos[c + d*x]^3*Csc[c/2]*Csc[c/2 + (d*x)
/2]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(80640*d) +
 (979*Cos[c + d*x]^3*Csc[c/2]*Csc[c/2 + (d*x)/2]^5*Sec[c/2 + (d*x)/2]^6*(a
+ a*Sec[c + d*x])^3*Sin[(d*x)/2])/(53760*d) + (5*Cos[c + d*x]^3*Csc[c/2]*Cs
c[c/2 + (d*x)/2]^7*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2]
)/(2016*d) + (Cos[c + d*x]^3*Csc[c/2]*Csc[c/2 + (d*x)/2]^9*Sec[c/2 + (d*x)/
2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(4608*d) - (35*Cos[c + d*x]^3*Sec
[c/2]*Sec[c/2 + (d*x)/2]^7*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(1536*d) -
(Cos[c + d*x]^3*Sec[c/2]*Sec[c/2 + (d*x)/2]^9*(a + a*Sec[c + d*x])^3*Sin[(d
*x)/2])/(1536*d) + (Cos[c + d*x]*Sec[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c +
 d*x])^3*Sin[d*x])/(16*d) + (Cos[c + d*x]^2*Sec[c]*Sec[c/2 + (d*x)/2]^6*(a
+ a*Sec[c + d*x])^3*(Sin[c] + 6*Sin[d*x]))/(16*d) - (Cos[c + d*x]^3*Sec[c/2
 + (d*x)/2]^8*(a + a*Sec[c + d*x])^3*Tan[c/2])/(1536*d)
```

fricas [A] time = 0.89, size = 375, normalized size = 1.62

$$-\frac{15872\,a^3\cos(dx+c)^8 - 36906\,a^3\cos(dx+c)^7 - 8322\,a^3\cos(dx+c)^6 + 73402\,a^3\cos(dx+c)^5 - 33342\,a^3\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/1260*(15872*a^3*cos(d*x + c)^8 - 36906*a^3*cos(d*x + c)^7 - 8322*a^3*cos
(d*x + c)^6 + 73402*a^3*cos(d*x + c)^5 - 33342*a^3*cos(d*x + c)^4 - 34746*a
^3*cos(d*x + c)^3 + 26702*a^3*cos(d*x + c)^2 - 1890*a^3*cos(d*x + c) - 630*
a^3 - 5355*(a^3*cos(d*x + c)^7 - 3*a^3*cos(d*x + c)^6 + 2*a^3*cos(d*x + c)^
5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2)*log(s
in(d*x + c) + 1)*sin(d*x + c) + 5355*(a^3*cos(d*x + c)^7 - 3*a^3*cos(d*x +
c)^6 + 2*a^3*cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^3 +
 a^3*cos(d*x + c)^2)*log(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^
7 - 3*d*cos(d*x + c)^6 + 2*d*cos(d*x + c)^5 + 2*d*cos(d*x + c)^4 - 3*d*cos(
d*x + c)^3 + d*cos(d*x + c)^2)*sin(d*x + c))
```

giac [A] time = 0.51, size = 202, normalized size = 0.87

$$-\frac{105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 171360\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 171360\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + 3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20160*(105*a^3*tan(1/2*d*x + 1/2*c)^3 - 171360*a^3*log(abs(tan(1/2*d*x +
 1/2*c) + 1)) + 171360*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 3780*a^3*ta
n(1/2*d*x + 1/2*c) + 20160*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*tan(1/2*d*
x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + (220185*a^3*tan(1/2*d*x + 1/2*
```

```
c)^8 + 26880*a^3*tan(1/2*d*x + 1/2*c)^6 + 4347*a^3*tan(1/2*d*x + 1/2*c)^4 +
 540*a^3*tan(1/2*d*x + 1/2*c)^2 + 35*a^3)/tan(1/2*d*x + 1/2*c)^9)/d
```

maple [B] time = 1.13, size = 446, normalized size = 1.92

$$-\frac{3968a^3\cot(dx+c)}{315d}-\frac{a^3\cot(dx+c)\left(\csc^8(dx+c)\right)}{9d}-\frac{8a^3\cot(dx+c)\left(\csc^6(dx+c)\right)}{63d}-\frac{16a^3\cot(dx+c)\left(\csc^4(dx\right.}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^10*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -3968/315*a^3*cot(d*x+c)/d-1/9/d*a^3*cot(d*x+c)*csc(d*x+c)^8-8/63/d*a^3*cot
(d*x+c)*csc(d*x+c)^6-16/105/d*a^3*cot(d*x+c)*csc(d*x+c)^4-64/315/d*a^3*cot(
d*x+c)*csc(d*x+c)^2-1/3/d*a^3/sin(d*x+c)^9-3/7/d*a^3/sin(d*x+c)^7-3/5/d*a^3
/sin(d*x+c)^5-1/d*a^3/sin(d*x+c)^3-17/2/d*a^3/sin(d*x+c)+17/2/d*a^3*ln(sec(
d*x+c)+tan(d*x+c))-1/3/d*a^3/sin(d*x+c)^9/cos(d*x+c)-10/21/d*a^3/sin(d*x+c)
^7/cos(d*x+c)-16/21/d*a^3/sin(d*x+c)^5/cos(d*x+c)-32/21/d*a^3/sin(d*x+c)^3/
cos(d*x+c)+128/21/d*a^3/sin(d*x+c)/cos(d*x+c)-1/9/d*a^3/sin(d*x+c)^9/cos(d*
x+c)^2-11/63/d*a^3/sin(d*x+c)^7/cos(d*x+c)^2-11/35/d*a^3/sin(d*x+c)^5/cos(d
*x+c)^2-11/15/d*a^3/sin(d*x+c)^3/cos(d*x+c)^2+11/6/d*a^3/sin(d*x+c)/cos(d*x
+c)^2
```

maxima [A] time = 0.37, size = 308, normalized size = 1.33

$$-\frac{a^3\left(\frac{2\left(3465\sin(dx+c)^{10}-2310\sin(dx+c)^8-462\sin(dx+c)^6-198\sin(dx+c)^4-110\sin(dx+c)^2-70\right)}{\sin(dx+c)^{11}-\sin(dx+c)^9}-3465\log(\sin(dx+c)+1)+3465\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/1260*(a^3*(2*(3465*sin(d*x + c)^10 - 2310*sin(d*x + c)^8 - 462*sin(d*x +
 c)^6 - 198*sin(d*x + c)^4 - 110*sin(d*x + c)^2 - 70)/(sin(d*x + c)^11 - si
n(d*x + c)^9) - 3465*log(sin(d*x + c) + 1) + 3465*log(sin(d*x + c) - 1)) +
6*a^3*(2*(315*sin(d*x + c)^8 + 105*sin(d*x + c)^6 + 63*sin(d*x + c)^4 + 45*
sin(d*x + c)^2 + 35)/sin(d*x + c)^9 - 315*log(sin(d*x + c) + 1) + 315*log(s
in(d*x + c) - 1)) + 60*a^3*((315*tan(d*x + c)^8 + 210*tan(d*x + c)^6 + 126*
tan(d*x + c)^4 + 45*tan(d*x + c)^2 + 7)/tan(d*x + c)^9 - 63*tan(d*x + c)) +
 4*(315*tan(d*x + c)^8 + 420*tan(d*x + c)^6 + 378*tan(d*x + c)^4 + 180*tan(
d*x + c)^2 + 35)*a^3/tan(d*x + c)^9)/d
```

mupad [B] time = 1.04, size = 204, normalized size = 0.88

$$\frac{17\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}-\frac{a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{192\,d}-\frac{3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{16\,d}-\frac{1019\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}-\frac{5282\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}}{3}+\dots}{d\left(64\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^3/sin(c + d*x)^10,x)
```

```
[Out] (17*a^3*atanh(tan(c/2 + (d*x)/2)))/d - (a^3*tan(c/2 + (d*x)/2)^3)/(192*d) -
 (3*a^3*tan(c/2 + (d*x)/2))/(16*d) - ((94*a^3*tan(c/2 + (d*x)/2)^2)/63 + (3
302*a^3*tan(c/2 + (d*x)/2)^4)/315 + (6242*a^3*tan(c/2 + (d*x)/2)^6)/105 + (
8132*a^3*tan(c/2 + (d*x)/2)^8)/15 - (5282*a^3*tan(c/2 + (d*x)/2)^10)/3 + 10
19*a^3*tan(c/2 + (d*x)/2)^12 + a^3/9)/(d*(64*tan(c/2 + (d*x)/2)^9 - 128*tan
(c/2 + (d*x)/2)^11 + 64*tan(c/2 + (d*x)/2)^13))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**10*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.57 $\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{\sin^8(c+dx)}{8ad}-\frac{\cos^9(c+dx)}{9ad}+\frac{3\cos^7(c+dx)}{7ad}-\frac{3\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}$$

```
[Out] 1/3*cos(d*x+c)^3/a/d-3/5*cos(d*x+c)^5/a/d+3/7*cos(d*x+c)^7/a/d-1/9*cos(d*x+
c)^9/a/d+1/8*sin(d*x+c)^8/a/d
```

Rubi [A] time = 0.16, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2835, 2564, 30, 2565, 270}

$$\frac{\sin^8(c+dx)}{8ad}-\frac{\cos^9(c+dx)}{9ad}+\frac{3\cos^7(c+dx)}{7ad}-\frac{3\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^9/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cos[c + d*x]^3/(3*a*d) - (3*Cos[c + d*x]^5)/(5*a*d) + (3*Cos[c + d*x]^7)/(7
*a*d) - Cos[c + d*x]^9/(9*a*d) + Sin[c + d*x]^8/(8*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^9(c+dx)}{-a-a\cos(c+dx)}\,dx\\ &= \frac{\int \cos(c+dx)\sin^7(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^7(c+dx)\,dx}{a}\\ &= \frac{\operatorname{Subst}\left(\int x^7\,dx, x, \sin(c+dx)\right)}{ad} + \frac{\operatorname{Subst}\left(\int x^2\left(1-x^2\right)^3\,dx, x, \cos(c+dx)\right)}{ad}\\ &= \frac{\sin^8(c+dx)}{8ad} + \frac{\operatorname{Subst}\left(\int \left(x^2-3x^4+3x^6-x^8\right)dx, x, \cos(c+dx)\right)}{ad}\\ &= \frac{\cos^3(c+dx)}{3ad} - \frac{3\cos^5(c+dx)}{5ad} + \frac{3\cos^7(c+dx)}{7ad} - \frac{\cos^9(c+dx)}{9ad} + \frac{\sin^8(c+dx)}{8ad}\end{aligned}$$

Mathematica [A] time = 4.87, size = 62, normalized size = 0.68

$$\frac{\sin^{10}\left(\frac{1}{2}(c+dx)\right)(6995\cos(c+dx)+3650\cos(2(c+dx))+1085\cos(3(c+dx))+140\cos(4(c+dx))+4258)}{315ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^9/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((4258 + 6995*Cos[c + d*x] + 3650*Cos[2*(c + d*x)] + 1085*Cos[3*(c + d*x)]
+ 140*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]^10)/(315*a*d)
```

fricas [A] time = 0.82, size = 89, normalized size = 0.98

$$-\frac{280\cos(dx+c)^9-315\cos(dx+c)^8-1080\cos(dx+c)^7+1260\cos(dx+c)^6+1512\cos(dx+c)^5-189}{2520\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2520*(280*cos(d*x + c)^9 - 315*cos(d*x + c)^8 - 1080*cos(d*x + c)^7 + 12
60*cos(d*x + c)^6 + 1512*cos(d*x + c)^5 - 1890*cos(d*x + c)^4 - 840*cos(d*x
 + c)^3 + 1260*cos(d*x + c)^2)/(a*d)
```

giac [A] time = 0.28, size = 141, normalized size = 1.55

$$\frac{32\left(\frac{9(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{36(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{84(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{126(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{630(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}-1\right)}{315\,ad\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 32/315*(9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 36*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + 84*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 126*(c
```

```
os(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 630*(cos(d*x + c) - 1)^5/(cos(d*x
 + c) + 1)^5 - 1)/(a*d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^9)
```

maple [A] time = 0.54, size = 89, normalized size = 0.98

$$\frac{-\frac{1}{2\sec(dx+c)^6}-\frac{1}{2\sec(dx+c)^2}+\frac{3}{7\sec(dx+c)^7}-\frac{1}{9\sec(dx+c)^9}+\frac{1}{3\sec(dx+c)^3}+\frac{3}{4\sec(dx+c)^4}+\frac{1}{8\sec(dx+c)^8}-\frac{3}{5\sec(dx+c)^5}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^9/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/2/sec(d*x+c)^6-1/2/sec(d*x+c)^2+3/7/sec(d*x+c)^7-1/9/sec(d*x+c)^9
+1/3/sec(d*x+c)^3+3/4/sec(d*x+c)^4+1/8/sec(d*x+c)^8-3/5/sec(d*x+c)^5)
```

maxima [A] time = 0.33, size = 89, normalized size = 0.98

$$-\frac{280 \cos(dx+c)^9 - 315 \cos(dx+c)^8 - 1080 \cos(dx+c)^7 + 1260 \cos(dx+c)^6 + 1512 \cos(dx+c)^5 - 1890 \mathrm{co}}{2520\, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2520*(280*cos(d*x + c)^9 - 315*cos(d*x + c)^8 - 1080*cos(d*x + c)^7 + 12
60*cos(d*x + c)^6 + 1512*cos(d*x + c)^5 - 1890*cos(d*x + c)^4 - 840*cos(d*x
 + c)^3 + 1260*cos(d*x + c)^2)/(a*d)
```

mupad [B] time = 0.09, size = 110, normalized size = 1.21

$$-\frac{\frac{\cos(c+dx)^2}{2a}-\frac{\cos(c+dx)^3}{3a}-\frac{3\cos(c+dx)^4}{4a}+\frac{3\cos(c+dx)^5}{5a}+\frac{\cos(c+dx)^6}{2a}-\frac{3\cos(c+dx)^7}{7a}-\frac{\cos(c+dx)^8}{8a}+\frac{\cos(c+dx)^9}{9a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^9/(a + a/cos(c + d*x)),x)
```

```
[Out] -(cos(c + d*x)^2/(2*a) - cos(c + d*x)^3/(3*a) - (3*cos(c + d*x)^4)/(4*a) +
(3*cos(c + d*x)^5)/(5*a) + cos(c + d*x)^6/(2*a) - (3*cos(c + d*x)^7)/(7*a)
- cos(c + d*x)^8/(8*a) + cos(c + d*x)^9/(9*a))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**9/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.58 $\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{\sin^6(c+dx)}{6ad}+\frac{\cos^7(c+dx)}{7ad}-\frac{2\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}$$

```
[Out] 1/3*cos(d*x+c)^3/a/d-2/5*cos(d*x+c)^5/a/d+1/7*cos(d*x+c)^7/a/d+1/6*sin(d*x+
c)^6/a/d
```

Rubi [A] time = 0.15, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2835, 2564, 30, 2565, 270}

$$\frac{\sin^6(c+dx)}{6ad}+\frac{\cos^7(c+dx)}{7ad}-\frac{2\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cos[c + d*x]^3/(3*a*d) - (2*Cos[c + d*x]^5)/(5*a*d) + Cos[c + d*x]^7/(7*a*d
) + Sin[c + d*x]^6/(6*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^7(c+dx)}{-a-a\cos(c+dx)}\,dx\\ &= \frac{\int \cos(c+dx)\sin^5(c+dx)\,dx}{a}-\frac{\int \cos^2(c+dx)\sin^5(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int x^5\,dx,x,\sin(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx,x,\cos(c+dx)\right)}{ad}\\ &= \frac{\sin^6(c+dx)}{6ad}+\frac{\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx,x,\cos(c+dx)\right)}{ad}\\ &= \frac{\cos^3(c+dx)}{3ad}-\frac{2\cos^5(c+dx)}{5ad}+\frac{\cos^7(c+dx)}{7ad}+\frac{\sin^6(c+dx)}{6ad}\end{aligned}$$

Mathematica [A] time = 1.72, size = 52, normalized size = 0.71

$$\frac{4\sin^8\left(\frac{1}{2}(c+dx)\right)(197\cos(c+dx)+85\cos(2(c+dx))+15\cos(3(c+dx))+123)}{105ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + a*Sec[c + d*x]),x]
```

```
[Out] (4*(123 + 197*Cos[c + d*x] + 85*Cos[2*(c + d*x)] + 15*Cos[3*(c + d*x)])*Sin
[(c + d*x)/2]^8)/(105*a*d)
```

fricas [A] time = 0.48, size = 69, normalized size = 0.95

$$\frac{30\,\cos\left(dx+c\right)^{7}-35\,\cos\left(dx+c\right)^{6}-84\,\cos\left(dx+c\right)^{5}+105\,\cos\left(dx+c\right)^{4}+70\,\cos\left(dx+c\right)^{3}-105\,\cos\left(dx+c\right)^{2}}{210\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/210*(30*cos(d*x + c)^7 - 35*cos(d*x + c)^6 - 84*cos(d*x + c)^5 + 105*cos(
d*x + c)^4 + 70*cos(d*x + c)^3 - 105*cos(d*x + c)^2)/(a*d)
```

giac [A] time = 0.84, size = 119, normalized size = 1.63

$$\frac{16\left(\frac{7\,(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{21\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{35\,(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{140\,(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-1\right)}{105\,ad\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 16/105*(7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 21*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + 35*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 140*(c
os(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 1)/(a*d*((cos(d*x + c) - 1)/(cos(
d*x + c) + 1) - 1)^7)
```

maple [A] time = 0.46, size = 70, normalized size = 0.96

$$-\frac{\frac{1}{6\sec(dx+c)^6}+\frac{1}{2\sec(dx+c)^2}-\frac{1}{7\sec(dx+c)^7}-\frac{1}{3\sec(dx+c)^3}-\frac{1}{2\sec(dx+c)^4}+\frac{2}{5\sec(dx+c)^5}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/d/a*(1/6/sec(d*x+c)^6+1/2/sec(d*x+c)^2-1/7/sec(d*x+c)^7-1/3/sec(d*x+c)^3
-1/2/sec(d*x+c)^4+2/5/sec(d*x+c)^5)
```

maxima [A] time = 0.34, size = 69, normalized size = 0.95

$$\frac{30\cos(dx+c)^7-35\cos(dx+c)^6-84\cos(dx+c)^5+105\cos(dx+c)^4+70\cos(dx+c)^3-105\cos(dx+}{210\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/210*(30*cos(d*x + c)^7 - 35*cos(d*x + c)^6 - 84*cos(d*x + c)^5 + 105*cos(
d*x + c)^4 + 70*cos(d*x + c)^3 - 105*cos(d*x + c)^2)/(a*d)
```

mupad [B] time = 0.06, size = 84, normalized size = 1.15

$$-\frac{\frac{\cos(c+d\,x)^2}{2\,a}-\frac{\cos(c+d\,x)^3}{3\,a}-\frac{\cos(c+d\,x)^4}{2\,a}+\frac{2\cos(c+d\,x)^5}{5\,a}+\frac{\cos(c+d\,x)^6}{6\,a}-\frac{\cos(c+d\,x)^7}{7\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a + a/cos(c + d*x)),x)
```

```
[Out] -(cos(c + d*x)^2/(2*a) - cos(c + d*x)^3/(3*a) - cos(c + d*x)^4/(2*a) + (2*c
os(c + d*x)^5)/(5*a) + cos(c + d*x)^6/(6*a) - cos(c + d*x)^7/(7*a))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.59 $\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{\sin^4(c+dx)}{4ad}-\frac{\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}$$

```
[Out] 1/3*cos(d*x+c)^3/a/d-1/5*cos(d*x+c)^5/a/d+1/4*sin(d*x+c)^4/a/d
```

Rubi [A] time = 0.15, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2835, 2564, 30, 2565, 14}

$$\frac{\sin^4(c+dx)}{4ad}-\frac{\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cos[c + d*x]^3/(3*a*d) - Cos[c + d*x]^5/(5*a*d) + Sin[c + d*x]^4/(4*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
```

```
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^5(c+dx)}{-a-a\cos(c+dx)}\,dx\\ &= \frac{\int \cos(c+dx)\sin^3(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^3(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int x^3\,dx, x, \sin(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int x^2\left(1-x^2\right)\,dx, x, \cos(c+dx)\right)}{ad}\\ &= \frac{\sin^4(c+dx)}{4ad} + \frac{\text{Subst}\left(\int \left(x^2-x^4\right)\,dx, x, \cos(c+dx)\right)}{ad}\\ &= \frac{\cos^3(c+dx)}{3ad} - \frac{\cos^5(c+dx)}{5ad} + \frac{\sin^4(c+dx)}{4ad}\end{aligned}$$

Mathematica [A] time = 0.40, size = 42, normalized size = 0.76

$$\frac{2\sin^6\left(\frac{1}{2}(c+dx)\right)(21\cos(c+dx)+6\cos(2(c+dx))+13)}{15ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + a*Sec[c + d*x]),x]
[Out] (2*(13 + 21*Cos[c + d*x] + 6*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]^6)/(15*a*d)
```

fricas [A] time = 0.53, size = 49, normalized size = 0.89

$$-\frac{12\cos(dx+c)^5 - 15\cos(dx+c)^4 - 20\cos(dx+c)^3 + 30\cos(dx+c)^2}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -1/60*(12*cos(d*x + c)^5 - 15*cos(d*x + c)^4 - 20*cos(d*x + c)^3 + 30*cos(d
*x + c)^2)/(a*d)
```

giac [A] time = 0.22, size = 97, normalized size = 1.76

$$\frac{4\left(\frac{5(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{10(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{30(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - 1\right)}{15\,ad\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] 4/15*(5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 10*(cos(d*x + c) - 1)^2/(co
s(d*x + c) + 1)^2 + 30*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 1)/(a*d*
((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)
```

maple [A] time = 0.44, size = 49, normalized size = 0.89

$$\frac{-\frac{1}{2\sec(dx+c)^2} + \frac{1}{3\sec(dx+c)^3} + \frac{1}{4\sec(dx+c)^4} - \frac{1}{5\sec(dx+c)^5}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d/a*(-1/2/sec(d*x+c)^2+1/3/sec(d*x+c)^3+1/4/sec(d*x+c)^4-1/5/sec(d*x+c)^5
)
```

maxima [A] time = 0.32, size = 49, normalized size = 0.89

$$-\frac{12\cos\left(dx+c\right)^{5}-15\cos\left(dx+c\right)^{4}-20\cos\left(dx+c\right)^{3}+30\cos\left(dx+c\right)^{2}}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(12*cos(d*x + c)^5 - 15*cos(d*x + c)^4 - 20*cos(d*x + c)^3 + 30*cos(d
*x + c)^2)/(a*d)
```

mupad [B] time = 0.07, size = 58, normalized size = 1.05

$$-\frac{\frac{\cos(c+d\,x)^2}{2\,a}-\frac{\cos(c+d\,x)^3}{3\,a}-\frac{\cos(c+d\,x)^4}{4\,a}+\frac{\cos(c+d\,x)^5}{5\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + a/cos(c + d*x)),x)
```

```
[Out] -(cos(c + d*x)^2/(2*a) - cos(c + d*x)^3/(3*a) - cos(c + d*x)^4/(4*a) + cos(
c + d*x)^5/(5*a))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.60 $\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\sin^2(c+dx)}{2ad}+\frac{\cos^3(c+dx)}{3ad}$$

```
[Out] 1/3*cos(d*x+c)^3/a/d+1/2*sin(d*x+c)^2/a/d
```

Rubi [A] time = 0.13, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2835, 2564, 30, 2565}

$$\frac{\sin^2(c+dx)}{2ad}+\frac{\cos^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cos[c + d*x]^3/(3*a*d) + Sin[c + d*x]^2/(2*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^3(c+dx)}{-a-a\cos(c+dx)}\,dx\\ &= \frac{\int \cos(c+dx)\sin(c+dx)\,dx}{a}-\frac{\int \cos^2(c+dx)\sin(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}(\int x\,dx,x,\sin(c+dx))}{ad}+\frac{\text{Subst}\left(\int x^2\,dx,x,\cos(c+dx)\right)}{ad}\\ &= \frac{\cos^3(c+dx)}{3ad}+\frac{\sin^2(c+dx)}{2ad}\end{aligned}$$

Mathematica [A] time = 0.13, size = 32, normalized size = 0.86

$$\frac{2\sin^4\left(\frac{1}{2}(c+dx)\right)(2\cos(c+dx)+1)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sec[c + d*x]),x]
[Out] (2*(1 + 2*Cos[c + d*x])*Sin[(c + d*x)/2]^4)/(3*a*d)
```

fricas [A] time = 0.67, size = 29, normalized size = 0.78

$$\frac{2\,\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)^2}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] 1/6*(2*cos(d*x + c)^3 - 3*cos(d*x + c)^2)/(a*d)
```

giac [A] time = 1.00, size = 32, normalized size = 0.86

$$\frac{\frac{2\,\cos\left(dx+c\right)^3}{d}-\frac{3\,\cos\left(dx+c\right)^2}{d}}{6\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] 1/6*(2*cos(d*x + c)^3/d - 3*cos(d*x + c)^2/d)/a
```

maple [A] time = 0.43, size = 30, normalized size = 0.81

$$-\frac{\frac{1}{2\sec\left(dx+c\right)^2}-\frac{1}{3\sec\left(dx+c\right)^3}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sec(d*x+c)),x)
[Out] -1/d/a*(1/2/sec(d*x+c)^2-1/3/sec(d*x+c)^3)
```

maxima [A] time = 0.32, size = 29, normalized size = 0.78

$$\frac{2\,\cos\left(dx+c\right)^3-3\,\cos\left(dx+c\right)^2}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*cos(d*x + c)^3 - 3*cos(d*x + c)^2)/(a*d)
```

mupad [B] time = 0.88, size = 26, normalized size = 0.70

$$\frac{\cos\left(c+d\,x\right)^2\,\left(2\,\cos\left(c+d\,x\right)-3\right)}{6\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + a/cos(c + d*x)),x)
```

```
[Out] (cos(c + d*x)^2*(2*cos(c + d*x) - 3))/(6*a*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.61 $\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{\log(\cos(c+dx)+1)}{ad}-\frac{\cos(c+dx)}{ad}$$

```
[Out] -cos(d*x+c)/a/d+ln(1+cos(d*x+c))/a/d
```

Rubi [A] time = 0.07, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 2833, 12, 43}

$$\frac{\log(\cos(c+dx)+1)}{ad}-\frac{\cos(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(Cos[c + d*x]/(a*d)) + Log[1 + Cos[c + d*x]]/(a*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)\sin(c+dx)}{-a-a\cos(c+dx)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x}{a(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{-a+x}\,dx, x, -a\cos(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(1-\frac{a}{a-x}\right)dx, x, -a\cos(c+dx)\right)}{a^2d}$$
$$= -\frac{\cos(c+dx)}{ad} + \frac{\log(1+\cos(c+dx))}{ad}$$

Mathematica [A] time = 0.09, size = 28, normalized size = 0.90

$$-\frac{\cos(c+dx) - 2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sec[c + d*x]),x]
[Out] -((Cos[c + d*x] - 2*Log[Cos[(c + d*x)/2]])/(a*d))
```

fricas [A] time = 0.83, size = 28, normalized size = 0.90

$$-\frac{\cos(dx+c) - \log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -(cos(d*x + c) - log(1/2*cos(d*x + c) + 1/2))/(a*d)
```

giac [A] time = 0.22, size = 34, normalized size = 1.10

$$-\frac{\cos(dx+c)}{ad} + \frac{\log(|-\cos(dx+c)-1|)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] -cos(d*x + c)/(a*d) + log(abs(-cos(d*x + c) - 1))/(a*d)
```

maple [A] time = 0.06, size = 49, normalized size = 1.58

$$-\frac{1}{da\sec(dx+c)} - \frac{\ln(\sec(dx+c))}{da} + \frac{\ln(1+\sec(dx+c))}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sec(d*x+c)),x)
[Out] -1/d/a/sec(d*x+c)-1/d/a*ln(sec(d*x+c))+1/d/a*ln(1+sec(d*x+c))
```

maxima [A] time = 0.32, size = 30, normalized size = 0.97

$$-\frac{\frac{\cos(dx+c)}{a} - \frac{\log(\cos(dx+c)+1)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(cos(d*x + c)/a - log(cos(d*x + c) + 1)/a)/d
```

mupad [**B**] time = 0.05, size = 25, normalized size = 0.81

$$\frac{\ln\left(\cos\left(c+d\,x\right)+1\right)-\cos\left(c+d\,x\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + a/cos(c + d*x)),x)
```

```
[Out] (log(cos(c + d*x) + 1) - cos(c + d*x))/(a*d)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)/(sec(c + d*x) + 1), x)/a
```

3.62 $\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=58

$$-\frac{\csc^2(c+dx)}{2ad}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] -1/2*arctanh(cos(d*x+c))/a/d+1/2*cot(d*x+c)*csc(d*x+c)/a/d-1/2*csc(d*x+c)^2
/a/d
```

Rubi [A] time = 0.10, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3872, 2706, 2606, 30, 2611, 3770}

$$-\frac{\csc^2(c+dx)}{2ad}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -ArcTanh[Cos[c + d*x]]/(2*a*d) + (Cot[c + d*x]*Csc[c + d*x])/(2*a*d) - Csc[
c + d*x]^2/(2*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
```

```
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)}{-a-a\cos(c+dx)}\,dx\\ &= -\frac{\int \cot^2(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^2(c+dx)\,dx}{a}\\ &= \frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\int \csc(c+dx)\,dx}{2a}-\frac{\text{Subst}(\int x\,dx,x,\csc(c+dx))}{ad}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}-\frac{\csc^2(c+dx)}{2ad}\end{aligned}$$

Mathematica [A] time = 0.11, size = 67, normalized size = 1.16

$$-\frac{\sec(c+dx)\left(2\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+1\right)}{2ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sec[c + d*x]),x]
[Out] -1/2*((1 + 2*Cos[(c + d*x)/2]^2*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/
2]]))*Sec[c + d*x])/(a*d*(1 + Sec[c + d*x]))
```

fricas [A] time = 0.87, size = 60, normalized size = 1.03

$$-\frac{(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-(\cos(dx+c)+1)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2}{4\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] -1/4*((cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - (cos(d*x + c) + 1)*l
og(-1/2*cos(d*x + c) + 1/2) + 2)/(a*d*cos(d*x + c) + a*d)
```

giac [A] time = 0.89, size = 56, normalized size = 0.97

$$\frac{\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}+\frac{\cos(dx+c)-1}{a(\cos(dx+c)+1)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] 1/4*(log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + (cos(d*x + c) -
1)/(a*(cos(d*x + c) + 1)))/d
```

maple [A] time = 0.48, size = 54, normalized size = 0.93

$$\frac{\ln(-1+\cos(dx+c))}{4ad}-\frac{1}{2ad\,(1+\cos(dx+c))}-\frac{\ln(1+\cos(dx+c))}{4da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/4/a/d*ln(-1+cos(d*x+c))-1/2/a/d/(1+cos(d*x+c))-1/4*ln(1+cos(d*x+c))/d/a
```

maxima [A] time = 0.32, size = 47, normalized size = 0.81

$$-\frac{\frac{\log(\cos(dx+c)+1)}{a} - \frac{\log(\cos(dx+c)-1)}{a} + \frac{2}{a\cos(dx+c)+a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(log(cos(d*x + c) + 1)/a - log(cos(d*x + c) - 1)/a + 2/(a*cos(d*x + c)
 + a))/d
```

mupad [B] time = 0.92, size = 33, normalized size = 0.57

$$-\frac{1}{2\,d\,\left(a+a\,\cos\left(c+d\,x\right)\right)}-\frac{\mathrm{atanh}\left(\cos\left(c+d\,x\right)\right)}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + a/cos(c + d*x))),x)
```

```
[Out] - 1/(2*d*(a + a*cos(c + d*x))) - atanh(cos(c + d*x))/(2*a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)/(sec(c + d*x) + 1), x)/a
```

3.63 $\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=82

$$-\frac{\csc^4(c+dx)}{4ad}-\frac{\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\cot(c+dx)\csc(c+dx)}{8ad}$$

```
[Out] -1/8*arctanh(cos(d*x+c))/a/d-1/8*cot(d*x+c)*csc(d*x+c)/a/d+1/4*cot(d*x+c)*c
sc(d*x+c)^3/a/d-1/4*csc(d*x+c)^4/a/d
```

Rubi [A] time = 0.16, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2835, 2606, 30, 2611, 3768, 3770}

$$-\frac{\csc^4(c+dx)}{4ad}-\frac{\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\cot(c+dx)\csc(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] -ArcTanh[Cos[c + d*x]]/(8*a*d) - (Cot[c + d*x]*Csc[c + d*x])/(8*a*d) + (Cot
[c + d*x]*Csc[c + d*x]^3)/(4*a*d) - Csc[c + d*x]^4/(4*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc^2(c+dx)}{-a-a\cos(c+dx)}\,dx\\ &= -\frac{\int \cot^2(c+dx)\csc^3(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^4(c+dx)\,dx}{a}\\ &= \frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{\int \csc^3(c+dx)\,dx}{4a}-\frac{\text{Subst}\left(\int x^3\,dx,x,\csc(c+dx)\right)}{ad}\\ &= -\frac{\cot(c+dx)\csc(c+dx)}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\csc^4(c+dx)}{4ad}+\frac{\int \csc(c+dx)}{8a}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot(c+dx)\csc(c+dx)}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\csc^4(}{4}\end{aligned}$$

Mathematica [A] time = 0.39, size = 91, normalized size = 1.11

$$-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(2\csc^2\left(\frac{1}{2}(c+dx)\right)+\sec^4\left(\frac{1}{2}(c+dx)\right)-4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\cos\left(\frac{1}{2}\right.\right.\right.}{16ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/16*(Cos[(c + d*x)/2]^2*(2*Csc[(c + d*x)/2]^2 + 4*Log[Cos[(c + d*x)/2]] -
 4*Log[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^4)*Sec[c + d*x])/(a*d*(1 + Sec[
c + d*x]))
```

fricas [A] time = 0.73, size = 138, normalized size = 1.68

$$\frac{2\cos(dx+c)^2-\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+\left(\cos(dx+c)^3+\right.}{16\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2-ad\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) -
1)*log(1/2*cos(d*x + c) + 1/2) + (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x
 + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) + 2*cos(d*x + c) + 4)/(a*d*cos(d*x
+ c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d)
```

giac [A] time = 0.24, size = 129, normalized size = 1.57

$$-\frac{\frac{2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)(\cos(dx+c)+1)}{a(\cos(dx+c)-1)}-\frac{2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}-\frac{\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)*(cos(d*x + c) + 1)/(a*
(cos(d*x + c) - 1)) - 2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a
 - (2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2)/a^2)/d
```

maple [A] time = 0.65, size = 72, normalized size = 0.88

$$\frac{1}{8ad\left(-1+\cos\left(dx+c\right)\right)}+\frac{\ln\left(-1+\cos\left(dx+c\right)\right)}{16ad}-\frac{1}{8ad\left(1+\cos\left(dx+c\right)\right)^{2}}-\frac{\ln\left(1+\cos\left(dx+c\right)\right)}{16da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/8/a/d/(-1+cos(d*x+c))+1/16/a/d*ln(-1+cos(d*x+c))-1/8/a/d/(1+cos(d*x+c))^2
-1/16*ln(1+cos(d*x+c))/d/a
```

maxima [A] time = 0.36, size = 86, normalized size = 1.05

$$\frac{\frac{2\left(\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)+2\right)}{a\cos\left(dx+c\right)^{3}+a\cos\left(dx+c\right)^{2}-a\cos\left(dx+c\right)-a}-\frac{\log\left(\cos\left(dx+c\right)+1\right)}{a}+\frac{\log\left(\cos\left(dx+c\right)-1\right)}{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(d*x + c)^2 + cos(d*x + c) + 2)/(a*cos(d*x + c)^3 + a*cos(d*x +
 c)^2 - a*cos(d*x + c) - a) - log(cos(d*x + c) + 1)/a + log(cos(d*x + c) -
1)/a)/d
```

mupad [B] time = 0.94, size = 75, normalized size = 0.91

$$-\frac{\mathrm{atanh}\left(\cos\left(c+d\,x\right)\right)}{8\,a\,d}-\frac{\frac{\cos\left(c+d\,x\right)^2}{8}+\frac{\cos\left(c+d\,x\right)}{8}+\frac{1}{4}}{d\,\left(-a\,\cos\left(c+d\,x\right)^3-a\,\cos\left(c+d\,x\right)^2+a\,\cos\left(c+d\,x\right)+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + a/cos(c + d*x))),x)
```

```
[Out] - atanh(cos(c + d*x))/(8*a*d) - (cos(c + d*x)/8 + cos(c + d*x)^2/8 + 1/4)/(
d*(a + a*cos(c + d*x) - a*cos(c + d*x)^2 - a*cos(c + d*x)^3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^{3}{\left(c+dx\right)}}{\sec{\left(c+dx\right)}+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**3/(sec(c + d*x) + 1), x)/a
```

3.64 $\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=106

$$-\frac{\csc^6(c+dx)}{6ad}-\frac{\tanh^{-1}(\cos(c+dx))}{16ad}+\frac{\cot(c+dx)\csc^5(c+dx)}{6ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{24ad}-\frac{\cot(c+dx)\csc(c+dx)}{16ad}$$

```
[Out] -1/16*arctanh(cos(d*x+c))/a/d-1/16*cot(d*x+c)*csc(d*x+c)/a/d-1/24*cot(d*x+c
)*csc(d*x+c)^3/a/d+1/6*cot(d*x+c)*csc(d*x+c)^5/a/d-1/6*csc(d*x+c)^6/a/d
```

Rubi [A] time = 0.17, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2835, 2606, 30, 2611, 3768, 3770}

$$-\frac{\csc^6(c+dx)}{6ad}-\frac{\tanh^{-1}(\cos(c+dx))}{16ad}+\frac{\cot(c+dx)\csc^5(c+dx)}{6ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{24ad}-\frac{\cot(c+dx)\csc(c+dx)}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] -ArcTanh[Cos[c + d*x]]/(16*a*d) - (Cot[c + d*x]*Csc[c + d*x])/(16*a*d) - (C
ot[c + d*x]*Csc[c + d*x]^3)/(24*a*d) + (Cot[c + d*x]*Csc[c + d*x]^5)/(6*a*d
) - Csc[c + d*x]^6/(6*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx = -\int \frac{\cot(c+dx)\csc^4(c+dx)}{-a-a\cos(c+dx)}\,dx$$
$$= -\frac{\int \cot^2(c+dx)\csc^5(c+dx)\,dx}{a} + \frac{\int \cot(c+dx)\csc^6(c+dx)\,dx}{a}$$
$$= \frac{\cot(c+dx)\csc^5(c+dx)}{6ad} + \frac{\int \csc^5(c+dx)\,dx}{6a} - \frac{\text{Subst}\left(\int x^5\,dx, x, \csc(c+dx)\right)}{ad}$$
$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{24ad} + \frac{\cot(c+dx)\csc^5(c+dx)}{6ad} - \frac{\csc^6(c+dx)}{6ad} + \frac{\int \csc^3(c+dx)\,d}{8a}$$
$$= -\frac{\cot(c+dx)\csc(c+dx)}{16ad} - \frac{\cot(c+dx)\csc^3(c+dx)}{24ad} + \frac{\cot(c+dx)\csc^5(c+dx)}{6ad} - \frac{\csc^6(}{6}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{16ad} - \frac{\cot(c+dx)\csc(c+dx)}{16ad} - \frac{\cot(c+dx)\csc^3(c+dx)}{24ad} + \frac{\cot(c+}{}$$

Mathematica [A] time = 0.50, size = 122, normalized size = 1.15

$$-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(3\csc^4\left(\frac{1}{2}(c+dx)\right)+12\csc^2\left(\frac{1}{2}(c+dx)\right)+2\sec^6\left(\frac{1}{2}(c+dx)\right)+3\sec^4\left(\frac{1}{2}(c+dx)\right)\right.}{192ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/192*(Cos[(c + d*x)/2]^2*(12*Csc[(c + d*x)/2]^2 + 3*Csc[(c + d*x)/2]^4 +
24*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]) + 3*Sec[(c + d*x)/2]^4 +
 2*Sec[(c + d*x)/2]^6)*Sec[c + d*x])/(a*d*(1 + Sec[c + d*x]))
```

fricas [B] time = 0.91, size = 217, normalized size = 2.05

$$\frac{6\cos(dx+c)^4+6\cos(dx+c)^3-10\cos(dx+c)^2-3\left(\cos(dx+c)^5+\cos(dx+c)^4-2\cos(dx+c)^3-2\cos(\right.}{96\left(ad\cos(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(6*cos(d*x + c)^4 + 6*cos(d*x + c)^3 - 10*cos(d*x + c)^2 - 3*(cos(d*x
+ c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c
) + 1)*log(1/2*cos(d*x + c) + 1/2) + 3*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2
*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*log(-1/2*cos(d*x + c
```

```
) + 1/2) - 10*cos(d*x + c) - 16)/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4 -
 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d)
```

giac [A] time = 0.29, size = 182, normalized size = 1.72

$$\frac{\frac{3\left(\frac{6(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^2}{a(\cos(dx+c)-1)^2}+\frac{12\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}+\frac{\frac{12a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{9a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{2a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{a^3}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/384*(3*(6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 6*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^2/(a*(cos(d*x + c) - 1)^2) + 1
2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + (12*a^2*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 9*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^
2 + 2*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^3)/d
```

maple [A] time = 0.56, size = 108, normalized size = 1.02

$$-\frac{1}{32ad\left(-1+\cos\left(dx+c\right)\right)^{2}}+\frac{1}{16ad\left(-1+\cos\left(dx+c\right)\right)}+\frac{\ln\left(-1+\cos\left(dx+c\right)\right)}{32ad}-\frac{1}{24ad\left(1+\cos\left(dx+c\right)\right)^{3}}-\frac{}{32ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/32/a/d/(-1+cos(d*x+c))^2+1/16/a/d/(-1+cos(d*x+c))+1/32/a/d*ln(-1+cos(d*x
+c))-1/24/a/d/(1+cos(d*x+c))^3-1/32/a/d/(1+cos(d*x+c))^2-1/32*ln(1+cos(d*x+
c))/d/a
```

maxima [A] time = 0.35, size = 130, normalized size = 1.23

$$\frac{\frac{2\left(3\cos(dx+c)^4+3\cos(dx+c)^3-5\cos(dx+c)^2-5\cos(dx+c)-8\right)}{a\cos(dx+c)^5+a\cos(dx+c)^4-2a\cos(dx+c)^3-2a\cos(dx+c)^2+a\cos(dx+c)+a}-\frac{3\log(\cos(dx+c)+1)}{a}+\frac{3\log(\cos(dx+c)-1)}{a}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(2*(3*cos(d*x + c)^4 + 3*cos(d*x + c)^3 - 5*cos(d*x + c)^2 - 5*cos(d*x
 + c) - 8)/(a*cos(d*x + c)^5 + a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 - 2*a*
cos(d*x + c)^2 + a*cos(d*x + c) + a) - 3*log(cos(d*x + c) + 1)/a + 3*log(co
s(d*x + c) - 1)/a)/d
```

mupad [B] time = 1.01, size = 115, normalized size = 1.08

$$-\frac{\mathrm{atanh}\left(\cos\left(c+d\,x\right)\right)}{16\,a\,d}-\frac{-\frac{\cos(c+d\,x)^4}{16}-\frac{\cos(c+d\,x)^3}{16}+\frac{5\cos(c+d\,x)^2}{48}+\frac{5\cos(c+d\,x)}{48}+\frac{1}{6}}{d\,\left(a\cos\left(c+d\,x\right)^5+a\cos\left(c+d\,x\right)^4-2\,a\cos\left(c+d\,x\right)^3-2\,a\cos\left(c+d\,x\right)^2+a\,\cos(c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^5*(a + a/cos(c + d*x))),x)
```

```
[Out] - atanh(cos(c + d*x))/(16*a*d) - ((5*cos(c + d*x))/48 + (5*cos(c + d*x)^2)/
48 - cos(c + d*x)^3/16 - cos(c + d*x)^4/16 + 1/6)/(d*(a + a*cos(c + d*x) -
2*a*cos(c + d*x)^2 - 2*a*cos(c + d*x)^3 + a*cos(c + d*x)^4 + a*cos(c + d*x)
^5))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^5(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+a*sec(d*x+c)),x)
[Out] Integral(csc(c + d*x)**5/(sec(c + d*x) + 1), x)/a
```

3.65 $\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{\sin^7(c+dx)}{7ad}+\frac{\sin^5(c+dx)\cos^3(c+dx)}{8ad}+\frac{5\sin^3(c+dx)\cos^3(c+dx)}{48ad}+\frac{5\sin(c+dx)\cos^3(c+dx)}{64ad}-\frac{5\sin(c+d}{12}$$

```
[Out] -5/128*x/a-5/128*cos(d*x+c)*sin(d*x+c)/a/d+5/64*cos(d*x+c)^3*sin(d*x+c)/a/d
+5/48*cos(d*x+c)^3*sin(d*x+c)^3/a/d+1/8*cos(d*x+c)^3*sin(d*x+c)^5/a/d+1/7*s
in(d*x+c)^7/a/d
```

Rubi [A] time = 0.21, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2839, 2564, 30, 2568, 2635, 8}

$$\frac{\sin^7(c+dx)}{7ad}+\frac{\sin^5(c+dx)\cos^3(c+dx)}{8ad}+\frac{5\sin^3(c+dx)\cos^3(c+dx)}{48ad}+\frac{5\sin(c+dx)\cos^3(c+dx)}{64ad}-\frac{5\sin(c+d}{12}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-5*x)/(128*a) - (5*Cos[c + d*x]*Sin[c + d*x])/(128*a*d) + (5*Cos[c + d*x]^
3*Sin[c + d*x])/(64*a*d) + (5*Cos[c + d*x]^3*Sin[c + d*x]^3)/(48*a*d) + (Co
s[c + d*x]^3*Sin[c + d*x]^5)/(8*a*d) + Sin[c + d*x]^7/(7*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
```

```
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^8(c+dx)}{-a-a\cos(c+dx)}\,dx\\
&= \frac{\int \cos(c+dx)\sin^6(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^6(c+dx)\,dx}{a}\\
&= \frac{\cos^3(c+dx)\sin^5(c+dx)}{8ad} - \frac{5\int \cos^2(c+dx)\sin^4(c+dx)\,dx}{8a} + \frac{\text{Subst}\left(\int x^6\,dx, x, \sin(c\right.}{ad}\\
&= \frac{5\cos^3(c+dx)\sin^3(c+dx)}{48ad} + \frac{\cos^3(c+dx)\sin^5(c+dx)}{8ad} + \frac{\sin^7(c+dx)}{7ad} - \frac{5\int \cos^2(c+}{}\\
&= \frac{5\cos^3(c+dx)\sin(c+dx)}{64ad} + \frac{5\cos^3(c+dx)\sin^3(c+dx)}{48ad} + \frac{\cos^3(c+dx)\sin^5(c+dx)}{8ad} +\\
&= -\frac{5\cos(c+dx)\sin(c+dx)}{128ad} + \frac{5\cos^3(c+dx)\sin(c+dx)}{64ad} + \frac{5\cos^3(c+dx)\sin^3(c+dx)}{48ad}\\
&= -\frac{5x}{128a} - \frac{5\cos(c+dx)\sin(c+dx)}{128ad} + \frac{5\cos^3(c+dx)\sin(c+dx)}{64ad} + \frac{5\cos^3(c+dx)\sin^3(}{48ad}
\end{aligned}$$

Mathematica [A] time = 1.31, size = 132, normalized size = 1.06

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(1680\sin(c+dx)+336\sin(2(c+dx))-1008\sin(3(c+dx))+168\sin(4(c+dx))+\right.}{10752ad(\sec(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(1176*c - 840*d*x + 1680*Sin[c + d*x] + 33
6*Sin[2*(c + d*x)] - 1008*Sin[3*(c + d*x)] + 168*Sin[4*(c + d*x)] + 336*Sin
[5*(c + d*x)] - 112*Sin[6*(c + d*x)] - 48*Sin[7*(c + d*x)] + 21*Sin[8*(c +
d*x)] - 1176*Tan[c/2]))/(10752*a*d*(1 + Sec[c + d*x]))
```

fricas [A] time = 0.86, size = 91, normalized size = 0.73

$$-\frac{105\,dx-\left(336\,\cos\left(dx+c\right)^7-384\,\cos\left(dx+c\right)^6-952\,\cos\left(dx+c\right)^5+1152\,\cos\left(dx+c\right)^4+826\,\cos\left(dx+c\right)^3-\right.}{2688\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2688*(105*d*x - (336*cos(d*x + c)^7 - 384*cos(d*x + c)^6 - 952*cos(d*x +
 c)^5 + 1152*cos(d*x + c)^4 + 826*cos(d*x + c)^3 - 1152*cos(d*x + c)^2 - 10
5*cos(d*x + c) + 384)*sin(d*x + c))/(a*d)
```

giac [A] time = 0.30, size = 139, normalized size = 1.11

$$-\frac{\frac{105\,(dx+c)}{a}+\frac{2\left(105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+805\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+2681\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-44099\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-5053\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-2681\,\tan\cdots}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{8}a}}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2688*(105*(d*x + c)/a + 2*(105*tan(1/2*d*x + 1/2*c)^15 + 805*tan(1/2*d*x
 + 1/2*c)^13 + 2681*tan(1/2*d*x + 1/2*c)^11 - 44099*tan(1/2*d*x + 1/2*c)^9
- 5053*tan(1/2*d*x + 1/2*c)^7 - 2681*tan(1/2*d*x + 1/2*c)^5 - 805*tan(1/2*d
*x + 1/2*c)^3 - 105*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^8*a
))/d
```

maple [B] time = 0.44, size = 290, normalized size = 2.32

$$\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{115\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{383\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^8}+\frac{5053\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\cdots\right.\right.}{1344ad\left(1+\tan^2\left(\frac{dx}{2}+\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^8/(a+a*sec(d*x+c)),x)
```

```
[Out] 5/64/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)+115/192/a/d/(1+tan(1
/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^3+383/192/a/d/(1+tan(1/2*d*x+1/2*c)^2
)^8*tan(1/2*d*x+1/2*c)^5+5053/1344/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d
*x+1/2*c)^7+44099/1344/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^9-
383/192/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^11-115/192/a/d/(1
+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^13-5/64/a/d/(1+tan(1/2*d*x+1/2*
c)^2)^8*tan(1/2*d*x+1/2*c)^15-5/64/a/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.48, size = 360, normalized size = 2.88

$$\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2681\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5053\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{44099\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{2681\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{805\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{105\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}}{a+\frac{8a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{56a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{70a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{56a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{8a\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{a\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}}-\frac{105\arctan\left(\cdots\right.}{a}}{1344\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/1344*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 2681*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5053*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 + 44099*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 2681*s
in(d*x + c)^11/(cos(d*x + c) + 1)^11 - 805*sin(d*x + c)^13/(cos(d*x + c) +
1)^13 - 105*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)/(a + 8*a*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 28*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 56*a*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
+ 56*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a*sin(d*x + c)^12/(cos(d*
x + c) + 1)^12 + 8*a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + a*sin(d*x + c)
^16/(cos(d*x + c) + 1)^16) - 105*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)
/d
```

mupad [B] time = 3.90, size = 132, normalized size = 1.06

$$-\frac{-\frac{5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{15}}{64}-\frac{115\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{13}}{192}-\frac{383\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}}{192}+\frac{44099\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{9}}{1344}+\frac{5053\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{1344}+\frac{383\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{192}+\frac{115\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{192}}{a\,d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)^{8}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8/(a + a/cos(c + d*x)),x)
```

```
[Out] ((5*tan(c/2 + (d*x)/2))/64 + (115*tan(c/2 + (d*x)/2)^3)/192 + (383*tan(c/2
+ (d*x)/2)^5)/192 + (5053*tan(c/2 + (d*x)/2)^7)/1344 + (44099*tan(c/2 + (d*
x)/2)^9)/1344 - (383*tan(c/2 + (d*x)/2)^11)/192 - (115*tan(c/2 + (d*x)/2)^1
3)/192 - (5*tan(c/2 + (d*x)/2)^15)/64)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^8) -
 (5*x)/(128*a)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**8/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.66 $\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=99

$$\frac{\sin^5(c+dx)}{5ad}+\frac{\sin^3(c+dx)\cos^3(c+dx)}{6ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{8ad}-\frac{\sin(c+dx)\cos(c+dx)}{16ad}-\frac{x}{16a}$$

```
[Out] -1/16*x/a-1/16*cos(d*x+c)*sin(d*x+c)/a/d+1/8*cos(d*x+c)^3*sin(d*x+c)/a/d+1/
6*cos(d*x+c)^3*sin(d*x+c)^3/a/d+1/5*sin(d*x+c)^5/a/d
```

Rubi [A] time = 0.18, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3872, 2839, 2564, 30, 2568, 2635, 8}

$$\frac{\sin^5(c+dx)}{5ad}+\frac{\sin^3(c+dx)\cos^3(c+dx)}{6ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{8ad}-\frac{\sin(c+dx)\cos(c+dx)}{16ad}-\frac{x}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] -x/(16*a) - (Cos[c + d*x]*Sin[c + d*x])/(16*a*d) + (Cos[c + d*x]^3*Sin[c +
d*x])/(8*a*d) + (Cos[c + d*x]^3*Sin[c + d*x]^3)/(6*a*d) + Sin[c + d*x]^5/(5
*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
```

```
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^6(c+dx)}{-a-a\cos(c+dx)}\,dx\\
&= \frac{\int \cos(c+dx)\sin^4(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^4(c+dx)\,dx}{a}\\
&= \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} - \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{2a} + \frac{\text{Subst}\left(\int x^4\,dx, x, \sin(c+\right.}{ad}\\
&= \frac{\cos^3(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} + \frac{\sin^5(c+dx)}{5ad} - \frac{\int \cos^2(c+dx)}{8a}\\
&= -\frac{\cos(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} + \text{sin}\\
&= -\frac{x}{16a} - \frac{\cos(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad}\end{aligned}$$

Mathematica [A] time = 0.74, size = 112, normalized size = 1.13

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(120\sin(c+dx)+15\sin(2(c+dx))-60\sin(3(c+dx))+15\sin(4(c+dx))+12\sin\right.}{480ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(75*c - 60*d*x + 120*Sin[c + d*x] + 15*Sin
[2*(c + d*x)] - 60*Sin[3*(c + d*x)] + 15*Sin[4*(c + d*x)] + 12*Sin[5*(c + d
*x)] - 5*Sin[6*(c + d*x)] - 75*Tan[c/2]))/(480*a*d*(1 + Sec[c + d*x]))
```

fricas [A] time = 0.89, size = 70, normalized size = 0.71

$$-\frac{15\,dx+\left(40\,\cos(dx+c)^5-48\,\cos(dx+c)^4-70\,\cos(dx+c)^3+96\,\cos(dx+c)^2+15\,\cos(dx+c)-48\right)\sin(}{240\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(15*d*x + (40*cos(d*x + c)^5 - 48*cos(d*x + c)^4 - 70*cos(d*x + c)^3
 + 96*cos(d*x + c)^2 + 15*cos(d*x + c) - 48)*sin(d*x + c))/(a*d)
```

giac [A] time = 0.45, size = 113, normalized size = 1.14

$$-\frac{\frac{15\,(dx+c)}{a}+\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+85\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-1338\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-198\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-85\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}a}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(15*(d*x + c)/a + 2*(15*tan(1/2*d*x + 1/2*c)^11 + 85*tan(1/2*d*x + 1
/2*c)^9 - 1338*tan(1/2*d*x + 1/2*c)^7 - 198*tan(1/2*d*x + 1/2*c)^5 - 85*tan
(1/2*d*x + 1/2*c)^3 - 15*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1
)^6*a))/d
```

maple [B] time = 0.43, size = 222, normalized size = 2.24

$$-\frac{\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}-\frac{17\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{223\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{33\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/8/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11-17/24/a/d/(1+tan(
1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9+223/20/a/d/(1+tan(1/2*d*x+1/2*c)^2
)^6*tan(1/2*d*x+1/2*c)^7+33/20/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1
/2*c)^5+17/24/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3+1/8/a/d/(
1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)-1/8/a/d*arctan(tan(1/2*d*x+1/2
*c))
```

maxima [B] time = 0.44, size = 278, normalized size = 2.81

$$\frac{\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{85\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{198\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{1338\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{85\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{15\,\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a+\frac{6\,a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15\,a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20\,a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15\,a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6\,a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{15\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/120*((15*sin(d*x + c)/(cos(d*x + c) + 1) + 85*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 198*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1338*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7 - 85*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 15*sin(d*x +
c)^11/(cos(d*x + c) + 1)^11)/(a + 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
 15*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + 15*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10 + a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) - 15*arcta
n(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

mupad [B] time = 3.66, size = 106, normalized size = 1.07

$$\frac{-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}-\frac{17\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9}{24}+\frac{223\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{20}+\frac{33\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{20}+\frac{17\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{a\,d\,{\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}^6}-\frac{x}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + a/cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)/8 + (17*tan(c/2 + (d*x)/2)^3)/24 + (33*tan(c/2 + (d*x)/
2)^5)/20 + (223*tan(c/2 + (d*x)/2)^7)/20 - (17*tan(c/2 + (d*x)/2)^9)/24 - t
an(c/2 + (d*x)/2)^11/8)/(a*d*(tan(c/2 + (d*x)/2)^2 + 1)^6) - x/(16*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^6(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**6/(sec(c + d*x) + 1), x)/a
```

3.67 $\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{\sin^3(c+dx)}{3ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\sin(c+dx)\cos(c+dx)}{8ad}-\frac{x}{8a}$$

```
[Out] -1/8*x/a-1/8*cos(d*x+c)*sin(d*x+c)/a/d+1/4*cos(d*x+c)^3*sin(d*x+c)/a/d+1/3*
sin(d*x+c)^3/a/d
```

Rubi [A] time = 0.15, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2839, 2564, 30, 2568, 2635, 8}

$$\frac{\sin^3(c+dx)}{3ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\sin(c+dx)\cos(c+dx)}{8ad}-\frac{x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] -x/(8*a) - (Cos[c + d*x]*Sin[c + d*x])/(8*a*d) + (Cos[c + d*x]^3*Sin[c + d*
x])/(4*a*d) + Sin[c + d*x]^3/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
```

```
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^4(c+dx)}{-a-a\cos(c+dx)}\,dx\\
&= \frac{\int \cos(c+dx)\sin^2(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{a}\\
&= \frac{\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\int \cos^2(c+dx)\,dx}{4a} + \frac{\text{Subst}\left(\int x^2\,dx, x, \sin(c+dx)\right)}{ad}\\
&= -\frac{\cos(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{4ad} + \frac{\sin^3(c+dx)}{3ad} - \frac{\int 1\,dx}{8a}\\
&= -\frac{x}{8a} - \frac{\cos(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{4ad} + \frac{\sin^3(c+dx)}{3ad}
\end{aligned}$$

Mathematica [A] time = 0.65, size = 83, normalized size = 1.14

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(24\sin(c+dx)-8\sin(3(c+dx))+3\left(\sin(4(c+dx))+4c-4\tan\left(\frac{c}{2}\right)-4dx\right)\right)}{48ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(24*Sin[c + d*x] - 8*Sin[3*(c + d*x)] + 3*
(4*c - 4*d*x + Sin[4*(c + d*x)] - 4*Tan[c/2])))/(48*a*d*(1 + Sec[c + d*x]))
```

fricas [A] time = 1.01, size = 51, normalized size = 0.70

$$-\frac{3\,dx-\left(6\,\cos\left(dx+c\right)^3-8\,\cos\left(dx+c\right)^2-3\,\cos\left(dx+c\right)+8\right)\sin\left(dx+c\right)}{24\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(3*d*x - (6*cos(d*x + c)^3 - 8*cos(d*x + c)^2 - 3*cos(d*x + c) + 8)*s
in(d*x + c))/(a*d)
```

giac [A] time = 0.20, size = 87, normalized size = 1.19

$$-\frac{\frac{3\,(dx+c)}{a}+\frac{2\left(3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-53\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-11\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4 a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(3*(d*x + c)/a + 2*(3*tan(1/2*d*x + 1/2*c)^7 - 53*tan(1/2*d*x + 1/2*c
)^5 - 11*tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1
/2*c)^2 + 1)^4*a))/d
```

maple [B] time = 0.42, size = 154, normalized size = 2.11

$$-\frac{\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{53\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{11\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}+\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+53/12/a/d/(1+tan(1
/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5+11/12/a/d/(1+tan(1/2*d*x+1/2*c)^2)^
4*tan(1/2*d*x+1/2*c)^3+1/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c
)-1/4/a/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.43, size = 196, normalized size = 2.68

$$\frac{\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{11\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{53\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a+\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*((3*sin(d*x + c)/(cos(d*x + c) + 1) + 11*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 53*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a*sin
(d*x + c)^8/(cos(d*x + c) + 1)^8) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1
))/a)/d
```

mupad [B] time = 0.98, size = 55, normalized size = 0.75

$$\frac{\sin(c+d\,x)}{4\,a\,d}-\frac{x}{8\,a}-\frac{\sin(3\,c+3\,d\,x)}{12\,a\,d}+\frac{\sin(4\,c+4\,d\,x)}{32\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + a/cos(c + d*x)),x)
```

```
[Out] sin(c + d*x)/(4*a*d) - x/(8*a) - sin(3*c + 3*d*x)/(12*a*d) + sin(4*c + 4*d*
x)/(32*a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sin^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

3.68 $\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=44

$$\frac{\sin(c+dx)}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{x}{2a}$$

```
[Out] -1/2*x/a+sin(d*x+c)/a/d-1/2*cos(d*x+c)*sin(d*x+c)/a/d
```

Rubi [A] time = 0.11, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2839, 2637, 2635, 8}

$$\frac{\sin(c+dx)}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] -x/(2*a) + Sin[c + d*x]/(a*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)\sin^2(c+dx)}{-a-a\cos(c+dx)}\,dx$$
$$= \frac{\int \cos(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\,dx}{a}$$
$$= \frac{\sin(c+dx)}{ad} - \frac{\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\int 1\,dx}{2a}$$
$$= -\frac{x}{2a} + \frac{\sin(c+dx)}{ad} - \frac{\cos(c+dx)\sin(c+dx)}{2ad}$$

Mathematica [A] time = 0.28, size = 68, normalized size = 1.55

$$-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-4\sin(c+dx)+\sin(2(c+dx))-c+\tan\left(\frac{c}{2}\right)+2dx\right)}{2ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/2*(Cos[(c + d*x)/2]^2*Sec[c + d*x]*(-c + 2*d*x - 4*Sin[c + d*x] + Sin[2*
(c + d*x)] + Tan[c/2]))/(a*d*(1 + Sec[c + d*x]))
```

fricas [A] time = 1.69, size = 27, normalized size = 0.61

$$-\frac{dx + (\cos(dx+c) - 2)\sin(dx+c)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(d*x + (cos(d*x + c) - 2)*sin(d*x + c))/(a*d)
```

giac [A] time = 0.32, size = 58, normalized size = 1.32

$$-\frac{\frac{dx+c}{a} - \frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2 a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)/a - 2*(3*tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))/((t
an(1/2*d*x + 1/2*c)^2 + 1)^2*a))/d
```

maple [B] time = 0.33, size = 85, normalized size = 1.93

$$\frac{3\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2} + \frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2} - \frac{\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sec(d*x+c)),x)
```

```
[Out] 3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+1/a/d/(1+tan(1/2*d*x+
1/2*c)^2)^2*tan(1/2*d*x+1/2*c)-1/a/d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.45, size = 112, normalized size = 2.55

$$\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] ((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/
(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4) - arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

mupad [B] time = 0.96, size = 30, normalized size = 0.68

$$-\frac{\sin(2c+2dx)-4\sin(c+dx)+2dx}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + a/cos(c + d*x)),x)
```

```
[Out] -(sin(2*c + 2*d*x) - 4*sin(c + d*x) + 2*d*x)/(4*a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sin^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

3.69 $\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^3(c+dx)}{3ad}$$

```
[Out] 1/3*cot(d*x+c)^3/a/d-1/3*csc(d*x+c)^3/a/d
```

Rubi [A] time = 0.13, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2839, 2606, 30, 2607}

$$\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^3/(3*a*d) - Csc[c + d*x]^3/(3*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc(c+dx)}{-a-a\cos(c+dx)}\,dx\\ &= -\frac{\int \cot^2(c+dx)\csc^2(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^3(c+dx)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int x^2\,dx,x,-\cot(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^2\,dx,x,\csc(c+dx)\right)}{ad}\\ &= \frac{\cot^3(c+dx)}{3ad}-\frac{\csc^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [A] time = 0.22, size = 66, normalized size = 1.78

$$\frac{\csc(c)(2\sin(c+dx)+\sin(2(c+dx))+2\sin(c+2dx)-6\sin(c)+4\sin(dx))\csc(2(c+dx))}{6ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Csc[c]*Csc[2*(c + d*x)]*(-6*Sin[c] + 4*Sin[d*x] + 2*Sin[c + d*x] + Sin[2*(
c + d*x)] + 2*Sin[c + 2*d*x]))/(6*a*d*(1 + Sec[c + d*x]))
```

fricas [A] time = 0.71, size = 41, normalized size = 1.11

$$-\frac{\cos\left(dx+c\right)^2+\cos\left(dx+c\right)+1}{3\left(ad\cos\left(dx+c\right)+ad\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(cos(d*x + c)^2 + cos(d*x + c) + 1)/((a*d*cos(d*x + c) + a*d)*sin(d*x
+ c))
```

giac [A] time = 0.54, size = 37, normalized size = 1.00

$$-\frac{\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{a}+\frac{3}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(tan(1/2*d*x + 1/2*c)^3/a + 3/(a*tan(1/2*d*x + 1/2*c)))/d
```

maple [A] time = 0.47, size = 36, normalized size = 0.97

$$\frac{-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{4da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/4/d/a*(-1/3*tan(1/2*d*x+1/2*c)^3-1/tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.34, size = 49, normalized size = 1.32

$$-\frac{\frac{3\,(\cos(dx+c)+1)}{a\sin(dx+c)}+\frac{\sin(dx+c)^3}{a(\cos(dx+c)+1)^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*(cos(d*x + c) + 1)/(a*sin(d*x + c)) + sin(d*x + c)^3/(a*(cos(d*x +
 c) + 1)^3))/d
```

mupad [B] time = 0.93, size = 32, normalized size = 0.86

$$-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3}{12\,a\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + a/cos(c + d*x))),x)
```

```
[Out] -(tan(c/2 + (d*x)/2)^4 + 3)/(12*a*d*tan(c/2 + (d*x)/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

3.70 $\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^5(c+dx)}{5ad}$$

```
[Out] 1/3*cot(d*x+c)^3/a/d+1/5*cot(d*x+c)^5/a/d-1/5*csc(d*x+c)^5/a/d
```

Rubi [A] time = 0.14, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2839, 2606, 30, 2607, 14}

$$\frac{\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^3/(3*a*d) + Cot[c + d*x]^5/(5*a*d) - Csc[c + d*x]^5/(5*a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
```

```
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc^3(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= -\frac{\int \cot^2(c+dx)\csc^4(c+dx)\,dx}{a} + \frac{\int \cot(c+dx)\csc^5(c+dx)\,dx}{a} \\
&= -\frac{\text{Subst}\left(\int x^4\,dx, x, \csc(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int x^2\left(1+x^2\right)\,dx, x, -\cot(c+dx)\right)}{ad} \\
&= -\frac{\csc^5(c+dx)}{5ad} - \frac{\text{Subst}\left(\int \left(x^2+x^4\right)\,dx, x, -\cot(c+dx)\right)}{ad} \\
&= \frac{\cot^3(c+dx)}{3ad} + \frac{\cot^5(c+dx)}{5ad} - \frac{\csc^5(c+dx)}{5ad}
\end{aligned}
$$

Mathematica [B] time = 0.54, size = 116, normalized size = 2.11

$$
-\frac{\csc(c)(-54\sin(c+dx)-18\sin(2(c+dx))+18\sin(3(c+dx))+9\sin(4(c+dx))-32\sin(c+2dx)+32\sin(2}{960ad(\sec(c+dx)+1)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/960*(Csc[c]*Csc[c + d*x]^3*Sec[c + d*x]*(240*Sin[c] - 96*Sin[d*x] - 54*S
in[c + d*x] - 18*Sin[2*(c + d*x)] + 18*Sin[3*(c + d*x)] + 9*Sin[4*(c + d*x)
] - 32*Sin[c + 2*d*x] + 32*Sin[2*c + 3*d*x] + 16*Sin[3*c + 4*d*x]))/(a*d*(1
 + Sec[c + d*x]))
```

fricas [A] time = 0.67, size = 89, normalized size = 1.62

$$
-\frac{2\cos(dx+c)^4+2\cos(dx+c)^3-3\cos(dx+c)^2-3\cos(dx+c)-3}{15\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2-ad\cos(dx+c)-ad\right)\sin(dx+c)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 3*cos(d*x + c)^2 - 3*cos(d*x +
 c) - 3)/((a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d
)*sin(d*x + c))
```

giac [A] time = 0.24, size = 74, normalized size = 1.35

$$
-\frac{\frac{5\left(6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}+\frac{3a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+10a^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}{a^5}}{240d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(5*(6*tan(1/2*d*x + 1/2*c)^2 + 1)/(a*tan(1/2*d*x + 1/2*c)^3) + (3*a^
4*tan(1/2*d*x + 1/2*c)^5 + 10*a^4*tan(1/2*d*x + 1/2*c)^3)/a^5)/d
```

maple [A] time = 0.57, size = 62, normalized size = 1.13

$$-\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{2}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{16da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/16/d/a*(-1/5*tan(1/2*d*x+1/2*c)^5-2/3*tan(1/2*d*x+1/2*c)^3-1/3/tan(1/2*d*
x+1/2*c)^3-2/tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.33, size = 96, normalized size = 1.75

$$-\frac{\frac{\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a}+\frac{5\left(\frac{6\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^3}{a\sin(dx+c)^3}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*((10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5)/a + 5*(6*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x +
c) + 1)^3/(a*sin(d*x + c)^3))/d
```

mupad [B] time = 1.09, size = 60, normalized size = 1.09

$$-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+30\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+5}{240\,a\,d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + a/cos(c + d*x))),x)
```

```
[Out] -(30*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^6 + 3*tan(c/2 + (d*x)/2)^
8 + 5)/(240*a*d*tan(c/2 + (d*x)/2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

3.71 $\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{\cot^7(c+dx)}{7ad}+\frac{2\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^7(c+dx)}{7ad}$$

```
[Out] 1/3*cot(d*x+c)^3/a/d+2/5*cot(d*x+c)^5/a/d+1/7*cot(d*x+c)^7/a/d-1/7*csc(d*x+
c)^7/a/d
```

Rubi [A] time = 0.15, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2839, 2606, 30, 2607, 270}

$$\frac{\cot^7(c+dx)}{7ad}+\frac{2\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^7(c+dx)}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^3/(3*a*d) + (2*Cot[c + d*x]^5)/(5*a*d) + Cot[c + d*x]^7/(7*a*d
) - Csc[c + d*x]^7/(7*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
```

```
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc^5(c+dx)}{-a-a\cos(c+dx)}\,dx\\
&= -\frac{\int \cot^2(c+dx)\csc^6(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^7(c+dx)\,dx}{a}\\
&= -\frac{\text{Subst}\left(\int x^6\,dx, x, \csc(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^2\left(1+x^2\right)^2\,dx, x, -\cot(c+dx)\right)}{ad}\\
&= -\frac{\csc^7(c+dx)}{7ad}-\frac{\text{Subst}\left(\int \left(x^2+2x^4+x^6\right)\,dx, x, -\cot(c+dx)\right)}{ad}\\
&= \frac{\cot^3(c+dx)}{3ad}+\frac{2\cot^5(c+dx)}{5ad}+\frac{\cot^7(c+dx)}{7ad}-\frac{\csc^7(c+dx)}{7ad}
\end{aligned}$$

Mathematica [B] time = 0.66, size = 158, normalized size = 2.16

$$\frac{\csc(c)(1500\sin(c+dx)+375\sin(2(c+dx))-750\sin(3(c+dx))-300\sin(4(c+dx))+150\sin(5(c+dx))+75}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Csc[c]*Csc[c + d*x]^5*Sec[c + d*x]*(-8960*Sin[c] + 2560*Sin[d*x] + 1500*Si
n[c + d*x] + 375*Sin[2*(c + d*x)] - 750*Sin[3*(c + d*x)] - 300*Sin[4*(c + d
*x)] + 150*Sin[5*(c + d*x)] + 75*Sin[6*(c + d*x)] + 640*Sin[c + 2*d*x] - 12
80*Sin[2*c + 3*d*x] - 512*Sin[3*c + 4*d*x] + 256*Sin[4*c + 5*d*x] + 128*Sin
[5*c + 6*d*x]))/(53760*a*d*(1 + Sec[c + d*x]))
```

fricas [B] time = 0.88, size = 131, normalized size = 1.79

$$-\frac{8\cos(dx+c)^6+8\cos(dx+c)^5-20\cos(dx+c)^4-20\cos(dx+c)^3+15\cos(dx+c)^2+15\cos(dx+c)+}{105\left(ad\cos(dx+c)^5+ad\cos(dx+c)^4-2\,ad\cos(dx+c)^3-2\,ad\cos(dx+c)^2+ad\cos(dx+c)+ad\right)\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/105*(8*cos(d*x + c)^6 + 8*cos(d*x + c)^5 - 20*cos(d*x + c)^4 - 20*cos(d*
x + c)^3 + 15*cos(d*x + c)^2 + 15*cos(d*x + c) + 15)/((a*d*cos(d*x + c)^5 +
 a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos
(d*x + c) + a*d)*sin(d*x + c))
```

giac [A] time = 0.25, size = 103, normalized size = 1.41

$$-\frac{\frac{7\left(75\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+20\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}+\frac{15a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+84a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+175a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}{a^7}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6720*(7*(75*tan(1/2*d*x + 1/2*c)^4 + 20*tan(1/2*d*x + 1/2*c)^2 + 3)/(a*t
an(1/2*d*x + 1/2*c)^5) + (15*a^6*tan(1/2*d*x + 1/2*c)^7 + 84*a^6*tan(1/2*d*
x + 1/2*c)^5 + 175*a^6*tan(1/2*d*x + 1/2*c)^3)/a^7)/d
```

maple [A] time = 0.55, size = 88, normalized size = 1.21

$$-\frac{\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{4\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{5\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{4}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}-\frac{5}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{64da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/64/d/a*(-1/7*tan(1/2*d*x+1/2*c)^7-4/5*tan(1/2*d*x+1/2*c)^5-5/3*tan(1/2*d*
x+1/2*c)^3-4/3/tan(1/2*d*x+1/2*c)^3-1/5/tan(1/2*d*x+1/2*c)^5-5/tan(1/2*d*x+
1/2*c))
```

maxima [B] time = 0.34, size = 136, normalized size = 1.86

$$-\frac{\frac{\frac{175\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{84\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{15\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{a}+\frac{7\left(\frac{20\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{75\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+3\right)(\cos(dx+c)+1)^{5}}{a\sin(dx+c)^{5}}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6720*((175*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a + 7*(20*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + 75*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3)*
(cos(d*x + c) + 1)^5/(a*sin(d*x + c)^5))/d
```

mupad [B] time = 1.16, size = 153, normalized size = 2.10

$$-\frac{21\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+140\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+525\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{8}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+175\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{6720\,a\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^6*(a + a/cos(c + d*x))),x)
```

```
[Out] -(21*cos(c/2 + (d*x)/2)^12 + 15*sin(c/2 + (d*x)/2)^12 + 84*cos(c/2 + (d*x)/
2)^2*sin(c/2 + (d*x)/2)^10 + 175*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^8
+ 525*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4 + 140*cos(c/2 + (d*x)/2)^10
*sin(c/2 + (d*x)/2)^2)/(6720*a*d*cos(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^{6}{(c+dx)}}{\sec{(c+dx)}+1}\, dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**6/(sec(c + d*x) + 1), x)/a
```

3.72 $\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{\cot^9(c+dx)}{9ad}+\frac{3\cot^7(c+dx)}{7ad}+\frac{3\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^9(c+dx)}{9ad}$$

```
[Out] 1/3*cot(d*x+c)^3/a/d+3/5*cot(d*x+c)^5/a/d+3/7*cot(d*x+c)^7/a/d+1/9*cot(d*x+
c)^9/a/d-1/9*csc(d*x+c)^9/a/d
```

Rubi [A] time = 0.15, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2839, 2606, 30, 2607, 270}

$$\frac{\cot^9(c+dx)}{9ad}+\frac{3\cot^7(c+dx)}{7ad}+\frac{3\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^9(c+dx)}{9ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^3/(3*a*d) + (3*Cot[c + d*x]^5)/(5*a*d) + (3*Cot[c + d*x]^7)/(7
*a*d) + Cot[c + d*x]^9/(9*a*d) - Csc[c + d*x]^9/(9*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
```

```
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc^7(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= -\frac{\int \cot^2(c+dx)\csc^8(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^9(c+dx)\,dx}{a} \\
&= -\frac{\text{Subst}\left(\int x^8\,dx, x, \csc(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^2\left(1+x^2\right)^3\,dx, x, -\cot(c+dx)\right)}{ad} \\
&= -\frac{\csc^9(c+dx)}{9ad}-\frac{\text{Subst}\left(\int \left(x^2+3x^4+3x^6+x^8\right)dx, x, -\cot(c+dx)\right)}{ad} \\
&= \frac{\cot^3(c+dx)}{3ad}+\frac{3\cot^5(c+dx)}{5ad}+\frac{3\cot^7(c+dx)}{7ad}+\frac{\cot^9(c+dx)}{9ad}-\frac{\csc^9(c+dx)}{9ad}
\end{aligned}
$$

Mathematica [B] time = 1.06, size = 200, normalized size = 2.20

$$-\frac{\csc(c)(-85750\sin(c+dx)-17150\sin(2(c+dx))+51450\sin(3(c+dx))+17150\sin(4(c+dx))-17150\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/5160960*(Csc[c]*Csc[c + d*x]^7*Sec[c + d*x]*(645120*Sin[c] - 143360*Sin[
d*x] - 85750*Sin[c + d*x] - 17150*Sin[2*(c + d*x)] + 51450*Sin[3*(c + d*x)]
 + 17150*Sin[4*(c + d*x)] - 17150*Sin[5*(c + d*x)] - 7350*Sin[6*(c + d*x)]
+ 2450*Sin[7*(c + d*x)] + 1225*Sin[8*(c + d*x)] - 28672*Sin[c + 2*d*x] + 86
016*Sin[2*c + 3*d*x] + 28672*Sin[3*c + 4*d*x] - 28672*Sin[4*c + 5*d*x] - 12
288*Sin[5*c + 6*d*x] + 4096*Sin[6*c + 7*d*x] + 2048*Sin[7*c + 8*d*x]))/(a*d
*(1 + Sec[c + d*x]))
```

fricas [B] time = 0.76, size = 177, normalized size = 1.95

$$-\frac{16\cos(dx+c)^8+16\cos(dx+c)^7-56\cos(dx+c)^6-56\cos(dx+c)^5+70\cos(dx+c)^4+70\cos(dx+}{315\left(ad\cos(dx+c)^7+ad\cos(dx+c)^6-3\,ad\cos(dx+c)^5-3\,ad\cos(dx+c)^4+3\,ad\cos(dx+c)^3+3\,ad\,\text{c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/315*(16*cos(d*x + c)^8 + 16*cos(d*x + c)^7 - 56*cos(d*x + c)^6 - 56*cos(
d*x + c)^5 + 70*cos(d*x + c)^4 + 70*cos(d*x + c)^3 - 35*cos(d*x + c)^2 - 35
*cos(d*x + c) - 35)/((a*d*cos(d*x + c)^7 + a*d*cos(d*x + c)^6 - 3*a*d*cos(d
*x + c)^5 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x + c)^3 + 3*a*d*cos(d*x + c
)^2 - a*d*cos(d*x + c) - a*d)*sin(d*x + c))
```

giac [A] time = 0.28, size = 132, normalized size = 1.45

$$-\frac{\frac{3\left(1470\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+490\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+126\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7}+\frac{35a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+270a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+882a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{a^9}}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/80640*(3*(1470*tan(1/2*d*x + 1/2*c)^6 + 490*tan(1/2*d*x + 1/2*c)^4 + 126
*tan(1/2*d*x + 1/2*c)^2 + 15)/(a*tan(1/2*d*x + 1/2*c)^7) + (35*a^8*tan(1/2*
d*x + 1/2*c)^9 + 270*a^8*tan(1/2*d*x + 1/2*c)^7 + 882*a^8*tan(1/2*d*x + 1/2
*c)^5 + 1470*a^8*tan(1/2*d*x + 1/2*c)^3)/a^9)/d
```

maple [A] time = 0.60, size = 114, normalized size = 1.25

$$\frac{-\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{6\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{14\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{14\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{1}{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}-\frac{14}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{6}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}-\frac{14}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{256da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/256/d/a*(-1/9*tan(1/2*d*x+1/2*c)^9-6/7*tan(1/2*d*x+1/2*c)^7-14/5*tan(1/2*
d*x+1/2*c)^5-14/3*tan(1/2*d*x+1/2*c)^3-1/7/tan(1/2*d*x+1/2*c)^7-14/3/tan(1/
2*d*x+1/2*c)^3-6/5/tan(1/2*d*x+1/2*c)^5-14/tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.40, size = 176, normalized size = 1.93

$$-\frac{\frac{\frac{1470\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{882\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{270\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{35\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}}{a}+\frac{3\left(\frac{126\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{490\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{1470\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+15\right)(\cos(dx+c)+1)^{7}}{a\sin(dx+c)^{7}}}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/80640*((1470*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 882*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5 + 270*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x +
 c)^9/(cos(d*x + c) + 1)^9)/a + 3*(126*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
+ 490*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1470*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + 15)*(cos(d*x + c) + 1)^7/(a*sin(d*x + c)^7))/d
```

mupad [B] time = 1.45, size = 201, normalized size = 2.21

$$-\frac{45\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}+378\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1470\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+4410\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{80}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^8*(a + a/cos(c + d*x))),x)
```

```
[Out] -(45*cos(c/2 + (d*x)/2)^16 + 35*sin(c/2 + (d*x)/2)^16 + 270*cos(c/2 + (d*x)
/2)^2*sin(c/2 + (d*x)/2)^14 + 882*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^1
2 + 1470*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 + 4410*cos(c/2 + (d*x)/
2)^10*sin(c/2 + (d*x)/2)^6 + 1470*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/2)^
4 + 378*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2)/(80640*a*d*cos(c/2 + (d
*x)/2)^9*sin(c/2 + (d*x)/2)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.73 $\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=109

$$\frac{\cot^{11}(c+dx)}{11ad}+\frac{4\cot^{9}(c+dx)}{9ad}+\frac{6\cot^{7}(c+dx)}{7ad}+\frac{4\cot^{5}(c+dx)}{5ad}+\frac{\cot^{3}(c+dx)}{3ad}-\frac{\csc^{11}(c+dx)}{11ad}$$

```
[Out] 1/3*cot(d*x+c)^3/a/d+4/5*cot(d*x+c)^5/a/d+6/7*cot(d*x+c)^7/a/d+4/9*cot(d*x+
c)^9/a/d+1/11*cot(d*x+c)^11/a/d-1/11*csc(d*x+c)^11/a/d
```

Rubi [A] time = 0.16, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2839, 2606, 30, 2607, 270}

$$\frac{\cot^{11}(c+dx)}{11ad}+\frac{4\cot^{9}(c+dx)}{9ad}+\frac{6\cot^{7}(c+dx)}{7ad}+\frac{4\cot^{5}(c+dx)}{5ad}+\frac{\cot^{3}(c+dx)}{3ad}-\frac{\csc^{11}(c+dx)}{11ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^10/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^3/(3*a*d) + (4*Cot[c + d*x]^5)/(5*a*d) + (6*Cot[c + d*x]^7)/(7
*a*d) + (4*Cot[c + d*x]^9)/(9*a*d) + Cot[c + d*x]^11/(11*a*d) - Csc[c + d*x
]^11/(11*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc^9(c+dx)}{-a-a\cos(c+dx)}\,dx\\ &= -\frac{\int \cot^2(c+dx)\csc^{10}(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^{11}(c+dx)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int x^{10}\,dx,x,\csc(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^2\left(1+x^2\right)^4\,dx,x,-\cot(c+dx)\right)}{ad}\\ &= -\frac{\csc^{11}(c+dx)}{11ad}-\frac{\text{Subst}\left(\int \left(x^2+4x^4+6x^6+4x^8+x^{10}\right)dx,x,-\cot(c+dx)\right)}{ad}\\ &= \frac{\cot^3(c+dx)}{3ad}+\frac{4\cot^5(c+dx)}{5ad}+\frac{6\cot^7(c+dx)}{7ad}+\frac{4\cot^9(c+dx)}{9ad}+\frac{\cot^{11}(c+dx)}{11ad}-\text{csc}\end{aligned}$$

Mathematica [B] time = 1.58, size = 242, normalized size = 2.22

$$\frac{\csc(c)(5000940\sin(c+dx)+833490\sin(2(c+dx))-3333960\sin(3(c+dx))-952560\sin(4(c+dx))+142884}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^10/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Csc[c]*Csc[c + d*x]^9*Sec[c + d*x]*(-45416448*Sin[c] + 8257536*Sin[d*x] +
5000940*Sin[c + d*x] + 833490*Sin[2*(c + d*x)] - 3333960*Sin[3*(c + d*x)] -
 952560*Sin[4*(c + d*x)] + 1428840*Sin[5*(c + d*x)] + 535815*Sin[6*(c + d*x
)] - 357210*Sin[7*(c + d*x)] - 158760*Sin[8*(c + d*x)] + 39690*Sin[9*(c + d
*x)] + 19845*Sin[10*(c + d*x)] + 1376256*Sin[c + 2*d*x] - 5505024*Sin[2*c +
 3*d*x] - 1572864*Sin[3*c + 4*d*x] + 2359296*Sin[4*c + 5*d*x] + 884736*Sin[
5*c + 6*d*x] - 589824*Sin[6*c + 7*d*x] - 262144*Sin[7*c + 8*d*x] + 65536*Si
n[8*c + 9*d*x] + 32768*Sin[9*c + 10*d*x]))/(454164480*a*d*(1 + Sec[c + d*x]
))
```

fricas [B] time = 0.60, size = 219, normalized size = 2.01

$$-\frac{128\cos(dx+c)^{10}+128\cos(dx+c)^9-576\cos(dx+c)^8-576\cos(dx+c)^7+1008\cos(dx+c)^6+1008\cos}{3465\left(ad\cos(dx+c)^9+ad\cos(dx+c)^8-4\,ad\cos(dx+c)^7-4\,ad\cos(dx+c)^6+6\,ad\cos(dx+c)^5+6\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3465*(128*cos(d*x + c)^10 + 128*cos(d*x + c)^9 - 576*cos(d*x + c)^8 - 57
6*cos(d*x + c)^7 + 1008*cos(d*x + c)^6 + 1008*cos(d*x + c)^5 - 840*cos(d*x
+ c)^4 - 840*cos(d*x + c)^3 + 315*cos(d*x + c)^2 + 315*cos(d*x + c) + 315)/
((a*d*cos(d*x + c)^9 + a*d*cos(d*x + c)^8 - 4*a*d*cos(d*x + c)^7 - 4*a*d*co
s(d*x + c)^6 + 6*a*d*cos(d*x + c)^5 + 6*a*d*cos(d*x + c)^4 - 4*a*d*cos(d*x
+ c)^3 - 4*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d)*sin(d*x + c))
```

giac [A] time = 0.29, size = 161, normalized size = 1.48

$$-\frac{\frac{11\left(13230\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^8+5040\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6+1701\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+360\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+35\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9}+\frac{315a^{10}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+3080a^{10}\tan\left(\frac{1}{2}dx\right.}{}}{3548160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3548160*(11*(13230*tan(1/2*d*x + 1/2*c)^8 + 5040*tan(1/2*d*x + 1/2*c)^6
+ 1701*tan(1/2*d*x + 1/2*c)^4 + 360*tan(1/2*d*x + 1/2*c)^2 + 35)/(a*tan(1/2
*d*x + 1/2*c)^9) + (315*a^10*tan(1/2*d*x + 1/2*c)^11 + 3080*a^10*tan(1/2*d*
x + 1/2*c)^9 + 13365*a^10*tan(1/2*d*x + 1/2*c)^7 + 33264*a^10*tan(1/2*d*x +
 1/2*c)^5 + 48510*a^10*tan(1/2*d*x + 1/2*c)^3)/a^11)/d
```

maple [A] time = 0.59, size = 140, normalized size = 1.28

$$\frac{-\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}-\frac{8\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}-\frac{27\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{48\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-14\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{8}{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}-\frac{1}{9\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{9}}}{1024da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^10/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/1024/d/a*(-1/11*tan(1/2*d*x+1/2*c)^11-8/9*tan(1/2*d*x+1/2*c)^9-27/7*tan(1
/2*d*x+1/2*c)^7-48/5*tan(1/2*d*x+1/2*c)^5-14*tan(1/2*d*x+1/2*c)^3-8/7/tan(1
/2*d*x+1/2*c)^7-1/9/tan(1/2*d*x+1/2*c)^9-16/tan(1/2*d*x+1/2*c)^3-27/5/tan(1
/2*d*x+1/2*c)^5-42/tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.34, size = 216, normalized size = 1.98

$$-\frac{\frac{\frac{48510\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{33264\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{13365\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{3080\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}+\frac{315\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a}+\frac{11\left(\frac{360\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{1701\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{5040\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{13230}{(\cos(}\right.}{a\sin(dx+c)^{9}}}{3548160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3548160*((48510*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 33264*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5 + 13365*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3080*
sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 315*sin(d*x + c)^11/(cos(d*x + c) + 1
)^11)/a + 11*(360*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1701*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 5040*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 13230*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 + 35)*(cos(d*x + c) + 1)^9/(a*sin(d*x + c
)^9))/d
```

mupad [B] time = 3.11, size = 139, normalized size = 1.28

$$-\frac{63\cos(c+d\,x)+\frac{21\cos(2\,c+2\,d\,x)}{2}-42\cos(3\,c+3\,d\,x)-12\cos(4\,c+4\,d\,x)+18\cos(5\,c+5\,d\,x)+\frac{27\cos(6\,c+}{4}}{3548160\,a\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}\sin\left(\frac{c}{2}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^10*(a + a/cos(c + d*x))),x)
```

```
[Out] -(63*cos(c + d*x) + (21*cos(2*c + 2*d*x))/2 - 42*cos(3*c + 3*d*x) - 12*cos(
4*c + 4*d*x) + 18*cos(5*c + 5*d*x) + (27*cos(6*c + 6*d*x))/4 - (9*cos(7*c +
 7*d*x))/2 - 2*cos(8*c + 8*d*x) + cos(9*c + 9*d*x)/2 + cos(10*c + 10*d*x)/4
 + 693/2)/(3548160*a*d*cos(c/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^9)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**10/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.74 $\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=137

$$-\frac{(a-a\cos(c+dx))^{11}}{11a^{13}d}+\frac{4(a-a\cos(c+dx))^{10}}{5a^{12}d}-\frac{25(a-a\cos(c+dx))^{9}}{9a^{11}d}+\frac{19(a-a\cos(c+dx))^{8}}{4a^{10}d}-\frac{4(a-a\cos(c}{a^9d}$$

```
[Out] 4/3*(a-a*cos(d*x+c))^6/a^8/d-4*(a-a*cos(d*x+c))^7/a^9/d+19/4*(a-a*cos(d*x+c
))^8/a^10/d-25/9*(a-a*cos(d*x+c))^9/a^11/d+4/5*(a-a*cos(d*x+c))^10/a^12/d-1
/11*(a-a*cos(d*x+c))^11/a^13/d
```

Rubi [A] time = 0.19, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2836, 12, 88}

$$-\frac{(a-a\cos(c+dx))^{11}}{11a^{13}d}+\frac{4(a-a\cos(c+dx))^{10}}{5a^{12}d}-\frac{25(a-a\cos(c+dx))^{9}}{9a^{11}d}+\frac{19(a-a\cos(c+dx))^{8}}{4a^{10}d}-\frac{4(a-a\cos(c}{a^9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^11/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*(a - a*Cos[c + d*x])^6)/(3*a^8*d) - (4*(a - a*Cos[c + d*x])^7)/(a^9*d) +
 (19*(a - a*Cos[c + d*x])^8)/(4*a^10*d) - (25*(a - a*Cos[c + d*x])^9)/(9*a^
11*d) + (4*(a - a*Cos[c + d*x])^10)/(5*a^12*d) - (a - a*Cos[c + d*x])^11/(1
1*a^13*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^{11}(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^5x^2(-a+x)^3}{a^2}\,dx, x, -a\cos(c+dx)\right)}{a^{11}d}$$
$$= \frac{\text{Subst}\left(\int (-a-x)^5x^2(-a+x)^3\,dx, x, -a\cos(c+dx)\right)}{a^{13}d}$$
$$= \frac{\text{Subst}\left(\int \left(-8a^5(-a-x)^5 - 28a^4(-a-x)^6 - 38a^3(-a-x)^7 - 25a^2(-a-x)^8 - 8a(-a\right.\right.}{a^{13}d}$$
$$= \frac{4(a-a\cos(c+dx))^6}{3a^8d} - \frac{4(a-a\cos(c+dx))^7}{a^9d} + \frac{19(a-a\cos(c+dx))^8}{4a^{10}d} - \frac{25(a-a\,\mathrm{co}}{9a}$$

Mathematica [A] time = 5.63, size = 72, normalized size = 0.53

$$\frac{4\sin^{12}\left(\frac{1}{2}(c+dx)\right)(4038\cos(c+dx)+2586\cos(2(c+dx))+1189\cos(3(c+dx))+342\cos(4(c+dx))+45\cos(}{495a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^11/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*(2360 + 4038*Cos[c + d*x] + 2586*Cos[2*(c + d*x)] + 1189*Cos[3*(c + d*x)
] + 342*Cos[4*(c + d*x)] + 45*Cos[5*(c + d*x)])*Sin[(c + d*x)/2]^12)/(495*a
^2*d)
```

fricas [A] time = 1.51, size = 89, normalized size = 0.65

$$\frac{180\cos(dx+c)^{11} - 396\cos(dx+c)^{10} - 440\cos(dx+c)^{9} + 1485\cos(dx+c)^{8} - 1980\cos(dx+c)^{6} + 792\cos}{1980\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/1980*(180*cos(d*x + c)^11 - 396*cos(d*x + c)^10 - 440*cos(d*x + c)^9 + 14
85*cos(d*x + c)^8 - 1980*cos(d*x + c)^6 + 792*cos(d*x + c)^5 + 990*cos(d*x
+ c)^4 - 660*cos(d*x + c)^3)/(a^2*d)
```

giac [A] time = 0.89, size = 185, normalized size = 1.35

$$-\frac{64\left(\frac{11(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{55(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{165(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{330(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} + \frac{462(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5} + \frac{198(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6} + \frac{99(}{(}\right.}{495\,a^2d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -64/495*(11*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 55*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 165*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 330
*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 462*(cos(d*x + c) - 1)^5/(cos(
d*x + c) + 1)^5 + 198*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 990*(cos(
d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 1)/(a^2*d*((cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - 1)^11)
```

maple [A] time = 0.69, size = 88, normalized size = 0.64

$$-\frac{\frac{1}{\sec(dx+c)^{6}}-\frac{1}{11\sec(dx+c)^{11}}+\frac{2}{9\sec(dx+c)^{9}}+\frac{1}{3\sec(dx+c)^{3}}-\frac{1}{2\sec(dx+c)^{4}}-\frac{3}{4\sec(dx+c)^{8}}+\frac{1}{5\sec(dx+c)^{10}}-\frac{2}{5\sec(dx+c)^{5}}}{d\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^11/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/d/a^2*(1/sec(d*x+c)^6-1/11/sec(d*x+c)^11+2/9/sec(d*x+c)^9+1/3/sec(d*x+c)
^3-1/2/sec(d*x+c)^4-3/4/sec(d*x+c)^8+1/5/sec(d*x+c)^10-2/5/sec(d*x+c)^5)
```

maxima [A] time = 0.33, size = 89, normalized size = 0.65

$$\frac{180\cos(dx+c)^{11}-396\cos(dx+c)^{10}-440\cos(dx+c)^{9}+1485\cos(dx+c)^{8}-1980\cos(dx+c)^{6}+792\ldots}{1980\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/1980*(180*cos(d*x + c)^11 - 396*cos(d*x + c)^10 - 440*cos(d*x + c)^9 + 14
85*cos(d*x + c)^8 - 1980*cos(d*x + c)^6 + 792*cos(d*x + c)^5 + 990*cos(d*x
+ c)^4 - 660*cos(d*x + c)^3)/(a^2*d)
```

mupad [B] time = 0.09, size = 109, normalized size = 0.80

$$-\frac{\frac{\cos(c+d\,x)^{3}}{3\,a^{2}}-\frac{\cos(c+d\,x)^{4}}{2\,a^{2}}-\frac{2\cos(c+d\,x)^{5}}{5\,a^{2}}+\frac{\cos(c+d\,x)^{6}}{a^{2}}-\frac{3\cos(c+d\,x)^{8}}{4\,a^{2}}+\frac{2\cos(c+d\,x)^{9}}{9\,a^{2}}+\frac{\cos(c+d\,x)^{10}}{5\,a^{2}}-\frac{\cos(c+d\,x)^{11}}{11\,a^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^11/(a + a/cos(c + d*x))^2,x)
```

```
[Out] -(cos(c + d*x)^3/(3*a^2) - cos(c + d*x)^4/(2*a^2) - (2*cos(c + d*x)^5)/(5*a
^2) + cos(c + d*x)^6/a^2 - (3*cos(c + d*x)^8)/(4*a^2) + (2*cos(c + d*x)^9)/
(9*a^2) + cos(c + d*x)^10/(5*a^2) - cos(c + d*x)^11/(11*a^2))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**11/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.75 $\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2} dx$

Optimal. Leaf size=114

$$\frac{(a-a\cos(c+dx))^9}{9a^{11}d}-\frac{3(a-a\cos(c+dx))^8}{4a^{10}d}+\frac{13(a-a\cos(c+dx))^7}{7a^9d}-\frac{2(a-a\cos(c+dx))^6}{a^8d}+\frac{4(a-a\cos(c+dx))^5}{5a^7d}$$

```
[Out] 4/5*(a-a*cos(d*x+c))^5/a^7/d-2*(a-a*cos(d*x+c))^6/a^8/d+13/7*(a-a*cos(d*x+c
))^7/a^9/d-3/4*(a-a*cos(d*x+c))^8/a^10/d+1/9*(a-a*cos(d*x+c))^9/a^11/d
```

Rubi [A] time = 0.18, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3872, 2836, 12, 88}

$$\frac{(a-a\cos(c+dx))^9}{9a^{11}d}-\frac{3(a-a\cos(c+dx))^8}{4a^{10}d}+\frac{13(a-a\cos(c+dx))^7}{7a^9d}-\frac{2(a-a\cos(c+dx))^6}{a^8d}+\frac{4(a-a\cos(c+dx))^5}{5a^7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^9/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*(a - a*Cos[c + d*x])^5)/(5*a^7*d) - (2*(a - a*Cos[c + d*x])^6)/(a^8*d) +
 (13*(a - a*Cos[c + d*x])^7)/(7*a^9*d) - (3*(a - a*Cos[c + d*x])^8)/(4*a^10
*d) + (a - a*Cos[c + d*x])^9/(9*a^11*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\sin^9(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^4x^2(-a+x)^2}{a^2}\,dx, x, -a\cos(c+dx)\right)}{a^9d} \\
&= \frac{\text{Subst}\left(\int (-a-x)^4x^2(-a+x)^2\,dx, x, -a\cos(c+dx)\right)}{a^{11}d} \\
&= \frac{\text{Subst}\left(\int \left(4a^4(-a-x)^4+12a^3(-a-x)^5+13a^2(-a-x)^6+6a(-a-x)^7+(-a-x\right.\right.}{a^{11}d} \\
&= \frac{4(a-a\cos(c+dx))^5}{5a^7d} - \frac{2(a-a\cos(c+dx))^6}{a^8d} + \frac{13(a-a\cos(c+dx))^7}{7a^9d} - \frac{3(a-a}{}
\end{aligned}
$$

Mathematica [A] time = 3.95, size = 62, normalized size = 0.54

$$\frac{2\sin^{10}\left(\frac{1}{2}(c+dx)\right)(1615\cos(c+dx)+970\cos(2(c+dx))+385\cos(3(c+dx))+70\cos(4(c+dx))+992)}{315a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^9/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*(992 + 1615*Cos[c + d*x] + 970*Cos[2*(c + d*x)] + 385*Cos[3*(c + d*x)] +
 70*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]^10)/(315*a^2*d)
```

fricas [A] time = 0.60, size = 79, normalized size = 0.69

$$-\frac{140\cos(dx+c)^9-315\cos(dx+c)^8-180\cos(dx+c)^7+840\cos(dx+c)^6-252\cos(dx+c)^5-630\cos}{1260\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/1260*(140*cos(d*x + c)^9 - 315*cos(d*x + c)^8 - 180*cos(d*x + c)^7 + 840
*cos(d*x + c)^6 - 252*cos(d*x + c)^5 - 630*cos(d*x + c)^4 + 420*cos(d*x + c
)^3)/(a^2*d)
```

giac [A] time = 0.29, size = 141, normalized size = 1.24

$$-\frac{64\left(\frac{9(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{36(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{84(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{126(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{210(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}-1\right)}{315\,a^2d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -64/315*(9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 36*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 + 84*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 126*(
cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 210*(cos(d*x + c) - 1)^6/(cos(d*
x + c) + 1)^6 - 1)/(a^2*d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^9)
```

maple [A] time = 0.63, size = 79, normalized size = 0.69

$$\frac{-\frac{2}{3\sec(dx+c)^6}+\frac{1}{7\sec(dx+c)^7}-\frac{1}{9\sec(dx+c)^9}-\frac{1}{3\sec(dx+c)^3}+\frac{1}{2\sec(dx+c)^4}+\frac{1}{4\sec(dx+c)^8}+\frac{1}{5\sec(dx+c)^5}}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^9/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(-2/3/sec(d*x+c)^6+1/7/sec(d*x+c)^7-1/9/sec(d*x+c)^9-1/3/sec(d*x+c)
^3+1/2/sec(d*x+c)^4+1/4/sec(d*x+c)^8+1/5/sec(d*x+c)^5)
```

maxima [A] time = 0.33, size = 79, normalized size = 0.69

$$-\frac{140\,\cos\left(dx+c\right)^{9}-315\,\cos\left(dx+c\right)^{8}-180\,\cos\left(dx+c\right)^{7}+840\,\cos\left(dx+c\right)^{6}-252\,\cos\left(dx+c\right)^{5}-630\,\cos\left(d\right.}{1260\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/1260*(140*cos(d*x + c)^9 - 315*cos(d*x + c)^8 - 180*cos(d*x + c)^7 + 840
*cos(d*x + c)^6 - 252*cos(d*x + c)^5 - 630*cos(d*x + c)^4 + 420*cos(d*x + c
)^3)/(a^2*d)
```

mupad [B] time = 0.91, size = 96, normalized size = 0.84

$$\frac{\frac{\cos(c+d\,x)^4}{2\,a^2}-\frac{\cos(c+d\,x)^3}{3\,a^2}+\frac{\cos(c+d\,x)^5}{5\,a^2}-\frac{2\cos(c+d\,x)^6}{3\,a^2}+\frac{\cos(c+d\,x)^7}{7\,a^2}+\frac{\cos(c+d\,x)^8}{4\,a^2}-\frac{\cos(c+d\,x)^9}{9\,a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^9/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (cos(c + d*x)^4/(2*a^2) - cos(c + d*x)^3/(3*a^2) + cos(c + d*x)^5/(5*a^2) -
 (2*cos(c + d*x)^6)/(3*a^2) + cos(c + d*x)^7/(7*a^2) + cos(c + d*x)^8/(4*a^
2) - cos(c + d*x)^9/(9*a^2))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**9/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.76 $\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=73

$$\frac{\cos^7(c+dx)}{7a^2d}-\frac{\cos^6(c+dx)}{3a^2d}+\frac{\cos^4(c+dx)}{2a^2d}-\frac{\cos^3(c+dx)}{3a^2d}$$

```
[Out] -1/3*cos(d*x+c)^3/a^2/d+1/2*cos(d*x+c)^4/a^2/d-1/3*cos(d*x+c)^6/a^2/d+1/7*c
os(d*x+c)^7/a^2/d
```

Rubi [A] time = 0.16, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2836, 12, 75}

$$\frac{\cos^7(c+dx)}{7a^2d}-\frac{\cos^6(c+dx)}{3a^2d}+\frac{\cos^4(c+dx)}{2a^2d}-\frac{\cos^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -Cos[c + d*x]^3/(3*a^2*d) + Cos[c + d*x]^4/(2*a^2*d) - Cos[c + d*x]^6/(3*a^
2*d) + Cos[c + d*x]^7/(7*a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^7(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^3x^2(-a+x)}{a^2}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int (-a-x)^3x^2(-a+x)\,dx, x, -a\cos(c+dx)\right)}{a^9 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^4x^2+2a^3x^3-2ax^5-x^6\right)\,dx, x, -a\cos(c+dx)\right)}{a^9 d}$$
$$= -\frac{\cos^3(c+dx)}{3a^2d} + \frac{\cos^4(c+dx)}{2a^2d} - \frac{\cos^6(c+dx)}{3a^2d} + \frac{\cos^7(c+dx)}{7a^2d}$$

Mathematica [**A**] time = 1.97, size = 53, normalized size = 0.73

$$\frac{4\sin^8\left(\frac{1}{2}(c+dx)\right)(17\cos(c+dx)+10\cos(2(c+dx))+3(\cos(3(c+dx))+4))}{21a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*(17*Cos[c + d*x] + 10*Cos[2*(c + d*x)] + 3*(4 + Cos[3*(c + d*x)]))*Sin[(
c + d*x)/2]^8)/(21*a^2*d)
```

fricas [**A**] time = 0.75, size = 49, normalized size = 0.67

$$\frac{6\,\cos\left(dx+c\right)^7 - 14\,\cos\left(dx+c\right)^6 + 21\,\cos\left(dx+c\right)^4 - 14\,\cos\left(dx+c\right)^3}{42\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/42*(6*cos(d*x + c)^7 - 14*cos(d*x + c)^6 + 21*cos(d*x + c)^4 - 14*cos(d*x
 + c)^3)/(a^2*d)
```

giac [**B**] time = 0.28, size = 141, normalized size = 1.93

$$-\frac{8\left(\frac{7(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{21(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{35(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{14(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} + \frac{42(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5} - 1\right)}{21\,a^2d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -8/21*(7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 21*(cos(d*x + c) - 1)^2/(c
os(d*x + c) + 1)^2 + 35*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 14*(cos
(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 42*(cos(d*x + c) - 1)^5/(cos(d*x +
c) + 1)^5 - 1)/(a^2*d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7)
```

maple [**A**] time = 0.60, size = 50, normalized size = 0.68

$$-\frac{\frac{1}{3\sec(dx+c)^6} - \frac{1}{7\sec(dx+c)^7} + \frac{1}{3\sec(dx+c)^3} - \frac{1}{2\sec(dx+c)^4}}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/d/a^2*(1/3/sec(d*x+c)^6-1/7/sec(d*x+c)^7+1/3/sec(d*x+c)^3-1/2/sec(d*x+c)
^4)
```

maxima [A] time = 0.32, size = 49, normalized size = 0.67

$$\frac{6 \cos(dx+c)^{7} - 14 \cos(dx+c)^{6} + 21 \cos(dx+c)^{4} - 14 \cos(dx+c)^{3}}{42 a^{2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/42*(6*cos(d*x + c)^7 - 14*cos(d*x + c)^6 + 21*cos(d*x + c)^4 - 14*cos(d*x
 + c)^3)/(a^2*d)
```

mupad [B] time = 0.92, size = 58, normalized size = 0.79

$$-\frac{\frac{\cos(c+d\,x)^3}{3\,a^2} - \frac{\cos(c+d\,x)^4}{2\,a^2} + \frac{\cos(c+d\,x)^6}{3\,a^2} - \frac{\cos(c+d\,x)^7}{7\,a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a + a/cos(c + d*x))^2,x)
```

```
[Out] -(cos(c + d*x)^3/(3*a^2) - cos(c + d*x)^4/(2*a^2) + cos(c + d*x)^6/(3*a^2)
- cos(c + d*x)^7/(7*a^2))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.77 $\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$-\frac{\cos^5(c+dx)}{5a^2d}+\frac{\cos^4(c+dx)}{2a^2d}-\frac{\cos^3(c+dx)}{3a^2d}$$

```
[Out] -1/3*cos(d*x+c)^3/a^2/d+1/2*cos(d*x+c)^4/a^2/d-1/5*cos(d*x+c)^5/a^2/d
```

Rubi [A] time = 0.15, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2836, 12, 43}

$$-\frac{\cos^5(c+dx)}{5a^2d}+\frac{\cos^4(c+dx)}{2a^2d}-\frac{\cos^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -Cos[c + d*x]^3/(3*a^2*d) + Cos[c + d*x]^4/(2*a^2*d) - Cos[c + d*x]^5/(5*a^
2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\sin^5(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)^2x^2}{a^2}\,dx, x, -a\cos(c+dx)\right)}{a^5 d} \\
&= \frac{\text{Subst}\left(\int (-a-x)^2x^2\,dx, x, -a\cos(c+dx)\right)}{a^7 d} \\
&= \frac{\text{Subst}\left(\int \left(a^2x^2+2ax^3+x^4\right)\,dx, x, -a\cos(c+dx)\right)}{a^7 d} \\
&= -\frac{\cos^3(c+dx)}{3a^2d} + \frac{\cos^4(c+dx)}{2a^2d} - \frac{\cos^5(c+dx)}{5a^2d}
\end{aligned}
$$

Mathematica [A] time = 0.60, size = 42, normalized size = 0.76

$$\frac{4\sin^6\left(\frac{1}{2}(c+dx)\right)(3\cos(c+dx)+3\cos(2(c+dx))+4)}{15a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*(4 + 3*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]^6)/(15*a^2*d)
```

fricas [A] time = 1.46, size = 39, normalized size = 0.71

$$-\frac{6\cos(dx+c)^5 - 15\cos(dx+c)^4 + 10\cos(dx+c)^3}{30\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/30*(6*cos(d*x + c)^5 - 15*cos(d*x + c)^4 + 10*cos(d*x + c)^3)/(a^2*d)
```

giac [B] time = 0.25, size = 119, normalized size = 2.16

$$-\frac{8\left(\frac{10(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{20(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{15(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{15(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} - 2\right)}{15\,a^2d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -8/15*(10*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 20*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + 15*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 15*(co
s(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 2)/(a^2*d*((cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - 1)^5)
```

maple [A] time = 0.61, size = 39, normalized size = 0.71

$$\frac{-\frac{1}{3\sec(dx+c)^3} + \frac{1}{2\sec(dx+c)^4} - \frac{1}{5\sec(dx+c)^5}}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(-1/3/sec(d*x+c)^3+1/2/sec(d*x+c)^4-1/5/sec(d*x+c)^5)
```

maxima [A] time = 0.32, size = 39, normalized size = 0.71

$$-\frac{6 \cos(dx+c)^5 - 15 \cos(dx+c)^4 + 10 \cos(dx+c)^3}{30 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(6*cos(d*x + c)^5 - 15*cos(d*x + c)^4 + 10*cos(d*x + c)^3)/(a^2*d)
```

mupad [B] time = 0.06, size = 36, normalized size = 0.65

$$-\frac{\cos(c+dx)^3 \left(6 \cos(c+dx)^2 - 15 \cos(c+dx) + 10\right)}{30 a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + a/cos(c + d*x))^2,x)
```

```
[Out] -(cos(c + d*x)^3*(6*cos(c + d*x)^2 - 15*cos(c + d*x) + 10))/(30*a^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.78 $\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=66

$$\frac{\cos^3(c+dx)}{3a^2d} - \frac{\cos^2(c+dx)}{a^2d} + \frac{2\cos(c+dx)}{a^2d} - \frac{2\log(\cos(c+dx)+1)}{a^2d}$$

```
[Out] 2*cos(d*x+c)/a^2/d-cos(d*x+c)^2/a^2/d+1/3*cos(d*x+c)^3/a^2/d-2*ln(1+cos(d*x
+c))/a^2/d
```

Rubi [A] time = 0.16, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2836, 12, 77}

$$\frac{\cos^3(c+dx)}{3a^2d} - \frac{\cos^2(c+dx)}{a^2d} + \frac{2\cos(c+dx)}{a^2d} - \frac{2\log(\cos(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*Cos[c + d*x])/(a^2*d) - Cos[c + d*x]^2/(a^2*d) + Cos[c + d*x]^3/(3*a^2*d
) - (2*Log[1 + Cos[c + d*x]])/(a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)x^2}{a^2(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)x^2}{-a+x}\,dx, x, -a\cos(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a^2+\frac{2a^3}{a-x}-2ax-x^2\right)dx, x, -a\cos(c+dx)\right)}{a^5 d}$$
$$= \frac{2\cos(c+dx)}{a^2 d} - \frac{\cos^2(c+dx)}{a^2 d} + \frac{\cos^3(c+dx)}{3a^2 d} - \frac{2\log(1+\cos(c+dx))}{a^2 d}$$

Mathematica [A] time = 0.24, size = 51, normalized size = 0.77

$$\frac{27\cos(c+dx)-6\cos(2(c+dx))+\cos(3(c+dx))-48\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-22}{12a^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-22 + 27*Cos[c + d*x] - 6*Cos[2*(c + d*x)] + Cos[3*(c + d*x)] - 48*Log[Cos
[(c + d*x)/2]])/(12*a^2*d)
```

fricas [A] time = 0.45, size = 48, normalized size = 0.73

$$\frac{\cos(dx+c)^3 - 3\cos(dx+c)^2 + 6\cos(dx+c) - 6\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{3a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^3 - 3*cos(d*x + c)^2 + 6*cos(d*x + c) - 6*log(1/2*cos(d*x
 + c) + 1/2))/(a^2*d)
```

giac [A] time = 1.42, size = 75, normalized size = 1.14

$$-\frac{2\log(|-\cos(dx+c)-1|)}{a^2 d} + \frac{a^4 d^2\cos(dx+c)^3 - 3a^4 d^2\cos(dx+c)^2 + 6a^4 d^2\cos(dx+c)}{3a^6 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*log(abs(-cos(d*x + c) - 1))/(a^2*d) + 1/3*(a^4*d^2*cos(d*x + c)^3 - 3*a^
4*d^2*cos(d*x + c)^2 + 6*a^4*d^2*cos(d*x + c))/(a^6*d^3)
```

maple [A] time = 0.65, size = 82, normalized size = 1.24

$$\frac{1}{3d\,a^2\sec(dx+c)^3} - \frac{1}{d\,a^2\sec(dx+c)^2} + \frac{2}{d\,a^2\sec(dx+c)} + \frac{2\ln(\sec(dx+c))}{d\,a^2} - \frac{2\ln(1+\sec(dx+c))}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/3/d/a^2/sec(d*x+c)^3-1/d/a^2/sec(d*x+c)^2+2/d/a^2/sec(d*x+c)+2/d/a^2*ln(s
ec(d*x+c))-2/d/a^2*ln(1+sec(d*x+c))
```

maxima [A] time = 0.33, size = 51, normalized size = 0.77

$$\frac{\frac{\cos(dx+c)^3-3\,\cos(dx+c)^2+6\,\cos(dx+c)}{a^2}-\frac{6\,\log(\cos(dx+c)+1)}{a^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((cos(d*x + c)^3 - 3*cos(d*x + c)^2 + 6*cos(d*x + c))/a^2 - 6*log(cos(d
*x + c) + 1)/a^2)/d
```

mupad [B] time = 0.06, size = 56, normalized size = 0.85

$$-\frac{\frac{2\,\ln(\cos(c+d\,x)+1)}{a^2}-\frac{2\,\cos(c+d\,x)}{a^2}+\frac{\cos(c+d\,x)^2}{a^2}-\frac{\cos(c+d\,x)^3}{3\,a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + a/cos(c + d*x))^2,x)
```

```
[Out] -((2*log(cos(c + d*x) + 1))/a^2 - (2*cos(c + d*x))/a^2 + cos(c + d*x)^2/a^2
 - cos(c + d*x)^3/(3*a^2))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.79 $\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=52

$$-\frac{\cos(c+dx)}{a^2d}+\frac{1}{d\left(a^2\cos(c+dx)+a^2\right)}+\frac{2\log(\cos(c+dx)+1)}{a^2d}$$

```
[Out] -cos(d*x+c)/a^2/d+1/d/(a^2+a^2*cos(d*x+c))+2*ln(1+cos(d*x+c))/a^2/d
```

Rubi [A] time = 0.10, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 2833, 12, 43}

$$-\frac{\cos(c+dx)}{a^2d}+\frac{1}{d\left(a^2\cos(c+dx)+a^2\right)}+\frac{2\log(\cos(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Cos[c + d*x]/(a^2*d)) + 1/(d*(a^2 + a^2*Cos[c + d*x])) + (2*Log[1 + Cos[c
 + d*x]])/(a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{a^2(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a^2}{(a-x)^2}-\frac{2a}{a-x}\right)dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= -\frac{\cos(c+dx)}{a^2d} + \frac{1}{d\left(a^2+a^2\cos(c+dx)\right)} + \frac{2\log(1+\cos(c+dx))}{a^2d}$$

Mathematica [A] time = 0.24, size = 64, normalized size = 1.23

$$-\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\cos(2(c+dx)) - 8\cos(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) - 3\right)}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/4*((-3 + Cos[2*(c + d*x)] - 8*Log[Cos[(c + d*x)/2]] - 8*Cos[c + d*x]*Log
[Cos[(c + d*x)/2]])*Sec[(c + d*x)/2]^2)/(a^2*d)
```

fricas [A] time = 0.67, size = 58, normalized size = 1.12

$$-\frac{\cos(dx+c)^2 - 2(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) + \cos(dx+c) - 1}{a^2d\cos(dx+c) + a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(cos(d*x + c)^2 - 2*(cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + cos(d
*x + c) - 1)/(a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 0.20, size = 52, normalized size = 1.00

$$-\frac{\cos(dx+c)}{a^2d} + \frac{2\log(|-\cos(dx+c)-1|)}{a^2d} + \frac{1}{a^2d(\cos(dx+c)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -cos(d*x + c)/(a^2*d) + 2*log(abs(-cos(d*x + c) - 1))/(a^2*d) + 1/(a^2*d*(c
os(d*x + c) + 1))
```

maple [A] time = 0.17, size = 68, normalized size = 1.31

$$-\frac{1}{d\,a^2\sec(dx+c)} - \frac{2\ln(\sec(dx+c))}{d\,a^2} - \frac{1}{a^2d(1+\sec(dx+c))} + \frac{2\ln(1+\sec(dx+c))}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/d/a^2/sec(d*x+c)-2/d/a^2*ln(sec(d*x+c))-1/a^2/d/(1+sec(d*x+c))+2/d/a^2*l
n(1+sec(d*x+c))
```

maxima [A] time = 0.37, size = 46, normalized size = 0.88

$$\frac{\frac{1}{a^2\cos(dx+c)+a^2} - \frac{\cos(dx+c)}{a^2} + \frac{2\log(\cos(dx+c)+1)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (1/(a^2*cos(d*x + c) + a^2) - cos(d*x + c)/a^2 + 2*log(cos(d*x + c) + 1)/a^
2)/d
```

mupad [B] time = 0.92, size = 46, normalized size = 0.88

$$\frac{2\ln(\cos(c+d\,x)+1)}{a^2\,d} - \frac{\cos(c+d\,x)^2-2}{a^2\,d\,\left(\cos(c+d\,x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*log(cos(c + d*x) + 1))/(a^2*d) - (cos(c + d*x)^2 - 2)/(a^2*d*(cos(c + d*
x) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.80 $\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=60

$$-\frac{3}{4d\left(a^2\cos(c+dx)+a^2\right)}-\frac{\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{1}{4d(a\cos(c+dx)+a)^2}$$

```
[Out] -1/4*arctanh(cos(d*x+c))/a^2/d+1/4/d/(a+a*cos(d*x+c))^2-3/4/d/(a^2+a^2*cos(
d*x+c))
```

Rubi [A] time = 0.13, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3872, 2836, 12, 88, 206}

$$-\frac{3}{4d\left(a^2\cos(c+dx)+a^2\right)}-\frac{\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{1}{4d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -ArcTanh[Cos[c + d*x]]/(4*a^2*d) + 1/(4*d*(a + a*Cos[c + d*x])^2) - 3/(4*d*
(a^2 + a^2*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)\cot(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{a\,\text{Subst}\left(\int \frac{x^2}{a^2(-a-x)(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(-a-x)(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a}{2(a-x)^3} - \frac{3}{4(a-x)^2} + \frac{1}{4(a^2-x^2)}\right)dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{1}{4d(a+a\cos(c+dx))^2} - \frac{3}{4d\left(a^2+a^2\cos(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, -a\cos(c+\right.}{4ad}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{4a^2d} + \frac{1}{4d(a+a\cos(c+dx))^2} - \frac{3}{4d\left(a^2+a^2\cos(c+dx)\right)}$$

Mathematica [A] time = 0.21, size = 83, normalized size = 1.38

$$-\frac{\sec^2(c+dx)\left(6\cos^2\left(\frac{1}{2}(c+dx)\right)+4\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-1\right)}{4a^2d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/4*((-1 + 6*Cos[(c + d*x)/2]^2 + 4*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)/
2]] - Log[Sin[(c + d*x)/2]]))*Sec[c + d*x]^2)/(a^2*d*(1 + Sec[c + d*x])^2)
```

fricas [A] time = 1.46, size = 106, normalized size = 1.77

$$-\frac{\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\log\left(-\frac{1}{2}\cos\right.}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*((cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - (
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 6*cos(d
*x + c) + 4)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 1.14, size = 87, normalized size = 1.45

$$\frac{\frac{2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2}+\frac{\frac{4a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^4}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 + (4*a^2*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2)/a^4)/d
```

maple [A] time = 0.57, size = 72, normalized size = 1.20

$$\frac{\ln(-1+\cos(dx+c))}{8d\,a^2}+\frac{1}{4d\,a^2\,(1+\cos(dx+c))^2}-\frac{3}{4d\,a^2\,(1+\cos(dx+c))}-\frac{\ln(1+\cos(dx+c))}{8a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/8/d/a^2*ln(-1+cos(d*x+c))+1/4/d/a^2/(1+cos(d*x+c))^2-3/4/d/a^2/(1+cos(d*x
+c))-1/8*ln(1+cos(d*x+c))/a^2/d
```

maxima [A] time = 0.40, size = 74, normalized size = 1.23

$$-\frac{\frac{2\,(3\,\cos(dx+c)+2)}{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2}+\frac{\log(\cos(dx+c)+1)}{a^2}-\frac{\log(\cos(dx+c)-1)}{a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(2*(3*cos(d*x + c) + 2)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2
) + log(cos(d*x + c) + 1)/a^2 - log(cos(d*x + c) - 1)/a^2)/d
```

mupad [B] time = 0.10, size = 60, normalized size = 1.00

$$-\frac{\frac{3\,\cos(c+d\,x)}{4}+\frac{1}{2}}{d\,\left(a^2\cos(c+d\,x)^2+2\,a^2\,\cos(c+d\,x)+a^2\right)}-\frac{\mathrm{atanh}\left(\cos\left(c+d\,x\right)\right)}{4\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] - ((3*cos(c + d*x))/4 + 1/2)/(d*(2*a^2*cos(c + d*x) + a^2 + a^2*cos(c + d*x
)^2)) - atanh(cos(c + d*x))/(4*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.81 $\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=42

$$-\frac{2a\cos(c+dx)+a}{6d(1-\cos(c+dx))(a\cos(c+dx)+a)^3}$$

```
[Out] 1/6*(-a-2*a*cos(d*x+c))/d/(1-cos(d*x+c))/(a+a*cos(d*x+c))^3
```

Rubi [A] time = 0.13, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2836, 12, 81}

$$-\frac{2a\cos(c+dx)+a}{6d(1-\cos(c+dx))(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a + 2*a*Cos[c + d*x])/(6*d*(1 - Cos[c + d*x])*(a + a*Cos[c + d*x])^3)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)\csc(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{a^3\operatorname{Subst}\left(\int \frac{x^2}{a^2(-a-x)^2(-a+x)^4}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a\operatorname{Subst}\left(\int \frac{x^2}{(-a-x)^2(-a+x)^4}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{a+2a\cos(c+dx)}{6d(1-\cos(c+dx))(a+a\cos(c+dx))^3}$$

Mathematica [A] time = 0.10, size = 38, normalized size = 0.90

$$-\frac{(2\cos(c+dx)+1)\csc^2(c+dx)}{6a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/6*((1 + 2*Cos[c + d*x])*Csc[c + d*x]^2)/(a^2*d*(1 + Cos[c + d*x])^2)
```

fricas [A] time = 0.66, size = 60, normalized size = 1.43

$$\frac{2\cos(dx+c)+1}{6\left(a^2d\cos(dx+c)^4+2a^2d\cos(dx+c)^3-2a^2d\cos(dx+c)-a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*cos(d*x + c) + 1)/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 - 2
*a^2*d*cos(d*x + c) - a^2*d)
```

giac [B] time = 0.30, size = 82, normalized size = 1.95

$$\frac{\frac{3(\cos(dx+c)+1)}{a^2(\cos(dx+c)-1)}+\frac{\frac{6a^4(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a^4(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{a^6}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/96*(3*(cos(d*x + c) + 1)/(a^2*(cos(d*x + c) - 1)) + (6*a^4*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - a^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^
6)/d
```

maple [A] time = 0.72, size = 57, normalized size = 1.36

$$\frac{\frac{1}{-16+16\cos(dx+c)}+\frac{1}{12(1+\cos(dx+c))^3}-\frac{1}{8(1+\cos(dx+c))^2}-\frac{1}{16(1+\cos(dx+c))}}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(1/16/(-1+cos(d*x+c))+1/12/(1+cos(d*x+c))^3-1/8/(1+cos(d*x+c))^2-1/
16/(1+cos(d*x+c)))
```

maxima [A] time = 0.58, size = 59, normalized size = 1.40

$$\frac{2\cos(dx+c)+1}{6\left(a^2\cos(dx+c)^4+2a^2\cos(dx+c)^3-2a^2\cos(dx+c)-a^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*cos(d*x + c) + 1)/((a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^3 - 2*a^
2*cos(d*x + c) - a^2)*d)
```

mupad [B] time = 0.94, size = 58, normalized size = 1.38

$$-\frac{\frac{\cos(c+d\,x)}{3}+\frac{1}{6}}{d\,\left(-a^2\cos(c+d\,x)^4-2\,a^2\cos(c+d\,x)^3+2\,a^2\,\cos(c+d\,x)+a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + a/cos(c + d*x))^2),x)
```

```
[Out] -(cos(c + d*x)/3 + 1/6)/(d*(2*a^2*cos(c + d*x) + a^2 - 2*a^2*cos(c + d*x)^3
 - a^2*cos(c + d*x)^4))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc^3(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.82 $\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=146

$$\frac{a^2}{32d(a\cos(c+dx)+a)^4}-\frac{1}{64d\left(a^2-a^2\cos(c+dx)\right)}-\frac{1}{32d\left(a^2\cos(c+dx)+a^2\right)}+\frac{\tanh^{-1}(\cos(c+dx))}{64a^2d}-\frac{}{48d(a\,c}$$

```
[Out] 1/64*arctanh(cos(d*x+c))/a^2/d-1/64/d/(a-a*cos(d*x+c))^2+1/32*a^2/d/(a+a*co
s(d*x+c))^4-1/48*a/d/(a+a*cos(d*x+c))^3-1/32/d/(a+a*cos(d*x+c))^2-1/64/d/(a
^2-a^2*cos(d*x+c))-1/32/d/(a^2+a^2*cos(d*x+c))
```

Rubi [A] time = 0.22, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2836, 12, 88, 206}

$$\frac{a^2}{32d(a\cos(c+dx)+a)^4}-\frac{1}{64d\left(a^2-a^2\cos(c+dx)\right)}-\frac{1}{32d\left(a^2\cos(c+dx)+a^2\right)}+\frac{\tanh^{-1}(\cos(c+dx))}{64a^2d}-\frac{}{48d(a\,c}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(64*a^2*d) - 1/(64*d*(a - a*Cos[c + d*x])^2) + a^2/(3
2*d*(a + a*Cos[c + d*x])^4) - a/(48*d*(a + a*Cos[c + d*x])^3) - 1/(32*d*(a
+ a*Cos[c + d*x])^2) - 1/(64*d*(a^2 - a^2*Cos[c + d*x])) - 1/(32*d*(a^2 + a
^2*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)\csc^3(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$

$$= \frac{a^5\,\text{Subst}\left(\int \frac{x^2}{a^2(-a-x)^3(-a+x)^5}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^3\,\text{Subst}\left(\int \frac{x^2}{(-a-x)^3(-a+x)^5}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{8a(a-x)^5} - \frac{1}{16a^2(a-x)^4} - \frac{1}{16a^3(a-x)^3} - \frac{1}{32a^4(a-x)^2} + \frac{1}{32a^3(a+x)^3} + \frac{1}{64a^4(a+x)^2} - \frac{}{64}\right.\right.}{d}$$

$$= -\frac{1}{64d(a-a\cos(c+dx))^2} + \frac{a^2}{32d(a+a\cos(c+dx))^4} - \frac{a}{48d(a+a\cos(c+dx))^3} - \frac{}{32}$$

$$= \frac{\tanh^{-1}(\cos(c+dx))}{64a^2d} - \frac{1}{64d(a-a\cos(c+dx))^2} + \frac{a^2}{32d(a+a\cos(c+dx))^4} - \frac{}{48d(a+}$$

Mathematica [A] time = 0.79, size = 152, normalized size = 1.04

$$-\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(6\csc^4\left(\frac{1}{2}(c+dx)\right)+12\csc^2\left(\frac{1}{2}(c+dx)\right)-3\sec^8\left(\frac{1}{2}(c+dx)\right)+4\sec^6\left(\frac{1}{2}(c+dx)\right)\right.}{384a^2d(\sec(c+dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/384*(Cos[(c + d*x)/2]^4*(12*Csc[(c + d*x)/2]^2 + 6*Csc[(c + d*x)/2]^4 +
24*(-Log[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]]) + 24*Sec[(c + d*x)/2]^2
 + 12*Sec[(c + d*x)/2]^4 + 4*Sec[(c + d*x)/2]^6 - 3*Sec[(c + d*x)/2]^8)*Sec
[c + d*x]^2)/(a^2*d*(1 + Sec[c + d*x])^2)
```

fricas [B] time = 0.64, size = 283, normalized size = 1.94

$$-\frac{6\cos(dx+c)^5 + 12\cos(dx+c)^4 - 4\cos(dx+c)^3 - 20\cos(dx+c)^2 - 3\left(\cos(dx+c)^6 + 2\cos(dx+c)^5 - \text{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/384*(6*cos(d*x + c)^5 + 12*cos(d*x + c)^4 - 4*cos(d*x + c)^3 - 20*cos(d*
x + c)^2 - 3*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(d*
x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2)
 + 3*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3
 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 70*c
os(d*x + c) + 32)/(a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a^2*d*co
s(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(
d*x + c) + a^2*d)
```

giac [A] time = 0.30, size = 207, normalized size = 1.42

$$\frac{\frac{6\left(\frac{4(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^2}{a^2(\cos(dx+c)-1)^2} - \frac{12\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2} + \frac{\frac{48a^6(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{6a^6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{8a^6(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} + \frac{3a^6(\cos(dx+}{(\cos(dx+c)}}{a^8}}{1536d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1536*(6*(4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^2/(a^2*(cos(d*x + c) - 1)^2)
- 12*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 + (48*a^6*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 6*a^6*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - 8*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 3*a^6*(cos(d*x +
 c) - 1)^4/(cos(d*x + c) + 1)^4)/a^8)/d
```

maple [A] time = 0.70, size = 144, normalized size = 0.99

$$-\frac{1}{64d\,a^2\left(-1+\cos\left(dx+c\right)\right)^2}+\frac{1}{64d\,a^2\left(-1+\cos\left(dx+c\right)\right)}-\frac{\ln\left(-1+\cos\left(dx+c\right)\right)}{128d\,a^2}+\frac{1}{32d\,a^2\left(1+\cos\left(dx+c\right)\right)^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/64/d/a^2/(-1+cos(d*x+c))^2+1/64/d/a^2/(-1+cos(d*x+c))-1/128/d/a^2*ln(-1+
cos(d*x+c))+1/32/d/a^2/(1+cos(d*x+c))^4-1/48/d/a^2/(1+cos(d*x+c))^3-1/32/d/
a^2/(1+cos(d*x+c))^2-1/32/d/a^2/(1+cos(d*x+c))+1/128*ln(1+cos(d*x+c))/a^2/d
```

maxima [A] time = 0.37, size = 167, normalized size = 1.14

$$-\frac{\frac{2\left(3\,\cos(dx+c)^5+6\,\cos(dx+c)^4-2\,\cos(dx+c)^3-10\,\cos(dx+c)^2+35\,\cos(dx+c)+16\right)}{a^2\cos(dx+c)^6+2\,a^2\cos(dx+c)^5-a^2\cos(dx+c)^4-4\,a^2\cos(dx+c)^3-a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2}-\frac{3\,\log(\cos(dx+c)+1)}{a^2}+\frac{3\,\log(\cos(dx+c)-}{a^2}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/384*(2*(3*cos(d*x + c)^5 + 6*cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 10*cos(
d*x + c)^2 + 35*cos(d*x + c) + 16)/(a^2*cos(d*x + c)^6 + 2*a^2*cos(d*x + c)
^5 - a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^3 - a^2*cos(d*x + c)^2 + 2*a^2
*cos(d*x + c) + a^2) - 3*log(cos(d*x + c) + 1)/a^2 + 3*log(cos(d*x + c) - 1
)/a^2)/d
```

mupad [B] time = 1.05, size = 152, normalized size = 1.04

$$\frac{\mathrm{atanh}\left(\cos\left(c+d\,x\right)\right)}{64\,a^2\,d}-\frac{\frac{\cos(c+d\,x)^5}{64}+\frac{\cos(c+d\,x)^4}{32}-\frac{\cos(c+d\,x)^3}{96}-\frac{5\cos(c+d\,x)^2}{96}+\frac{35\,\cos(c+d}{192}}{d\,\left(a^2\cos\left(c+d\,x\right)^6+2\,a^2\cos\left(c+d\,x\right)^5-a^2\cos\left(c+d\,x\right)^4-4\,a^2\cos\left(c+d\,x\right)^3-a^2\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^5*(a + a/cos(c + d*x))^2),x)
```

```
[Out] atanh(cos(c + d*x))/(64*a^2*d) - ((35*cos(c + d*x))/192 - (5*cos(c + d*x)^2
)/96 - cos(c + d*x)^3/96 + cos(c + d*x)^4/32 + cos(c + d*x)^5/64 + 1/12)/(d
*(2*a^2*cos(c + d*x) + a^2 - a^2*cos(c + d*x)^2 - 4*a^2*cos(c + d*x)^3 - a^
2*cos(c + d*x)^4 + 2*a^2*cos(c + d*x)^5 + a^2*cos(c + d*x)^6))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc^5\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.83 $\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=167

$$\frac{2\sin^7(c+dx)}{7a^2d}-\frac{2\sin^5(c+dx)}{5a^2d}-\frac{\sin^3(c+dx)\cos^5(c+dx)}{8a^2d}-\frac{\sin(c+dx)\cos^5(c+dx)}{16a^2d}-\frac{\sin^3(c+dx)\cos^3(c+dx)}{6a^2d}-$$

```
[Out] 11/128*x/a^2+11/128*cos(d*x+c)*sin(d*x+c)/a^2/d-7/64*cos(d*x+c)^3*sin(d*x+c
)/a^2/d-1/16*cos(d*x+c)^5*sin(d*x+c)/a^2/d-1/6*cos(d*x+c)^3*sin(d*x+c)^3/a^
2/d-1/8*cos(d*x+c)^5*sin(d*x+c)^3/a^2/d-2/5*sin(d*x+c)^5/a^2/d+2/7*sin(d*x+
c)^7/a^2/d
```

Rubi [A] time = 0.44, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3872, 2875, 2873, 2568, 2635, 8, 2564, 14}

$$\frac{2\sin^7(c+dx)}{7a^2d}-\frac{2\sin^5(c+dx)}{5a^2d}-\frac{\sin^3(c+dx)\cos^5(c+dx)}{8a^2d}-\frac{\sin^3(c+dx)\cos^3(c+dx)}{6a^2d}-\frac{\sin(c+dx)\cos^5(c+dx)}{16a^2d}-$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (11*x)/(128*a^2) + (11*Cos[c + d*x]*Sin[c + d*x])/(128*a^2*d) - (7*Cos[c +
d*x]^3*Sin[c + d*x])/(64*a^2*d) - (Cos[c + d*x]^5*Sin[c + d*x])/(16*a^2*d)
- (Cos[c + d*x]^3*Sin[c + d*x]^3)/(6*a^2*d) - (Cos[c + d*x]^5*Sin[c + d*x]^
3)/(8*a^2*d) - (2*Sin[c + d*x]^5)/(5*a^2*d) + (2*Sin[c + d*x]^7)/(7*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\sin^8(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\int \cos^2(c+dx)(-a+a\cos(c+dx))^2\sin^4(c+dx)\,dx}{a^4} \\
&= \frac{\int \left(a^2\cos^2(c+dx)\sin^4(c+dx) - 2a^2\cos^3(c+dx)\sin^4(c+dx) + a^2\cos^4(c+dx)\right.}{a^4} \\
&= \frac{\int \cos^2(c+dx)\sin^4(c+dx)\,dx}{a^2} + \frac{\int \cos^4(c+dx)\sin^4(c+dx)\,dx}{a^2} - \frac{2\int \cos^3(c+}{} \\
&= -\frac{\cos^3(c+dx)\sin^3(c+dx)}{6a^2d} - \frac{\cos^5(c+dx)\sin^3(c+dx)}{8a^2d} + \frac{3\int \cos^4(c+dx)\sin^2(c}{8a^2} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{8a^2d} - \frac{\cos^5(c+dx)\sin(c+dx)}{16a^2d} - \frac{\cos^3(c+dx)\sin^3(c+dx}{6a^2d} \\
&= \frac{\cos(c+dx)\sin(c+dx)}{16a^2d} - \frac{7\cos^3(c+dx)\sin(c+dx)}{64a^2d} - \frac{\cos^5(c+dx)\sin(c+dx)}{16a^2d} - \\
&= \frac{x}{16a^2} + \frac{11\cos(c+dx)\sin(c+dx)}{128a^2d} - \frac{7\cos^3(c+dx)\sin(c+dx)}{64a^2d} - \frac{\cos^5(c+dx)\sin}{16a^2d} \\
&= \frac{11x}{128a^2} + \frac{11\cos(c+dx)\sin(c+dx)}{128a^2d} - \frac{7\cos^3(c+dx)\sin(c+dx)}{64a^2d} - \frac{\cos^5(c+dx)s}{16a^2}
\end{aligned}
$$

Mathematica [A] time = 3.14, size = 131, normalized size = 0.78

$$
\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(-10080\sin(c+dx) - 1680\sin(2(c+dx)) + 3360\sin(3(c+dx)) - 2520\sin(4(c\right.}{26880a^2d(}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(9240*d*x - 10080*Sin[c + d*x] - 1680*Si
n[2*(c + d*x)] + 3360*Sin[3*(c + d*x)] - 2520*Sin[4*(c + d*x)] + 672*Sin[5*
```

```
(c + d*x)] + 560*Sin[6*(c + d*x)] - 480*Sin[7*(c + d*x)] + 105*Sin[8*(c + d
*x)] + 980*Tan[c/2]))/(26880*a^2*d*(1 + Sec[c + d*x])^2)
```

fricas [A] time = 0.70, size = 90, normalized size = 0.54

$$\frac{1155\,dx + \left(1680\,\cos\left(dx+c\right)^{7} - 3840\,\cos\left(dx+c\right)^{6} - 280\,\cos\left(dx+c\right)^{5} + 6144\,\cos\left(dx+c\right)^{4} - 3710\,\cos\left(dx+c\right.}{13440\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/13440*(1155*d*x + (1680*cos(d*x + c)^7 - 3840*cos(d*x + c)^6 - 280*cos(d*
x + c)^5 + 6144*cos(d*x + c)^4 - 3710*cos(d*x + c)^3 - 768*cos(d*x + c)^2 +
 1155*cos(d*x + c) - 1536)*sin(d*x + c))/(a^2*d)
```

giac [A] time = 0.26, size = 139, normalized size = 0.83

$$\frac{\frac{1155\,(dx+c)}{a^{2}} + \frac{2\left(1155\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15} + 8855\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13} - 142541\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} + 31007\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9} - 55583\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7} - 29491\,\mathrm{ta}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{8}a^{2}}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/13440*(1155*(d*x + c)/a^2 + 2*(1155*tan(1/2*d*x + 1/2*c)^15 + 8855*tan(1/
2*d*x + 1/2*c)^13 - 142541*tan(1/2*d*x + 1/2*c)^11 + 31007*tan(1/2*d*x + 1/
2*c)^9 - 55583*tan(1/2*d*x + 1/2*c)^7 - 29491*tan(1/2*d*x + 1/2*c)^5 - 8855
*tan(1/2*d*x + 1/2*c)^3 - 1155*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)
^2 + 1)^8*a^2))/d
```

maple [A] time = 0.66, size = 290, normalized size = 1.74

$$-\frac{11\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{8}} - \frac{253\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{8}} - \frac{4213\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{960d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{8}} - \frac{55583\left(\tan^{7}\left(\frac{dx}{2}\right.\right.}{6720d\,a^{2}\left(1+\tan^{2}\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^8/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -11/64/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)-253/192/d/a^2/(1
+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^3-4213/960/d/a^2/(1+tan(1/2*d*x
+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^5-55583/6720/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)
^8*tan(1/2*d*x+1/2*c)^7+31007/6720/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2
*d*x+1/2*c)^9-20363/960/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)
^11+253/192/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^13+11/64/d/
a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^15+11/64/d/a^2*arctan(tan
(1/2*d*x+1/2*c))
```

maxima [B] time = 0.44, size = 378, normalized size = 2.26

$$-\frac{\frac{\frac{1155\sin(dx+c)}{\cos(dx+c)+1} + \frac{8855\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}} + \frac{29491\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}} + \frac{55583\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}} - \frac{31007\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}} + \frac{142541\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}} - \frac{8855\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}} - \frac{1155\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}}{a^{2} + \frac{8a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}} + \frac{28a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}} + \frac{56a^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}} + \frac{70a^{2}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}} + \frac{56a^{2}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{28a^{2}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}} + \frac{8a^{2}\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}} + \frac{a^{2}\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}} - \frac{1155\,\mathrm{arct}}{}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6720*((1155*sin(d*x + c)/(cos(d*x + c) + 1) + 8855*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 29491*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 55583*sin(d*x
+ c)^7/(cos(d*x + c) + 1)^7 - 31007*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 1
42541*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 8855*sin(d*x + c)^13/(cos(d*x
 + c) + 1)^13 - 1155*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)/(a^2 + 8*a^2*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 + 56*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^2*sin(d*x + c)^8/(c
os(d*x + c) + 1)^8 + 56*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a^2*
sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 8*a^2*sin(d*x + c)^14/(cos(d*x + c)
 + 1)^14 + a^2*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 1155*arctan(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 3.93, size = 133, normalized size = 0.80

$$\frac{11\,x}{128\,a^2}-\frac{-\frac{11\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64}-\frac{253\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{192}+\frac{20363\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{960}-\frac{31007\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{6720}+\frac{55583\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{6720}+\frac{4213\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{960}}{a^2\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (11*x)/(128*a^2) - ((11*tan(c/2 + (d*x)/2))/64 + (253*tan(c/2 + (d*x)/2)^3)
/192 + (4213*tan(c/2 + (d*x)/2)^5)/960 + (55583*tan(c/2 + (d*x)/2)^7)/6720
- (31007*tan(c/2 + (d*x)/2)^9)/6720 + (20363*tan(c/2 + (d*x)/2)^11)/960 - (
253*tan(c/2 + (d*x)/2)^13)/192 - (11*tan(c/2 + (d*x)/2)^15)/64)/(a^2*d*(tan
(c/2 + (d*x)/2)^2 + 1)^8)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**8/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.84 $\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=104

$$-\frac{\sin^3(c+dx)(a-a\cos(c+dx))^3}{6a^5d}-\frac{\sin^5(c+dx)}{10a^2d}-\frac{\sin^3(c+dx)\cos(c+dx)}{8a^2d}-\frac{3\sin(c+dx)\cos(c+dx)}{16a^2d}+\frac{3x}{16a^2}$$

```
[Out] 3/16*x/a^2-3/16*cos(d*x+c)*sin(d*x+c)/a^2/d-1/8*cos(d*x+c)*sin(d*x+c)^3/a^2
/d-1/6*(a-a*cos(d*x+c))^3*sin(d*x+c)^3/a^5/d-1/10*sin(d*x+c)^5/a^2/d
```

Rubi [A] time = 0.31, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2875, 2870, 2669, 2635, 8}

$$-\frac{\sin^5(c+dx)}{10a^2d}-\frac{\sin^3(c+dx)(a-a\cos(c+dx))^3}{6a^5d}-\frac{\sin^3(c+dx)\cos(c+dx)}{8a^2d}-\frac{3\sin(c+dx)\cos(c+dx)}{16a^2d}+\frac{3x}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (3*x)/(16*a^2) - (3*Cos[c + d*x]*Sin[c + d*x])/(16*a^2*d) - (Cos[c + d*x]*S
in[c + d*x]^3)/(8*a^2*d) - ((a - a*Cos[c + d*x])^3*Sin[c + d*x]^3)/(6*a^5*d
) - Sin[c + d*x]^5/(10*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2870

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(2*b*f*g*(m + 1)), x] + Dist[a/(2*g^2)
, Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[m - p, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\sin^6(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\int \cos^2(c+dx)(-a+a\cos(c+dx))^2\sin^2(c+dx)\,dx}{a^4} \\
&= -\frac{(a-a\cos(c+dx))^3\sin^3(c+dx)}{6a^5d} - \frac{\int(-a+a\cos(c+dx))\sin^4(c+dx)\,dx}{2a^3} \\
&= -\frac{(a-a\cos(c+dx))^3\sin^3(c+dx)}{6a^5d} - \frac{\sin^5(c+dx)}{10a^2d} + \frac{\int \sin^4(c+dx)\,dx}{2a^2} \\
&= -\frac{\cos(c+dx)\sin^3(c+dx)}{8a^2d} - \frac{(a-a\cos(c+dx))^3\sin^3(c+dx)}{6a^5d} - \frac{\sin^5(c+dx)}{10a^2d} + \frac{3}{} \\
&= -\frac{3\cos(c+dx)\sin(c+dx)}{16a^2d} - \frac{\cos(c+dx)\sin^3(c+dx)}{8a^2d} - \frac{(a-a\cos(c+dx))^3\sin^3}{6a^5d} \\
&= \frac{3x}{16a^2} - \frac{3\cos(c+dx)\sin(c+dx)}{16a^2d} - \frac{\cos(c+dx)\sin^3(c+dx)}{8a^2d} - \frac{(a-a\cos(c+dx)}{6a^5c}
\end{aligned}$$

Mathematica [A] time = 0.95, size = 111, normalized size = 1.07

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(-480\sin(c+dx)+30\sin(2(c+dx))+80\sin(3(c+dx))-90\sin(4(c+dx))+\right.}{480a^2d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(360*d*x - 480*Sin[c + d*x] + 30*Sin[2*(
c + d*x)] + 80*Sin[3*(c + d*x)] - 90*Sin[4*(c + d*x)] + 48*Sin[5*(c + d*x)]
 - 10*Sin[6*(c + d*x)] + 25*Tan[c/2]))/(480*a^2*d*(1 + Sec[c + d*x])^2)
```

fricas [A] time = 1.43, size = 71, normalized size = 0.68

$$\frac{45\,dx - \left(40\,\cos(dx+c)^5 - 96\,\cos(dx+c)^4 + 50\,\cos(dx+c)^3 + 32\,\cos(dx+c)^2 - 45\,\cos(dx+c) + 64\right)\sin}{240\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(45*d*x - (40*cos(d*x + c)^5 - 96*cos(d*x + c)^4 + 50*cos(d*x + c)^3
+ 32*cos(d*x + c)^2 - 45*cos(d*x + c) + 64)*sin(d*x + c))/(a^2*d)
```

giac [A] time = 0.26, size = 113, normalized size = 1.09

$$\frac{\frac{45\,(dx+c)}{a^2} + \frac{2\left(45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} - 1025\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9 - 174\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 594\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 255\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 45\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6 a^2}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(45*(d*x + c)/a^2 + 2*(45*tan(1/2*d*x + 1/2*c)^11 - 1025*tan(1/2*d*x
+ 1/2*c)^9 - 174*tan(1/2*d*x + 1/2*c)^7 - 594*tan(1/2*d*x + 1/2*c)^5 - 255*
tan(1/2*d*x + 1/2*c)^3 - 45*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2
+ 1)^6*a^2))/d
```

maple [B] time = 0.63, size = 222, normalized size = 2.13

$$\frac{3\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}-\frac{205\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}-\frac{29\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{6}}-\frac{99\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 3/8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11-205/24/d/a^2/(1+
tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9-29/20/d/a^2/(1+tan(1/2*d*x+1/2
*c)^2)^6*tan(1/2*d*x+1/2*c)^7-99/20/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/
2*d*x+1/2*c)^5-17/8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3-3
/8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)+3/8/d/a^2*arctan(tan
(1/2*d*x+1/2*c))
```

maxima [B] time = 0.55, size = 292, normalized size = 2.81

$$-\frac{\frac{\frac{45\sin(dx+c)}{\cos(dx+c)+1}+\frac{255\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{594\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{174\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{1025\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{45\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^{2}+\frac{6a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{15a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{20a^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{15a^{2}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}+\frac{6a^{2}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^{2}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{45\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{2}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/120*((45*sin(d*x + c)/(cos(d*x + c) + 1) + 255*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 594*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 174*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 1025*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 45*sin(d*x
 + c)^11/(cos(d*x + c) + 1)^11)/(a^2 + 6*a^2*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 15*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a^2*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 + 15*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a^2*s
in(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^2*sin(d*x + c)^12/(cos(d*x + c) +
1)^12) - 45*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 3.75, size = 107, normalized size = 1.03

$$\frac{3\,x}{16\,a^{2}}-\frac{-\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{205\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{24}+\frac{29\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}+\frac{99\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}+\frac{17\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{8}+\frac{3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{a^{2}\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (3*x)/(16*a^2) - ((3*tan(c/2 + (d*x)/2))/8 + (17*tan(c/2 + (d*x)/2)^3)/8 +
(99*tan(c/2 + (d*x)/2)^5)/20 + (29*tan(c/2 + (d*x)/2)^7)/20 + (205*tan(c/2
+ (d*x)/2)^9)/24 - (3*tan(c/2 + (d*x)/2)^11)/8)/(a^2*d*(tan(c/2 + (d*x)/2)^
2 + 1)^6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^6(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**6/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.85 $\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=87

$$\frac{2\sin^3(c+dx)}{3a^2d}-\frac{2\sin(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}+\frac{7\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{7x}{8a^2}$$

```
[Out] 7/8*x/a^2-2*sin(d*x+c)/a^2/d+7/8*cos(d*x+c)*sin(d*x+c)/a^2/d+1/4*cos(d*x+c)
^3*sin(d*x+c)/a^2/d+2/3*sin(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.23, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2869, 2757, 2635, 8, 2633}

$$\frac{2\sin^3(c+dx)}{3a^2d}-\frac{2\sin(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}+\frac{7\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{7x}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (7*x)/(8*a^2) - (2*Sin[c + d*x])/(a^2*d) + (7*Cos[c + d*x]*Sin[c + d*x])/(8
*a^2*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a^2*d) + (2*Sin[c + d*x]^3)/(3*a
^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
```

```
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^4(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{\int \cos^2(c+dx)(-a+a\cos(c+dx))^2\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cos^2(c+dx)-2a^2\cos^3(c+dx)+a^2\cos^4(c+dx)\right)dx}{a^4}$$
$$= \frac{\int \cos^2(c+dx)\,dx}{a^2}+\frac{\int \cos^4(c+dx)\,dx}{a^2}-\frac{2\int \cos^3(c+dx)\,dx}{a^2}$$
$$= \frac{\cos(c+dx)\sin(c+dx)}{2a^2d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d}+\frac{\int 1\,dx}{2a^2}+\frac{3\int \cos^2(c+dx)\,d}{4a^2}$$
$$= \frac{x}{2a^2}-\frac{2\sin(c+dx)}{a^2d}+\frac{7\cos(c+dx)\sin(c+dx)}{8a^2d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d}+\frac{2\,\mathrm{s}}{}$$
$$= \frac{7x}{8a^2}-\frac{2\sin(c+dx)}{a^2d}+\frac{7\cos(c+dx)\sin(c+dx)}{8a^2d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d}+\frac{2\,\mathrm{s}}{}$$

Mathematica [A] time = 0.59, size = 91, normalized size = 1.05

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(-144\sin(c+dx)+48\sin(2(c+dx))-16\sin(3(c+dx))+3\sin(4(c+dx))+2\right.}{24a^2d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(84*d*x - 144*Sin[c + d*x] + 48*Sin[2*(c
 + d*x)] - 16*Sin[3*(c + d*x)] + 3*Sin[4*(c + d*x)] + 2*Tan[c/2]))/(24*a^2*
d*(1 + Sec[c + d*x])^2)
```

fricas [A] time = 0.99, size = 50, normalized size = 0.57

$$\frac{21\,dx+\left(6\,\cos\left(dx+c\right)^3-16\,\cos\left(dx+c\right)^2+21\,\cos\left(dx+c\right)-32\right)\sin\left(dx+c\right)}{24\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(21*d*x + (6*cos(d*x + c)^3 - 16*cos(d*x + c)^2 + 21*cos(d*x + c) - 32
)*sin(d*x + c))/(a^2*d)
```

giac [A] time = 0.25, size = 87, normalized size = 1.00

$$\frac{\frac{21\,(dx+c)}{a^2}-\frac{2\left(75\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+83\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+77\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(21*(d*x + c)/a^2 - 2*(75*tan(1/2*d*x + 1/2*c)^7 + 83*tan(1/2*d*x + 1/
2*c)^5 + 77*tan(1/2*d*x + 1/2*c)^3 + 21*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x
 + 1/2*c)^2 + 1)^4*a^2))/d
```

maple [A] time = 0.67, size = 154, normalized size = 1.77

$$-\frac{25\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{83\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{77\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\frac{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -25/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7-83/12/d/a^2/(1+
tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-77/12/d/a^2/(1+tan(1/2*d*x+1/2
*c)^2)^4*tan(1/2*d*x+1/2*c)^3-7/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*
d*x+1/2*c)+7/4/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.56, size = 206, normalized size = 2.37

$$-\frac{\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{83\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{75\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}}{a^{2}+\frac{4a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{6a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{4a^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{a^{2}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}}}-\frac{21\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{2}}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/12*((21*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 83*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 75*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)/(a^2 + 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^2*s
in(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1
)^6 + a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 21*arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a^2)/d
```

mupad [B] time = 4.51, size = 81, normalized size = 0.93

$$\frac{7x}{8a^{2}}-\frac{\frac{25\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{7}}{4}+\frac{83\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{5}}{12}+\frac{77\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{3}}{12}+\frac{7\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4}}{a^{2}d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}+1\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (7*x)/(8*a^2) - ((7*tan(c/2 + (d*x)/2))/4 + (77*tan(c/2 + (d*x)/2)^3)/12 +
(83*tan(c/2 + (d*x)/2)^5)/12 + (25*tan(c/2 + (d*x)/2)^7)/4)/(a^2*d*(tan(c/2
 + (d*x)/2)^2 + 1)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\sin^{4}(c+dx)}{\sec^{2}(c+dx)+2\sec(c+dx)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.86 $\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=69

$$\frac{2\sin(c+dx)}{a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{2\sin(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{5x}{2a^2}$$

```
[Out] -5/2*x/a^2+2*sin(d*x+c)/a^2/d-1/2*cos(d*x+c)*sin(d*x+c)/a^2/d+2*sin(d*x+c)/
a^2/d/(1+cos(d*x+c))
```

Rubi [A] time = 0.32, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2874, 2950, 2709, 2637, 2635, 8, 2648}

$$\frac{2\sin(c+dx)}{a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{2\sin(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{5x}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-5*x)/(2*a^2) + (2*Sin[c + d*x])/(a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*
a^2*d) + (2*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
```

```
, 0])
```

Rule 2950

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^n*c^n,
 Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
0] && IntegerQ[n]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(-a-a\cos(c+dx))^2}\,dx\\
&= \frac{\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))}{-a-a\cos(c+dx)}\,dx}{a^2}\\
&= \frac{\int(-a+a\cos(c+dx))^2\cot^2(c+dx)\,dx}{a^4}\\
&= \frac{\int\left(-2+2\cos(c+dx)-\cos^2(c+dx)+\frac{2}{1+\cos(c+dx)}\right)dx}{a^2}\\
&= -\frac{2x}{a^2}-\frac{\int\cos^2(c+dx)\,dx}{a^2}+\frac{2\int\cos(c+dx)\,dx}{a^2}+\frac{2\int\frac{1}{1+\cos(c+dx)}\,dx}{a^2}\\
&= -\frac{2x}{a^2}+\frac{2\sin(c+dx)}{a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}+\frac{2\sin(c+dx)}{a^2d(1+\cos(c+dx))}-\frac{\int 1\,dx}{2a^2}\\
&= -\frac{5x}{2a^2}+\frac{2\sin(c+dx)}{a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}+\frac{2\sin(c+dx)}{a^2d(1+\cos(c+dx))}
\end{aligned}$$

Mathematica [A] time = 0.36, size = 121, normalized size = 1.75

$$-\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-25\sin\left(c+\frac{dx}{2}\right)-21\sin\left(c+\frac{3dx}{2}\right)-21\sin\left(2c+\frac{3dx}{2}\right)+3\sin\left(2c+\frac{5dx}{2}\right)+3\sin\left(3c+\right.\right.}{48a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/48*(Sec[c/2]*Sec[(c + d*x)/2]*(60*d*x*Cos[(d*x)/2] + 60*d*x*Cos[c + (d*x
)/2] - 119*Sin[(d*x)/2] - 25*Sin[c + (d*x)/2] - 21*Sin[c + (3*d*x)/2] - 21*
Sin[2*c + (3*d*x)/2] + 3*Sin[2*c + (5*d*x)/2] + 3*Sin[3*c + (5*d*x)/2]))/(a
^2*d)
```

fricas [A] time = 0.88, size = 61, normalized size = 0.88

$$-\frac{5\,dx\cos(dx+c)+5\,dx+\left(\cos(dx+c)^2-3\,\cos(dx+c)-8\right)\sin(dx+c)}{2\left(a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(5*d*x*cos(d*x + c) + 5*d*x + (cos(d*x + c)^2 - 3*cos(d*x + c) - 8)*si
n(d*x + c))/(a^2*d*cos(d*x + c) + a^2*d)
```

giac [A] time = 0.73, size = 75, normalized size = 1.09

$$-\frac{\frac{5\,(dx+c)}{a^2}-\frac{4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}-\frac{2\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*(d*x + c)/a^2 - 4*tan(1/2*d*x + 1/2*c)/a^2 - 2*(5*tan(1/2*d*x + 1/2
*c)^3 + 3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

maple [A] time = 0.63, size = 103, normalized size = 1.49

$$\frac{2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^2}+\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^2\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{5\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*tan(1/2*d*x+1/2*c)+5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1
/2*c)^3+3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)-5/d/a^2*arcta
n(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.51, size = 140, normalized size = 2.03

$$\frac{\frac{\frac{3\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2+\frac{2\,a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{5\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{2\,\sin(dx+c)}{a^2(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] ((3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4) - 5*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 2*sin(
d*x + c)/(a^2*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 1.03, size = 91, normalized size = 1.32

$$\frac{4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-5\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(c+d\,x)+10\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,a^2\,d\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (4*sin(c/2 + (d*x)/2) - 5*cos(c/2 + (d*x)/2)*(c + d*x) + 10*cos(c/2 + (d*x)
/2)^2*sin(c/2 + (d*x)/2) - 4*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2))/(2*a^
2*d*cos(c/2 + (d*x)/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
[Out] Integral(sin(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.87 $\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=73

$$-\frac{2\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{2\csc^3(c+dx)}{3a^2d}$$

```
[Out] -1/3*cot(d*x+c)^3/a^2/d-2/5*cot(d*x+c)^5/a^2/d-2/3*csc(d*x+c)^3/a^2/d+2/5*c
sc(d*x+c)^5/a^2/d
```

Rubi [A] time = 0.20, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2711, 2607, 30, 2606, 14}

$$-\frac{2\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{2\csc^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -Cot[c + d*x]^3/(3*a^2*d) - (2*Cot[c + d*x]^5)/(5*a^2*d) - (2*Csc[c + d*x]^
3)/(3*a^2*d) + (2*Csc[c + d*x]^5)/(5*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
```

```
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{\int \left(a^2\cot^4(c+dx)\csc^2(c+dx) - 2a^2\cot^3(c+dx)\csc^3(c+dx) + a^2\cot^2(c+dx)\csc^4\right.}{a^4}$$
$$= \frac{\int \cot^4(c+dx)\csc^2(c+dx)\,dx}{a^2} + \frac{\int \cot^2(c+dx)\csc^4(c+dx)\,dx}{a^2} - \frac{2\int \cot^3(c+dx)\,c}{a^2}$$
$$= \frac{\text{Subst}\left(\int x^4\,dx, x, -\cot(c+dx)\right)}{a^2d} + \frac{\text{Subst}\left(\int x^2\left(1+x^2\right)dx, x, -\cot(c+dx)\right)}{a^2d} + \frac{2\,\text{Su}}{}$$
$$= -\frac{\cot^5(c+dx)}{5a^2d} + \frac{\text{Subst}\left(\int \left(x^2+x^4\right)dx, x, -\cot(c+dx)\right)}{a^2d} + \frac{2\,\text{Subst}\left(\int \left(-x^2+x^4\right)d\right.}{a^2d}$$
$$= -\frac{\cot^3(c+dx)}{3a^2d} - \frac{2\cot^5(c+dx)}{5a^2d} - \frac{2\csc^3(c+dx)}{3a^2d} + \frac{2\csc^5(c+dx)}{5a^2d}$$

Mathematica [A] time = 0.48, size = 105, normalized size = 1.44

$$\frac{\csc(c)(55\sin(c+dx) + 44\sin(2(c+dx)) + 11\sin(3(c+dx)) - 60\sin(2c+dx) + 16\sin(c+2dx) + 4\sin(2c+3d}{240a^2d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Csc[c]*Csc[c + d*x]*Sec[c + d*x]^2*(-80*Sin[c] + 80*Sin[d*x] + 55*Sin[c +
d*x] + 44*Sin[2*(c + d*x)] + 11*Sin[3*(c + d*x)] - 60*Sin[2*c + d*x] + 16*S
in[c + 2*d*x] + 4*Sin[2*c + 3*d*x]))/(240*a^2*d*(1 + Sec[c + d*x])^2)
```

fricas [A] time = 0.59, size = 71, normalized size = 0.97

$$-\frac{\cos(dx+c)^3 + 2\cos(dx+c)^2 + 8\cos(dx+c) + 4}{15\left(a^2d\cos(dx+c)^2 + 2a^2d\cos(dx+c) + a^2d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + 8*cos(d*x + c) + 4)/((a^2*d*cos(
d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c))
```

giac [A] time = 0.26, size = 74, normalized size = 1.01

$$-\frac{\frac{15}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)} - \frac{3a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 5a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 15a^8\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{10}}}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/120*(15/(a^2*tan(1/2*d*x + 1/2*c)) - (3*a^8*tan(1/2*d*x + 1/2*c)^5 - 5*a
^8*tan(1/2*d*x + 1/2*c)^3 - 15*a^8*tan(1/2*d*x + 1/2*c))/a^10)/d
```

maple [A] time = 0.62, size = 60, normalized size = 0.82

$$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{8d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/8/d/a^2*(1/5*tan(1/2*d*x+1/2*c)^5-1/3*tan(1/2*d*x+1/2*c)^3-tan(1/2*d*x+1/
2*c)-1/tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.52, size = 90, normalized size = 1.23

$$-\frac{\frac{\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^2}+\frac{15\,(\cos(dx+c)+1)}{a^2\,\sin(dx+c)}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/120*((15*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^2 + 15*(cos(d*x + c) +
1)/(a^2*sin(d*x + c)))/d
```

mupad [B] time = 0.98, size = 71, normalized size = 0.97

$$-\frac{8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+14\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-3}{120\,a^2\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + a/cos(c + d*x))^2),x)
```

```
[Out] -(14*cos(c/2 + (d*x)/2)^2 - 4*cos(c/2 + (d*x)/2)^4 + 8*cos(c/2 + (d*x)/2)^6
 - 3)/(120*a^2*d*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.88 $\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=91

$$-\frac{2\cot^7(c+dx)}{7a^2d}-\frac{3\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}-\frac{2\csc^5(c+dx)}{5a^2d}$$

```
[Out] -1/3*cot(d*x+c)^3/a^2/d-3/5*cot(d*x+c)^5/a^2/d-2/7*cot(d*x+c)^7/a^2/d-2/5*c
sc(d*x+c)^5/a^2/d+2/7*csc(d*x+c)^7/a^2/d
```

Rubi [A] time = 0.34, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2875, 2873, 2607, 14, 2606, 270}

$$-\frac{2\cot^7(c+dx)}{7a^2d}-\frac{3\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}-\frac{2\csc^5(c+dx)}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -Cot[c + d*x]^3/(3*a^2*d) - (3*Cot[c + d*x]^5)/(5*a^2*d) - (2*Cot[c + d*x]^
7)/(7*a^2*d) - (2*Csc[c + d*x]^5)/(5*a^2*d) + (2*Csc[c + d*x]^7)/(7*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
```

```
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{\int(-a+a\cos(c+dx))^2\cot^2(c+dx)\csc^6(c+dx)\,dx}{a^4}$$
$$= \frac{\int\left(a^2\cot^4(c+dx)\csc^4(c+dx)-2a^2\cot^3(c+dx)\csc^5(c+dx)+a^2\cot^2(c+dx)\,\mathrm{c}\right.}{a^4}$$
$$= \frac{\int\cot^4(c+dx)\csc^4(c+dx)\,dx}{a^2}+\frac{\int\cot^2(c+dx)\csc^6(c+dx)\,dx}{a^2}-\frac{2\int\cot^3(c+d}{}$$
$$= \frac{\mathrm{Subst}\left(\int x^4\left(1+x^2\right)dx,x,-\cot(c+dx)\right)}{a^2d}+\frac{\mathrm{Subst}\left(\int x^2\left(1+x^2\right)^2dx,x,-\cot(c\right.}{a^2d}$$
$$= \frac{\mathrm{Subst}\left(\int\left(x^4+x^6\right)dx,x,-\cot(c+dx)\right)}{a^2d}+\frac{\mathrm{Subst}\left(\int\left(x^2+2x^4+x^6\right)dx,x,-\cot(\right.}{a^2d}$$
$$= -\frac{\cot^3(c+dx)}{3a^2d}-\frac{3\cot^5(c+dx)}{5a^2d}-\frac{2\cot^7(c+dx)}{7a^2d}-\frac{2\csc^5(c+dx)}{5a^2d}+\frac{2\csc^7(c+d}{7a^2d}$$

Mathematica [A] time = 0.70, size = 149, normalized size = 1.64

$$-\frac{\csc(c)(-714\sin(c+dx)-408\sin(2(c+dx))+153\sin(3(c+dx))+204\sin(4(c+dx))+51\sin(5(c+dx))+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/13440*(Csc[c]*Csc[c + d*x]^3*Sec[c + d*x]^2*(1344*Sin[c] - 1456*Sin[d*x]
 - 714*Sin[c + d*x] - 408*Sin[2*(c + d*x)] + 153*Sin[3*(c + d*x)] + 204*Sin
[4*(c + d*x)] + 51*Sin[5*(c + d*x)] + 1680*Sin[2*c + d*x] + 128*Sin[c + 2*d
*x] - 48*Sin[2*c + 3*d*x] - 64*Sin[3*c + 4*d*x] - 16*Sin[4*c + 5*d*x]))/(a^
2*d*(1 + Sec[c + d*x])^2)
```

fricas [A] time = 0.79, size = 108, normalized size = 1.19

$$\frac{2\cos(dx+c)^5+4\cos(dx+c)^4-\cos(dx+c)^3-6\cos(dx+c)^2+24\cos(dx+c)+12}{105\left(a^2d\cos(dx+c)^4+2a^2d\cos(dx+c)^3-2a^2d\cos(dx+c)-a^2d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(2*cos(d*x + c)^5 + 4*cos(d*x + c)^4 - cos(d*x + c)^3 - 6*cos(d*x + c
)^2 + 24*cos(d*x + c) + 12)/((a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3
 - 2*a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))
```

giac [A] time = 0.38, size = 105, normalized size = 1.15

$$-\frac{\frac{35\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}-\frac{15a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+21a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-70a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-210a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{14}}}{3360d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3360*(35*(3*tan(1/2*d*x + 1/2*c)^2 + 1)/(a^2*tan(1/2*d*x + 1/2*c)^3) - (
15*a^12*tan(1/2*d*x + 1/2*c)^7 + 21*a^12*tan(1/2*d*x + 1/2*c)^5 - 70*a^12*t
an(1/2*d*x + 1/2*c)^3 - 210*a^12*tan(1/2*d*x + 1/2*c))/a^14)/d
```

maple [A] time = 0.80, size = 86, normalized size = 0.95

$$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{2\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{32d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/32/d/a^2*(1/7*tan(1/2*d*x+1/2*c)^7+1/5*tan(1/2*d*x+1/2*c)^5-2/3*tan(1/2*d
*x+1/2*c)^3-2*tan(1/2*d*x+1/2*c)-1/3/tan(1/2*d*x+1/2*c)^3-1/tan(1/2*d*x+1/2
*c))
```

maxima [A] time = 0.48, size = 134, normalized size = 1.47

$$-\frac{\frac{\frac{210\sin(dx+c)}{\cos(dx+c)+1}+\frac{70\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^2}+\frac{35\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^3}{a^2\sin(dx+c)^3}}{3360d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3360*((210*sin(d*x + c)/(cos(d*x + c) + 1) + 70*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^2 + 35*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)*(c
os(d*x + c) + 1)^3/(a^2*sin(d*x + c)^3))/d
```

mupad [B] time = 1.07, size = 121, normalized size = 1.33

$$-\frac{64\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-96\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+24\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+4\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+54\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-15}{3360\,a^2\,d\,\left(\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + a/cos(c + d*x))^2),x)
```

```
[Out] -(54*cos(c/2 + (d*x)/2)^2 + 4*cos(c/2 + (d*x)/2)^4 + 24*cos(c/2 + (d*x)/2)^
6 - 96*cos(c/2 + (d*x)/2)^8 + 64*cos(c/2 + (d*x)/2)^10 - 15)/(3360*a^2*d*(c
os(c/2 + (d*x)/2)^7*sin(c/2 + (d*x)/2) - cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*
x)/2)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.89 $\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$-\frac{2\cot^9(c+dx)}{9a^2d}-\frac{5\cot^7(c+dx)}{7a^2d}-\frac{4\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}-\frac{2\csc^7(c+dx)}{7a^2d}$$

```
[Out] -1/3*cot(d*x+c)^3/a^2/d-4/5*cot(d*x+c)^5/a^2/d-5/7*cot(d*x+c)^7/a^2/d-2/9*c
ot(d*x+c)^9/a^2/d-2/7*csc(d*x+c)^7/a^2/d+2/9*csc(d*x+c)^9/a^2/d
```

Rubi [A] time = 0.35, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2875, 2873, 2607, 270, 2606, 14}

$$-\frac{2\cot^9(c+dx)}{9a^2d}-\frac{5\cot^7(c+dx)}{7a^2d}-\frac{4\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}-\frac{2\csc^7(c+dx)}{7a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -Cot[c + d*x]^3/(3*a^2*d) - (4*Cot[c + d*x]^5)/(5*a^2*d) - (5*Cot[c + d*x]^
7)/(7*a^2*d) - (2*Cot[c + d*x]^9)/(9*a^2*d) - (2*Csc[c + d*x]^7)/(7*a^2*d)
+ (2*Csc[c + d*x]^9)/(9*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cot^2(c+dx)\csc^4(c+dx)}{(-a-a\cos(c+dx))^2}\,dx\\
&= \frac{\int (-a+a\cos(c+dx))^2\cot^2(c+dx)\csc^8(c+dx)\,dx}{a^4}\\
&= \frac{\int \left(a^2\cot^4(c+dx)\csc^6(c+dx)-2a^2\cot^3(c+dx)\csc^7(c+dx)+a^2\cot^2(c+dx)\,c\right.}{a^4}\\
&= \frac{\int \cot^4(c+dx)\csc^6(c+dx)\,dx}{a^2}+\frac{\int \cot^2(c+dx)\csc^8(c+dx)\,dx}{a^2}-\frac{2\int \cot^3(c+d}{}\\
&= \frac{\operatorname{Subst}\left(\int x^4\left(1+x^2\right)^2\,dx,x,-\cot(c+dx)\right)}{a^2d}+\frac{\operatorname{Subst}\left(\int x^2\left(1+x^2\right)^3\,dx,x,-\cot(c\right.}{a^2d}\\
&= \frac{\operatorname{Subst}\left(\int \left(x^4+2x^6+x^8\right)\,dx,x,-\cot(c+dx)\right)}{a^2d}+\frac{\operatorname{Subst}\left(\int \left(x^2+3x^4+3x^6+x^8\right)\right.}{a^2d}\\
&= -\frac{\cot^3(c+dx)}{3a^2d}-\frac{4\cot^5(c+dx)}{5a^2d}-\frac{5\cot^7(c+dx)}{7a^2d}-\frac{2\cot^9(c+dx)}{9a^2d}-\frac{2\csc^7(c+dx}{7a^2d}\end{aligned}$$

Mathematica [A] time = 1.02, size = 191, normalized size = 1.75

$$\frac{\csc(c)(25875\sin(c+dx)+11500\sin(2(c+dx))-10925\sin(3(c+dx))-9200\sin(4(c+dx))+575\sin(5(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Csc[c]*Csc[c + d*x]^5*Sec[c + d*x]^2*(-61440*Sin[c] + 84480*Sin[d*x] + 258
75*Sin[c + d*x] + 11500*Sin[2*(c + d*x)] - 10925*Sin[3*(c + d*x)] - 9200*Si
n[4*(c + d*x)] + 575*Sin[5*(c + d*x)] + 2300*Sin[6*(c + d*x)] + 575*Sin[7*(
c + d*x)] - 107520*Sin[2*c + d*x] - 10240*Sin[c + 2*d*x] + 9728*Sin[2*c + 3
*d*x] + 8192*Sin[3*c + 4*d*x] - 512*Sin[4*c + 5*d*x] - 2048*Sin[5*c + 6*d*x
] - 512*Sin[6*c + 7*d*x]))/(1290240*a^2*d*(1 + Sec[c + d*x])^2)
```

fricas [A] time = 1.30, size = 169, normalized size = 1.55

$$\frac{8\cos(dx+c)^7+16\cos(dx+c)^6-12\cos(dx+c)^5-40\cos(dx+c)^4-5\cos(dx+c)^3+30\cos(dx+}{315\left(a^2d\cos(dx+c)^6+2a^2d\cos(dx+c)^5-a^2d\cos(dx+c)^4-4a^2d\cos(dx+c)^3-a^2d\cos(dx+c)^2+2a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/315*(8*cos(d*x + c)^7 + 16*cos(d*x + c)^6 - 12*cos(d*x + c)^5 - 40*cos(d*
x + c)^4 - 5*cos(d*x + c)^3 + 30*cos(d*x + c)^2 - 40*cos(d*x + c) - 20)/((a
^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a^2*d*cos(d*x + c)^4 - 4*a^2
*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*si
n(d*x + c))
```

giac [A] time = 0.35, size = 134, normalized size = 1.23

$$-\frac{\frac{63\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)}{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}-\frac{35\,a^{16}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+135\,a^{16}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+63\,a^{16}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-525\,a^{16}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-1}{a^{18}}}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/40320*(63*(5*tan(1/2*d*x + 1/2*c)^4 + 5*tan(1/2*d*x + 1/2*c)^2 + 1)/(a^2
*tan(1/2*d*x + 1/2*c)^5) - (35*a^16*tan(1/2*d*x + 1/2*c)^9 + 135*a^16*tan(1
/2*d*x + 1/2*c)^7 + 63*a^16*tan(1/2*d*x + 1/2*c)^5 - 525*a^16*tan(1/2*d*x +
 1/2*c)^3 - 1575*a^16*tan(1/2*d*x + 1/2*c))/a^18)/d
```

maple [A] time = 0.72, size = 112, normalized size = 1.03

$$\frac{\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{3\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{5\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{128d\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/128/d/a^2*(1/9*tan(1/2*d*x+1/2*c)^9+3/7*tan(1/2*d*x+1/2*c)^7+1/5*tan(1/2*
d*x+1/2*c)^5-5/3*tan(1/2*d*x+1/2*c)^3-5*tan(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/
2*c)^3-1/5/tan(1/2*d*x+1/2*c)^5-1/tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.47, size = 174, normalized size = 1.60

$$-\frac{\frac{\frac{1575\sin(dx+c)}{\cos(dx+c)+1}+\frac{525\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{63\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{135\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{35\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}}{a^{2}}+\frac{63\left(\frac{5\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{5\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+1\right)(\cos(dx+c)+1)^{5}}{a^{2}\sin(dx+c)^{5}}}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/40320*((1575*sin(d*x + c)/(cos(d*x + c) + 1) + 525*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 - 63*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 135*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 - 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^2 + 63*(
5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 5*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 + 1)*(cos(d*x + c) + 1)^5/(a^2*sin(d*x + c)^5))/d
```

mupad [B] time = 2.18, size = 106, normalized size = 0.97

$$-\frac{\frac{375\cos(c+d\,x)}{8}-\frac{5\cos(2\,c+2\,d\,x)}{2}+\frac{19\cos(3\,c+3\,d\,x)}{8}+2\,\cos\left(4\,c+4\,d\,x\right)-\frac{\cos(5\,c+5\,d\,x)}{8}-\frac{\cos(6\,c+6\,d\,x)}{2}-\frac{\cos(7\,c+7\,d\,x)}{8}+15}{40320\,a^{2}\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^6*(a + a/cos(c + d*x))^2),x)
```

```
[Out] -((375*cos(c + d*x))/8 - (5*cos(2*c + 2*d*x))/2 + (19*cos(3*c + 3*d*x))/8 +
 2*cos(4*c + 4*d*x) - cos(5*c + 5*d*x)/8 - cos(6*c + 6*d*x)/2 - cos(7*c + 7
*d*x)/8 + 15)/(40320*a^2*d*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^6(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**6/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

3.90 $\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=125

$$-\frac{2\cot^{11}(c+dx)}{11a^2d}-\frac{7\cot^9(c+dx)}{9a^2d}-\frac{9\cot^7(c+dx)}{7a^2d}-\frac{\cot^5(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{2\csc^{11}(c+dx)}{11a^2d}-\frac{2\csc^9(c+dx)}{9a^2d}$$

```
[Out] -1/3*cot(d*x+c)^3/a^2/d-cot(d*x+c)^5/a^2/d-9/7*cot(d*x+c)^7/a^2/d-7/9*cot(d
*x+c)^9/a^2/d-2/11*cot(d*x+c)^11/a^2/d-2/9*csc(d*x+c)^9/a^2/d+2/11*csc(d*x+
c)^11/a^2/d
```

Rubi [A] time = 0.37, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2875, 2873, 2607, 270, 2606, 14}

$$-\frac{2\cot^{11}(c+dx)}{11a^2d}-\frac{7\cot^9(c+dx)}{9a^2d}-\frac{9\cot^7(c+dx)}{7a^2d}-\frac{\cot^5(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{2\csc^{11}(c+dx)}{11a^2d}-\frac{2\csc^9(c+dx)}{9a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -Cot[c + d*x]^3/(3*a^2*d) - Cot[c + d*x]^5/(a^2*d) - (9*Cot[c + d*x]^7)/(7*
a^2*d) - (7*Cot[c + d*x]^9)/(9*a^2*d) - (2*Cot[c + d*x]^11)/(11*a^2*d) - (2
*Csc[c + d*x]^9)/(9*a^2*d) + (2*Csc[c + d*x]^11)/(11*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)\csc^6(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{\int (-a+a\cos(c+dx))^2\cot^2(c+dx)\csc^{10}(c+dx)\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cot^4(c+dx)\csc^8(c+dx) - 2a^2\cot^3(c+dx)\csc^9(c+dx) + a^2\cot^2(c+dx)\,c\right.}{a^4}$$
$$= \frac{\int \cot^4(c+dx)\csc^8(c+dx)\,dx}{a^2} + \frac{\int \cot^2(c+dx)\csc^{10}(c+dx)\,dx}{a^2} - \frac{2\int \cot^3(c+a}{}$$
$$= \frac{\mathrm{Subst}\left(\int x^4\left(1+x^2\right)^3\,dx, x, -\cot(c+dx)\right)}{a^2 d} + \frac{\mathrm{Subst}\left(\int x^2\left(1+x^2\right)^4\,dx, x, -\cot(c\right.}{a^2 d}$$
$$= \frac{\mathrm{Subst}\left(\int \left(x^4+3x^6+3x^8+x^{10}\right)dx, x, -\cot(c+dx)\right)}{a^2 d} + \frac{\mathrm{Subst}\left(\int \left(x^2+4x^4+6x\right.\right.}{}$$
$$= -\frac{\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+dx)}{a^2d} - \frac{9\cot^7(c+dx)}{7a^2d} - \frac{7\cot^9(c+dx)}{9a^2d} - \frac{2\cot^{11}(c+dx)}{11a^2d}$$

Mathematica [A] time = 1.50, size = 233, normalized size = 1.86

$$-\frac{\csc(c)(-218834\sin(c+dx) - 79576\sin(2(c+dx)) + 119364\sin(3(c+dx)) + 79576\sin(4(c+dx)) - 28420\,s}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/22708224*(Csc[c]*Csc[c + d*x]^7*Sec[c + d*x]^2*(630784*Sin[c] - 1103872*
Sin[d*x] - 218834*Sin[c + d*x] - 79576*Sin[2*(c + d*x)] + 119364*Sin[3*(c +
 d*x)] + 79576*Sin[4*(c + d*x)] - 28420*Sin[5*(c + d*x)] - 34104*Sin[6*(c +
 d*x)] - 1421*Sin[7*(c + d*x)] + 5684*Sin[8*(c + d*x)] + 1421*Sin[9*(c + d*
x)] + 1419264*Sin[2*c + d*x] + 114688*Sin[c + 2*d*x] - 172032*Sin[2*c + 3*d
*x] - 114688*Sin[3*c + 4*d*x] + 40960*Sin[4*c + 5*d*x] + 49152*Sin[5*c + 6*
d*x] + 2048*Sin[6*c + 7*d*x] - 8192*Sin[7*c + 8*d*x] - 2048*Sin[8*c + 9*d*x
]))/(a^2*d*(1 + Sec[c + d*x])^2)
```

fricas [A] time = 1.54, size = 204, normalized size = 1.63

$$\frac{16\cos(dx+c)^9 + 32\cos(dx+c)^8 - 40\cos(dx+c)^7 - 112\cos(dx+c)^6 + 14\cos(dx+c)^5 + 140\cos(dx+}{693\left(a^2d\cos(dx+c)^8 + 2a^2d\cos(dx+c)^7 - 2a^2d\cos(dx+c)^6 - 6a^2d\cos(dx+c)^5 + 6a^2d\cos(dx+c)^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/693*(16*cos(d*x + c)^9 + 32*cos(d*x + c)^8 - 40*cos(d*x + c)^7 - 112*cos(
d*x + c)^6 + 14*cos(d*x + c)^5 + 140*cos(d*x + c)^4 + 35*cos(d*x + c)^3 - 7
0*cos(d*x + c)^2 + 56*cos(d*x + c) + 28)/((a^2*d*cos(d*x + c)^8 + 2*a^2*d*c
os(d*x + c)^7 - 2*a^2*d*cos(d*x + c)^6 - 6*a^2*d*cos(d*x + c)^5 + 6*a^2*d*c
os(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 - 2*a^2*d*cos(d*x + c) - a^2*d)*sin(
d*x + c))
```

giac [A] time = 0.35, size = 134, normalized size = 1.07

$$-\frac{\frac{33\left(56\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+3\right)}{a^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}}-\frac{63a^{20}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+385a^{20}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+792a^{20}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-3234a^{20}\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{a^{22}}}{354816d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/354816*(33*(56*tan(1/2*d*x + 1/2*c)^4 + 21*tan(1/2*d*x + 1/2*c)^2 + 3)/(
a^2*tan(1/2*d*x + 1/2*c)^7) - (63*a^20*tan(1/2*d*x + 1/2*c)^11 + 385*a^20*t
an(1/2*d*x + 1/2*c)^9 + 792*a^20*tan(1/2*d*x + 1/2*c)^7 - 3234*a^20*tan(1/2
*d*x + 1/2*c)^3 - 9702*a^20*tan(1/2*d*x + 1/2*c))/a^22)/d
```

maple [A] time = 0.84, size = 112, normalized size = 0.90

$$\frac{\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}+\frac{5\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{8\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}-\frac{14\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}-\frac{8}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{1}{\tan\left(\frac{dx}{2}\right.}}{512d\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/512/d/a^2*(1/11*tan(1/2*d*x+1/2*c)^11+5/9*tan(1/2*d*x+1/2*c)^9+8/7*tan(1/
2*d*x+1/2*c)^7-14/3*tan(1/2*d*x+1/2*c)^3-14*tan(1/2*d*x+1/2*c)-1/7/tan(1/2*
d*x+1/2*c)^7-8/3/tan(1/2*d*x+1/2*c)^3-1/tan(1/2*d*x+1/2*c)^5)
```

maxima [A] time = 0.43, size = 174, normalized size = 1.39

$$-\frac{\frac{\frac{9702\sin(dx+c)}{\cos(dx+c)+1}+\frac{3234\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{792\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}-\frac{385\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}-\frac{63\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^{2}}+\frac{33\left(\frac{21\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{56\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+3\right)(\cos(dx+c)+1)^{7}}{a^{2}\sin(dx+c)^{7}}}{354816d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/354816*((9702*sin(d*x + c)/(cos(d*x + c) + 1) + 3234*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 - 792*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 385*sin(d*x +
c)^9/(cos(d*x + c) + 1)^9 - 63*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/a^2 +
 33*(21*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 56*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + 3)*(cos(d*x + c) + 1)^7/(a^2*sin(d*x + c)^7))/d
```

mupad [B] time = 1.69, size = 201, normalized size = 1.61

$$-\frac{99\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}+693\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+1848\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}+9702\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{354}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^8*(a + a/cos(c + d*x))^2),x)
```

```
[Out] -(99*cos(c/2 + (d*x)/2)^18 - 63*sin(c/2 + (d*x)/2)^18 - 385*cos(c/2 + (d*x)
/2)^2*sin(c/2 + (d*x)/2)^16 - 792*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^1
4 + 3234*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^10 + 9702*cos(c/2 + (d*x)/
2)^10*sin(c/2 + (d*x)/2)^8 + 1848*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^
4 + 693*cos(c/2 + (d*x)/2)^16*sin(c/2 + (d*x)/2)^2)/(354816*a^2*d*cos(c/2 +
 (d*x)/2)^11*sin(c/2 + (d*x)/2)^7)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.91 $\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=139

$$-\frac{(a-a\cos(c+dx))^{11}}{11a^{14}d}+\frac{7(a-a\cos(c+dx))^{10}}{10a^{13}d}-\frac{19(a-a\cos(c+dx))^9}{9a^{12}d}+\frac{25(a-a\cos(c+dx))^8}{8a^{11}d}-\frac{16(a-a\cos(c+}{7a^{10}d}$$

```
[Out] 2/3*(a-a*cos(d*x+c))^6/a^9/d-16/7*(a-a*cos(d*x+c))^7/a^10/d+25/8*(a-a*cos(d
*x+c))^8/a^11/d-19/9*(a-a*cos(d*x+c))^9/a^12/d+7/10*(a-a*cos(d*x+c))^10/a^1
3/d-1/11*(a-a*cos(d*x+c))^11/a^14/d
```

Rubi [A] time = 0.19, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2836, 12, 88}

$$-\frac{(a-a\cos(c+dx))^{11}}{11a^{14}d}+\frac{7(a-a\cos(c+dx))^{10}}{10a^{13}d}-\frac{19(a-a\cos(c+dx))^9}{9a^{12}d}+\frac{25(a-a\cos(c+dx))^8}{8a^{11}d}-\frac{16(a-a\cos(c+}{7a^{10}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^11/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (2*(a - a*Cos[c + d*x])^6)/(3*a^9*d) - (16*(a - a*Cos[c + d*x])^7)/(7*a^10*
d) + (25*(a - a*Cos[c + d*x])^8)/(8*a^11*d) - (19*(a - a*Cos[c + d*x])^9)/(
9*a^12*d) + (7*(a - a*Cos[c + d*x])^10)/(10*a^13*d) - (a - a*Cos[c + d*x])^
11/(11*a^14*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^{11}(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^5x^3(-a+x)^2}{a^3}\,dx, x, -a\cos(c+dx)\right)}{a^{11}d}$$
$$= \frac{\text{Subst}\left(\int (-a-x)^5x^3(-a+x)^2\,dx, x, -a\cos(c+dx)\right)}{a^{14}d}$$
$$= \frac{\text{Subst}\left(\int \left(-4a^5(-a-x)^5 - 16a^4(-a-x)^6 - 25a^3(-a-x)^7 - 19a^2(-a-x)^8 - 7a(-\right.\right.}{a^{14}d}$$
$$= \frac{2(a-a\cos(c+dx))^6}{3a^9d} - \frac{16(a-a\cos(c+dx))^7}{7a^{10}d} + \frac{25(a-a\cos(c+dx))^8}{8a^{11}d} - \frac{19(a-}{}$$

Mathematica [A] time = 4.64, size = 120, normalized size = 0.86

$$\frac{2273040\cos(c+dx) - 1496880\cos(2(c+dx)) + 535920\cos(3(c+dx)) + 110880\cos(4(c+dx)) - 293832\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^11/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-1615571 + 2273040*Cos[c + d*x] - 1496880*Cos[2*(c + d*x)] + 535920*Cos[3*
(c + d*x)] + 110880*Cos[4*(c + d*x)] - 293832*Cos[5*(c + d*x)] + 212520*Cos
[6*(c + d*x)] - 67320*Cos[7*(c + d*x)] - 27720*Cos[8*(c + d*x)] + 40040*Cos
[9*(c + d*x)] - 16632*Cos[10*(c + d*x)] + 2520*Cos[11*(c + d*x)])/(28385280
*a^3*d)
```

fricas [A] time = 0.74, size = 89, normalized size = 0.64

$$\frac{2520\cos(dx+c)^{11} - 8316\cos(dx+c)^{10} + 3080\cos(dx+c)^9 + 17325\cos(dx+c)^8 - 19800\cos(dx+c)^7 -}{27720\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/27720*(2520*cos(d*x + c)^11 - 8316*cos(d*x + c)^10 + 3080*cos(d*x + c)^9
+ 17325*cos(d*x + c)^8 - 19800*cos(d*x + c)^7 - 4620*cos(d*x + c)^6 + 16632
*cos(d*x + c)^5 - 6930*cos(d*x + c)^4)/(a^3*d)
```

giac [A] time = 0.41, size = 207, normalized size = 1.49

$$\frac{32\left(\frac{209(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{1045(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{3135(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{6270(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} + \frac{8778(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5} - \frac{13398(\cos(dx+c)-}{(\cos(dx+c)+1)}\right.}{3465\,a^3d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 32/3465*(209*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1045*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 + 3135*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 -
 6270*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 8778*(cos(d*x + c) - 1)^5
/(cos(d*x + c) + 1)^5 - 13398*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 2
310*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 9240*(cos(d*x + c) - 1)^8/(
cos(d*x + c) + 1)^8 - 19)/(a^3*d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1
)^11)
```

maple [A] time = 0.89, size = 90, normalized size = 0.65

$$-\frac{\frac{1}{6\sec(dx+c)^{6}}+\frac{3}{10\sec(dx+c)^{10}}-\frac{1}{11\sec(dx+c)^{11}}+\frac{5}{7\sec(dx+c)^{7}}-\frac{1}{9\sec(dx+c)^{9}}+\frac{1}{4\sec(dx+c)^{4}}-\frac{5}{8\sec(dx+c)^{8}}-\frac{3}{5\sec(dx+c)^{5}}}{d\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^11/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/d/a^3*(1/6/sec(d*x+c)^6+3/10/sec(d*x+c)^10-1/11/sec(d*x+c)^11+5/7/sec(d*
x+c)^7-1/9/sec(d*x+c)^9+1/4/sec(d*x+c)^4-5/8/sec(d*x+c)^8-3/5/sec(d*x+c)^5)
```

maxima [A] time = 0.53, size = 89, normalized size = 0.64

$$\frac{2520\cos\left(dx+c\right)^{11}-8316\cos\left(dx+c\right)^{10}+3080\cos\left(dx+c\right)^{9}+17325\cos\left(dx+c\right)^{8}-19800\cos\left(dx+c\right)^{7}-46}{27720\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/27720*(2520*cos(d*x + c)^11 - 8316*cos(d*x + c)^10 + 3080*cos(d*x + c)^9
+ 17325*cos(d*x + c)^8 - 19800*cos(d*x + c)^7 - 4620*cos(d*x + c)^6 + 16632
*cos(d*x + c)^5 - 6930*cos(d*x + c)^4)/(a^3*d)
```

mupad [B] time = 0.09, size = 110, normalized size = 0.79

$$-\frac{\frac{\cos(c+d\,x)^{4}}{4\,a^{3}}-\frac{3\cos(c+d\,x)^{5}}{5\,a^{3}}+\frac{\cos(c+d\,x)^{6}}{6\,a^{3}}+\frac{5\cos(c+d\,x)^{7}}{7\,a^{3}}-\frac{5\cos(c+d\,x)^{8}}{8\,a^{3}}-\frac{\cos(c+d\,x)^{9}}{9\,a^{3}}+\frac{3\cos(c+d\,x)^{10}}{10\,a^{3}}-\frac{\cos(c+d\,x)^{11}}{11\,a^{3}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^11/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(cos(c + d*x)^4/(4*a^3) - (3*cos(c + d*x)^5)/(5*a^3) + cos(c + d*x)^6/(6*a
^3) + (5*cos(c + d*x)^7)/(7*a^3) - (5*cos(c + d*x)^8)/(8*a^3) - cos(c + d*x
)^9/(9*a^3) + (3*cos(c + d*x)^10)/(10*a^3) - cos(c + d*x)^11/(11*a^3))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**11/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.92 $\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=109

$$-\frac{\cos^9(c+dx)}{9a^3d}+\frac{3\cos^8(c+dx)}{8a^3d}-\frac{2\cos^7(c+dx)}{7a^3d}-\frac{\cos^6(c+dx)}{3a^3d}+\frac{3\cos^5(c+dx)}{5a^3d}-\frac{\cos^4(c+dx)}{4a^3d}$$

```
[Out] -1/4*cos(d*x+c)^4/a^3/d+3/5*cos(d*x+c)^5/a^3/d-1/3*cos(d*x+c)^6/a^3/d-2/7*c
os(d*x+c)^7/a^3/d+3/8*cos(d*x+c)^8/a^3/d-1/9*cos(d*x+c)^9/a^3/d
```

Rubi [A] time = 0.18, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2836, 12, 75}

$$-\frac{\cos^9(c+dx)}{9a^3d}+\frac{3\cos^8(c+dx)}{8a^3d}-\frac{2\cos^7(c+dx)}{7a^3d}-\frac{\cos^6(c+dx)}{3a^3d}+\frac{3\cos^5(c+dx)}{5a^3d}-\frac{\cos^4(c+dx)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^9/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -Cos[c + d*x]^4/(4*a^3*d) + (3*Cos[c + d*x]^5)/(5*a^3*d) - Cos[c + d*x]^6/(
3*a^3*d) - (2*Cos[c + d*x]^7)/(7*a^3*d) + (3*Cos[c + d*x]^8)/(8*a^3*d) - Co
s[c + d*x]^9/(9*a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^9(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^4x^3(-a+x)}{a^3}\,dx, x, -a\cos(c+dx)\right)}{a^9d}$$
$$= \frac{\text{Subst}\left(\int (-a-x)^4x^3(-a+x)\,dx, x, -a\cos(c+dx)\right)}{a^{12}d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^5x^3-3a^4x^4-2a^3x^5+2a^2x^6+3ax^7+x^8\right)dx, x, -a\cos(c+dx)\right)}{a^{12}d}$$
$$= -\frac{\cos^4(c+dx)}{4a^3d}+\frac{3\cos^5(c+dx)}{5a^3d}-\frac{\cos^6(c+dx)}{3a^3d}-\frac{2\cos^7(c+dx)}{7a^3d}+\frac{3\cos^8(c+dx)}{8a^3d}-$$

Mathematica [A] time = 3.10, size = 100, normalized size = 0.92

$$-\frac{-52920\cos(c+dx)+37800\cos(2(c+dx))-18480\cos(3(c+dx))+3780\cos(4(c+dx))+3024\cos(5(c+dx)}{322560a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^9/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/322560*(34771 - 52920*Cos[c + d*x] + 37800*Cos[2*(c + d*x)] - 18480*Cos[
3*(c + d*x)] + 3780*Cos[4*(c + d*x)] + 3024*Cos[5*(c + d*x)] - 4200*Cos[6*(
c + d*x)] + 2700*Cos[7*(c + d*x)] - 945*Cos[8*(c + d*x)] + 140*Cos[9*(c + d
*x)])/(a^3*d)
```

fricas [A] time = 1.30, size = 69, normalized size = 0.63

$$-\frac{280\cos(dx+c)^9-945\cos(dx+c)^8+720\cos(dx+c)^7+840\cos(dx+c)^6-1512\cos(dx+c)^5+630\cos(}{2520\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2520*(280*cos(d*x + c)^9 - 945*cos(d*x + c)^8 + 720*cos(d*x + c)^7 + 840
*cos(d*x + c)^6 - 1512*cos(d*x + c)^5 + 630*cos(d*x + c)^4)/(a^3*d)
```

giac [A] time = 0.44, size = 185, normalized size = 1.70

$$\frac{32\left(\frac{36(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{144(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{336(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{504(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{630(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}-\frac{105(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}+\frac{315}{(c}\right.}{315\,a^3d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 32/315*(36*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 144*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 336*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 504
*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 630*(cos(d*x + c) - 1)^5/(cos(
d*x + c) + 1)^5 - 105*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 315*(cos(
d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 4)/(a^3*d*((cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - 1)^9)
```

maple [A] time = 0.72, size = 69, normalized size = 0.63

$$\frac{-\frac{1}{3\sec(dx+c)^6}-\frac{2}{7\sec(dx+c)^7}-\frac{1}{9\sec(dx+c)^9}-\frac{1}{4\sec(dx+c)^4}+\frac{3}{8\sec(dx+c)^8}+\frac{3}{5\sec(dx+c)^5}}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^9/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*(-1/3/sec(d*x+c)^6-2/7/sec(d*x+c)^7-1/9/sec(d*x+c)^9-1/4/sec(d*x+c)
^4+3/8/sec(d*x+c)^8+3/5/sec(d*x+c)^5)
```

maxima [A] time = 0.51, size = 69, normalized size = 0.63

$$-\frac{280 \cos(dx+c)^9 - 945 \cos(dx+c)^8 + 720 \cos(dx+c)^7 + 840 \cos(dx+c)^6 - 1512 \cos(dx+c)^5 + 630 \cos(dx+c)^4}{2520\, a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2520*(280*cos(d*x + c)^9 - 945*cos(d*x + c)^8 + 720*cos(d*x + c)^7 + 840
*cos(d*x + c)^6 - 1512*cos(d*x + c)^5 + 630*cos(d*x + c)^4)/(a^3*d)
```

mupad [B] time = 0.92, size = 84, normalized size = 0.77

$$-\frac{\frac{\cos(c+d\,x)^4}{4\,a^3}-\frac{3\cos(c+d\,x)^5}{5\,a^3}+\frac{\cos(c+d\,x)^6}{3\,a^3}+\frac{2\cos(c+d\,x)^7}{7\,a^3}-\frac{3\cos(c+d\,x)^8}{8\,a^3}+\frac{\cos(c+d\,x)^9}{9\,a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^9/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(cos(c + d*x)^4/(4*a^3) - (3*cos(c + d*x)^5)/(5*a^3) + cos(c + d*x)^6/(3*a
^3) + (2*cos(c + d*x)^7)/(7*a^3) - (3*cos(c + d*x)^8)/(8*a^3) + cos(c + d*x
)^9/(9*a^3))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**9/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.93 $\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3} dx$

Optimal. Leaf size=73

$$\frac{\cos^7(c+dx)}{7a^3d} - \frac{\cos^6(c+dx)}{2a^3d} + \frac{3\cos^5(c+dx)}{5a^3d} - \frac{\cos^4(c+dx)}{4a^3d}$$

```
[Out] -1/4*cos(d*x+c)^4/a^3/d+3/5*cos(d*x+c)^5/a^3/d-1/2*cos(d*x+c)^6/a^3/d+1/7*c
os(d*x+c)^7/a^3/d
```

Rubi [A] time = 0.16, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3872, 2836, 12, 43}

$$\frac{\cos^7(c+dx)}{7a^3d} - \frac{\cos^6(c+dx)}{2a^3d} + \frac{3\cos^5(c+dx)}{5a^3d} - \frac{\cos^4(c+dx)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -Cos[c + d*x]^4/(4*a^3*d) + (3*Cos[c + d*x]^5)/(5*a^3*d) - Cos[c + d*x]^6/(
2*a^3*d) + Cos[c + d*x]^7/(7*a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^7(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^3x^3}{a^3}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int (-a-x)^3x^3\,dx, x, -a\cos(c+dx)\right)}{a^{10} d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^3x^3-3a^2x^4-3ax^5-x^6\right)\,dx, x, -a\cos(c+dx)\right)}{a^{10} d}$$
$$= -\frac{\cos^4(c+dx)}{4a^3d}+\frac{3\cos^5(c+dx)}{5a^3d}-\frac{\cos^6(c+dx)}{2a^3d}+\frac{\cos^7(c+dx)}{7a^3d}$$

Mathematica [A] time = 1.76, size = 80, normalized size = 1.10

$$\frac{4060\cos(c+dx)-3220\cos(2(c+dx))+2100\cos(3(c+dx))-1120\cos(4(c+dx))+476\cos(5(c+dx))-14}{8960a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-2421 + 4060*Cos[c + d*x] - 3220*Cos[2*(c + d*x)] + 2100*Cos[3*(c + d*x)]
- 1120*Cos[4*(c + d*x)] + 476*Cos[5*(c + d*x)] - 140*Cos[6*(c + d*x)] + 20*
Cos[7*(c + d*x)])/(8960*a^3*d)
```

fricas [A] time = 0.67, size = 49, normalized size = 0.67

$$\frac{20\cos(dx+c)^7-70\cos(dx+c)^6+84\cos(dx+c)^5-35\cos(dx+c)^4}{140\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/140*(20*cos(d*x + c)^7 - 70*cos(d*x + c)^6 + 84*cos(d*x + c)^5 - 35*cos(d
*x + c)^4)/(a^3*d)
```

giac [B] time = 1.27, size = 163, normalized size = 2.23

$$\frac{4\left(\frac{91(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{273(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{455(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{490(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}+\frac{210(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}-\frac{140(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6}-13\right.}{35\,a^3d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 4/35*(91*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 273*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + 455*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 490*(
cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 210*(cos(d*x + c) - 1)^5/(cos(d*
x + c) + 1)^5 - 140*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 13)/(a^3*d*
((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7)
```

maple [A] time = 0.64, size = 50, normalized size = 0.68

$$-\frac{\frac{1}{2\sec(dx+c)^6}-\frac{1}{7\sec(dx+c)^7}+\frac{1}{4\sec(dx+c)^4}-\frac{3}{5\sec(dx+c)^5}}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/d/a^3*(1/2/sec(d*x+c)^6-1/7/sec(d*x+c)^7+1/4/sec(d*x+c)^4-3/5/sec(d*x+c)
^5)
```

maxima [A] time = 0.32, size = 49, normalized size = 0.67

$$\frac{20 \cos(dx+c)^7 - 70 \cos(dx+c)^6 + 84 \cos(dx+c)^5 - 35 \cos(dx+c)^4}{140\, a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/140*(20*cos(d*x + c)^7 - 70*cos(d*x + c)^6 + 84*cos(d*x + c)^5 - 35*cos(d
*x + c)^4)/(a^3*d)
```

mupad [B] time = 0.07, size = 58, normalized size = 0.79

$$-\frac{\frac{\cos(c+d\,x)^4}{4\,a^3} - \frac{3\cos(c+d\,x)^5}{5\,a^3} + \frac{\cos(c+d\,x)^6}{2\,a^3} - \frac{\cos(c+d\,x)^7}{7\,a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(cos(c + d*x)^4/(4*a^3) - (3*cos(c + d*x)^5)/(5*a^3) + cos(c + d*x)^6/(2*a
^3) - cos(c + d*x)^7/(7*a^3))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.94 $\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=102

$$-\frac{\cos^5(c+dx)}{5a^3d}+\frac{3\cos^4(c+dx)}{4a^3d}-\frac{4\cos^3(c+dx)}{3a^3d}+\frac{2\cos^2(c+dx)}{a^3d}-\frac{4\cos(c+dx)}{a^3d}+\frac{4\log(\cos(c+dx)+1)}{a^3d}$$

```
[Out] -4*cos(d*x+c)/a^3/d+2*cos(d*x+c)^2/a^3/d-4/3*cos(d*x+c)^3/a^3/d+3/4*cos(d*x
+c)^4/a^3/d-1/5*cos(d*x+c)^5/a^3/d+4*ln(1+cos(d*x+c))/a^3/d
```

Rubi [A] time = 0.18, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2836, 12, 88}

$$-\frac{\cos^5(c+dx)}{5a^3d}+\frac{3\cos^4(c+dx)}{4a^3d}-\frac{4\cos^3(c+dx)}{3a^3d}+\frac{2\cos^2(c+dx)}{a^3d}-\frac{4\cos(c+dx)}{a^3d}+\frac{4\log(\cos(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-4*Cos[c + d*x])/(a^3*d) + (2*Cos[c + d*x]^2)/(a^3*d) - (4*Cos[c + d*x]^3)
/(3*a^3*d) + (3*Cos[c + d*x]^4)/(4*a^3*d) - Cos[c + d*x]^5/(5*a^3*d) + (4*L
og[1 + Cos[c + d*x]])/(a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^5(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^2x^3}{a^3(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^2x^3}{-a+x}\,dx, x, -a\cos(c+dx)\right)}{a^8 d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^4 - \frac{4a^5}{a-x} + 4a^3x + 4a^2x^2 + 3ax^3 + x^4\right)dx, x, -a\cos(c+dx)\right)}{a^8 d}$$
$$= -\frac{4\cos(c+dx)}{a^3d} + \frac{2\cos^2(c+dx)}{a^3d} - \frac{4\cos^3(c+dx)}{3a^3d} + \frac{3\cos^4(c+dx)}{4a^3d} - \frac{\cos^5(c+dx)}{5a^3d}$$

Mathematica [A] time = 0.96, size = 73, normalized size = 0.72

$$\frac{-4920\cos(c+dx)+1320\cos(2(c+dx))-380\cos(3(c+dx))+90\cos(4(c+dx))-12\cos(5(c+dx))+7680\log}{960a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3857 - 4920*Cos[c + d*x] + 1320*Cos[2*(c + d*x)] - 380*Cos[3*(c + d*x)] +
90*Cos[4*(c + d*x)] - 12*Cos[5*(c + d*x)] + 7680*Log[Cos[(c + d*x)/2]])/(96
0*a^3*d)
```

fricas [A] time = 0.82, size = 70, normalized size = 0.69

$$-\frac{12\cos(dx+c)^5 - 45\cos(dx+c)^4 + 80\cos(dx+c)^3 - 120\cos(dx+c)^2 + 240\cos(dx+c) - 240\log\left(\frac{1}{2}\cos\right.}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(12*cos(d*x + c)^5 - 45*cos(d*x + c)^4 + 80*cos(d*x + c)^3 - 120*cos(
d*x + c)^2 + 240*cos(d*x + c) - 240*log(1/2*cos(d*x + c) + 1/2))/(a^3*d)
```

giac [A] time = 0.42, size = 172, normalized size = 1.69

$$-\frac{\frac{60\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3} + \frac{\frac{85(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{20(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{200(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} + \frac{205(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} - \frac{137(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5} - 29}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/15*(60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (85*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) - 20*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 - 200*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 205*(cos(d*x + c)
 - 1)^4/(cos(d*x + c) + 1)^4 - 137*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^
5 - 29)/(a^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5))/d
```

maple [A] time = 0.72, size = 114, normalized size = 1.12

$$-\frac{1}{5d\,a^3\sec(dx+c)^5} + \frac{3}{4d\,a^3\sec(dx+c)^4} - \frac{4}{3d\,a^3\sec(dx+c)^3} + \frac{2}{d\,a^3\sec(dx+c)^2} - \frac{4}{d\,a^3\sec(dx+c)} - \frac{4\ln(\sec(dx}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/5/d/a^3/sec(d*x+c)^5+3/4/d/a^3/sec(d*x+c)^4-4/3/d/a^3/sec(d*x+c)^3+2/d/a
^3/sec(d*x+c)^2-4/d/a^3/sec(d*x+c)-4/d/a^3*ln(sec(d*x+c))+4/d/a^3*ln(1+sec(
d*x+c))
```

maxima [A] time = 0.33, size = 73, normalized size = 0.72

$$-\frac{\frac{12 \cos\left(dx+c\right)^{5}-45 \cos\left(dx+c\right)^{4}+80 \cos\left(dx+c\right)^{3}-120 \cos\left(dx+c\right)^{2}+240 \cos\left(dx+c\right)}{a^{3}}-\frac{240 \log\left(\cos\left(dx+c\right)+1\right)}{a^{3}}}{60 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*((12*cos(d*x + c)^5 - 45*cos(d*x + c)^4 + 80*cos(d*x + c)^3 - 120*cos
(d*x + c)^2 + 240*cos(d*x + c))/a^3 - 240*log(cos(d*x + c) + 1)/a^3)/d
```

mupad [B] time = 0.90, size = 82, normalized size = 0.80

$$\frac{\frac{4 \ln\left(\cos\left(c+d\,x\right)+1\right)}{a^3}-\frac{4 \cos\left(c+d\,x\right)}{a^3}+\frac{2 \cos\left(c+d\,x\right)^2}{a^3}-\frac{4 \cos\left(c+d\,x\right)^3}{3\,a^3}+\frac{3 \cos\left(c+d\,x\right)^4}{4\,a^3}-\frac{\cos\left(c+d\,x\right)^5}{5\,a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((4*log(cos(c + d*x) + 1))/a^3 - (4*cos(c + d*x))/a^3 + (2*cos(c + d*x)^2)/
a^3 - (4*cos(c + d*x)^3)/(3*a^3) + (3*cos(c + d*x)^4)/(4*a^3) - cos(c + d*x
)^5/(5*a^3))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.95 $\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=89

$$\frac{\cos^3(c+dx)}{3a^3d}-\frac{3\cos^2(c+dx)}{2a^3d}+\frac{5\cos(c+dx)}{a^3d}-\frac{2}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{7\log(\cos(c+dx)+1)}{a^3d}$$

```
[Out] 5*cos(d*x+c)/a^3/d-3/2*cos(d*x+c)^2/a^3/d+1/3*cos(d*x+c)^3/a^3/d-2/d/(a^3+a
^3*cos(d*x+c))-7*ln(1+cos(d*x+c))/a^3/d
```

Rubi [A] time = 0.18, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3872, 2836, 12, 77}

$$\frac{\cos^3(c+dx)}{3a^3d}-\frac{3\cos^2(c+dx)}{2a^3d}+\frac{5\cos(c+dx)}{a^3d}-\frac{2}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{7\log(\cos(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (5*Cos[c + d*x])/(a^3*d) - (3*Cos[c + d*x]^2)/(2*a^3*d) + Cos[c + d*x]^3/(3
*a^3*d) - 2/(d*(a^3 + a^3*Cos[c + d*x])) - (7*Log[1 + Cos[c + d*x]])/(a^3*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^3(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)x^3}{a^3(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)x^3}{(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^6d}$$
$$= \frac{\text{Subst}\left(\int \left(-5a^2 - \frac{2a^4}{(a-x)^2} + \frac{7a^3}{a-x} - 3ax - x^2\right)dx, x, -a\cos(c+dx)\right)}{a^6d}$$
$$= \frac{5\cos(c+dx)}{a^3d} - \frac{3\cos^2(c+dx)}{2a^3d} + \frac{\cos^3(c+dx)}{3a^3d} - \frac{2}{d\left(a^3+a^3\cos(c+dx)\right)} - \frac{7\log(}{}$$

Mathematica [A] time = 0.44, size = 99, normalized size = 1.11

$$-\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-184\cos(2(c+dx)) + 28\cos(3(c+dx)) - 4\cos(4(c+dx)) + 1344\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + \right.}{24a^3d(\cos(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/24*(Cos[(c + d*x)/2]^4*(389 - 184*Cos[2*(c + d*x)] + 28*Cos[3*(c + d*x)]
 - 4*Cos[4*(c + d*x)] + 1344*Log[Cos[(c + d*x)/2]] + Cos[c + d*x]*(-19 + 13
44*Log[Cos[(c + d*x)/2]])))/(a^3*d*(1 + Cos[c + d*x])^3)
```

fricas [A] time = 0.80, size = 82, normalized size = 0.92

$$\frac{4\cos(dx+c)^4 - 14\cos(dx+c)^3 + 42\cos(dx+c)^2 - 84(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) + 69\cos}{12\left(a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*cos(d*x + c)^4 - 14*cos(d*x + c)^3 + 42*cos(d*x + c)^2 - 84*(cos(d*
x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 69*cos(d*x + c) - 15)/(a^3*d*cos(
d*x + c) + a^3*d)
```

giac [A] time = 0.64, size = 94, normalized size = 1.06

$$-\frac{7\log(|-\cos(dx+c)-1|)}{a^3d} - \frac{2}{a^3d(\cos(dx+c)+1)} + \frac{2a^6d^5\cos(dx+c)^3 - 9a^6d^5\cos(dx+c)^2 + 30a^6d^5\cos(}{6a^9d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -7*log(abs(-cos(d*x + c) - 1))/(a^3*d) - 2/(a^3*d*(cos(d*x + c) + 1)) + 1/6
*(2*a^6*d^5*cos(d*x + c)^3 - 9*a^6*d^5*cos(d*x + c)^2 + 30*a^6*d^5*cos(d*x
+ c))/(a^9*d^6)
```

maple [A] time = 0.73, size = 100, normalized size = 1.12

$$\frac{1}{3d\,a^3\sec(dx+c)^3} - \frac{3}{2d\,a^3\sec(dx+c)^2} + \frac{5}{d\,a^3\sec(dx+c)} + \frac{7\ln(\sec(dx+c))}{d\,a^3} + \frac{2}{d\,a^3(1+\sec(dx+c))} - \frac{7\ln(1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/3/d/a^3/sec(d*x+c)^3-3/2/d/a^3/sec(d*x+c)^2+5/d/a^3/sec(d*x+c)+7/d/a^3*ln
(sec(d*x+c))+2/d/a^3/(1+sec(d*x+c))-7/d/a^3*ln(1+sec(d*x+c))
```

maxima [A] time = 0.34, size = 72, normalized size = 0.81

$$-\frac{\frac{12}{a^3\cos(dx+c)+a^3}-\frac{2\cos(dx+c)^3-9\cos(dx+c)^2+30\cos(dx+c)}{a^3}+\frac{42\log(\cos(dx+c)+1)}{a^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/6*(12/(a^3*cos(d*x + c) + a^3) - (2*cos(d*x + c)^3 - 9*cos(d*x + c)^2 +
30*cos(d*x + c))/a^3 + 42*log(cos(d*x + c) + 1)/a^3)/d
```

mupad [B] time = 0.89, size = 75, normalized size = 0.84

$$-\frac{\frac{2}{a^3\cos(c+dx)+a^3}+\frac{7\ln(\cos(c+dx)+1)}{a^3}-\frac{5\cos(c+dx)}{a^3}+\frac{3\cos(c+dx)^2}{2a^3}-\frac{\cos(c+dx)^3}{3a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(2/(a^3*cos(c + d*x) + a^3) + (7*log(cos(c + d*x) + 1))/a^3 - (5*cos(c + d
*x))/a^3 + (3*cos(c + d*x)^2)/(2*a^3) - cos(c + d*x)^3/(3*a^3))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.96 $\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=75

$$-\frac{\cos(c+dx)}{a^3d}+\frac{3}{d\left(a^3\cos(c+dx)+a^3\right)}+\frac{3\log(\cos(c+dx)+1)}{a^3d}-\frac{1}{2ad(a\cos(c+dx)+a)^2}$$

```
[Out] -cos(d*x+c)/a^3/d-1/2/a/d/(a+a*cos(d*x+c))^2+3/d/(a^3+a^3*cos(d*x+c))+3*ln(
1+cos(d*x+c))/a^3/d
```

Rubi [A] time = 0.12, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 2833, 12, 43}

$$-\frac{\cos(c+dx)}{a^3d}+\frac{3}{d\left(a^3\cos(c+dx)+a^3\right)}+\frac{3\log(\cos(c+dx)+1)}{a^3d}-\frac{1}{2ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Cos[c + d*x]/(a^3*d)) - 1/(2*a*d*(a + a*Cos[c + d*x])^2) + 3/(d*(a^3 + a^
3*Cos[c + d*x])) + (3*Log[1 + Cos[c + d*x]])/(a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{a^3(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(1-\frac{a^3}{(a-x)^3}+\frac{3a^2}{(a-x)^2}-\frac{3a}{a-x}\right)dx, x, -a\cos(c+dx)\right)}{a^4 d}$$
$$= -\frac{\cos(c+dx)}{a^3 d} - \frac{1}{2ad(a+a\cos(c+dx))^2} + \frac{3}{d\left(a^3+a^3\cos(c+dx)\right)} + \frac{3\log(1+\cos(c}{a^3 d}$$

Mathematica [A] time = 0.36, size = 103, normalized size = 1.37

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-2\cos(3(c+dx))+72\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\cos(2(c+dx))\left(24\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-5\right)+}{4a^3 d(\cos(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(21 - 2*Cos[3*(c + d*x)] + 72*Log[Cos[(c + d*x)/2]] + C
os[2*(c + d*x)]*(-5 + 24*Log[Cos[(c + d*x)/2]]) + Cos[c + d*x]*(22 + 96*Log
[Cos[(c + d*x)/2]])))/(4*a^3*d*(1 + Cos[c + d*x])^3)
```

fricas [A] time = 0.56, size = 96, normalized size = 1.28

$$-\frac{2\cos(dx+c)^3+4\cos(dx+c)^2-6\left(\cos(dx+c)^2+2\cos(dx+c)+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-4\cos(dx+}{2\left(a^3 d\cos(dx+c)^2+2a^3 d\cos(dx+c)+a^3 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - 6*(cos(d*x + c)^2 + 2*cos(d*x +
 c) + 1)*log(1/2*cos(d*x + c) + 1/2) - 4*cos(d*x + c) - 5)/(a^3*d*cos(d*x +
 c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 0.29, size = 63, normalized size = 0.84

$$-\frac{\cos(dx+c)}{a^3 d} + \frac{3\log(|-\cos(dx+c)-1|)}{a^3 d} + \frac{6\cos(dx+c)+5}{2a^3 d(\cos(dx+c)+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -cos(d*x + c)/(a^3*d) + 3*log(abs(-cos(d*x + c) - 1))/(a^3*d) + 1/2*(6*cos(
d*x + c) + 5)/(a^3*d*(cos(d*x + c) + 1)^2)
```

maple [A] time = 0.26, size = 86, normalized size = 1.15

$$-\frac{1}{d\,a^3\sec(dx+c)} - \frac{3\ln(\sec(dx+c))}{d\,a^3} - \frac{1}{2a^3 d(1+\sec(dx+c))^2} - \frac{2}{d\,a^3(1+\sec(dx+c))} + \frac{3\ln(1+\sec(dx+c))}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/d/a^3/sec(d*x+c)-3/d/a^3*ln(sec(d*x+c))-1/2/a^3/d/(1+sec(d*x+c))^2-2/d/a
^3/(1+sec(d*x+c))+3/d/a^3*ln(1+sec(d*x+c))
```

maxima [A] time = 0.51, size = 71, normalized size = 0.95

$$\frac{\frac{6\cos(dx+c)+5}{a^3\cos(dx+c)^2+2a^3\cos(dx+c)+a^3} - \frac{2\cos(dx+c)}{a^3} + \frac{6\log(\cos(dx+c)+1)}{a^3}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((6*cos(d*x + c) + 5)/(a^3*cos(d*x + c)^2 + 2*a^3*cos(d*x + c) + a^3) -
 2*cos(d*x + c)/a^3 + 6*log(cos(d*x + c) + 1)/a^3)/d
```

mupad [B] time = 0.08, size = 59, normalized size = 0.79

$$\frac{3\ln(\cos(c+dx)+1)}{a^3 d} - \frac{\cos(c+dx)}{a^3 d} + \frac{3\cos(c+dx)+\frac{5}{2}}{a^3 d(\cos(c+dx)+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (3*log(cos(c + d*x) + 1))/(a^3*d) - cos(c + d*x)/(a^3*d) + (3*cos(c + d*x)
+ 5/2)/(a^3*d*(cos(c + d*x) + 1)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x)
 + 1), x)/a**3
```

3.97 $\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3}\, dx$

Optimal. Leaf size=82

$$-\frac{7}{8d\left(a^3\cos(c+dx)+a^3\right)}-\frac{\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{5}{8ad(a\cos(c+dx)+a)^2}-\frac{1}{6d(a\cos(c+dx)+a)^3}$$

```
[Out] -1/8*arctanh(cos(d*x+c))/a^3/d-1/6/d/(a+a*cos(d*x+c))^3+5/8/a/d/(a+a*cos(d*
x+c))^2-7/8/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.15, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3872, 2836, 12, 88, 206}

$$-\frac{7}{8d\left(a^3\cos(c+dx)+a^3\right)}-\frac{\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{5}{8ad(a\cos(c+dx)+a)^2}-\frac{1}{6d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -ArcTanh[Cos[c + d*x]]/(8*a^3*d) - 1/(6*d*(a + a*Cos[c + d*x])^3) + 5/(8*a*
d*(a + a*Cos[c + d*x])^2) - 7/(8*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cos^2(c+dx)\cot(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= \frac{a\,\text{Subst}\left(\int \frac{x^3}{a^3(-a-x)(-a+x)^4}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{(-a-x)(-a+x)^4}\,dx, x, -a\cos(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a^2}{2(a-x)^4}+\frac{5a}{4(a-x)^3}-\frac{7}{8(a-x)^2}+\frac{1}{8(a^2-x^2)}\right)dx, x, -a\cos(c+dx)\right)}{a^2 d}$$
$$= -\frac{1}{6d(a+a\cos(c+dx))^3}+\frac{5}{8ad(a+a\cos(c+dx))^2}-\frac{7}{8d\left(a^3+a^3\cos(c+dx)\right)}+$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{1}{6d(a+a\cos(c+dx))^3}+\frac{5}{8ad(a+a\cos(c+dx))^2}-\frac{}{8d\left(\right.}$$

Mathematica [A] time = 0.35, size = 97, normalized size = 1.18

$$-\frac{\sec^3(c+dx)\left(42\cos^4\left(\frac{1}{2}(c+dx)\right)-15\cos^2\left(\frac{1}{2}(c+dx)\right)+12\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\right.\right.\right.}{12a^3d(\sec(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/12*((2 - 15*Cos[(c + d*x)/2]^2 + 42*Cos[(c + d*x)/2]^4 + 12*Cos[(c + d*x
)/2]^6*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]))*Sec[c + d*x]^3)/(a^
3*d*(1 + Sec[c + d*x])^3)
```

fricas [B] time = 0.67, size = 151, normalized size = 1.84

$$-\frac{42\cos(dx+c)^2+3\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3\left(\cos\right.}{48\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/48*(42*cos(d*x + c)^2 + 3*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x
 + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^3 + 3*cos(d*x + c)
^2 + 3*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 54*cos(d*x + c) + 2
0)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) +
a^3*d)
```

giac [A] time = 0.30, size = 113, normalized size = 1.38

$$\frac{\frac{6\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3}+\frac{\frac{18a^6(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9a^6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{2a^6(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{a^9}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(6*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 + (18*a^6*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^6*(cos(d*x + c) - 1)^2/(cos(d*x +
c) + 1)^2 + 2*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^9)/d
```

maple [A] time = 0.69, size = 90, normalized size = 1.10

$$\frac{\ln(-1+\cos(dx+c))}{16d\,a^3}-\frac{1}{6d\,a^3(1+\cos(dx+c))^3}+\frac{5}{8d\,a^3(1+\cos(dx+c))^2}-\frac{7}{8d\,a^3(1+\cos(dx+c))}-\frac{\ln(1+\cos(}{16a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/16/d/a^3*ln(-1+cos(d*x+c))-1/6/d/a^3/(1+cos(d*x+c))^3+5/8/d/a^3/(1+cos(d*
x+c))^2-7/8/d/a^3/(1+cos(d*x+c))-1/16*ln(1+cos(d*x+c))/a^3/d
```

maxima [A] time = 0.32, size = 98, normalized size = 1.20

$$-\frac{\frac{2\left(21\cos(dx+c)^2+27\cos(dx+c)+10\right)}{a^3\cos(dx+c)^3+3a^3\cos(dx+c)^2+3a^3\cos(dx+c)+a^3}+\frac{3\log(\cos(dx+c)+1)}{a^3}-\frac{3\log(\cos(dx+c)-1)}{a^3}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/48*(2*(21*cos(d*x + c)^2 + 27*cos(d*x + c) + 10)/(a^3*cos(d*x + c)^3 + 3
*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3) + 3*log(cos(d*x + c) + 1)/a
^3 - 3*log(cos(d*x + c) - 1)/a^3)/d
```

mupad [B] time = 0.11, size = 83, normalized size = 1.01

$$-\frac{\frac{7\cos(c+d\,x)^2}{8}+\frac{9\cos(c+d\,x)}{8}+\frac{5}{12}}{d\left(a^3\cos(c+d\,x)^3+3a^3\cos(c+d\,x)^2+3a^3\cos(c+d\,x)+a^3\right)}-\frac{\operatorname{atanh}(\cos(c+d\,x))}{8a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + a/cos(c + d*x))^3),x)
```

```
[Out] - ((9*cos(c + d*x))/8 + (7*cos(c + d*x)^2)/8 + 5/12)/(d*(3*a^3*cos(c + d*x)
 + a^3 + 3*a^3*cos(c + d*x)^2 + a^3*cos(c + d*x)^3)) - atanh(cos(c + d*x))/
(8*a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x)
 + 1), x)/a**3
```

3.98 $\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=126

$$-\frac{1}{32d\left(a^3-a^3\cos(c+dx)\right)}-\frac{1}{16d\left(a^3\cos(c+dx)+a^3\right)}+\frac{\tanh^{-1}(\cos(c+dx))}{32a^3d}-\frac{a}{16d(a\cos(c+dx)+a)^4}+\frac{}{6d(a\,c}$$

```
[Out] 1/32*arctanh(cos(d*x+c))/a^3/d-1/16*a/d/(a+a*cos(d*x+c))^4+1/6/d/(a+a*cos(d
*x+c))^3-3/32/a/d/(a+a*cos(d*x+c))^2-1/32/d/(a^3-a^3*cos(d*x+c))-1/16/d/(a^
3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.13, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2707, 88, 206}

$$-\frac{1}{32d\left(a^3-a^3\cos(c+dx)\right)}-\frac{1}{16d\left(a^3\cos(c+dx)+a^3\right)}+\frac{\tanh^{-1}(\cos(c+dx))}{32a^3d}-\frac{a}{16d(a\cos(c+dx)+a)^4}+\frac{}{6d(a\,c}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(32*a^3*d) - a/(16*d*(a + a*Cos[c + d*x])^4) + 1/(6*d
*(a + a*Cos[c + d*x])^3) - 3/(32*a*d*(a + a*Cos[c + d*x])^2) - 1/(32*d*(a^3
 - a^3*Cos[c + d*x])) - 1/(16*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cot^3(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{(-a-x)^2(-a+x)^5}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a}{4(a-x)^5} + \frac{1}{2(a-x)^4} - \frac{3}{16a(a-x)^3} - \frac{1}{16a^2(a-x)^2} + \frac{1}{32a^2(a+x)^2} - \frac{1}{32a^2(a^2-x^2)}\right) dx, x, -\right.}{d}$$
$$= -\frac{a}{16d(a+a\cos(c+dx))^4} + \frac{1}{6d(a+a\cos(c+dx))^3} - \frac{3}{32ad(a+a\cos(c+dx))^2} - \frac{}{32}$$
$$= \frac{\tanh^{-1}(\cos(c+dx))}{32a^3d} - \frac{a}{16d(a+a\cos(c+dx))^4} + \frac{1}{6d(a+a\cos(c+dx))^3} - \frac{}{32ad(a+}$$

Mathematica [A] time = 0.65, size = 138, normalized size = 1.10

$$-\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(12\csc^2\left(\frac{1}{2}(c+dx)\right)+3\sec^8\left(\frac{1}{2}(c+dx)\right)-16\sec^6\left(\frac{1}{2}(c+dx)\right)+18\sec^4\left(\frac{1}{2}(c+d\right.\right.}{96a^3d(\sec(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/96*(Cos[(c + d*x)/2]^6*(12*Csc[(c + d*x)/2]^2 + 24*(-Log[Cos[(c + d*x)/2
]] + Log[Sin[(c + d*x)/2]]) + 24*Sec[(c + d*x)/2]^2 + 18*Sec[(c + d*x)/2]^4
 - 16*Sec[(c + d*x)/2]^6 + 3*Sec[(c + d*x)/2]^8)*Sec[c + d*x]^3)/(a^3*d*(1
+ Sec[c + d*x])^3)
```

fricas [B] time = 0.76, size = 240, normalized size = 1.90

$$-\frac{6\cos(dx+c)^4+18\cos(dx+c)^3-50\cos(dx+c)^2-3\left(\cos(dx+c)^5+3\cos(dx+c)^4+2\cos(dx+c)^3-2\right.}{192\left(a^3d\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/192*(6*cos(d*x + c)^4 + 18*cos(d*x + c)^3 - 50*cos(d*x + c)^2 - 3*(cos(d
*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(
d*x + c) - 1)*log(1/2*cos(d*x + c) + 1/2) + 3*(cos(d*x + c)^5 + 3*cos(d*x +
 c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*log(-1/2*
cos(d*x + c) + 1/2) - 54*cos(d*x + c) - 16)/(a^3*d*cos(d*x + c)^5 + 3*a^3*d
*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3*a^3*d
*cos(d*x + c) - a^3*d)
```

giac [A] time = 0.69, size = 182, normalized size = 1.44

$$\frac{\frac{12\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)(\cos(dx+c)+1)}{a^3(\cos(dx+c)-1)} - \frac{12\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3} + \frac{\frac{24a^9(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{12a^9(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{4a^9(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{3a^9(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}}{a^{12}}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/768*(12*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)*(cos(d*x + c) + 1)/(a
^3*(cos(d*x + c) - 1)) - 12*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1
))/a^3 + (24*a^9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12*a^9*(cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2 - 4*a^9*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
 1)^3 - 3*a^9*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/a^12)/d
```

maple [A] time = 0.86, size = 126, normalized size = 1.00

$$\frac{1}{32d\,a^3\left(-1+\cos\left(dx+c\right)\right)}-\frac{\ln\left(-1+\cos\left(dx+c\right)\right)}{64d\,a^3}-\frac{1}{16d\,a^3\left(1+\cos\left(dx+c\right)\right)^4}+\frac{1}{6d\,a^3\left(1+\cos\left(dx+c\right)\right)^3}-\frac{}{32d\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/32/d/a^3/(-1+cos(d*x+c))-1/64/d/a^3*ln(-1+cos(d*x+c))-1/16/d/a^3/(1+cos(d
*x+c))^4+1/6/d/a^3/(1+cos(d*x+c))^3-3/32/d/a^3/(1+cos(d*x+c))^2-1/16/d/a^3/
(1+cos(d*x+c))+1/64*ln(1+cos(d*x+c))/a^3/d
```

maxima [A] time = 0.40, size = 146, normalized size = 1.16

$$-\frac{\frac{2\left(3\,\cos(dx+c)^4+9\,\cos(dx+c)^3-25\,\cos(dx+c)^2-27\,\cos(dx+c)-8\right)}{a^3\cos(dx+c)^5+3\,a^3\cos(dx+c)^4+2\,a^3\cos(dx+c)^3-2\,a^3\cos(dx+c)^2-3\,a^3\cos(dx+c)-a^3}-\frac{3\,\log(\cos(dx+c)+1)}{a^3}+\frac{3\,\log(\cos(dx+c)-1)}{a^3}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/192*(2*(3*cos(d*x + c)^4 + 9*cos(d*x + c)^3 - 25*cos(d*x + c)^2 - 27*cos
(d*x + c) - 8)/(a^3*cos(d*x + c)^5 + 3*a^3*cos(d*x + c)^4 + 2*a^3*cos(d*x +
 c)^3 - 2*a^3*cos(d*x + c)^2 - 3*a^3*cos(d*x + c) - a^3) - 3*log(cos(d*x +
c) + 1)/a^3 + 3*log(cos(d*x + c) - 1)/a^3)/d
```

mupad [B] time = 0.17, size = 130, normalized size = 1.03

$$\frac{\mathrm{atanh}\left(\cos\left(c+d\,x\right)\right)}{32\,a^3\,d}-\frac{-\frac{\cos(c+d\,x)^4}{32}-\frac{3\cos(c+d\,x)^3}{32}+\frac{25\cos(c+d\,x)^2}{96}+\frac{9\,\cos(c+d\,x)}{32}+\frac{1}{12}}{d\,\left(-a^3\cos\left(c+d\,x\right)^5-3\,a^3\cos\left(c+d\,x\right)^4-2\,a^3\cos\left(c+d\,x\right)^3+2\,a^3\cos\left(c+d\,x\right)^2+3\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + a/cos(c + d*x))^3),x)
```

```
[Out] atanh(cos(c + d*x))/(32*a^3*d) - ((9*cos(c + d*x))/32 + (25*cos(c + d*x)^2)
/96 - (3*cos(c + d*x)^3)/32 - cos(c + d*x)^4/32 + 1/12)/(d*(3*a^3*cos(c + d
*x) + a^3 + 2*a^3*cos(c + d*x)^2 - 2*a^3*cos(c + d*x)^3 - 3*a^3*cos(c + d*x
)^4 - a^3*cos(c + d*x)^5))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc^3\left(c+dx\right)}{\sec^3\left(c+dx\right)+3\sec^2\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.99 $\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=128

$$-\frac{3}{128d\left(a^3\cos(c+dx)+a^3\right)}+\frac{3\tanh^{-1}(\cos(c+dx))}{128a^3d}-\frac{a^2}{40d(a\cos(c+dx)+a)^5}+\frac{3a}{64d(a\cos(c+dx)+a)^4}-\frac{}{128ad(a}$$

```
[Out] 3/128*arctanh(cos(d*x+c))/a^3/d-1/128/a/d/(a-a*cos(d*x+c))^2-1/40*a^2/d/(a+
a*cos(d*x+c))^5+3/64*a/d/(a+a*cos(d*x+c))^4-1/64/a/d/(a+a*cos(d*x+c))^2-3/1
28/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A] time = 0.21, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2836, 12, 88, 206}

$$-\frac{a^2}{40d(a\cos(c+dx)+a)^5}-\frac{3}{128d\left(a^3\cos(c+dx)+a^3\right)}+\frac{3\tanh^{-1}(\cos(c+dx))}{128a^3d}+\frac{3a}{64d(a\cos(c+dx)+a)^4}-\frac{}{128ad(a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(128*a^3*d) - 1/(128*a*d*(a - a*Cos[c + d*x])^2)
- a^2/(40*d*(a + a*Cos[c + d*x])^5) + (3*a)/(64*d*(a + a*Cos[c + d*x])^4) -
 1/(64*a*d*(a + a*Cos[c + d*x])^2) - 3/(128*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cot^3(c+dx)\csc^2(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$

$$= \frac{a^5\,\text{Subst}\left(\int \frac{x^3}{a^3(-a-x)^3(-a+x)^6}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^2\,\text{Subst}\left(\int \frac{x^3}{(-a-x)^3(-a+x)^6}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^2\,\text{Subst}\left(\int \left(-\frac{1}{8(a-x)^6}+\frac{3}{16a(a-x)^5}-\frac{1}{32a^3(a-x)^3}-\frac{3}{128a^4(a-x)^2}+\frac{1}{64a^3(a+x)^3}-\frac{3}{128a^4(a^2-x^2}\right.\right.}{d}$$

$$= -\frac{1}{128ad(a-a\cos(c+dx))^2}-\frac{a^2}{40d(a+a\cos(c+dx))^5}+\frac{3a}{64d(a+a\cos(c+dx))^4}$$

$$= \frac{3\tanh^{-1}(\cos(c+dx))}{128a^3d}-\frac{1}{128ad(a-a\cos(c+dx))^2}-\frac{a^2}{40d(a+a\cos(c+dx))^5}+$$

Mathematica [A] time = 5.29, size = 137, normalized size = 1.07

$$-\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(60\cos^8\left(\frac{1}{2}(c+dx)\right)-15\cos^2\left(\frac{1}{2}(c+dx)\right)+10\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\cot^4\left(\frac{1}{2}(c+d\right.\right.\right.}{640a^3d(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/640*((4 - 15*Cos[(c + d*x)/2]^2 + 60*Cos[(c + d*x)/2]^8 + 10*Cos[(c + d*
x)/2]^6*(2 + Cot[(c + d*x)/2]^4) - 120*Cos[(c + d*x)/2]^10*(Log[Cos[(c + d*
x)/2]] - Log[Sin[(c + d*x)/2]]))*Sec[(c + d*x)/2]^4*Sec[c + d*x]^3)/(a^3*d*
(1 + Sec[c + d*x])^3)
```

fricas [B] time = 0.85, size = 317, normalized size = 2.48

$$-\frac{30\cos(dx+c)^6+90\cos(dx+c)^5+40\cos(dx+c)^4-120\cos(dx+c)^3+122\cos(dx+c)^2-15\left(\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/1280*(30*cos(d*x + c)^6 + 90*cos(d*x + c)^5 + 40*cos(d*x + c)^4 - 120*co
s(d*x + c)^3 + 122*cos(d*x + c)^2 - 15*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 +
 cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*
cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 15*(cos(d*x + c)^7 + 3*cos(
d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x
 + c)^2 + 3*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 126*cos(d*x +
c) + 32)/(a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a^3*d*cos(d*x + c
)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^
2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 1.07, size = 232, normalized size = 1.81

$$\frac{\frac{10\left(\frac{2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^2}{a^3(\cos(dx+c)-1)^2}-\frac{60\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3}+\frac{\frac{60a^{12}(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{30a^{12}(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{20a^{12}(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-}{a^{15}}}{5120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/5120*(10*(2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^2/(a^3*(cos(d*x + c) - 1)^2)
 - 60*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 + (60*a^12*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 30*a^12*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 - 20*a^12*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 5*a^12*(co
s(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 4*a^12*(cos(d*x + c) - 1)^5/(cos(d
*x + c) + 1)^5)/a^15)/d
```

maple [A] time = 0.80, size = 126, normalized size = 0.98

$$-\frac{1}{128d\,a^3\,(-1+\cos(dx+c))^2}-\frac{3\ln(-1+\cos(dx+c))}{256d\,a^3}-\frac{1}{40d\,a^3\,(1+\cos(dx+c))^5}+\frac{3}{64d\,a^3\,(1+\cos(dx+c))^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/128/d/a^3/(-1+cos(d*x+c))^2-3/256/d/a^3*ln(-1+cos(d*x+c))-1/40/d/a^3/(1+
cos(d*x+c))^5+3/64/d/a^3/(1+cos(d*x+c))^4-1/64/d/a^3/(1+cos(d*x+c))^2-3/128
/d/a^3/(1+cos(d*x+c))+3/256*ln(1+cos(d*x+c))/a^3/d
```

maxima [A] time = 0.38, size = 188, normalized size = 1.47

$$-\frac{\frac{2\left(15\cos(dx+c)^6+45\cos(dx+c)^5+20\cos(dx+c)^4-60\cos(dx+c)^3+61\cos(dx+c)^2+63\cos(dx+c)+16\right)}{a^3\cos(dx+c)^7+3a^3\cos(dx+c)^6+a^3\cos(dx+c)^5-5a^3\cos(dx+c)^4-5a^3\cos(dx+c)^3+a^3\cos(dx+c)^2+3a^3\cos(dx+c)+a^3}-\frac{15\log(\cos(dx+c)+1)}{a^3}+\frac{15}{}}{1280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/1280*(2*(15*cos(d*x + c)^6 + 45*cos(d*x + c)^5 + 20*cos(d*x + c)^4 - 60*
cos(d*x + c)^3 + 61*cos(d*x + c)^2 + 63*cos(d*x + c) + 16)/(a^3*cos(d*x + c
)^7 + 3*a^3*cos(d*x + c)^6 + a^3*cos(d*x + c)^5 - 5*a^3*cos(d*x + c)^4 - 5*
a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3) - 15*lo
g(cos(d*x + c) + 1)/a^3 + 15*log(cos(d*x + c) - 1)/a^3)/d
```

mupad [B] time = 1.09, size = 173, normalized size = 1.35

$$\frac{3\,\mathrm{atanh}\left(\cos\left(c+d\,x\right)\right)}{128\,a^3\,d}-\frac{\frac{3\cos(c+d\,x)^6}{128}+\frac{9\cos(c+d\,x)^5}{128}+\frac{\cos(c+d\,x)^4}{32}-\frac{3\cos(c+d\,x)^3}{32}+\frac{61\cos(c}{64}}{d\,\left(a^3\cos\left(c+d\,x\right)^7+3\,a^3\cos\left(c+d\,x\right)^6+a^3\cos\left(c+d\,x\right)^5-5\,a^3\cos\left(c+d\,x\right)^4-5\,a^3\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^5*(a + a/cos(c + d*x))^3),x)
```

```
[Out] (3*atanh(cos(c + d*x)))/(128*a^3*d) - ((63*cos(c + d*x))/640 + (61*cos(c +
d*x)^2)/640 - (3*cos(c + d*x)^3)/32 + cos(c + d*x)^4/32 + (9*cos(c + d*x)^5
)/128 + (3*cos(c + d*x)^6)/128 + 1/40)/(d*(3*a^3*cos(c + d*x) + a^3 + a^3*c
os(c + d*x)^2 - 5*a^3*cos(c + d*x)^3 - 5*a^3*cos(c + d*x)^4 + a^3*cos(c + d
*x)^5 + 3*a^3*cos(c + d*x)^6 + a^3*cos(c + d*x)^7))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.100 $\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=157

$$\frac{3\sin^7(c+dx)}{7a^3d}-\frac{7\sin^5(c+dx)}{5a^3d}+\frac{4\sin^3(c+dx)}{3a^3d}+\frac{\sin(c+dx)\cos^7(c+dx)}{8a^3d}+\frac{23\sin(c+dx)\cos^5(c+dx)}{48a^3d}-\frac{29\sin(c+}{}$$

```
[Out] -29/128*x/a^3-29/128*cos(d*x+c)*sin(d*x+c)/a^3/d-29/192*cos(d*x+c)^3*sin(d*
x+c)/a^3/d+23/48*cos(d*x+c)^5*sin(d*x+c)/a^3/d+1/8*cos(d*x+c)^7*sin(d*x+c)/
a^3/d+4/3*sin(d*x+c)^3/a^3/d-7/5*sin(d*x+c)^5/a^3/d+3/7*sin(d*x+c)^7/a^3/d
```

Rubi [A] time = 0.46, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3872, 2875, 2873, 2564, 14, 2568, 2635, 8, 270}

$$\frac{3\sin^7(c+dx)}{7a^3d}-\frac{7\sin^5(c+dx)}{5a^3d}+\frac{4\sin^3(c+dx)}{3a^3d}+\frac{\sin(c+dx)\cos^7(c+dx)}{8a^3d}+\frac{23\sin(c+dx)\cos^5(c+dx)}{48a^3d}-\frac{29\sin(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-29*x)/(128*a^3) - (29*Cos[c + d*x]*Sin[c + d*x])/(128*a^3*d) - (29*Cos[c
+ d*x]^3*Sin[c + d*x])/(192*a^3*d) + (23*Cos[c + d*x]^5*Sin[c + d*x])/(48*a
^3*d) + (Cos[c + d*x]^7*Sin[c + d*x])/(8*a^3*d) + (4*Sin[c + d*x]^3)/(3*a^3
*d) - (7*Sin[c + d*x]^5)/(5*a^3*d) + (3*Sin[c + d*x]^7)/(7*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cos^3(c+dx)\sin^8(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= -\frac{\int \cos^3(c+dx)(-a+a\cos(c+dx))^3\sin^2(c+dx)\,dx}{a^6} \\
&= -\frac{\int \left(-a^3\cos^3(c+dx)\sin^2(c+dx)+3a^3\cos^4(c+dx)\sin^2(c+dx)-3a^3\cos^5(c+\right.}{a^6} \\
&= \frac{\int \cos^3(c+dx)\sin^2(c+dx)\,dx}{a^3}-\frac{\int \cos^6(c+dx)\sin^2(c+dx)\,dx}{a^3}-\frac{3\int \cos^4(c+d}{} \\
&= \frac{\cos^5(c+dx)\sin(c+dx)}{2a^3d}+\frac{\cos^7(c+dx)\sin(c+dx)}{8a^3d}-\frac{\int \cos^6(c+dx)\,dx}{8a^3}-\frac{\int \cos}{} \\
&= -\frac{\cos^3(c+dx)\sin(c+dx)}{8a^3d}+\frac{23\cos^5(c+dx)\sin(c+dx)}{48a^3d}+\frac{\cos^7(c+dx)\sin(c+d}{8a^3d} \\
&= -\frac{3\cos(c+dx)\sin(c+dx)}{16a^3d}-\frac{29\cos^3(c+dx)\sin(c+dx)}{192a^3d}+\frac{23\cos^5(c+dx)\sin(c}{48a^3d} \\
&= -\frac{3x}{16a^3}-\frac{29\cos(c+dx)\sin(c+dx)}{128a^3d}-\frac{29\cos^3(c+dx)\sin(c+dx)}{192a^3d}+\frac{23\cos^5(c+}{4} \\
&= -\frac{29x}{128a^3}-\frac{29\cos(c+dx)\sin(c+dx)}{128a^3d}-\frac{29\cos^3(c+dx)\sin(c+dx)}{192a^3d}+\frac{23\cos^5(c+}{}
\end{aligned}
$$

Mathematica [A] time = 5.00, size = 131, normalized size = 0.83

$$
\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(38640\sin(c+dx)-6720\sin(2(c+dx))-3920\sin(3(c+dx))+5880\sin(4(c+\right.}{13440a^3d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(-24360*d*x + 38640*Sin[c + d*x] - 6720*
Sin[2*(c + d*x)] - 3920*Sin[3*(c + d*x)] + 5880*Sin[4*(c + d*x)] - 4368*Sin
[5*(c + d*x)] + 2240*Sin[6*(c + d*x)] - 720*Sin[7*(c + d*x)] + 105*Sin[8*(c
 + d*x)] + 294*Tan[c/2]))/(13440*a^3*d*(1 + Sec[c + d*x])^3)
```

fricas [A] time = 0.68, size = 91, normalized size = 0.58

$$-\frac{3045\,dx - \left(1680\,\cos\left(dx+c\right)^{7} - 5760\,\cos\left(dx+c\right)^{6} + 6440\,\cos\left(dx+c\right)^{5} - 1536\,\cos\left(dx+c\right)^{4} - 2030\,\cos\left(dx\right.\right.}{13440\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/13440*(3045*d*x - (1680*cos(d*x + c)^7 - 5760*cos(d*x + c)^6 + 6440*cos(
d*x + c)^5 - 1536*cos(d*x + c)^4 - 2030*cos(d*x + c)^3 + 2432*cos(d*x + c)^
2 - 3045*cos(d*x + c) + 4864)*sin(d*x + c))/(a^3*d)
```

giac [A] time = 0.80, size = 139, normalized size = 0.89

$$-\frac{\frac{3045\,(dx+c)}{a^{3}} + \frac{2\left(3045\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15} - 120015\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13} - 36939\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} - 218007\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9} - 146537\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7} - 77\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{8}a^{3}}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/13440*(3045*(d*x + c)/a^3 + 2*(3045*tan(1/2*d*x + 1/2*c)^15 - 120015*tan
(1/2*d*x + 1/2*c)^13 - 36939*tan(1/2*d*x + 1/2*c)^11 - 218007*tan(1/2*d*x +
 1/2*c)^9 - 146537*tan(1/2*d*x + 1/2*c)^7 - 77749*tan(1/2*d*x + 1/2*c)^5 -
23345*tan(1/2*d*x + 1/2*c)^3 - 3045*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1
/2*c)^2 + 1)^8*a^3))/d
```

maple [B] time = 0.68, size = 290, normalized size = 1.85

$$\frac{29\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{64d\,a^{3}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{8}} + \frac{667\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{192d\,a^{3}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{8}} + \frac{11107\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{960d\,a^{3}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{8}} + \frac{146537\left(\tan^{7}\left(\frac{dx}{2}\right.\right.}{6720d\,a^{3}\left(1+\tan^{2}\left(\frac{d}{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^8/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 29/64/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)+667/192/d/a^3/(1+
tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^3+11107/960/d/a^3/(1+tan(1/2*d*x
+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^5+146537/6720/d/a^3/(1+tan(1/2*d*x+1/2*c)^2
)^8*tan(1/2*d*x+1/2*c)^7+72669/2240/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/
2*d*x+1/2*c)^9+1759/320/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)
^11+1143/64/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^13-29/64/d/
a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^15-29/64/d/a^3*arctan(tan
(1/2*d*x+1/2*c))
```

maxima [B] time = 0.45, size = 378, normalized size = 2.41

$$\frac{\frac{\frac{3045\sin(dx+c)}{\cos(dx+c)+1} + \frac{23345\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}} + \frac{77749\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}} + \frac{146537\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}} + \frac{218007\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}} + \frac{36939\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}} + \frac{120015\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}} - \frac{3045\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}}{a^{3} + \frac{8a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}} + \frac{28a^{3}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}} + \frac{56a^{3}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}} + \frac{70a^{3}\sin(dx+c)^{8}}{(\cos(dx+c)+1)^{8}} + \frac{56a^{3}\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}} + \frac{28a^{3}\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}} + \frac{8a^{3}\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}} + \frac{a^{3}\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}} - \frac{3045\,\mathrm{ar}\ldots}{}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6720*((3045*sin(d*x + c)/(cos(d*x + c) + 1) + 23345*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 77749*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 146537*sin(d*x
 + c)^7/(cos(d*x + c) + 1)^7 + 218007*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 +
 36939*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 120015*sin(d*x + c)^13/(cos(
d*x + c) + 1)^13 - 3045*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)/(a^3 + 8*a^3
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^3*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 + 56*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^3*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8 + 56*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a
^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 8*a^3*sin(d*x + c)^14/(cos(d*x +
 c) + 1)^14 + a^3*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 3045*arctan(sin(
d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 3.87, size = 132, normalized size = 0.84

$$\frac{-\frac{29\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{15}}{64}+\frac{1143\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}}{64}+\frac{1759\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{320}+\frac{72669\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{2240}+\frac{146537\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{6720}+\frac{11107\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{960}+667\cdots}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8/(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((29*tan(c/2 + (d*x)/2))/64 + (667*tan(c/2 + (d*x)/2)^3)/192 + (11107*tan(c
/2 + (d*x)/2)^5)/960 + (146537*tan(c/2 + (d*x)/2)^7)/6720 + (72669*tan(c/2
+ (d*x)/2)^9)/2240 + (1759*tan(c/2 + (d*x)/2)^11)/320 + (1143*tan(c/2 + (d*
x)/2)^13)/64 - (29*tan(c/2 + (d*x)/2)^15)/64)/(a^3*d*(tan(c/2 + (d*x)/2)^2
+ 1)^8) - (29*x)/(128*a^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**8/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.101 $\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=129

$$\frac{3\sin^5(c+dx)}{5a^3d}-\frac{7\sin^3(c+dx)}{3a^3d}+\frac{4\sin(c+dx)}{a^3d}-\frac{\sin(c+dx)\cos^5(c+dx)}{6a^3d}-\frac{23\sin(c+dx)\cos^3(c+dx)}{24a^3d}-\frac{23\sin(c+}{1}$$

```
[Out] -23/16*x/a^3+4*sin(d*x+c)/a^3/d-23/16*cos(d*x+c)*sin(d*x+c)/a^3/d-23/24*cos
(d*x+c)^3*sin(d*x+c)/a^3/d-1/6*cos(d*x+c)^5*sin(d*x+c)/a^3/d-7/3*sin(d*x+c)
^3/a^3/d+3/5*sin(d*x+c)^5/a^3/d
```

Rubi [A] time = 0.29, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3872, 2869, 2757, 2633, 2635, 8}

$$\frac{3\sin^5(c+dx)}{5a^3d}-\frac{7\sin^3(c+dx)}{3a^3d}+\frac{4\sin(c+dx)}{a^3d}-\frac{\sin(c+dx)\cos^5(c+dx)}{6a^3d}-\frac{23\sin(c+dx)\cos^3(c+dx)}{24a^3d}-\frac{23\sin(c+}{1}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-23*x)/(16*a^3) + (4*Sin[c + d*x])/(a^3*d) - (23*Cos[c + d*x]*Sin[c + d*x]
)/(16*a^3*d) - (23*Cos[c + d*x]^3*Sin[c + d*x])/(24*a^3*d) - (Cos[c + d*x]^
5*Sin[c + d*x])/(6*a^3*d) - (7*Sin[c + d*x]^3)/(3*a^3*d) + (3*Sin[c + d*x]^
5)/(5*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^6(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= -\frac{\int \cos^3(c+dx)(-a+a\cos(c+dx))^3\,dx}{a^6}$$
$$= -\frac{\int \left(-a^3\cos^3(c+dx)+3a^3\cos^4(c+dx)-3a^3\cos^5(c+dx)+a^3\cos^6(c+dx)\right) d\ldots}{a^6}$$
$$= \frac{\int \cos^3(c+dx)\,dx}{a^3} - \frac{\int \cos^6(c+dx)\,dx}{a^3} - \frac{3\int \cos^4(c+dx)\,dx}{a^3} + \frac{3\int \cos^5(c+dx)\ldots}{a^3}$$
$$= -\frac{3\cos^3(c+dx)\sin(c+dx)}{4a^3d} - \frac{\cos^5(c+dx)\sin(c+dx)}{6a^3d} - \frac{5\int \cos^4(c+dx)\,dx}{6a^3} - \ldots$$
$$= \frac{4\sin(c+dx)}{a^3d} - \frac{9\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{23\cos^3(c+dx)\sin(c+dx)}{24a^3d} - \frac{\cos^5(c\ldots}{}$$
$$= -\frac{9x}{8a^3} + \frac{4\sin(c+dx)}{a^3d} - \frac{23\cos(c+dx)\sin(c+dx)}{16a^3d} - \frac{23\cos^3(c+dx)\sin(c+dx)}{24a^3d}\ldots$$
$$= -\frac{23x}{16a^3} + \frac{4\sin(c+dx)}{a^3d} - \frac{23\cos(c+dx)\sin(c+dx)}{16a^3d} - \frac{23\cos^3(c+dx)\sin(c+dx\ldots}{24a^3d}$$

Mathematica [A] time = 1.96, size = 111, normalized size = 0.86

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(5040\sin(c+dx)-1890\sin(2(c+dx))+760\sin(3(c+dx))-270\sin(4(c+dx\ldots\right.}{240a^3d(\sec(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(-2760*d*x + 5040*Sin[c + d*x] - 1890*Si
n[2*(c + d*x)] + 760*Sin[3*(c + d*x)] - 270*Sin[4*(c + d*x)] + 72*Sin[5*(c
+ d*x)] - 10*Sin[6*(c + d*x)] + 9*Tan[c/2]))/(240*a^3*d*(1 + Sec[c + d*x])^
3)
```

fricas [A] time = 0.72, size = 70, normalized size = 0.54

$$-\frac{345\,dx + \left(40\cos(dx+c)^5 - 144\cos(dx+c)^4 + 230\cos(dx+c)^3 - 272\cos(dx+c)^2 + 345\cos(dx+c) - \ldots\right.}{240\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(345*d*x + (40*cos(d*x + c)^5 - 144*cos(d*x + c)^4 + 230*cos(d*x + c
)^3 - 272*cos(d*x + c)^2 + 345*cos(d*x + c) - 544)*sin(d*x + c))/(a^3*d)
```

giac [A] time = 0.35, size = 113, normalized size = 0.88

$$-\frac{\frac{345(dx+c)}{a^3} - \frac{2\left(1575\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+3165\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+5814\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+4554\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+1955\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+345\tan\left(\ldots\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^6a^3}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/240*(345*(d*x + c)/a^3 - 2*(1575*tan(1/2*d*x + 1/2*c)^11 + 3165*tan(1/2*
d*x + 1/2*c)^9 + 5814*tan(1/2*d*x + 1/2*c)^7 + 4554*tan(1/2*d*x + 1/2*c)^5
+ 1955*tan(1/2*d*x + 1/2*c)^3 + 345*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1
/2*c)^2 + 1)^6*a^3))/d
```

maple [A] time = 0.73, size = 222, normalized size = 1.72

$$\frac{105\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{211\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{969\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^6}+\frac{759\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 105/8/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11+211/8/d/a^3/(1
+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9+969/20/d/a^3/(1+tan(1/2*d*x+1
/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7+759/20/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan
(1/2*d*x+1/2*c)^5+391/24/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c
)^3+23/8/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)-23/8/d/a^3*arc
tan(tan(1/2*d*x+1/2*c))
```

maxima [B] time = 0.50, size = 292, normalized size = 2.26

$$\frac{\frac{\frac{345\sin(dx+c)}{\cos(dx+c)+1}+\frac{1955\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{4554\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5814\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{3165\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{1575\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^3+\frac{6a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{345\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/120*((345*sin(d*x + c)/(cos(d*x + c) + 1) + 1955*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 4554*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5814*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 + 3165*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 1575*si
n(d*x + c)^11/(cos(d*x + c) + 1)^11)/(a^3 + 6*a^3*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a^3*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 15*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*
a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^3*sin(d*x + c)^12/(cos(d*x +
c) + 1)^12) - 345*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

mupad [B] time = 3.67, size = 106, normalized size = 0.82

$$\frac{\frac{105\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}}{8}+\frac{211\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}}{8}+\frac{969\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}}{20}+\frac{759\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}}{20}+\frac{391\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24}+\frac{23\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}}{a^3\,d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^6}-\frac{23\,x}{16\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + a/cos(c + d*x))^3,x)
```

```
[Out] ((23*tan(c/2 + (d*x)/2))/8 + (391*tan(c/2 + (d*x)/2)^3)/24 + (759*tan(c/2 +
 (d*x)/2)^5)/20 + (969*tan(c/2 + (d*x)/2)^7)/20 + (211*tan(c/2 + (d*x)/2)^9
)/8 + (105*tan(c/2 + (d*x)/2)^11)/8)/(a^3*d*(tan(c/2 + (d*x)/2)^2 + 1)^6) -
 (23*x)/(16*a^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.102 $\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3} dx$

Optimal. Leaf size=108

$$\frac{\sin^3(c+dx)}{a^3d} - \frac{7\sin(c+dx)}{a^3d} + \frac{\sin(c+dx)\cos^3(c+dx)}{4a^3d} + \frac{19\sin(c+dx)\cos(c+dx)}{8a^3d} - \frac{4\sin(c+dx)}{a^3d(\cos(c+dx)+1)} + \frac{51x}{8a^3}$$

```
[Out] 51/8*x/a^3-7*sin(d*x+c)/a^3/d+19/8*cos(d*x+c)*sin(d*x+c)/a^3/d+1/4*cos(d*x+
c)^3*sin(d*x+c)/a^3/d-4*sin(d*x+c)/a^3/d/(1+cos(d*x+c))+sin(d*x+c)^3/a^3/d
```

Rubi [A] time = 0.32, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2875, 2872, 2648, 2637, 2635, 8, 2633}

$$\frac{\sin^3(c+dx)}{a^3d} - \frac{7\sin(c+dx)}{a^3d} + \frac{\sin(c+dx)\cos^3(c+dx)}{4a^3d} + \frac{19\sin(c+dx)\cos(c+dx)}{8a^3d} - \frac{4\sin(c+dx)}{a^3d(\cos(c+dx)+1)} + \frac{51x}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (51*x)/(8*a^3) - (7*Sin[c + d*x])/(a^3*d) + (19*Cos[c + d*x]*Sin[c + d*x])/
(8*a^3*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a^3*d) - (4*Sin[c + d*x])/(a^3
*d*(1 + Cos[c + d*x])) + Sin[c + d*x]^3/(a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
```

```
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cos^3(c+dx)\sin^4(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= -\frac{\int \cos(c+dx)(-a+a\cos(c+dx))^3\cot^2(c+dx)\,dx}{a^6} \\
&= \frac{\int \left(4a+\frac{4a}{-1-\cos(c+dx)}-4a\cos(c+dx)+4a\cos^2(c+dx)-3a\cos^3(c+dx)+a\cos\ldots\right.}{a^4} \\
&= \frac{4x}{a^3}+\frac{\int \cos^4(c+dx)\,dx}{a^3}-\frac{3\int \cos^3(c+dx)\,dx}{a^3}+\frac{4\int \frac{1}{-1-\cos(c+dx)}\,dx}{a^3}-\frac{4\int \cos(c+\ldots}{a^3} \\
&= \frac{4x}{a^3}-\frac{4\sin(c+dx)}{a^3d}+\frac{2\cos(c+dx)\sin(c+dx)}{a^3d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^3d}-\frac{\ldots}{a^3d\ldots} \\
&= \frac{6x}{a^3}-\frac{7\sin(c+dx)}{a^3d}+\frac{19\cos(c+dx)\sin(c+dx)}{8a^3d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^3d}-\frac{\ldots}{a^3\ldots} \\
&= \frac{51x}{8a^3}-\frac{7\sin(c+dx)}{a^3d}+\frac{19\cos(c+dx)\sin(c+dx)}{8a^3d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^3d}-\frac{\ldots}{a\ldots}
\end{aligned}$$

Mathematica [A] time = 0.68, size = 173, normalized size = 1.60

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-997\sin\left(c+\frac{dx}{2}\right)-800\sin\left(c+\frac{3dx}{2}\right)-800\sin\left(2c+\frac{3dx}{2}\right)+160\sin\left(2c+\frac{5dx}{2}\right)+160\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(2040*d*x*Cos[(d*x)/2] + 2040*d*x*Cos[c + (d*x)/
2] - 3563*Sin[(d*x)/2] - 997*Sin[c + (d*x)/2] - 800*Sin[c + (3*d*x)/2] - 80
0*Sin[2*c + (3*d*x)/2] + 160*Sin[2*c + (5*d*x)/2] + 160*Sin[3*c + (5*d*x)/2
] - 35*Sin[3*c + (7*d*x)/2] - 35*Sin[4*c + (7*d*x)/2] + 5*Sin[4*c + (9*d*x)
/2] + 5*Sin[5*c + (9*d*x)/2]))/(640*a^3*d)
```

fricas [A] time = 0.64, size = 83, normalized size = 0.77

$$\frac{51\,dx\cos(dx+c)+51\,dx+\left(2\cos(dx+c)^4-6\cos(dx+c)^3+11\cos(dx+c)^2-29\cos(dx+c)-80\right)\sin(\ldots}{8\left(a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(51*d*x*cos(d*x + c) + 51*d*x + (2*cos(d*x + c)^4 - 6*cos(d*x + c)^3 +
11*cos(d*x + c)^2 - 29*cos(d*x + c) - 80)*sin(d*x + c))/(a^3*d*cos(d*x + c)
 + a^3*d)
```

giac [A] time = 0.30, size = 101, normalized size = 0.94

$$\frac{\frac{51\,(dx+c)}{a^3}-\frac{32\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}-\frac{2\left(77\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+149\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+123\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(51*(d*x + c)/a^3 - 32*tan(1/2*d*x + 1/2*c)/a^3 - 2*(77*tan(1/2*d*x + 1
/2*c)^7 + 149*tan(1/2*d*x + 1/2*c)^5 + 123*tan(1/2*d*x + 1/2*c)^3 + 35*tan(
1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^3))/d
```

maple [A] time = 0.75, size = 171, normalized size = 1.58

$$-\frac{4\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^3}-\frac{77\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{149\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{123\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4}-\frac{35\,\mathrm{ta}}{4d\,a^3\left(1+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -4/d/a^3*tan(1/2*d*x+1/2*c)-77/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d
*x+1/2*c)^7-149/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-123
/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-35/4/d/a^3/(1+tan(
1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)+51/4/d/a^3*arctan(tan(1/2*d*x+1/2*c)
)
```

maxima [B] time = 0.50, size = 227, normalized size = 2.10

$$-\frac{\frac{\frac{35\sin(dx+c)}{\cos(dx+c)+1}+\frac{123\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{149\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{77\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^3+\frac{4a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{51\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{16\sin(dx+c)}{a^3(\cos(dx+c)+1)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*((35*sin(d*x + c)/(cos(d*x + c) + 1) + 123*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 149*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 77*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7)/(a^3 + 4*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^3*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^3*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 + a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 51*arctan(sin(d*x + c)/(c
os(d*x + c) + 1))/a^3 + 16*sin(d*x + c)/(a^3*(cos(d*x + c) + 1)))/d
```

mupad [B] time = 2.66, size = 98, normalized size = 0.91

$$\frac{51\,x}{8\,a^3}-\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^3\,d}-\frac{\frac{77\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{4}+\frac{149\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{4}+\frac{123\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{4}+\frac{35\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}}{a^3\,d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (51*x)/(8*a^3) - (4*tan(c/2 + (d*x)/2))/(a^3*d) - ((35*tan(c/2 + (d*x)/2))/
4 + (123*tan(c/2 + (d*x)/2)^3)/4 + (149*tan(c/2 + (d*x)/2)^5)/4 + (77*tan(c
/2 + (d*x)/2)^7)/4)/(a^3*d*(tan(c/2 + (d*x)/2)^2 + 1)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.103 $\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=97

$$\frac{3\sin(c+dx)}{a^3d}-\frac{\sin(c+dx)\cos(c+dx)}{2a^3d}+\frac{19\sin(c+dx)}{3a^3d(\cos(c+dx)+1)}-\frac{2\sin(c+dx)}{3a^3d(\cos(c+dx)+1)^2}-\frac{11x}{2a^3}$$

```
[Out] -11/2*x/a^3+3*sin(d*x+c)/a^3/d-1/2*cos(d*x+c)*sin(d*x+c)/a^3/d-2/3*sin(d*x+
c)/a^3/d/(1+cos(d*x+c))^2+19/3*sin(d*x+c)/a^3/d/(1+cos(d*x+c))
```

Rubi [A] time = 0.31, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3872, 2874, 2966, 2637, 2635, 8, 2650, 2648}

$$\frac{3\sin(c+dx)}{a^3d}-\frac{\sin(c+dx)\cos(c+dx)}{2a^3d}+\frac{19\sin(c+dx)}{3a^3d(\cos(c+dx)+1)}-\frac{2\sin(c+dx)}{3a^3d(\cos(c+dx)+1)^2}-\frac{11x}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-11*x)/(2*a^3) + (3*Sin[c + d*x])/(a^3*d) - (Cos[c + d*x]*Sin[c + d*x])/(2
*a^3*d) - (2*Sin[c + d*x])/(3*a^3*d*(1 + Cos[c + d*x])^2) + (19*Sin[c + d*x
])/(3*a^3*d*(1 + Cos[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
```

```
, 0])
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^2(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= -\frac{\int \frac{\cos^3(c+dx)(-a+a\cos(c+dx))}{(-a-a\cos(c+dx))^2}\,dx}{a^2}$$
$$= -\frac{\int \left(\frac{5}{a}-\frac{3\cos(c+dx)}{a}+\frac{\cos^2(c+dx)}{a}+\frac{2}{a(1+\cos(c+dx))^2}-\frac{7}{a(1+\cos(c+dx))}\right)dx}{a^2}$$
$$= -\frac{5x}{a^3}-\frac{\int \cos^2(c+dx)\,dx}{a^3}-\frac{2\int \frac{1}{(1+\cos(c+dx))^2}\,dx}{a^3}+\frac{3\int \cos(c+dx)\,dx}{a^3}+\frac{7\int \frac{1}{1+\cos(}}{a^3}$$
$$= -\frac{5x}{a^3}+\frac{3\sin(c+dx)}{a^3d}-\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{2\sin(c+dx)}{3a^3d(1+\cos(c+dx))^2}+\frac{7}{a^3d(1}$$
$$= -\frac{11x}{2a^3}+\frac{3\sin(c+dx)}{a^3d}-\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{2\sin(c+dx)}{3a^3d(1+\cos(c+dx))^2}+\frac{}{3a^3d}$$

Mathematica [A] time = 0.47, size = 177, normalized size = 1.82

$$-\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(1326\sin\left(c+\frac{dx}{2}\right)-2012\sin\left(c+\frac{3dx}{2}\right)-498\sin\left(2c+\frac{3dx}{2}\right)-135\sin\left(2c+\frac{5dx}{2}\right)-1\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/960*(Sec[c/2]*Sec[(c + d*x)/2]^3*(1980*d*x*Cos[(d*x)/2] + 1980*d*x*Cos[c
 + (d*x)/2] + 660*d*x*Cos[c + (3*d*x)/2] + 660*d*x*Cos[2*c + (3*d*x)/2] - 3
216*Sin[(d*x)/2] + 1326*Sin[c + (d*x)/2] - 2012*Sin[c + (3*d*x)/2] - 498*Si
n[2*c + (3*d*x)/2] - 135*Sin[2*c + (5*d*x)/2] - 135*Sin[3*c + (5*d*x)/2] +
15*Sin[3*c + (7*d*x)/2] + 15*Sin[4*c + (7*d*x)/2]))/(a^3*d)
```

fricas [A] time = 0.71, size = 99, normalized size = 1.02

$$-\frac{33\,dx\cos(dx+c)^2+66\,dx\cos(dx+c)+33\,dx+\left(3\cos(dx+c)^3-12\cos(dx+c)^2-71\cos(dx+c)-52\right.}{6\left(a^3d\cos(dx+c)^2+2\,a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(33*d*x*cos(d*x + c)^2 + 66*d*x*cos(d*x + c) + 33*d*x + (3*cos(d*x + c
)^3 - 12*cos(d*x + c)^2 - 71*cos(d*x + c) - 52)*sin(d*x + c))/(a^3*d*cos(d*
x + c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)
```

giac [A] time = 0.32, size = 96, normalized size = 0.99

$$-\frac{\frac{33\,(dx+c)}{a^3}-\frac{6\left(7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^3}+\frac{2\left(a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-18\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^9}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(33*(d*x + c)/a^3 - 6*(7*tan(1/2*d*x + 1/2*c)^3 + 5*tan(1/2*d*x + 1/2*
c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) + 2*(a^6*tan(1/2*d*x + 1/2*c)^3 -
18*a^6*tan(1/2*d*x + 1/2*c))/a^9)/d
```

maple [A] time = 0.56, size = 122, normalized size = 1.26

$$-\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{3d\,a^3}+\frac{6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^3}+\frac{7\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^3\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{11\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/3/d/a^3*tan(1/2*d*x+1/2*c)^3+6/d/a^3*tan(1/2*d*x+1/2*c)+7/d/a^3/(1+tan(1
/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+5/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*
tan(1/2*d*x+1/2*c)-11/d/a^3*arctan(tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.59, size = 164, normalized size = 1.69

$$\frac{\frac{3\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{18\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^3}-\frac{33\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4) + (18*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x +
c)^3/(cos(d*x + c) + 1)^3)/a^3 - 33*arctan(sin(d*x + c)/(cos(d*x + c) + 1))
/a^3)/d
```

mupad [B] time = 1.06, size = 115, normalized size = 1.19

$$-\frac{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-38\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-42\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+12\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\sin\left(\frac{c}{2}+\frac{d}{2}\cdots\right.}{6\,a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + a/cos(c + d*x))^3,x)
```

```
[Out] -(2*sin(c/2 + (d*x)/2) - 38*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2) - 42*co
s(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2) + 12*cos(c/2 + (d*x)/2)^6*sin(c/2 + (
d*x)/2) + 33*cos(c/2 + (d*x)/2)^3*(c + d*x))/(6*a^3*d*cos(c/2 + (d*x)/2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sin^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.104 $\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=89

$$\frac{4\cot^7(c+dx)}{7a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}-\frac{4\csc^7(c+dx)}{7a^3d}+\frac{7\csc^5(c+dx)}{5a^3d}-\frac{\csc^3(c+dx)}{a^3d}$$

```
[Out] 3/5*cot(d*x+c)^5/a^3/d+4/7*cot(d*x+c)^7/a^3/d-csc(d*x+c)^3/a^3/d+7/5*csc(d*
x+c)^5/a^3/d-4/7*csc(d*x+c)^7/a^3/d
```

Rubi [A] time = 0.37, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2875, 2873, 2607, 30, 2606, 270, 14}

$$\frac{4\cot^7(c+dx)}{7a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}-\frac{4\csc^7(c+dx)}{7a^3d}+\frac{7\csc^5(c+dx)}{5a^3d}-\frac{\csc^3(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*Cot[c + d*x]^5)/(5*a^3*d) + (4*Cot[c + d*x]^7)/(7*a^3*d) - Csc[c + d*x]^
3/(a^3*d) + (7*Csc[c + d*x]^5)/(5*a^3*d) - (4*Csc[c + d*x]^7)/(7*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
```

```
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cos(c+dx)\cot^2(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= -\frac{\int (-a+a\cos(c+dx))^3\cot^3(c+dx)\csc^5(c+dx)\,dx}{a^6}$$
$$= \frac{\int \left(-a^3\cot^6(c+dx)\csc^2(c+dx)+3a^3\cot^5(c+dx)\csc^3(c+dx)-3a^3\cot^4(c+d\right.}{a^6}$$
$$= -\frac{\int \cot^6(c+dx)\csc^2(c+dx)\,dx}{a^3}+\frac{\int \cot^3(c+dx)\csc^5(c+dx)\,dx}{a^3}+\frac{3\int \cot^5(c+}{}$$
$$= -\frac{\text{Subst}\left(\int x^6\,dx,x,-\cot(c+dx)\right)}{a^3d}-\frac{\text{Subst}\left(\int x^4\left(-1+x^2\right)\,dx,x,\csc(c+dx)\right)}{a^3d}-$$
$$= \frac{\cot^7(c+dx)}{7a^3d}-\frac{\text{Subst}\left(\int \left(-x^4+x^6\right)\,dx,x,\csc(c+dx)\right)}{a^3d}-\frac{3\,\text{Subst}\left(\int \left(x^2-2x^4+\right.\right.}{a^3}$$
$$= \frac{3\cot^5(c+dx)}{5a^3d}+\frac{4\cot^7(c+dx)}{7a^3d}-\frac{\csc^3(c+dx)}{a^3d}+\frac{7\csc^5(c+dx)}{5a^3d}-\frac{4\csc^7(c+dx)}{7a^3d}$$

Mathematica [A] time = 0.64, size = 137, normalized size = 1.54

$$\frac{\csc(c)(602\sin(c+dx)+602\sin(2(c+dx))+258\sin(3(c+dx))+43\sin(4(c+dx))-560\sin(2c+dx)+168}{2240a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Csc[c]*Csc[c + d*x]*Sec[c + d*x]^3*(-840*Sin[c] + 448*Sin[d*x] + 602*Sin[c
 + d*x] + 602*Sin[2*(c + d*x)] + 258*Sin[3*(c + d*x)] + 43*Sin[4*(c + d*x)]
 - 560*Sin[2*c + d*x] + 168*Sin[c + 2*d*x] - 280*Sin[3*c + 2*d*x] - 48*Sin[
2*c + 3*d*x] - 8*Sin[3*c + 4*d*x]))/(2240*a^3*d*(1 + Sec[c + d*x])^3)
```

fricas [A] time = 0.55, size = 95, normalized size = 1.07

$$\frac{\cos(dx+c)^4+3\,\cos(dx+c)^3-15\,\cos(dx+c)^2-18\,\cos(dx+c)-6}{35\left(a^3d\cos(dx+c)^3+3\,a^3d\cos(dx+c)^2+3\,a^3d\cos(dx+c)+a^3d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/35*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 - 15*cos(d*x + c)^2 - 18*cos(d*x +
c) - 6)/((a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
 c) + a^3*d)*sin(d*x + c))
```

giac [A] time = 0.43, size = 73, normalized size = 0.82

$$-\frac{\frac{35}{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}+\frac{5\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-14\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+70\,a^{18}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{21}}}{560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/560*(35/(a^3*tan(1/2*d*x + 1/2*c)) + (5*a^18*tan(1/2*d*x + 1/2*c)^7 - 14
*a^18*tan(1/2*d*x + 1/2*c)^5 + 70*a^18*tan(1/2*d*x + 1/2*c))/a^21)/d
```

maple [A] time = 0.73, size = 60, normalized size = 0.67

$$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{16d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/16/d/a^3*(-1/7*tan(1/2*d*x+1/2*c)^7+2/5*tan(1/2*d*x+1/2*c)^5-2*tan(1/2*d*
x+1/2*c)-1/tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.40, size = 90, normalized size = 1.01

$$-\frac{\frac{\frac{70\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{14\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^3}+\frac{35\,(\cos(dx+c)+1)}{a^3\sin(dx+c)}}{560\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/560*((70*sin(d*x + c)/(cos(d*x + c) + 1) - 14*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^3 + 35*(cos(d*x + c) +
 1)/(a^3*sin(d*x + c)))/d
```

mupad [B] time = 1.01, size = 84, normalized size = 0.94

$$-\frac{-16\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+8\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+72\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-34\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+5}{560\,a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + a/cos(c + d*x))^3),x)
```

```
[Out] -(72*cos(c/2 + (d*x)/2)^4 - 34*cos(c/2 + (d*x)/2)^2 + 8*cos(c/2 + (d*x)/2)^
6 - 16*cos(c/2 + (d*x)/2)^8 + 5)/(560*a^3*d*cos(c/2 + (d*x)/2)^7*sin(c/2 +
(d*x)/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.105 $\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=103

$$\frac{4\cot^9(c+dx)}{9a^3d}+\frac{\cot^7(c+dx)}{a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}-\frac{4\csc^9(c+dx)}{9a^3d}+\frac{\csc^7(c+dx)}{a^3d}-\frac{3\csc^5(c+dx)}{5a^3d}$$

```
[Out] 3/5*cot(d*x+c)^5/a^3/d+cot(d*x+c)^7/a^3/d+4/9*cot(d*x+c)^9/a^3/d-3/5*csc(d*
x+c)^5/a^3/d+csc(d*x+c)^7/a^3/d-4/9*csc(d*x+c)^9/a^3/d
```

Rubi [A] time = 0.38, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2875, 2873, 2607, 14, 2606, 270}

$$\frac{4\cot^9(c+dx)}{9a^3d}+\frac{\cot^7(c+dx)}{a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}-\frac{4\csc^9(c+dx)}{9a^3d}+\frac{\csc^7(c+dx)}{a^3d}-\frac{3\csc^5(c+dx)}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*Cot[c + d*x]^5)/(5*a^3*d) + Cot[c + d*x]^7/(a^3*d) + (4*Cot[c + d*x]^9)/
(9*a^3*d) - (3*Csc[c + d*x]^5)/(5*a^3*d) + Csc[c + d*x]^7/(a^3*d) - (4*Csc[
c + d*x]^9)/(9*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cot^3(c+dx)\csc(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= -\frac{\int(-a+a\cos(c+dx))^3\cot^3(c+dx)\csc^7(c+dx)\,dx}{a^6}$$
$$= \frac{\int\left(-a^3\cot^6(c+dx)\csc^4(c+dx)+3a^3\cot^5(c+dx)\csc^5(c+dx)-3a^3\cot^4(c+d\right.}{a^6}$$
$$= -\frac{\int\cot^6(c+dx)\csc^4(c+dx)\,dx}{a^3}+\frac{\int\cot^3(c+dx)\csc^7(c+dx)\,dx}{a^3}+\frac{3\int\cot^5(c+}{}$$
$$= -\frac{\text{Subst}\left(\int x^6\left(-1+x^2\right)dx,x,\csc(c+dx)\right)}{a^3d}-\frac{\text{Subst}\left(\int x^6\left(1+x^2\right)dx,x,-\cot(c\right.}{a^3d}$$
$$= -\frac{\text{Subst}\left(\int\left(-x^6+x^8\right)dx,x,\csc(c+dx)\right)}{a^3d}-\frac{\text{Subst}\left(\int\left(x^6+x^8\right)dx,x,-\cot(c+d\right.}{a^3d}$$
$$= \frac{3\cot^5(c+dx)}{5a^3d}+\frac{\cot^7(c+dx)}{a^3d}+\frac{4\cot^9(c+dx)}{9a^3d}-\frac{3\csc^5(c+dx)}{5a^3d}+\frac{\csc^7(c+dx)}{a^3d}-$$

Mathematica [A] time = 0.87, size = 175, normalized size = 1.70

$$-\frac{\csc(c)(-1764\sin(c+dx)-1323\sin(2(c+dx))+98\sin(3(c+dx))+588\sin(4(c+dx))+294\sin(5(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/5760*(Csc[c]*Csc[2*(c + d*x)]^3*(5376*Sin[c] - 1152*Sin[d*x] - 1764*Sin[
c + d*x] - 1323*Sin[2*(c + d*x)] + 98*Sin[3*(c + d*x)] + 588*Sin[4*(c + d*x
)] + 294*Sin[5*(c + d*x)] + 49*Sin[6*(c + d*x)] + 3456*Sin[2*c + d*x] - 115
2*Sin[c + 2*d*x] + 2880*Sin[3*c + 2*d*x] - 128*Sin[2*c + 3*d*x] - 768*Sin[3
*c + 4*d*x] - 384*Sin[4*c + 5*d*x] - 64*Sin[5*c + 6*d*x]))/(a^3*d*(1 + Sec[
c + d*x])^3)
```

fricas [A] time = 0.70, size = 146, normalized size = 1.42

$$\frac{2\cos(dx+c)^6+6\cos(dx+c)^5+3\cos(dx+c)^4-7\cos(dx+c)^3+3\cos(dx+c)^2+6\cos(dx+c}{45\left(a^3d\cos(dx+c)^5+3a^3d\cos(dx+c)^4+2a^3d\cos(dx+c)^3-2a^3d\cos(dx+c)^2-3a^3d\cos(dx+c)-a^3d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/45*(2*cos(d*x + c)^6 + 6*cos(d*x + c)^5 + 3*cos(d*x + c)^4 - 7*cos(d*x +
c)^3 + 3*cos(d*x + c)^2 + 6*cos(d*x + c) + 2)/((a^3*d*cos(d*x + c)^5 + 3*a^
3*d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3*a^
3*d*cos(d*x + c) - a^3*d)*sin(d*x + c))
```

giac [A] time = 0.95, size = 73, normalized size = 0.71

$$-\frac{\frac{15}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}+\frac{5a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-27a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+135a^{24}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{27}}}{2880d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2880*(15/(a^3*tan(1/2*d*x + 1/2*c)^3) + (5*a^24*tan(1/2*d*x + 1/2*c)^9 -
 27*a^24*tan(1/2*d*x + 1/2*c)^5 + 135*a^24*tan(1/2*d*x + 1/2*c))/a^27)/d
```

maple [A] time = 0.81, size = 60, normalized size = 0.58

$$\frac{-\frac{\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}}{64d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/64/d/a^3*(-1/9*tan(1/2*d*x+1/2*c)^9+3/5*tan(1/2*d*x+1/2*c)^5-3*tan(1/2*d*
x+1/2*c)-1/3/tan(1/2*d*x+1/2*c)^3)
```

maxima [A] time = 0.40, size = 92, normalized size = 0.89

$$-\frac{\frac{\frac{135\sin(dx+c)}{\cos(dx+c)+1}-\frac{27\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^3}+\frac{15(\cos(dx+c)+1)^3}{a^3\sin(dx+c)^3}}{2880d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2880*((135*sin(d*x + c)/(cos(d*x + c) + 1) - 27*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 + 5*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^3 + 15*(cos(d*x + c)
 + 1)^3/(a^3*sin(d*x + c)^3))/d
```

mupad [B] time = 1.11, size = 105, normalized size = 1.02

$$-\frac{15\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}+135\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^8\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^4-27\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^4\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^8+5\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}}{2880a^3d\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^9\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + a/cos(c + d*x))^3),x)
```

```
[Out] -(15*cos(c/2 + (d*x)/2)^12 + 5*sin(c/2 + (d*x)/2)^12 - 27*cos(c/2 + (d*x)/2
)^4*sin(c/2 + (d*x)/2)^8 + 135*cos(c/2 + (d*x)/2)^8*sin(c/2 + (d*x)/2)^4)/(
2880*a^3*d*cos(c/2 + (d*x)/2)^9*sin(c/2 + (d*x)/2)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

3.106 $\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=127

$$\frac{4\cot^{11}(c+dx)}{11a^3d}+\frac{11\cot^9(c+dx)}{9a^3d}+\frac{10\cot^7(c+dx)}{7a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}-\frac{4\csc^{11}(c+dx)}{11a^3d}+\frac{7\csc^9(c+dx)}{9a^3d}-\frac{3\csc^7(c+dx)}{7a^3d}$$

```
[Out] 3/5*cot(d*x+c)^5/a^3/d+10/7*cot(d*x+c)^7/a^3/d+11/9*cot(d*x+c)^9/a^3/d+4/11
*cot(d*x+c)^11/a^3/d-3/7*csc(d*x+c)^7/a^3/d+7/9*csc(d*x+c)^9/a^3/d-4/11*csc
(d*x+c)^11/a^3/d
```

Rubi [A] time = 0.41, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2875, 2873, 2607, 270, 2606, 14}

$$\frac{4\cot^{11}(c+dx)}{11a^3d}+\frac{11\cot^9(c+dx)}{9a^3d}+\frac{10\cot^7(c+dx)}{7a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}-\frac{4\csc^{11}(c+dx)}{11a^3d}+\frac{7\csc^9(c+dx)}{9a^3d}-\frac{3\csc^7(c+dx)}{7a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*Cot[c + d*x]^5)/(5*a^3*d) + (10*Cot[c + d*x]^7)/(7*a^3*d) + (11*Cot[c +
d*x]^9)/(9*a^3*d) + (4*Cot[c + d*x]^11)/(11*a^3*d) - (3*Csc[c + d*x]^7)/(7*
a^3*d) + (7*Csc[c + d*x]^9)/(9*a^3*d) - (4*Csc[c + d*x]^11)/(11*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cot^3(c+dx)\csc^3(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= -\frac{\int (-a+a\cos(c+dx))^3\cot^3(c+dx)\csc^9(c+dx)\,dx}{a^6}$$
$$= \frac{\int \left(-a^3\cot^6(c+dx)\csc^6(c+dx)+3a^3\cot^5(c+dx)\csc^7(c+dx)-3a^3\cot^4(c+d\right.}{a^6}$$
$$= -\frac{\int \cot^6(c+dx)\csc^6(c+dx)\,dx}{a^3}+\frac{\int \cot^3(c+dx)\csc^9(c+dx)\,dx}{a^3}+\frac{3\int \cot^5(c+}{}$$
$$= -\frac{\operatorname{Subst}\left(\int x^8\left(-1+x^2\right)dx,x,\csc(c+dx)\right)}{a^3d}-\frac{\operatorname{Subst}\left(\int x^6\left(1+x^2\right)^2dx,x,-\cot(c\right.}{a^3d}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(-x^8+x^{10}\right)dx,x,\csc(c+dx)\right)}{a^3d}-\frac{\operatorname{Subst}\left(\int \left(x^6+2x^8+x^{10}\right)dx,x,-c\right.}{a^3d}$$
$$= \frac{3\cot^5(c+dx)}{5a^3d}+\frac{10\cot^7(c+dx)}{7a^3d}+\frac{11\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d}-\frac{3\csc^7(c+}{7a^3d}$$

Mathematica [A] time = 1.35, size = 223, normalized size = 1.76

$$\csc(c)(524150\sin(c+dx)+314490\sin(2(c+dx))-162010\sin(3(c+dx))-238250\sin(4(c+dx))-47650\,\mathrm{s}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Csc[c]*Csc[c + d*x]^5*Sec[c + d*x]^3*(-3886080*Sin[c] + 563200*Sin[d*x] +
524150*Sin[c + d*x] + 314490*Sin[2*(c + d*x)] - 162010*Sin[3*(c + d*x)] - 2
38250*Sin[4*(c + d*x)] - 47650*Sin[5*(c + d*x)] + 47650*Sin[6*(c + d*x)] +
28590*Sin[7*(c + d*x)] + 4765*Sin[8*(c + d*x)] - 2027520*Sin[2*c + d*x] + 1
486848*Sin[c + 2*d*x] - 2365440*Sin[3*c + 2*d*x] + 452608*Sin[2*c + 3*d*x]
+ 665600*Sin[3*c + 4*d*x] + 133120*Sin[4*c + 5*d*x] - 133120*Sin[5*c + 6*d*
x] - 79872*Sin[6*c + 7*d*x] - 13312*Sin[7*c + 8*d*x]))/(56770560*a^3*d*(1 +
 Sec[c + d*x])^3)
```

fricas [A] time = 0.68, size = 191, normalized size = 1.50

$$\frac{104\cos(dx+c)^8+312\cos(dx+c)^7+52\cos(dx+c)^6-676\cos(dx+c)^5-585\cos(dx+c)^4+325\cos}{3465\left(a^3d\cos(dx+c)^7+3a^3d\cos(dx+c)^6+a^3d\cos(dx+c)^5-5a^3d\cos(dx+c)^4-5a^3d\cos(dx+c)^3+a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3465*(104*cos(d*x + c)^8 + 312*cos(d*x + c)^7 + 52*cos(d*x + c)^6 - 676*c
os(d*x + c)^5 - 585*cos(d*x + c)^4 + 325*cos(d*x + c)^3 - 25*cos(d*x + c)^2
 - 150*cos(d*x + c) - 50)/((a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 +
 a^3*d*cos(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a
^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c))
```

giac [A] time = 0.46, size = 134, normalized size = 1.06

$$\frac{\frac{231\left(30\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-3\right)}{a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}}-\frac{315a^{30}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+770a^{30}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-990a^{30}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-4158a^{30}\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{a^{33}}}{887040d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/887040*(231*(30*tan(1/2*d*x + 1/2*c)^4 - 10*tan(1/2*d*x + 1/2*c)^2 - 3)/(
a^3*tan(1/2*d*x + 1/2*c)^5) - (315*a^30*tan(1/2*d*x + 1/2*c)^11 + 770*a^30*
tan(1/2*d*x + 1/2*c)^9 - 990*a^30*tan(1/2*d*x + 1/2*c)^7 - 4158*a^30*tan(1/
2*d*x + 1/2*c)^5 + 20790*a^30*tan(1/2*d*x + 1/2*c))/a^33)/d
```

maple [A] time = 0.80, size = 112, normalized size = 0.88

$$\frac{-\frac{\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}-\frac{2\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{9}+\frac{2\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{6\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-6\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2}{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{1}{5\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}+\frac{2}{\tan\left(\frac{dx}{2}+\right.}}{256d\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/256/d/a^3*(-1/11*tan(1/2*d*x+1/2*c)^11-2/9*tan(1/2*d*x+1/2*c)^9+2/7*tan(1
/2*d*x+1/2*c)^7+6/5*tan(1/2*d*x+1/2*c)^5-6*tan(1/2*d*x+1/2*c)-2/3/tan(1/2*d
*x+1/2*c)^3-1/5/tan(1/2*d*x+1/2*c)^5+2/tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.56, size = 174, normalized size = 1.37

$$-\frac{\frac{\frac{20790\sin(dx+c)}{\cos(dx+c)+1}-\frac{4158\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{990\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{770\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}+\frac{315\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^{3}}+\frac{231\left(\frac{10\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{30\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+3\right)(\cos(dx+c)+1)^{5}}{a^{3}\sin(dx+c)^{5}}}{887040d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/887040*((20790*sin(d*x + c)/(cos(d*x + c) + 1) - 4158*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5 - 990*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 770*sin(d*x +
 c)^9/(cos(d*x + c) + 1)^9 + 315*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/a^3
 + 231*(10*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 30*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 3)*(cos(d*x + c) + 1)^5/(a^3*sin(d*x + c)^5))/d
```

mupad [B] time = 1.43, size = 201, normalized size = 1.58

$$-\frac{693\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{16}+2310\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{14}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{2}-6930\cos\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}\sin\left(\frac{c}{2}+\frac{dx}{2}\right)^{4}+20790\cos\left(\frac{c}{2}+\right.}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^6*(a + a/cos(c + d*x))^3),x)
```

```
[Out] -(693*cos(c/2 + (d*x)/2)^16 + 315*sin(c/2 + (d*x)/2)^16 + 770*cos(c/2 + (d*
x)/2)^2*sin(c/2 + (d*x)/2)^14 - 990*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)
^12 - 4158*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^10 + 20790*cos(c/2 + (d*
x)/2)^10*sin(c/2 + (d*x)/2)^6 - 6930*cos(c/2 + (d*x)/2)^12*sin(c/2 + (d*x)/
2)^4 + 2310*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^2)/(887040*a^3*d*cos(c
/2 + (d*x)/2)^11*sin(c/2 + (d*x)/2)^5)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.107 $\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=145

$$\frac{4\cot^{13}(c+dx)}{13a^3d}+\frac{15\cot^{11}(c+dx)}{11a^3d}+\frac{7\cot^{9}(c+dx)}{3a^3d}+\frac{13\cot^{7}(c+dx)}{7a^3d}+\frac{3\cot^{5}(c+dx)}{5a^3d}-\frac{4\csc^{13}(c+dx)}{13a^3d}+\frac{7\csc^{11}(c+}{11a^3d}$$

```
[Out] 3/5*cot(d*x+c)^5/a^3/d+13/7*cot(d*x+c)^7/a^3/d+7/3*cot(d*x+c)^9/a^3/d+15/11
*cot(d*x+c)^11/a^3/d+4/13*cot(d*x+c)^13/a^3/d-1/3*csc(d*x+c)^9/a^3/d+7/11*c
sc(d*x+c)^11/a^3/d-4/13*csc(d*x+c)^13/a^3/d
```

Rubi [A] time = 0.42, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2875, 2873, 2607, 270, 2606, 14}

$$\frac{4\cot^{13}(c+dx)}{13a^3d}+\frac{15\cot^{11}(c+dx)}{11a^3d}+\frac{7\cot^{9}(c+dx)}{3a^3d}+\frac{13\cot^{7}(c+dx)}{7a^3d}+\frac{3\cot^{5}(c+dx)}{5a^3d}-\frac{4\csc^{13}(c+dx)}{13a^3d}+\frac{7\csc^{11}(c+}{11a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*Cot[c + d*x]^5)/(5*a^3*d) + (13*Cot[c + d*x]^7)/(7*a^3*d) + (7*Cot[c + d
*x]^9)/(3*a^3*d) + (15*Cot[c + d*x]^11)/(11*a^3*d) + (4*Cot[c + d*x]^13)/(1
3*a^3*d) - Csc[c + d*x]^9/(3*a^3*d) + (7*Csc[c + d*x]^11)/(11*a^3*d) - (4*C
sc[c + d*x]^13)/(13*a^3*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cot^3(c+dx)\csc^5(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= -\frac{\int (-a+a\cos(c+dx))^3\cot^3(c+dx)\csc^{11}(c+dx)\,dx}{a^6} \\
&= \frac{\int \left(-a^3\cot^6(c+dx)\csc^8(c+dx)+3a^3\cot^5(c+dx)\csc^9(c+dx)-3a^3\cot^4(c+d\right.}{a^6} \\
&= -\frac{\int \cot^6(c+dx)\csc^8(c+dx)\,dx}{a^3}+\frac{\int \cot^3(c+dx)\csc^{11}(c+dx)\,dx}{a^3}+\frac{3\int \cot^5(c+}{} \\
&= -\frac{\mathrm{Subst}\left(\int x^{10}\left(-1+x^2\right)dx,x,\csc(c+dx)\right)}{a^3d}-\frac{\mathrm{Subst}\left(\int x^6\left(1+x^2\right)^3 dx,x,-\cot(\right.}{a^3d} \\
&= -\frac{\mathrm{Subst}\left(\int \left(-x^{10}+x^{12}\right)dx,x,\csc(c+dx)\right)}{a^3d}-\frac{\mathrm{Subst}\left(\int \left(x^6+3x^8+3x^{10}+x^{12}\right)\right.}{a^3d} \\
&= \frac{3\cot^5(c+dx)}{5a^3d}+\frac{13\cot^7(c+dx)}{7a^3d}+\frac{7\cot^9(c+dx)}{3a^3d}+\frac{15\cot^{11}(c+dx)}{11a^3d}+\frac{4\cot^{13}(c}{13a^3}
\end{aligned}
$$

Mathematica [A] time = 1.93, size = 265, normalized size = 1.83

$$
-\frac{\csc(c)(-2764580\sin(c+dx)-1382290\sin(2(c+dx))+1275960\sin(3(c+dx))+1336720\sin(4(c+dx))-}{}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/984023040*(Csc[c]*Csc[c + d*x]^7*Sec[c + d*x]^3*(49201152*Sin[c] - 63365
12*Sin[d*x] - 2764580*Sin[c + d*x] - 1382290*Sin[2*(c + d*x)] + 1275960*Sin
[3*(c + d*x)] + 1336720*Sin[4*(c + d*x)] - 60760*Sin[5*(c + d*x)] - 524055*
Sin[6*(c + d*x)] - 167090*Sin[7*(c + d*x)] + 60760*Sin[8*(c + d*x)] + 45570
*Sin[9*(c + d*x)] + 7595*Sin[10*(c + d*x)] + 20500480*Sin[2*c + d*x] - 2366
8736*Sin[c + 2*d*x] + 30750720*Sin[3*c + 2*d*x] - 6537216*Sin[2*c + 3*d*x]
- 6848512*Sin[3*c + 4*d*x] + 311296*Sin[4*c + 5*d*x] + 2684928*Sin[5*c + 6*
d*x] + 856064*Sin[6*c + 7*d*x] - 311296*Sin[7*c + 8*d*x] - 233472*Sin[8*c +
 9*d*x] - 38912*Sin[9*c + 10*d*x]))/(a^3*d*(1 + Sec[c + d*x])^3)
```

fricas [A] time = 0.64, size = 214, normalized size = 1.48

$$
\frac{304\cos(dx+c)^{10}+912\cos(dx+c)^9-152\cos(dx+c)^8-2888\cos(dx+c)^7-1862\cos(dx+c)^6+2926}{15015\left(a^3d\cos(dx+c)^9+3a^3d\cos(dx+c)^8-8a^3d\cos(dx+c)^6-6a^3d\cos(dx+c)^5+\right.}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15015*(304*cos(d*x + c)^10 + 912*cos(d*x + c)^9 - 152*cos(d*x + c)^8 - 28
88*cos(d*x + c)^7 - 1862*cos(d*x + c)^6 + 2926*cos(d*x + c)^5 + 3325*cos(d*
x + c)^4 - 665*cos(d*x + c)^3 - 35*cos(d*x + c)^2 + 210*cos(d*x + c) + 70)/
((a^3*d*cos(d*x + c)^9 + 3*a^3*d*cos(d*x + c)^8 - 8*a^3*d*cos(d*x + c)^6 -
6*a^3*d*cos(d*x + c)^5 + 6*a^3*d*cos(d*x + c)^4 + 8*a^3*d*cos(d*x + c)^3 -
3*a^3*d*cos(d*x + c) - a^3*d)*sin(d*x + c))
```

giac [A] time = 2.02, size = 163, normalized size = 1.12

$$\frac{\frac{429\left(280\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}-35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}-28\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-5\right)}{a^{3}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}}-\frac{1155\,a^{36}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}+5460\,a^{36}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+5005\,a^{36}\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.}{}}{15375360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/15375360*(429*(280*tan(1/2*d*x + 1/2*c)^6 - 35*tan(1/2*d*x + 1/2*c)^4 - 2
8*tan(1/2*d*x + 1/2*c)^2 - 5)/(a^3*tan(1/2*d*x + 1/2*c)^7) - (1155*a^36*tan
(1/2*d*x + 1/2*c)^13 + 5460*a^36*tan(1/2*d*x + 1/2*c)^11 + 5005*a^36*tan(1/
2*d*x + 1/2*c)^9 - 17160*a^36*tan(1/2*d*x + 1/2*c)^7 - 42042*a^36*tan(1/2*d
*x + 1/2*c)^5 + 210210*a^36*tan(1/2*d*x + 1/2*c))/a^39)/d
```

maple [A] time = 0.82, size = 138, normalized size = 0.95

$$\frac{-\frac{\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{13}-\frac{4\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{11}-\frac{\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{8\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{14\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-14\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{7\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}-\frac{}{\tan(}}{1024d\,a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/1024/d/a^3*(-1/13*tan(1/2*d*x+1/2*c)^13-4/11*tan(1/2*d*x+1/2*c)^11-1/3*ta
n(1/2*d*x+1/2*c)^9+8/7*tan(1/2*d*x+1/2*c)^7+14/5*tan(1/2*d*x+1/2*c)^5-14*ta
n(1/2*d*x+1/2*c)-1/7/tan(1/2*d*x+1/2*c)^7-1/tan(1/2*d*x+1/2*c)^3-4/5/tan(1/
2*d*x+1/2*c)^5+8/tan(1/2*d*x+1/2*c))
```

maxima [A] time = 0.59, size = 214, normalized size = 1.48

$$-\frac{\frac{\frac{210210\sin(dx+c)}{\cos(dx+c)+1}-\frac{42042\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{17160\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}+\frac{5005\sin(dx+c)^{9}}{(\cos(dx+c)+1)^{9}}+\frac{5460\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{1155\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}}{a^{3}}+\frac{429\left(\frac{28\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{35\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{280\sin(d}{(\cos(dx+c}\right.}{a^{3}\sin(dx+c)^{7}}}{15375360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/15375360*((210210*sin(d*x + c)/(cos(d*x + c) + 1) - 42042*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 - 17160*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 5005*si
n(d*x + c)^9/(cos(d*x + c) + 1)^9 + 5460*sin(d*x + c)^11/(cos(d*x + c) + 1)
^11 + 1155*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)/a^3 + 429*(28*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 35*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 280*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5)*(cos(d*x + c) + 1)^7/(a^3*sin(d*x + c
)^7))/d
```

mupad [B] time = 2.13, size = 249, normalized size = 1.72

$$-\frac{2145\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{20}+12012\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{18}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+15015\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{16}\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-120120\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^8*(a + a/cos(c + d*x))^3),x)
```

```
[Out] -(2145*cos(c/2 + (d*x)/2)^20 + 1155*sin(c/2 + (d*x)/2)^20 + 5460*cos(c/2 +
(d*x)/2)^2*sin(c/2 + (d*x)/2)^18 + 5005*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x
)/2)^16 - 17160*cos(c/2 + (d*x)/2)^6*sin(c/2 + (d*x)/2)^14 - 42042*cos(c/2
+ (d*x)/2)^8*sin(c/2 + (d*x)/2)^12 + 210210*cos(c/2 + (d*x)/2)^12*sin(c/2 +
 (d*x)/2)^8 - 120120*cos(c/2 + (d*x)/2)^14*sin(c/2 + (d*x)/2)^6 + 15015*cos
(c/2 + (d*x)/2)^16*sin(c/2 + (d*x)/2)^4 + 12012*cos(c/2 + (d*x)/2)^18*sin(c
/2 + (d*x)/2)^2)/(15375360*a^3*d*cos(c/2 + (d*x)/2)^13*sin(c/2 + (d*x)/2)^7
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.108 $\int (a + a\sec(c + dx))(e\sin(c + dx))^{5/2}\,dx$

Optimal. Leaf size=157

$$-\frac{ae^{5/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{6ae^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{2ae(e\sin(c+d}{3d}$$

```
[Out] -a*e^(5/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d+a*e^(5/2)*arctanh((e*sin(
d*x+c))^(1/2)/e^(1/2))/d-2/3*a*e*(e*sin(d*x+c))^(3/2)/d-2/5*a*e*cos(d*x+c)*
(e*sin(d*x+c))^(3/2)/d-6/5*a*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/
2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x
+c))^(1/2)/d/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3872, 2838, 2564, 321, 329, 298, 203, 206, 2635, 2640, 2639}

$$-\frac{ae^{5/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{6ae^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{2ae(e\sin(c+d}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((a*e^(5/2)*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d) + (a*e^(5/2)*ArcTanh[
Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (6*a*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2
]*Sqrt[e*Sin[c + d*x]])/(5*d*Sqrt[Sin[c + d*x]]) - (2*a*e*(e*Sin[c + d*x])^
(3/2))/(3*d) - (2*a*e*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2))/(5*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^{5/2}\,dx = -\int (-a-a\cos(c+dx))\sec(c+dx)(e\sin(c+dx))^{5/2}\,dx$$
$$= a\int (e\sin(c+dx))^{5/2}\,dx + a\int \sec(c+dx)(e\sin(c+dx))^{5/2}\,dx$$
$$= -\frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} + \frac{a\,\text{Subst}\left(\int \frac{x^{5/2}}{1-\frac{x^2}{e^2}}\,dx, x, e\sin(c+a\right.}{de}$$
$$= -\frac{2ae(e\sin(c+dx))^{3/2}}{3d} - \frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} + \frac{(ae)\,\text{Subs}}{}$$
$$= \frac{6ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{2ae(e\sin(c+dx))^{3/2}}{3d} - 2a$$
$$= \frac{6ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{2ae(e\sin(c+dx))^{3/2}}{3d} - 2a$$
$$= -\frac{ae^{5/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + 6ae^2E\left(\frac{1}{2}\right.(c$$

Mathematica [A] time = 0.31, size = 106, normalized size = 0.68

$$-\frac{a(e\sin(c+dx))^{5/2}\left(10\sin^{\frac{3}{2}}(c+dx)+3\sin(2(c+dx))\sqrt{\sin(c+dx)}+18E\left(\frac{1}{4}(-2c-2dx+\pi)\middle|2\right)+15\tan^{-1}\left(\sqrt{s}\right.\right.}{15d\sin^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/15*(a*(e*Sin[c + d*x])^(5/2)*(15*ArcTan[Sqrt[Sin[c + d*x]]] - 15*ArcTanh
[Sqrt[Sin[c + d*x]]] + 18*EllipticE[(-2*c + Pi - 2*d*x)/4, 2] + 10*Sin[c +
d*x]^(3/2) + 3*Sqrt[Sin[c + d*x]]*Sin[2*(c + d*x)]))/(d*Sin[c + d*x]^(5/2))
```

fricas [F] time = 0.97, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(ae^2\cos(dx+c)^2 - ae^2 + \left(ae^2\cos(dx+c)^2 - ae^2\right)\sec(dx+c)\right)\sqrt{e\sin(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*e^2*cos(d*x + c)^2 - a*e^2 + (a*e^2*cos(d*x + c)^2 - a*e^2)*se
c(d*x + c))*sqrt(e*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)(e\sin(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(5/2), x)
```

maple [A] time = 3.71, size = 290, normalized size = 1.85

$$-\frac{2ae\left(e\sin\left(dx+c\right)\right)^{\frac{3}{2}}}{3d}-\frac{a\,e^{\frac{5}{2}}\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{d}+\frac{a\,e^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{d}+\frac{2a\,e^{3}\left(\sin^{4}\left(dx+c\right)\right)}{5d\cos\left(dx+c\right)\sqrt{e\sin\left(dx+c\right)}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/3*a*e*(e*sin(d*x+c))^(3/2)/d-a*e^(5/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/
2))/d+a*e^(5/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))/d+2/5/d*a*e^3/cos(d*x
+c)/(e*sin(d*x+c))^(1/2)*sin(d*x+c)^4-6/5/d*a*e^3/cos(d*x+c)/(e*sin(d*x+c))
^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ellipt
icE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3/5/d*a*e^3/cos(d*x+c)/(e*sin(d*x+c)
)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ellip
ticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2/5/d*a*e^3/cos(d*x+c)/(e*sin(d*x+c
))^(1/2)*sin(d*x+c)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(dx+c\right)+a\right)\left(e\sin\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(e\,\sin\left(c+d\,x\right)\right)}^{5/2}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x)),x)
```

```
[Out] int((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.109 $\int (a + a\sec(c + dx))(e\sin(c + dx))^{3/2}\,dx$

Optimal. Leaf size=154

$$\frac{ae^{3/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2ae^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{2ae\sqrt{e\sin(c+dx)}}{d}$$

```
[Out] a*e^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d+a*e^(3/2)*arctanh((e*sin(d
*x+c))^(1/2)/e^(1/2))/d-2/3*a*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c
)^(1/2)/d/(e*sin(d*x+c))^(1/2)-2*a*e*(e*sin(d*x+c))^(1/2)/d-2/3*a*e*cos(d*x
+c)*(e*sin(d*x+c))^(1/2)/d
```

Rubi [A] time = 0.20, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3872, 2838, 2564, 321, 329, 212, 206, 203, 2635, 2642, 2641}

$$\frac{ae^{3/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2ae^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{2ae\sqrt{e\sin(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*e^(3/2)*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (a*e^(3/2)*ArcTanh[Sqr
t[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*S
qrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (2*a*e*Sqrt[e*Sin[c + d*x]]
)/d - (2*a*e*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(3*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^{3/2}\,dx = -\int (-a-a\cos(c+dx))\sec(c+dx)(e\sin(c+dx))^{3/2}\,dx$$
$$= a\int (e\sin(c+dx))^{3/2}\,dx + a\int \sec(c+dx)(e\sin(c+dx))^{3/2}\,dx$$
$$= -\frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d} + \frac{a\,\text{Subst}\left(\int \frac{x^{3/2}}{1-\frac{x^2}{e^2}}\,dx, x, e\sin(c+dx\right.}{de}$$
$$= -\frac{2ae\sqrt{e\sin(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d} + \frac{(ae)\,\text{Subst}\left(\right.}{}$$
$$= \frac{2ae^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} - \frac{2ae\sqrt{e\sin(c+dx)}}{d} - \frac{2ae\,c}{}$$
$$= \frac{2ae^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} - \frac{2ae\sqrt{e\sin(c+dx)}}{d} - \frac{2ae\,c}{}$$
$$= \frac{ae^{3/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2ae^2F\left(\frac{1}{2}\left(c-\right.\right.}{3d}$$

Mathematica [A] time = 0.59, size = 170, normalized size = 1.10

$$\frac{a(e\sin(c+dx))^{3/2}\left(-24\sqrt{\sin(c+dx)} - 8F\left(\frac{1}{4}(-2c-2dx+\pi)\middle|2\right) - 3\log\left(1-\sqrt{\sin(c+dx)}\right) + 3\log\left(\sqrt{\sin(c+d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*(e*Sin[c + d*x])^(3/2)*(12*ArcTan[Sqrt[Sin[c + d*x]]] + 6*ArcTanh[Sqrt[S
in[c + d*x]]] - 8*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] - 3*Log[1 - Sqrt[Sin[
c + d*x]]] + 3*Log[1 + Sqrt[Sin[c + d*x]]] - 24*Sqrt[Sin[c + d*x]] - 8*Cos[
c + d*x]*Sec[2*(c + d*x)]*Sqrt[Sin[c + d*x]] + 16*Cos[c + d*x]*Sec[2*(c + d
*x)]*Sin[c + d*x]^(5/2)))/(12*d*Sin[c + d*x]^(3/2))
```

fricas [F] time = 1.89, size = 0, normalized size = 0.00

$$\text{integral}\left((ae\sec(dx+c)+ae)\sqrt{e\sin(dx+c)}\,\sin(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*e*sec(d*x + c) + a*e)*sqrt(e*sin(d*x + c))*sin(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)(e\sin(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(3/2), x)
```

maple [A] time = 3.63, size = 210, normalized size = 1.36

$$\frac{a\,e^{\frac{3}{2}}\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{d}+\frac{a\,e^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{d}-\frac{2ae\sqrt{e\sin\left(dx+c\right)}}{d}-\frac{a\,e^{2}\sqrt{-\sin\left(dx+c\right)+1}\,\sqrt{2\sin\left(dx\right.}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*sin(d*x+c))^(3/2),x)
```

```
[Out] a*e^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d+a*e^(3/2)*arctanh((e*sin(d
*x+c))^(1/2)/e^(1/2))/d-2*a*e*(e*sin(d*x+c))^(1/2)/d-1/3/d*a*e^2/cos(d*x+c)
/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+
c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2/3/d*a*e^2/cos(d*x+c
)/(e*sin(d*x+c))^(1/2)*sin(d*x+c)^3-2/3/d*a*e^2/cos(d*x+c)/(e*sin(d*x+c))^(
1/2)*sin(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(dx+c\right)+a\right)\left(e\sin\left(dx+c\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\sin\left(c+d\,x\right)\right)}^{3/2}\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x)),x)
```

```
[Out] int((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.110 $\int (a+a\sec(c+dx))\sqrt{e\sin(c+dx)}\,dx$

Optimal. Leaf size=104

$$-\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{a\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}$$

```
[Out] -a*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d+a*arctanh((e*sin(d*x+c))^
(1/2)/e^(1/2))*e^(1/2)/d-2*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+
1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))
^(1/2)/d/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3872, 2838, 2564, 329, 298, 203, 206, 2640, 2639}

$$-\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{a\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] -((a*Sqrt[e]*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d) + (a*Sqrt[e]*ArcTanh[
Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sq
rt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(c+dx))\sqrt{e\sin(c+dx)}\,dx &= -\int (-a-a\cos(c+dx))\sec(c+dx)\sqrt{e\sin(c+dx)}\,dx \\
&= a\int \sqrt{e\sin(c+dx)}\,dx + a\int \sec(c+dx)\sqrt{e\sin(c+dx)}\,dx \\
&= \frac{a\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{e^2}}\,dx, x, e\sin(c+dx)\right)}{de} + \frac{\left(a\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c}}{\sqrt{\sin(c+dx)}} \\
&= \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{(2a)\,\text{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{e^2}}\,dx, x,\right.}{de} \\
&= \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{(ae)\,\text{Subst}\left(\int \frac{1}{e-x^2}\,dx, x, \sqrt{}\right.}{d} \\
&= -\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{a\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2aE\left(\frac{1}{2}\left(c\right.\right.}{}
\end{aligned}$$

Mathematica [A] time = 0.12, size = 69, normalized size = 0.66

$$\frac{a\sqrt{e\sin(c+dx)}\left(-2E\left(\frac{1}{4}(-2c-2dx+\pi)\middle|2\right)-\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)+\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)\right)}{d\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (a*(-ArcTan[Sqrt[Sin[c + d*x]]] + ArcTanh[Sqrt[Sin[c + d*x]]] - 2*EllipticE
[(-2*c + Pi - 2*d*x)/4, 2])*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]])
```

fricas [F] time = 1.06, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)\sqrt{e\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a\sec(dx+c)+a)\sqrt{e\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c)), x)
```

maple [A] time = 3.64, size = 198, normalized size = 1.90

$$-\frac{a\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)\sqrt{e}}{d}+\frac{a\,\mathrm{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)\sqrt{e}}{d}-\frac{2ae\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}\,(dx\right.}{d\cos(dx+c)\sqrt{e\sin(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*sin(d*x+c))^(1/2),x)
```

```
[Out] -a*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d+a*arctanh((e*sin(d*x+c))^
(1/2)/e^(1/2))*e^(1/2)/d-2/d*a*e*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/
2)*sin(d*x+c)^(1/2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+
1)^(1/2),1/2*2^(1/2))+1/d*a*e*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^
(1/2),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a\sec(dx+c)+a)\sqrt{e\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{e\,\sin(c+d\,x)}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x)),x)
```

```
[Out] int((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\sqrt{e\sin(c+dx)}\,dx+\int\sqrt{e\sin(c+dx)}\sec(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(e*sin(c + d*x)), x) + Integral(sqrt(e*sin(c + d*x))*sec(c
+ d*x), x))
```

3.111 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=103

$$\frac{a\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{2a\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{e\sin(c+dx)}}$$

```
[Out] a*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(1/2)+a*arctanh((e*sin(d*x+c))^(
1/2)/e^(1/2))/d/e^(1/2)-2*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2
)/d/(e*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3872, 2838, 2564, 329, 212, 206, 203, 2642, 2641}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{2a\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (a*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*Sqrt[e]) + (a*ArcTanh[Sqrt[e*Si
n[c + d*x]]/Sqrt[e]])/(d*Sqrt[e]) + (2*a*EllipticF[(c - Pi/2 + d*x)/2, 2]*S
qrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+a\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx &= -\int \frac{(-a-a\cos(c+dx))\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx \\
&= a\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx + a\int \frac{\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx \\
&= \frac{a\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{e^2}\right)}\,dx, x, e\sin(c+dx)\right)}{de} + \frac{\left(a\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{\sqrt{e\sin(c+dx)}} \\
&= \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{1-\frac{x^4}{e^2}}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{de} \\
&= \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{a\,\mathrm{Subst}\left(\int \frac{1}{e-x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d} + \frac{a\,\mathrm{S}}{}\cdots \\
&= \frac{a\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+a}}{d\sqrt{e\sin(c+dx)}}\cdots
\end{aligned}
$$

Mathematica [A] time = 3.18, size = 193, normalized size = 1.87

$$
\frac{4a\cos\left(\frac{1}{2}(c+dx)\right)\left(4F\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{4}(c+dx)\right)}}\right)\middle|-1\right)+\sqrt{2}\left(\Pi\left(-1-\sqrt{2};\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{4}(c+dx)\right)}}\right)\middle|-1\right)-\Pi\left(1-\sqrt{\cdots}\right.\right.\right.}{d\sqrt{\tan\left(\frac{1}{4}(c+dx)\right)}\sqrt{1-\cdots}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (4*a*Cos[(c + d*x)/2]*(4*EllipticF[ArcSin[1/Sqrt[Tan[(c + d*x)/4]]], -1] +
Sqrt[2]*(EllipticPi[-1 - Sqrt[2], ArcSin[1/Sqrt[Tan[(c + d*x)/4]]], -1] - E
llipticPi[1 - Sqrt[2], ArcSin[1/Sqrt[Tan[(c + d*x)/4]]], -1] - EllipticPi[-
1 + Sqrt[2], ArcSin[1/Sqrt[Tan[(c + d*x)/4]]], -1] + EllipticPi[1 + Sqrt[2]
, ArcSin[1/Sqrt[Tan[(c + d*x)/4]]], -1])))/(d*Sqrt[1 - Cot[(c + d*x)/4]^2]*
Sqrt[e*Sin[c + d*x]]*Sqrt[Tan[(c + d*x)/4]])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(a\sec(dx+c)+a)\sqrt{e\sin(dx+c)}}{e\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c))/(e*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec(dx+c)+a}{\sqrt{e\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(e*sin(d*x + c)), x)
```

maple [A] time = 2.54, size = 122, normalized size = 1.18

$$\frac{a\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{a\,\mathrm{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{d\sqrt{e}}-\frac{a\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}\,(dx+c)\right)\mathrm{Ellipt}}{d\cos(dx+c)\sqrt{e\sin(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x)
```

```
[Out] a*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(1/2)+a*arctanh((e*sin(d*x+c))^(
1/2)/e^(1/2))/d/e^(1/2)-1/d*a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))/cos(d*x+c)/(e
*sin(d*x+c))^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec(dx+c)+a}{\sqrt{e\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(e*sin(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{\sqrt{e\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx+\int \frac{\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(e*sin(c + d*x)), x) + Integral(sec(c + d*x)/sqrt(e*sin(c
 + d*x)), x))
```

3.112 $\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=155

$$-\frac{a\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}-\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}-\frac{2a}{de\sqrt{e\sin(c+dx)}}-\frac{2a\,c}{de\sqrt{e}}$$

```
[Out] -a*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)+a*arctanh((e*sin(d*x+c))^
(1/2)/e^(1/2))/d/e^(3/2)-2*a/d/e/(e*sin(d*x+c))^(1/2)-2*a*cos(d*x+c)/d/e/(e
*sin(d*x+c))^(1/2)+2*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi
+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)
/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3872, 2838, 2564, 325, 329, 298, 203, 206, 2636, 2640, 2639}

$$-\frac{a\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}-\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}-\frac{2a}{de\sqrt{e\sin(c+dx)}}-\frac{2a\,c}{de\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((a*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(3/2))) + (a*ArcTanh[Sqrt[e
*Sin[c + d*x]]/Sqrt[e]])/(d*e^(3/2)) - (2*a)/(d*e*Sqrt[e*Sin[c + d*x]]) - (
2*a*Cos[c + d*x])/(d*e*Sqrt[e*Sin[c + d*x]]) - (2*a*EllipticE[(c - Pi/2 + d
*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*e^2*Sqrt[Sin[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = -\int \frac{(-a-a\cos(c+dx))\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx$$

$$= a\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx + a\int \frac{\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx$$

$$= -\frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{a\int\sqrt{e\sin(c+dx)}\,dx}{e^2} + \frac{a\,\text{Subst}\left(\int \frac{1}{x^{3/2}\left(1-\frac{x^2}{e^2}\right)}dx, x, e\sin(c+dx)\right)}{de}$$

$$= -\frac{2a}{de\sqrt{e\sin(c+dx)}} - \frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{e^2}}dx, x, e\sin(c+dx)\right)}{de^3} - \frac{(a\sqrt{e}}{}$$

$$= -\frac{2a}{de\sqrt{e\sin(c+dx)}} - \frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} + \frac{(2}{}$$

$$= -\frac{2a}{de\sqrt{e\sin(c+dx)}} - \frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} + \frac{a\,S}{}$$

$$= -\frac{a\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} - \frac{2a}{de\sqrt{e\sin(c+dx)}} - \frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx}}$$

Mathematica [A] time = 0.40, size = 143, normalized size = 0.92

$$-\frac{a\sin^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)\sec\left(\frac{1}{2}(c+dx)\right)\left(2\sqrt{\sin(c+dx)}\csc\left(\frac{1}{2}(c+dx)\right)-2\sec\left(\frac{1}{2}(c+dx)\right)E\left(\frac{1}{4}(-2c-2\right.\right.}{2d(e\sin(c+dx))^{3/}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] -1/2*(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]*(ArcTan[Sqrt[Sin[c + d*x]]]*Sec
[(c + d*x)/2] - ArcTanh[Sqrt[Sin[c + d*x]]]*Sec[(c + d*x)/2] - 2*EllipticE[
(-2*c + Pi - 2*d*x)/4, 2]*Sec[(c + d*x)/2] + 2*Csc[(c + d*x)/2]*Sqrt[Sin[c
+ d*x]])*Sin[c + d*x]^(3/2))/(d*(e*Sin[c + d*x])^(3/2))
```

fricas [F] time = 1.13, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(a\sec(dx+c)+a)\sqrt{e\sin(dx+c)}}{e^2\cos(dx+c)^2-e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c))/(e^2*cos(d*x + c)^2 - e
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec(dx+c)+a}{(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/(e*sin(d*x + c))^(3/2), x)
```

maple [A] time = 3.25, size = 247, normalized size = 1.59

$$-\frac{a\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{d\,e^{\frac{3}{2}}}+\frac{a\,\mathrm{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{d\,e^{\frac{3}{2}}}-\frac{2a}{de\sqrt{e\sin\left(dx+c\right)}}+\frac{2a\sqrt{-\sin\left(dx+c\right)+1}\,\sqrt{2\sin\left(dx+c\right)+}}{de\cos\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x)
```

```
[Out] -a*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)+a*arctanh((e*sin(d*x+c))^
(1/2)/e^(1/2))/d/e^(3/2)-2*a/d/e/(e*sin(d*x+c))^(1/2)+2/d*a/e/cos(d*x+c)/(e
*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^
(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/d*a/e/cos(d*x+c)/(e*si
n(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/
2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*a*cos(d*x+c)/d/e/(e*sin(d
*x+c))^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{{\left(e\,\sin\left(c+d\,x\right)\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{\left(e\sin\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx+\int \frac{\sec\left(c+dx\right)}{\left(e\sin\left(c+dx\right)\right)^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))**(3/2),x)
```

```
[Out] a*(Integral((e*sin(c + d*x))**(-3/2), x) + Integral(sec(c + d*x)/(e*sin(c +
 d*x))**(3/2), x))
```

3.113 $\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=160

$$\frac{a\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{2a\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3de^2\sqrt{e\sin(c+dx)}}-\frac{2a}{3de(e\sin(c+dx))^{3/2}}-\frac{2a}{3de(e$$

```
[Out] a*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)+a*arctanh((e*sin(d*x+c))^(
1/2)/e^(1/2))/d/e^(5/2)-2/3*a/d/e/(e*sin(d*x+c))^(3/2)-2/3*a*cos(d*x+c)/d/e
/(e*sin(d*x+c))^(3/2)-2/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2
)/d/e^2/(e*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3872, 2838, 2564, 325, 329, 212, 206, 203, 2636, 2642, 2641}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{2a\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3de^2\sqrt{e\sin(c+dx)}}-\frac{2a}{3de(e\sin(c+dx))^{3/2}}-\frac{2a}{3de(e$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (a*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(5/2)) + (a*ArcTanh[Sqrt[e*Si
n[c + d*x]]/Sqrt[e]])/(d*e^(5/2)) - (2*a)/(3*d*e*(e*Sin[c + d*x])^(3/2)) -
(2*a*Cos[c + d*x])/(3*d*e*(e*Sin[c + d*x])^(3/2)) + (2*a*EllipticF[(c - Pi/
2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = -\int \frac{(-a-a\cos(c+dx))\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx$$
$$= a\int \frac{1}{(e\sin(c+dx))^{5/2}}\,dx + a\int \frac{\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx$$
$$= -\frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}}dx}{3e^2} + \frac{a\,\text{Subst}\left(\int \frac{1}{x^{5/2}\left(1-\frac{x^2}{e^2}\right)}dx, x, e\sin(c+dx)\right)}{de}$$
$$= -\frac{2a}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{e^2}\right)}dx, x, e\sin(c+dx\right.}{de^3}$$
$$= -\frac{2a}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}}$$
$$= -\frac{2a}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}}$$
$$= \frac{a\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} - \frac{2a}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+}{3de(e\sin(c+}$$

Mathematica [A] time = 0.37, size = 120, normalized size = 0.75

$$-\frac{a\sqrt{\sin(c+dx)}\,(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(2F\left(\frac{1}{4}(-2c-2dx+\pi)\middle|2\right)-3\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)-3\tanh^{-1}\right.}{6de^2\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/6*(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(-3*ArcTan[Sqrt[Sin[c + d*x]]
] - 3*ArcTanh[Sqrt[Sin[c + d*x]]] + 2*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] +
 Csc[(c + d*x)/2]^2*Sqrt[Sin[c + d*x]])*Sqrt[Sin[c + d*x]])/(d*e^2*Sqrt[e*S
in[c + d*x]])
```

fricas [F] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(a\sec(dx+c)+a)\sqrt{e\sin(dx+c)}}{\left(e^3\cos(dx+c)^2-e^3\right)\sin(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c))/((e^3*cos(d*x + c)^2 -
e^3)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec(dx+c)+a}{(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)/(e*sin(d*x + c))^(5/2), x)
```

maple [A] time = 3.47, size = 212, normalized size = 1.32

$$\frac{a\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{d\,e^{\frac{5}{2}}}+\frac{a\,\mathrm{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)}{d\,e^{\frac{5}{2}}}-\frac{2a}{3de\,(e\sin(dx+c))^{\frac{3}{2}}}-\frac{a\left(\sqrt{\sin}\,(dx+c)\right)\sqrt{-\sin(dx+c)+1}\,\cdots}{3d\,e^{2}\cos\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x)
[Out] a*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)+a*arctanh((e*sin(d*x+c))^(
1/2)/e^(1/2))/d/e^(5/2)-2/3*a/d/e/(e*sin(d*x+c))^(3/2)-1/3/d*a/e^2*sin(d*x+
c)^(1/2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2/3/d*a/e^2*sin(d*x
+c)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)-2/3/d*a/e^2/sin(d*x+c)/cos(d*x+c)/(e*si
n(d*x+c))^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{{\left(e\,\sin\left(c+d\,x\right)\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(5/2),x)
[Out] int((a + a/cos(c + d*x))/(e*sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.114 $\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=194

$$-\frac{2a^2e^{5/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2e^{5/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}-\frac{9a^2e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{4a^2e(e\sin}{}$$

```
[Out] -2*a^2*e^(5/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d+2*a^2*e^(5/2)*arctanh
((e*sin(d*x+c))^(1/2)/e^(1/2))/d-4/3*a^2*e*(e*sin(d*x+c))^(3/2)/d-2/5*a^2*e
*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/d+a^2*e*sec(d*x+c)*(e*sin(d*x+c))^(3/2)/d+
9/5*a^2*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*E
llipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/sin(d*x+
c)^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3872, 2873, 2635, 2640, 2639, 2564, 321, 329, 298, 203, 206, 2566}

$$-\frac{2a^2e^{5/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2e^{5/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}-\frac{9a^2e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{4a^2e(e\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^2*e^(5/2)*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a^2*e^(5/2)*Ar
cTanh[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d - (9*a^2*e^2*EllipticE[(c - Pi/2 + d
*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*d*Sqrt[Sin[c + d*x]]) - (4*a^2*e*(e*Sin[
c + d*x])^(3/2))/(3*d) - (2*a^2*e*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2))/(5*d
) + (a^2*e*Sec[c + d*x]*(e*Sin[c + d*x])^(3/2))/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}\,dx = \int (-a-a\cos(c+dx))^2\sec^2(c+dx)(e\sin(c+dx))^{5/2}\,dx$$
$$= \int \left(a^2(e\sin(c+dx))^{5/2} + 2a^2\sec(c+dx)(e\sin(c+dx))^{5/2} + a^2\sec^2(\right.$$
$$= a^2\int (e\sin(c+dx))^{5/2}\,dx + a^2\int \sec^2(c+dx)(e\sin(c+dx))^{5/2}\,dx + ($$

$$= -\frac{2a^2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} + \frac{a^2e\sec(c+dx)(e\sin(c+dx))^{3}}{d}$$

$$= -\frac{4a^2e(e\sin(c+dx))^{3/2}}{3d} - \frac{2a^2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} + \frac{a^2e\,\mathrm{se}}{}$$

$$= -\frac{9a^2e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{4a^2e(e\sin(c+dx))^{3/2}}{3d}$$
$$= -\frac{9a^2e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{4a^2e(e\sin(c+dx))^{3/2}}{3d}$$
$$= -\frac{2a^2e^{5/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2a^2e^{5/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} - \frac{9a^2e^2E}{}$$

Mathematica [C] time = 17.79, size = 205, normalized size = 1.06

$$\frac{2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(e\sin(c+dx))^{5/2}\sec^4\left(\frac{1}{2}\sin^{-1}(\sin(c+dx))\right)\left(9\sin^{\frac{3}{2}}(c+dx)\sqrt{\cos^2(c+dx)}\ {}_2F_1\left(\frac{3}{4}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Cos[(c + d*x)/2]^4*Sec[c + d*x]*Sec[ArcSin[Sin[c + d*x]]/2]^4*(e*Sin
[c + d*x])^(5/2)*(-15*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] + 15*
ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] - 9*Sin[c + d*x]^(3/2) - 1
0*Sqrt[Cos[c + d*x]^2]*Sin[c + d*x]^(3/2) + 9*Sqrt[Cos[c + d*x]^2]*Hypergeo
metric2F1[3/4, 3/2, 7/4, Sin[c + d*x]^2]*Sin[c + d*x]^(3/2) + 3*Sin[c + d*x
]^(7/2)))/(15*d*Sin[c + d*x]^(5/2))
```

fricas [F] time = 1.91, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(a^2e^2\cos(dx+c)^2 - a^2e^2 + \left(a^2e^2\cos(dx+c)^2 - a^2e^2\right)\sec(dx+c)^2 + 2\left(a^2e^2\cos(dx+c)^2 - a^2e^2\right)\mathrm{sec}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*e^2*cos(d*x + c)^2 - a^2*e^2 + (a^2*e^2*cos(d*x + c)^2 - a^2
*e^2)*sec(d*x + c)^2 + 2*(a^2*e^2*cos(d*x + c)^2 - a^2*e^2)*sec(d*x + c))*s
qrt(e*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^2(e\sin(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2), x)

maple [A] time = 6.14, size = 265, normalized size = 1.37

$$-\frac{a^2\left(60\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)\sqrt{e\sin(dx+c)}\, e^{\frac{5}{2}}\cos(dx+c)-60\sqrt{e\sin(dx+c)}\,\operatorname{arctanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)e^{\frac{5}{2}}\cos(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(5/2),x)

[Out] -1/30/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*a^2*(60*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*(e*sin(d*x+c))^(1/2)*e^(5/2)*cos(d*x+c)-60*(e*sin(d*x+c))^(1/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*e^(5/2)*cos(d*x+c)-54*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*e^3+27*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*e^3-12*e^3*cos(d*x+c)^4-40*e^3*cos(d*x+c)^3+42*e^3*cos(d*x+c)^2+40*e^3*cos(d*x+c)-30*e^3)/d

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\sin\left(c+d\,x\right)\right)}^{5/2}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2,x)

[Out] int((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2, x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**2*(e*sin(d*x+c))**(5/2),x)

[Out] Timed out

3.115 $\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=192

$$\frac{2a^2e^{3/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}-\frac{a^2e^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{4a^2e\sqrt{e\sin(c}{d}$$

```
[Out] 2*a^2*e^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d+2*a^2*e^(3/2)*arctanh(
(e*sin(d*x+c))^(1/2)/e^(1/2))/d+1/3*a^2*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(
1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))
*sin(d*x+c)^(1/2)/d/(e*sin(d*x+c))^(1/2)-4*a^2*e*(e*sin(d*x+c))^(1/2)/d-2/3
*a^2*e*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/d+a^2*e*sec(d*x+c)*(e*sin(d*x+c))^(1
/2)/d
```

Rubi **[A]** time = 0.38, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3872, 2873, 2635, 2642, 2641, 2564, 321, 329, 212, 206, 203, 2566}

$$\frac{2a^2e^{3/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}-\frac{a^2e^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{4a^2e\sqrt{e\sin(c}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*e^(3/2)*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a^2*e^(3/2)*Arc
Tanh[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d - (a^2*e^2*EllipticF[(c - Pi/2 + d*x)
/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (4*a^2*e*Sqrt[e*Sin
[c + d*x]])/d - (2*a^2*e*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(3*d) + (a^2*e*
Sec[c + d*x]*Sqrt[e*Sin[c + d*x]])/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a + a\sec(c + dx))^2(e\sin(c + dx))^{3/2}\,dx = \int (-a - a\cos(c + dx))^2\sec^2(c + dx)(e\sin(c + dx))^{3/2}\,dx$$
$$= \int \left(a^2(e\sin(c + dx))^{3/2} + 2a^2\sec(c + dx)(e\sin(c + dx))^{3/2} + a^2\sec^2(\right.$$
$$= a^2\int (e\sin(c + dx))^{3/2}\,dx + a^2\int \sec^2(c + dx)(e\sin(c + dx))^{3/2}\,dx + ($$

$$= -\frac{2a^2e\cos(c + dx)\sqrt{e\sin(c + dx)}}{3d} + \frac{a^2e\sec(c + dx)\sqrt{e\sin(c + dx)}}{d} +$$

$$= -\frac{4a^2e\sqrt{e\sin(c + dx)}}{d} - \frac{2a^2e\cos(c + dx)\sqrt{e\sin(c + dx)}}{3d} + \frac{a^2e\sec(c}{}$$

$$= -\frac{a^2e^2F\left(\frac{1}{2}\left(c - \frac{\pi}{2} + dx\right)\middle|2\right)\sqrt{\sin(c + dx)}}{3d\sqrt{e\sin(c + dx)}} - \frac{4a^2e\sqrt{e\sin(c + dx)}}{d} - 2a$$
$$= -\frac{a^2e^2F\left(\frac{1}{2}\left(c - \frac{\pi}{2} + dx\right)\middle|2\right)\sqrt{\sin(c + dx)}}{3d\sqrt{e\sin(c + dx)}} - \frac{4a^2e\sqrt{e\sin(c + dx)}}{d} - 2a$$
$$= \frac{2a^2e^{3/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2a^2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} - \frac{a^2e^2F\left(\right.}{}$$

Mathematica [C] time = 15.39, size = 204, normalized size = 1.06

$$\frac{16a^2e\sin^4\left(\frac{1}{2}\sin^{-1}(\sin(c + dx))\right)\cos^4\left(\frac{1}{2}(c + dx)\right)\sec(c + dx)\sqrt{e\sin(c + dx)}\left(-\sqrt{\sin(c + dx)}\sqrt{\cos^2(c + dx)}\,{}_2F_1\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (16*a^2*e*Cos[(c + d*x)/2]^4*Sec[c + d*x]*Sqrt[e*Sin[c + d*x]]*(6*ArcTan[Sq
rt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] + 6*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt
[Cos[c + d*x]^2] + Sqrt[Sin[c + d*x]] - 12*Sqrt[Cos[c + d*x]^2]*Sqrt[Sin[c
+ d*x]] - Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/4, 1/2, 5/4, Sin[c + d*x
]^2]*Sqrt[Sin[c + d*x]] + 2*Sin[c + d*x]^(5/2))*Sin[ArcSin[Sin[c + d*x]]/2]
^4)/(3*d*Sin[c + d*x]^(9/2))
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2e\sec(dx + c)^2 + 2a^2e\sec(dx + c) + a^2e\right)\sqrt{e\sin(dx + c)}\sin(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*e*sec(d*x + c)^2 + 2*a^2*e*sec(d*x + c) + a^2*e)*sqrt(e*sin(d
*x + c))*sin(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx + c) + a)^2(e\sin(dx + c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2), x)
```

maple [A] time = 5.50, size = 201, normalized size = 1.05

$$a^2 \left(12 \cos(dx+c)\, e^{\frac{3}{2}} \sqrt{e \sin(dx+c)}\ \arctan\left(\frac{\sqrt{e \sin(dx+c)}}{\sqrt{e}}\right) + 12 \cos(dx+c)\, e^{\frac{3}{2}} \sqrt{e \sin(dx+c)}\ \operatorname{arctanh}\left(\frac{\sqrt{e \sin(}}{\sqrt{e}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/6/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*a^2*(12*cos(d*x+c)*e^(3/2)*(e*sin(d*x+c
))^(1/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))+12*cos(d*x+c)*e^(3/2)*(e*sin(
d*x+c))^(1/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))+(-sin(d*x+c)+1)^(1/2)*(
2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*
2^(1/2))*e^2-4*e^2*sin(d*x+c)*cos(d*x+c)^2-24*e^2*sin(d*x+c)*cos(d*x+c)+6*e
^2*sin(d*x+c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(dx+c) + a)^2 (e \sin(dx+c))^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\, \sin(c+d\,x))^{3/2} \left(a + \frac{a}{\cos(c+d\,x)}\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.116 $\int (a + a\sec(c + dx))^2 \sqrt{e\sin(c + dx)}\, dx$

Optimal. Leaf size=138

$$-\frac{2a^2\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de}+\frac{a^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\right|}{d\sqrt{\sin(c}}$$

```
[Out] a^2*sec(d*x+c)*(e*sin(d*x+c))^(3/2)/d/e-2*a^2*arctan((e*sin(d*x+c))^(1/2)/e
^(1/2))*e^(1/2)/d+2*a^2*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d-a^2
*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(co
s(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/d/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3872, 2873, 2640, 2639, 2564, 329, 298, 203, 206, 2571}

$$-\frac{2a^2\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de}+\frac{a^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\right|}{d\sqrt{\sin(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (-2*a^2*Sqrt[e]*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a^2*Sqrt[e]*Ar
cTanh[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (a^2*EllipticE[(c - Pi/2 + d*x)/2,
 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]]) + (a^2*Sec[c + d*x]*(e*Sin
[c + d*x])^(3/2))/(d*e)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] && !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}\,dx = \int (-a-a\cos(c+dx))^2\sec^2(c+dx)\sqrt{e\sin(c+dx)}\,dx$$

$$= \int \left(a^2\sqrt{e\sin(c+dx)} + 2a^2\sec(c+dx)\sqrt{e\sin(c+dx)} + a^2\sec^2(c+\right.$$

$$= a^2\int \sqrt{e\sin(c+dx)}\,dx + a^2\int \sec^2(c+dx)\sqrt{e\sin(c+dx)}\,dx + \left(2\right.$$

$$= \frac{a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de} - \frac{1}{2}a^2\int \sqrt{e\sin(c+dx)}\,dx + \frac{\left(2a^2\right)}{}$$

$$= \frac{2a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{a^2\sec(c+dx)(e\sin(c+}{de}$$

$$= \frac{a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{a^2\sec(c+dx)(e\sin(c+d}{de}$$

$$= -\frac{2a^2\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2a^2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{a^2E\left(\right.}{}$$

Mathematica [C] time = 2.18, size = 168, normalized size = 1.22

$$-\frac{2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{e\sin(c+dx)}\sec^4\left(\frac{1}{2}\sin^{-1}(\sin(c+dx))\right)\left(\sin^{\frac{3}{2}}(c+dx)\sqrt{\cos^2(c+dx)}\,{}_2F_1\left(\frac{3}{4},\right.\right.}{3d\sqrt{}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (-2*a^2*Cos[(c + d*x)/2]^4*Sec[c + d*x]*Sec[ArcSin[Sin[c + d*x]]/2]^4*Sqrt[
e*Sin[c + d*x]]*(3*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] - 3*ArcT
anh[Sqrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] - 3*Sin[c + d*x]^(3/2) + Sqrt[
Cos[c + d*x]^2]*Hypergeometric2F1[3/4, 3/2, 7/4, Sin[c + d*x]^2]*Sin[c + d*
x]^(3/2)))/(3*d*Sqrt[Sin[c + d*x]])
```

fricas [F] time = 1.94, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2\right)\sqrt{e\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*sqrt(e*sin(d*x + c
)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a\sec(dx+c)+a)^2\sqrt{e\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(e*sin(d*x + c)), x)
```

maple [A] time = 6.16, size = 220, normalized size = 1.59

$$-\frac{a^2\left(2\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}\,(dx+c)\right)\text{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)e-\sqrt{-\sin(d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*a^2*(2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))
*e-(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF(
(-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*e+4*cos(d*x+c)*e^(1/2)*(e*sin(d*x+c))^(1
/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))-4*cos(d*x+c)*e^(1/2)*(e*sin(d*x+c)
)^(1/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))+2*e*cos(d*x+c)^2-2*e)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a\sec(dx+c)+a)^2\sqrt{e\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(e*sin(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{e\,\sin\left(c+d\,x\right)}\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^{2} \left(\int \sqrt{e \sin{\left(c + d x \right)}}\, dx + \int 2 \sqrt{e \sin{\left(c + d x \right)}} \sec{\left(c + d x \right)}\, dx + \int \sqrt{e \sin{\left(c + d x \right)}} \sec^{2}{\left(c + d x \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*sin(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(sqrt(e*sin(c + d*x)), x) + Integral(2*sqrt(e*sin(c + d*x))*s
ec(c + d*x), x) + Integral(sqrt(e*sin(c + d*x))*sec(c + d*x)**2, x))
```

3.117 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=139

$$\frac{2a^2\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{de}+\frac{3a^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\right.\right.}{d\sqrt{e\sin(c+dx)}}$$

```
[Out] 2*a^2*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(1/2)+2*a^2*arctanh((e*sin(d
*x+c))^(1/2)/e^(1/2))/d/e^(1/2)-3*a^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/s
in(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d
*x+c)^(1/2)/d/(e*sin(d*x+c))^(1/2)+a^2*sec(d*x+c)*(e*sin(d*x+c))^(1/2)/d/e
```

Rubi [A] time = 0.31, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3872, 2873, 2642, 2641, 2564, 329, 212, 206, 203, 2571}

$$\frac{2a^2\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{de}+\frac{3a^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\right.\right.}{d\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (2*a^2*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*Sqrt[e]) + (2*a^2*ArcTanh[S
qrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*Sqrt[e]) + (3*a^2*EllipticF[(c - Pi/2 + d*
x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c + d*x]]) + (a^2*Sec[c + d*x]*S
qrt[e*Sin[c + d*x]])/(d*e)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(-a-a\cos(c+dx))^2\sec^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx$$
$$= \int \left(\frac{a^2}{\sqrt{e\sin(c+dx)}} + \frac{2a^2\sec(c+dx)}{\sqrt{e\sin(c+dx)}} + \frac{a^2\sec^2(c+dx)}{\sqrt{e\sin(c+dx)}}\right)dx$$
$$= a^2\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx + a^2\int \frac{\sec^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx + \left(2a^2\right)\int \frac{\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx$$
$$= \frac{a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{de} + \frac{1}{2}a^2\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{e^2}\right)}\right.}{de}$$
$$= \frac{2a^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{de} + \frac{\left(4a^2\right)\text{Sub}}{}$$
$$= \frac{3a^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{de} + \frac{\left(2a^2\right)\text{Sub}}{}$$
$$= \frac{2a^2\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{3a^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(}}{d\sqrt{e\sin(c+dx)}}$$

Mathematica [C] time = 71.89, size = 164, normalized size = 1.18

$$\frac{a^2\sqrt{\sin(c+dx)}\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sec^4\left(\frac{1}{2}\sin^{-1}(\sin(c+dx))\right)\left(3\sqrt{\sin(c+dx)}\sqrt{\cos^2(c+dx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\right.\right.}{d\sqrt{e\,\text{si}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (a^2*Cos[(c + d*x)/2]^4*Sec[c + d*x]*Sec[ArcSin[Sin[c + d*x]]/2]^4*(2*ArcTa
n[Sqrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] + 2*ArcTanh[Sqrt[Sin[c + d*x]]]*
Sqrt[Cos[c + d*x]^2] + Sqrt[Sin[c + d*x]] + 3*Sqrt[Cos[c + d*x]^2]*Hypergeo
metric2F1[1/4, 1/2, 5/4, Sin[c + d*x]^2]*Sqrt[Sin[c + d*x]])*Sqrt[Sin[c + d
*x]])/(d*Sqrt[e*Sin[c + d*x]])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2\right)\sqrt{e\sin\left(dx+c\right)}}{e\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*sqrt(e*sin(d*x + c
))/(e*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\sec\left(dx+c\right)+a\right)^2}{\sqrt{e\sin\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(e*sin(d*x + c)), x)
```

maple [A] time = 5.25, size = 163, normalized size = 1.17

$$\frac{a^2\left(-3\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}\,(dx+c)\right)\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)\sqrt{e}+4\,\mathrm{co}}{2\sqrt{e}\,\cos(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/2/e^(1/2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*a^2*(-3*(-sin(d*x+c)+1)^(1/2)*(
2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*
2^(1/2))*e^(1/2)+4*cos(d*x+c)*(e*sin(d*x+c))^(1/2)*arctanh((e*sin(d*x+c))^(
1/2)/e^(1/2))+4*cos(d*x+c)*(e*sin(d*x+c))^(1/2)*arctan((e*sin(d*x+c))^(1/2)
/e^(1/2))+2*e^(1/2)*sin(d*x+c))/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\sqrt{e\,\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx+\int \frac{2\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx+\int \frac{\sec^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(1/sqrt(e*sin(c + d*x)), x) + Integral(2*sec(c + d*x)/sqrt(e*
sin(c + d*x)), x) + Integral(sec(c + d*x)**2/sqrt(e*sin(c + d*x)), x))
```

3.118 $\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=224

$$-\frac{2a^2\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}+\frac{3a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de^3}-\frac{5a^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\cdots}}{de^2\sqrt{\sin(c+d\cdots}}$$

```
[Out] -2*a^2*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)+2*a^2*arctanh((e*sin(
d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)+3*a^2*sec(d*x+c)*(e*sin(d*x+c))^(3/2)/d/e^
3-4*a^2/d/e/(e*sin(d*x+c))^(1/2)-2*a^2*cos(d*x+c)/d/e/(e*sin(d*x+c))^(1/2)-
2*a^2*sec(d*x+c)/d/e/(e*sin(d*x+c))^(1/2)+5*a^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^
2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1
/2))*(e*sin(d*x+c))^(1/2)/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3872, 2873, 2636, 2640, 2639, 2564, 325, 329, 298, 203, 206, 2570, 2571}

$$-\frac{2a^2\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}+\frac{3a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de^3}-\frac{5a^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\cdots}}{de^2\sqrt{\sin(c+d\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^2*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(3/2)) + (2*a^2*ArcTanh[
Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(3/2)) - (4*a^2)/(d*e*Sqrt[e*Sin[c + d*
x]]) - (2*a^2*Cos[c + d*x])/(d*e*Sqrt[e*Sin[c + d*x]]) - (2*a^2*Sec[c + d*x
])/(d*e*Sqrt[e*Sin[c + d*x]]) - (5*a^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqr
t[e*Sin[c + d*x]])/(d*e^2*Sqrt[Sin[c + d*x]]) + (3*a^2*Sec[c + d*x]*(e*Sin[
c + d*x])^(3/2))/(d*e^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(-a-a\cos(c+dx))^2\sec^2(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx$$

$$= \int \left(\frac{a^2}{(e\sin(c+dx))^{3/2}} + \frac{2a^2\sec(c+dx)}{(e\sin(c+dx))^{3/2}} + \frac{a^2\sec^2(c+dx)}{(e\sin(c+dx))^{3/2}}\right)dx$$

$$= a^2\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx + a^2\int \frac{\sec^2(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx + \left(2a^2\right)\int \frac{\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,d$$

$$= -\frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\sec(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{a^2\int\sqrt{e\sin(c+dx)}\,dx}{e^2} + \frac{\left(3a^2\right)\int\sec^2(c+}{}$$

$$= -\frac{4a^2}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\sec(c+dx)}{de\sqrt{e\sin(c+dx)}} + \frac{3a^2\sec(c+dx)(e\sin(}{de^3}$$

$$= -\frac{4a^2}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\sec(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\right|}{de^2\sqrt{\sin(c}}$$

$$= -\frac{4a^2}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\sec(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{5a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\right|}{de^2\sqrt{\sin(c}}$$

$$= -\frac{2a^2\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} + \frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} - \frac{4a^2}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\cos(}{de\sqrt{e\sin}}$$

Mathematica [C] time = 11.06, size = 135, normalized size = 0.60

$$-\frac{2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\cot(c+dx)\sqrt{e\sin(c+dx)}\left(\sin^2(c+dx)\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};\sin^2(c+dx)\right)+6\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};\sin^2(c\right.\right.}{3de^2\sqrt{\cos^2(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^2*Cos[(c + d*x)/2]^4*Cot[c + d*x]*Sec[ArcSin[Sin[c + d*x]]/2]^4*Sqrt[
e*Sin[c + d*x]]*(6*Hypergeometric2F1[-1/4, 1, 3/4, Sin[c + d*x]^2] + 6*Hype
rgeometric2F1[-1/4, 3/2, 3/4, Sin[c + d*x]^2] + Hypergeometric2F1[3/4, 3/2,
 7/4, Sin[c + d*x]^2]*Sin[c + d*x]^2))/(3*d*e^2*Sqrt[Cos[c + d*x]^2])
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(a^2\sec(dx+c)^2+2a^2\sec(dx+c)+a^2\right)\sqrt{e\sin(dx+c)}}{e^2\cos(dx+c)^2-e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*sqrt(e*sin(d*x +
c))/(e^2*cos(d*x + c)^2 - e^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^2}{(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/(e*sin(d*x + c))^(3/2), x)
```

maple [A] time = 5.68, size = 238, normalized size = 1.06

$$-\frac{a^2\left(-10\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}\,(dx+c)\right)\operatorname{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)e^{\frac{3}{2}}+5\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x)
```

```
[Out] -1/2/e^(5/2)/(e*sin(d*x+c))^(1/2)/cos(d*x+c)*a^2*(-10*(-sin(d*x+c)+1)^(1/2)
*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/
2*2^(1/2))*e^(3/2)+5*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*e^(3/2)+10*e^(3/2)*cos
(d*x+c)^2+4*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*(e*sin(d*x+c))^(1/2)*e*cos
(d*x+c)-4*(e*sin(d*x+c))^(1/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*e*cos(
d*x+c)+8*e^(3/2)*cos(d*x+c)-2*e^(3/2))/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^2}{{\left(e\,\sin(c+d\,x)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{1}{(e\sin(c+dx))^{\frac{3}{2}}}\,dx+\int \frac{2\sec(c+dx)}{(e\sin(c+dx))^{\frac{3}{2}}}\,dx+\int \frac{\sec^2(c+dx)}{(e\sin(c+dx))^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
```

```
[Out] a**2*(Integral((e*sin(c + d*x))**(-3/2), x) + Integral(2*sec(c + d*x)/(e*si
n(c + d*x))**(3/2), x) + Integral(sec(c + d*x)**2/(e*sin(c + d*x))**(3/2),
x))
```

3.119 $\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=234

$$\frac{2a^2\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{5a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{3de^3}+\frac{7a^2\sqrt{\sin(c+dx)}F\left(\frac{1}{2}\left(c+dx\right.\right.}{3de^2\sqrt{e\sin(c+dx)}}$$

```
[Out] 2*a^2*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)+2*a^2*arctanh((e*sin(d
*x+c))^(1/2)/e^(1/2))/d/e^(5/2)-4/3*a^2/d/e/(e*sin(d*x+c))^(3/2)-2/3*a^2*co
s(d*x+c)/d/e/(e*sin(d*x+c))^(3/2)-2/3*a^2*sec(d*x+c)/d/e/(e*sin(d*x+c))^(3/
2)-7/3*a^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/d/e^2/(e*sin(d*
x+c))^(1/2)+5/3*a^2*sec(d*x+c)*(e*sin(d*x+c))^(1/2)/d/e^3
```

Rubi [A] time = 0.42, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3872, 2873, 2636, 2642, 2641, 2564, 325, 329, 212, 206, 203, 2570, 2571}

$$\frac{2a^2\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{5a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{3de^3}+\frac{7a^2\sqrt{\sin(c+dx)}F\left(\frac{1}{2}\left(c+dx\right.\right.}{3de^2\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(5/2)) + (2*a^2*ArcTanh[S
qrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(5/2)) - (4*a^2)/(3*d*e*(e*Sin[c + d*x])
^(3/2)) - (2*a^2*Cos[c + d*x])/(3*d*e*(e*Sin[c + d*x])^(3/2)) - (2*a^2*Sec[
c + d*x])/(3*d*e*(e*Sin[c + d*x])^(3/2)) + (7*a^2*EllipticF[(c - Pi/2 + d*x
)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*e^2*Sqrt[e*Sin[c + d*x]]) + (5*a^2*Sec[c +
 d*x]*Sqrt[e*Sin[c + d*x]])/(3*d*e^3)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(-a-a\cos(c+dx))^2\sec^2(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx$$

$$= \int \left(\frac{a^2}{(e\sin(c+dx))^{5/2}} + \frac{2a^2\sec(c+dx)}{(e\sin(c+dx))^{5/2}} + \frac{a^2\sec^2(c+dx)}{(e\sin(c+dx))^{5/2}}\right) dx$$

$$= a^2\int \frac{1}{(e\sin(c+dx))^{5/2}}\,dx + a^2\int \frac{\sec^2(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx + \left(2a^2\right)\int \frac{\sec(c+dx)}{(e\sin(c+dx))^{5/2}}$$

$$= -\frac{2a^2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\sec(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{a^2\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3e^2} + \frac{\left(5a^2\right)\int \frac{\sec^2(c}{\sqrt{e\sin(}}}{3e^2}$$

$$= -\frac{4a^2}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\sec(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{5a^2\sec(c+}{}$$

$$= -\frac{4a^2}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\sec(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{2a^2F\left(\frac{1}{2}\left(c-\right.\right.}{3d}$$

$$= -\frac{4a^2}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\sec(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{7a^2F\left(\frac{1}{2}\left(c-\right.\right.}{3d}$$

$$= \frac{2a^2\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} + \frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} - \frac{4a^2}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2 c}{3de(e\,si}$$

Mathematica [C] time = 48.88, size = 169, normalized size = 0.72

$$-\frac{a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{e\sin(c+dx)}\ \sec^4\left(\frac{1}{2}\sin^{-1}(\sin(c+dx))\right)\left(3\sqrt{\cos^2(c+dx)}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(c+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] -1/3*(a^2*Cos[(c + d*x)/2]^4*(3 + 4*Sqrt[Cos[c + d*x]^2]*Csc[c + d*x]^2*Hyp
ergeometric2F1[-3/4, 1, 1/4, Sin[c + d*x]^2] + 4*Sqrt[Cos[c + d*x]^2]*Csc[c
 + d*x]^2*Hypergeometric2F1[-3/4, 3/2, 1/4, Sin[c + d*x]^2] + 3*Sqrt[Cos[c
+ d*x]^2]*Hypergeometric2F1[1/4, 1/2, 5/4, Sin[c + d*x]^2])*Sec[c + d*x]*Se
c[ArcSin[Sin[c + d*x]]/2]^4*Sqrt[e*Sin[c + d*x]])/(d*e^3)
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2\right)\sqrt{e\sin\left(dx+c\right)}}{\left(e^3\cos\left(dx+c\right)^2-e^3\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

[Out] integral(-(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*sqrt(e*sin(d*x + c))/((e^3*cos(d*x + c)^2 - e^3)*sin(d*x + c)), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a \sec (dx + c) + a)^2}{(e \sin (dx + c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)^2/(e*sin(d*x + c))^(5/2), x)

maple [A] time = 6.25, size = 301, normalized size = 1.29

$$a^2 \left(7 \left(\sin^{\frac{7}{2}} (dx + c)\right) \sqrt{-\sin (dx + c) + 1}\, \sqrt{2 \sin (dx + c) + 2}\ \mathrm{EllipticF}\left(\sqrt{-\sin (dx + c) + 1}\,, \frac{\sqrt{2}}{2}\right) e^{\frac{7}{2}} - 14 e^{\frac{7}{2}} \left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x)

[Out] 1/6/e^(9/2)/(e*sin(d*x+c))^(3/2)/cos(d*x+c)/(cos(d*x+c)^2-1)*a^2*(7*sin(d*x+c)^(7/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*e^(7/2)-14*e^(7/2)*cos(d*x+c)^4-8*e^(7/2)*cos(d*x+c)^3+12*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*(e*sin(d*x+c))^(3/2)*e^2*cos(d*x+c)^3+12*(e*sin(d*x+c))^(3/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*e^2*cos(d*x+c)^3+20*e^(7/2)*cos(d*x+c)^2+8*e^(7/2)*cos(d*x+c)-12*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*(e*sin(d*x+c))^(3/2)*e^2*cos(d*x+c)-12*(e*sin(d*x+c))^(3/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*e^2*cos(d*x+c)-6*e^(7/2))/d

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2}{{\left(e\,\sin\left(c+d\,x\right)\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(5/2),x)

[Out] int((a + a/cos(c + d*x))^2/(e*sin(c + d*x))^(5/2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)

[Out] Timed out

3.120 $\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=139

$$-\frac{4e^4\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{21ad\sqrt{e\sin(c+dx)}}+\frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7ad}-\frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21ad}+\frac{2e(e\sin\cdots}{\cdots}$$

```
[Out] 2/5*e*(e*sin(d*x+c))^(5/2)/a/d+4/21*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)
/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin
(d*x+c)^(1/2)/a/d/(e*sin(d*x+c))^(1/2)-2/21*e^3*cos(d*x+c)*(e*sin(d*x+c))^(
1/2)/a/d+2/7*e^3*cos(d*x+c)^3*(e*sin(d*x+c))^(1/2)/a/d
```

Rubi [A] time = 0.28, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3872, 2839, 2564, 30, 2568, 2569, 2642, 2641}

$$\frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7ad}-\frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21ad}-\frac{4e^4\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{21ad\sqrt{e\sin(c+dx)}}+\frac{2e(e\sin\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(7/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a*d*Sqrt[e
*Sin[c + d*x]]) - (2*e^3*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(21*a*d) + (2*e
^3*Cos[c + d*x]^3*Sqrt[e*Sin[c + d*x]])/(7*a*d) + (2*e*(e*Sin[c + d*x])^(5/
2))/(5*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{-a-a\cos(c+dx)}\,dx\\
&= \frac{e^2\int \cos(c+dx)(e\sin(c+dx))^{3/2}\,dx}{a} - \frac{e^2\int \cos^2(c+dx)(e\sin(c+dx))^{3/2}\,dx}{a}\\
&= \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7ad} + \frac{e\,\mathrm{Subst}\left(\int x^{3/2}\,dx, x, e\sin(c+dx)\right)}{ad} - \frac{e^4\int \frac{\cos^2(c+}{\sqrt{e\sin(c}}}{7a}\\
&= -\frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21ad} + \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7ad} + \frac{2e(e\sin(c+dx}{5ad}\\
&= -\frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21ad} + \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7ad} + \frac{2e(e\sin(c+dx}{5ad}\\
&= -\frac{4e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21ad\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21ad} + \frac{2e^3\cos}{}\end{aligned}$$

Mathematica [A] time = 0.69, size = 122, normalized size = 0.88

$$\frac{e^3\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{e\sin(c+dx)}\left(40F\left(\frac{1}{4}(-2c-2dx+\pi)\middle|2\right)+\sqrt{\sin(c+dx)}\,(25\cos(c+dx)-42\right.}{105ad\sqrt{\sin(c+dx)}\,(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sin[c + d*x])^(7/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e^3*Cos[(c + d*x)/2]^2*Sec[c + d*x]*(40*EllipticF[(-2*c + Pi - 2*d*x)/4, 2
] + (42 + 25*Cos[c + d*x] - 42*Cos[2*(c + d*x)] + 15*Cos[3*(c + d*x)])*Sqrt
[Sin[c + d*x]])*Sqrt[e*Sin[c + d*x]])/(105*a*d*(1 + Sec[c + d*x])*Sqrt[Sin[
c + d*x]])
```

fricas [F] time = 1.13, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(e^{3}\cos\left(dx+c\right)^{2}-e^{3}\right)\sqrt{e\sin\left(dx+c\right)}\,\sin\left(dx+c\right)}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(-(e^3*cos(d*x + c)^2 - e^3)*sqrt(e*sin(d*x + c))*sin(d*x + c)/(a*s
ec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\sin\left(dx+c\right)\right)^{\frac{7}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(7/2)/(a*sec(d*x + c) + a), x)
```

maple [A] time = 3.28, size = 128, normalized size = 0.92

$$\frac{\frac{2e(e\sin(dx+c))^{\frac{5}{2}}}{5a}+\frac{2e^{4}\left(3\left(\sin^{5}(dx+c)\right)+\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}\,(dx+c)\right)\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-5\left(\sin^{3}(dx+c)\right)+2\sin(dx+c\cdots}{21a\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (2/5*e/a*(e*sin(d*x+c))^(5/2)+2/21*e^4*(3*sin(d*x+c)^5+(-sin(d*x+c)+1)^(1/2
)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1
/2*2^(1/2))-5*sin(d*x+c)^3+2*sin(d*x+c))/a/cos(d*x+c)/(e*sin(d*x+c))^(1/2))
/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\sin\left(dx+c\right)\right)^{\frac{7}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^(7/2)/(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)\,{\left(e\,\sin\left(c+d\,x\right)\right)}^{7/2}}{a\,\left(\cos\left(c+d\,x\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(7/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(7/2))/(a*(cos(c + d*x) + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.121 $\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=104

$$-\frac{4e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5ad\sqrt{\sin(c+dx)}}+\frac{2e(e\sin(c+dx))^{3/2}}{3ad}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5ad}$$

```
[Out] 2/3*e*(e*sin(d*x+c))^(3/2)/a/d-2/5*e*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/a/d+4/
5*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipti
cE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a/d/sin(d*x+c)^(
1/2)
```

Rubi [A] time = 0.22, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3872, 2839, 2564, 30, 2569, 2640, 2639}

$$-\frac{4e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5ad\sqrt{\sin(c+dx)}}+\frac{2e(e\sin(c+dx))^{3/2}}{3ad}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a*d*Sqrt[
Sin[c + d*x]]) + (2*e*(e*Sin[c + d*x])^(3/2))/(3*a*d) - (2*e*Cos[c + d*x]*(
e*Sin[c + d*x])^(3/2))/(5*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{-a-a\cos(c+dx)}\,dx$$
$$= \frac{e^2\int\cos(c+dx)\sqrt{e\sin(c+dx)}\,dx}{a} - \frac{e^2\int\cos^2(c+dx)\sqrt{e\sin(c+dx)}\,dx}{a}$$
$$= -\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5ad} + \frac{e\,\mathrm{Subst}\left(\int\sqrt{x}\,dx,x,e\sin(c+dx)\right)}{ad} - \frac{\left(2e^2\right)\int\sqrt{e}}{}$$
$$= \frac{2e(e\sin(c+dx))^{3/2}}{3ad} - \frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5ad} - \frac{\left(2e^2\sqrt{e\sin(c+dx)}\right)\int\sqrt{\sin}}{5a\sqrt{\sin(c+dx)}}$$
$$= -\frac{4e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5ad\sqrt{\sin(c+dx)}} + \frac{2e(e\sin(c+dx))^{3/2}}{3ad} - \frac{2e\cos(c+dx)(e\,s}{5ad}$$

Mathematica [C] time = 4.90, size = 232, normalized size = 2.23

$$\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(e\sin(c+dx))^{5/2}\left(\sqrt{\sin(c+dx)}\,(10\sin(c)\cos(dx)-3\sin(2c)\cos(2dx)+10\cos(\right.}{15ad\sin^{\frac{5}{2}}(c+dx)(}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sin[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]^2*Sec[c + d*x]*(e*Sin[c + d*x])^(5/2)*((2*Sqrt[2 - 2*E^
((2*I)*(c + d*x))]*(3*Hypergeometric2F1[-1/4, 1/2, 3/4, E^((2*I)*(c + d*x))
] + E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])*Se
c[c])/(E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x))]) +
 Sqrt[Sin[c + d*x]]*(10*Cos[d*x]*Sin[c] - 3*Cos[2*d*x]*Sin[2*c] + 10*Cos[c]
*Sin[d*x] - 3*Cos[2*c]*Sin[2*d*x] - 12*Tan[c])))/(15*a*d*(1 + Sec[c + d*x])
*Sin[c + d*x]^(5/2))
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\left(e^2\cos(dx+c)^2-e^2\right)\sqrt{e\sin(dx+c)}}{a\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(-(e^2*cos(d*x + c)^2 - e^2)*sqrt(e*sin(d*x + c))/(a*sec(d*x + c) +
 a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

maple [A] time = 3.49, size = 173, normalized size = 1.66

$$\frac{2e^3\left(6\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}\,(dx+c)\right)\text{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-3\sqrt{-\sin(d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 2/15/a/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^3*(6*(-sin(d*x+c)+1)^(1/2)*(2*sin(
d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2
))-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Elliptic
F((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3*cos(d*x+c)^4-5*cos(d*x+c)^3-3*cos(d*
x+c)^2+5*cos(d*x+c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)\,\left(e\,\sin(c+d\,x)\right)^{5/2}}{a\,\left(\cos(c+d\,x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(5/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(5/2))/(a*(cos(c + d*x) + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.122 $\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=102

$$-\frac{4e^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3ad\sqrt{e\sin(c+dx)}}+\frac{2e\sqrt{e\sin(c+dx)}}{ad}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3ad}$$

```
[Out] 4/3*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellip
ticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a/d/(e*sin(d*x+c))
^(1/2)+2*e*(e*sin(d*x+c))^(1/2)/a/d-2/3*e*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/a
/d
```

Rubi [A] time = 0.22, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3872, 2839, 2564, 30, 2569, 2642, 2641}

$$-\frac{4e^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3ad\sqrt{e\sin(c+dx)}}+\frac{2e\sqrt{e\sin(c+dx)}}{ad}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a*d*Sqrt[e*
Sin[c + d*x]]) + (2*e*Sqrt[e*Sin[c + d*x]])/(a*d) - (2*e*Cos[c + d*x]*Sqrt[
e*Sin[c + d*x]])/(3*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{-a-a\cos(c+dx)}\,dx$$
$$= \frac{e^2\int \frac{\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{a} - \frac{e^2\int \frac{\cos^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{a}$$
$$= -\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3ad} + \frac{e\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, e\sin(c+dx)\right)}{ad} - \frac{\left(2e^2\right)\int \frac{1}{\sqrt{e\sin(c+}}}{3a}$$
$$= \frac{2e\sqrt{e\sin(c+dx)}}{ad} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3ad} - \frac{\left(2e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3a\sqrt{e\sin(c+dx)}}$$
$$= -\frac{4e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3ad\sqrt{e\sin(c+dx)}} + \frac{2e\sqrt{e\sin(c+dx)}}{ad} - \frac{2e\cos(c+dx)\sqrt{e\sin(c}}{3ad}$$

Mathematica [A] time = 20.65, size = 69, normalized size = 0.68

$$-\frac{2(e\sin(c+dx))^{3/2}\left(\sqrt{\sin(c+dx)}(\cos(c+dx)-3)-2F\left(\frac{1}{4}(-2c-2dx+\pi)\middle|2\right)\right)}{3ad\sin^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sin[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*(-2*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + (-3 + Cos[c + d*x])*Sqrt[Sin[
c + d*x]])*(e*Sin[c + d*x])^(3/2))/(3*a*d*Sin[c + d*x]^(3/2))
```

fricas [F] time = 0.67, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{e\sin(dx+c)}\,e\sin(dx+c)}{a\sec(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*sin(d*x + c))*e*sin(d*x + c)/(a*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

maple [A] time = 3.28, size = 112, normalized size = 1.10

$$\frac{2e^2\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}\,(dx+c)\right)\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-\left(\cos^2(dx\right.\right.}{3a\cos(dx+c)\sqrt{e\sin(dx+c)}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 2/3/a/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^2*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-
cos(d*x+c)^2*sin(d*x+c)+3*cos(d*x+c)*sin(d*x+c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)\,\left(e\,\sin(c+d\,x)\right)^{3/2}}{a\,\left(\cos(c+d\,x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(3/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(3/2))/(a*(cos(c + d*x) + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.123 $\int \frac{\sqrt{e\sin(c+dx)}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=95

$$-\frac{2e}{ad\sqrt{e\sin(c+dx)}}+\frac{2e\cos(c+dx)}{ad\sqrt{e\sin(c+dx)}}+\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}}$$

```
[Out] -2*e/a/d/(e*sin(d*x+c))^(1/2)+2*e*cos(d*x+c)/a/d/(e*sin(d*x+c))^(1/2)-4*(si
n(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/
2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a/d/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3872, 2839, 2564, 30, 2567, 2640, 2639}

$$-\frac{2e}{ad\sqrt{e\sin(c+dx)}}+\frac{2e\cos(c+dx)}{ad\sqrt{e\sin(c+dx)}}+\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sin[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*e)/(a*d*Sqrt[e*Sin[c + d*x]]) + (2*e*Cos[c + d*x])/(a*d*Sqrt[e*Sin[c +
d*x]]) + (4*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a*d*Sqr
t[Sin[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{e\sin(c+dx)}}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{-a-a\cos(c+dx)}\,dx\\
&= \frac{e^2\int \frac{\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx}{a} - \frac{e^2\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx}{a}\\
&= \frac{2e\cos(c+dx)}{ad\sqrt{e\sin(c+dx)}} + \frac{2\int \sqrt{e\sin(c+dx)}\,dx}{a} + \frac{e\,\mathrm{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, e\sin(c+dx)\right)}{ad}\\
&= -\frac{2e}{ad\sqrt{e\sin(c+dx)}} + \frac{2e\cos(c+dx)}{ad\sqrt{e\sin(c+dx)}} + \frac{\left(2\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{a\sqrt{\sin(c+dx)}}\\
&= -\frac{2e}{ad\sqrt{e\sin(c+dx)}} + \frac{2e\cos(c+dx)}{ad\sqrt{e\sin(c+dx)}} + \frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}}
\end{aligned}
$$

Mathematica [C] time = 0.61, size = 249, normalized size = 2.62

$$
\frac{2\left(12e^{2ic}\sqrt{1-e^{2i(c+dx)}}\;{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};e^{2i(c+dx)}\right)+4e^{2i(c+dx)}\sqrt{1-e^{2i(c+dx)}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{2i(c+dx)}\right)+6e^{i(c+dx)}-9e^{2i(c+\cdots}\right.}{3a\left(1+ie^{ic}\right)\left(e^{ic}+i\right)d\left(-1+e^{i(c+dx)}\right)\left(1+e^{i(c+dx)}\right)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sin[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*(3 - 9*E^((2*I)*c) + 6*E^(I*(c + d*x)) - 9*E^((2*I)*(c + d*x)) + 3*E^((2
*I)*(2*c + d*x)) + 6*E^(I*(3*c + d*x)) + 12*E^((2*I)*c)*Sqrt[1 - E^((2*I)*(
c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, E^((2*I)*(c + d*x))] + 4*E^((2
*I)*(c + d*x))*Sqrt[1 - E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/
4, E^((2*I)*(c + d*x))])*Sqrt[e*Sin[c + d*x]])/(3*a*d*(1 + I*E^(I*c))*(I +
E^(I*c))*(-1 + E^(I*(c + d*x)))*(1 + E^(I*(c + d*x))))
```

fricas [F] time = 0.71, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\sqrt{e\sin(dx+c)}}{a\sec(dx+c)+a}, x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*sin(d*x + c))/(a*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\sin(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*sin(d*x + c))/(a*sec(d*x + c) + a), x)
```

maple [A] time = 3.89, size = 149, normalized size = 1.57

$$-\frac{2e\left(2\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}\,(dx+c)\right)\mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1}\,,\frac{\sqrt{2}}{2}\right)-\sqrt{-\sin(dx}{a\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -2/a/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e*(2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-(
-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-si
n(d*x+c)+1)^(1/2),1/2*2^(1/2))-cos(d*x+c)^2+cos(d*x+c))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\sin(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*sin(d*x + c))/(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)\,\sqrt{e\,\sin(c+d\,x)}}{a\,\left(\cos(c+d\,x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(1/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(1/2))/(a*(cos(c + d*x) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{e\sin(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(e*sin(c + d*x))/(sec(c + d*x) + 1), x)/a
```

3.124 $\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=101

$$-\frac{2e}{3ad(e\sin(c+dx))^{3/2}}+\frac{2e\cos(c+dx)}{3ad(e\sin(c+dx))^{3/2}}+\frac{4\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3ad\sqrt{e\sin(c+dx)}}$$

```
[Out] -2/3*e/a/d/(e*sin(d*x+c))^(3/2)+2/3*e*cos(d*x+c)/a/d/(e*sin(d*x+c))^(3/2)-4
/3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(
cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a/d/(e*sin(d*x+c))^(1/2
)
```

Rubi [A] time = 0.21, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3872, 2839, 2564, 30, 2567, 2642, 2641}

$$-\frac{2e}{3ad(e\sin(c+dx))^{3/2}}+\frac{2e\cos(c+dx)}{3ad(e\sin(c+dx))^{3/2}}+\frac{4\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3ad\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (-2*e)/(3*a*d*(e*Sin[c + d*x])^(3/2)) + (2*e*Cos[c + d*x])/(3*a*d*(e*Sin[c
+ d*x])^(3/2)) + (4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3
*a*d*Sqrt[e*Sin[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a + a\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = -\int \frac{\cos(c+dx)}{(-a - a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx$$
$$= \frac{e^2\int \frac{\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx}{a} - \frac{e^2\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx}{a}$$
$$= \frac{2e\cos(c+dx)}{3ad(e\sin(c+dx))^{3/2}} + \frac{2\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3a} + \frac{e\,\mathrm{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, e\sin\right.}{ad}$$
$$= -\frac{2e}{3ad(e\sin(c+dx))^{3/2}} + \frac{2e\cos(c+dx)}{3ad(e\sin(c+dx))^{3/2}} + \frac{\left(2\sqrt{\sin(c+dx)}\right)\int \frac{}{\sqrt{}}}{3a\sqrt{e\sin(c+}}$$
$$= -\frac{2e}{3ad(e\sin(c+dx))^{3/2}} + \frac{2e\cos(c+dx)}{3ad(e\sin(c+dx))^{3/2}} + \frac{4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right.}{3ad\sqrt{e\sin(c}}$$

Mathematica [A] time = 0.58, size = 77, normalized size = 0.76

$$\frac{2\cot\left(\frac{1}{2}(c+dx)\right)\left(\cos(c+dx) - 2\sin^{\frac{3}{2}}(c+dx)F\left(\frac{1}{4}(-2c-2dx+\pi)\middle|2\right) - 1\right)}{3ad(\cos(c+dx)+1)\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (2*Cot[(c + d*x)/2]*(-1 + Cos[c + d*x] - 2*EllipticF[(-2*c + Pi - 2*d*x)/4,
 2]*Sin[c + d*x]^(3/2)))/(3*a*d*(1 + Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\sin(dx+c)}}{(ae\sec(dx+c)+ae)\sin(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*sin(d*x + c))/((a*e*sec(d*x + c) + a*e)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)\sqrt{e\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c))), x)
```

maple [A] time = 3.34, size = 121, normalized size = 1.20

$$\frac{-\frac{2e}{3a(e\sin(dx+c))^{\frac{3}{2}}}-\frac{2\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{5}{2}}(dx+c)\right)\operatorname{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)+\sin^{3}(dx+c)-\sin(dx+c)\right)}{3a\sin(dx+c)^{2}\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x)
```

```
[Out] (-2/3/a*e/(e*sin(d*x+c))^(3/2)-2/3*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^
(1/2)*sin(d*x+c)^(5/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+sin(d*x
+c)^3-sin(d*x+c))/a/sin(d*x+c)^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)\sqrt{e\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)}{a\,\sqrt{e\,\sin(c+d\,x)}\,\left(\cos(c+d\,x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x))),x)
```

```
[Out] int(cos(c + d*x)/(a*(e*sin(c + d*x))^(1/2)*(cos(c + d*x) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{e\sin(c+dx)}\sec(c+dx)+\sqrt{e\sin(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*sin(c + d*x))*sec(c + d*x) + sqrt(e*sin(c + d*x))), x)/a
```

3.125 $\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5ade^2\sqrt{\sin(c+dx)}}-\frac{2e}{5ad(e\sin(c+dx))^{5/2}}+\frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}}-\frac{4\cos(c+dx)}{5ade\sqrt{e\sin(c+dx)}}$$

```
[Out] -2/5*e/a/d/(e*sin(d*x+c))^(5/2)+2/5*e*cos(d*x+c)/a/d/(e*sin(d*x+c))^(5/2)-4
/5*cos(d*x+c)/a/d/e/(e*sin(d*x+c))^(1/2)+4/5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^
(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2)
)*(e*sin(d*x+c))^(1/2)/a/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3872, 2839, 2564, 30, 2567, 2636, 2640, 2639}

$$-\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5ade^2\sqrt{\sin(c+dx)}}-\frac{2e}{5ad(e\sin(c+dx))^{5/2}}+\frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}}-\frac{4\cos(c+dx)}{5ade\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (-2*e)/(5*a*d*(e*Sin[c + d*x])^(5/2)) + (2*e*Cos[c + d*x])/(5*a*d*(e*Sin[c
+ d*x])^(5/2)) - (4*Cos[c + d*x])/(5*a*d*e*Sqrt[e*Sin[c + d*x]]) - (4*Ellip
ticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a*d*e^2*Sqrt[Sin[c + d
*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx &= -\int \frac{\cos(c+dx)}{(-a-a\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx \\
&= \frac{e^2\int \frac{\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx}{a} - \frac{e^2\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx}{a} \\
&= \frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}} + \frac{2\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx}{5a} + \frac{e\,\mathrm{Subst}\left(\int \frac{1}{x^{7/2}}\,dx, x\right.}{ad} \\
&= -\frac{2e}{5ad(e\sin(c+dx))^{5/2}} + \frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}} - \frac{4\cos(c+dx)}{5ade\sqrt{e\sin(c+}} \\
&= -\frac{2e}{5ad(e\sin(c+dx))^{5/2}} + \frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}} - \frac{4\cos(c+dx)}{5ade\sqrt{e\sin(c+}} \\
&= -\frac{2e}{5ad(e\sin(c+dx))^{5/2}} + \frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}} - \frac{4\cos(c+dx)}{5ade\sqrt{e\sin(c+}}
\end{aligned}
$$

Mathematica [C] time = 1.10, size = 124, normalized size = 0.92

$$
\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(\cos(c+dx)+i\sin(c+dx))\left(2\sqrt{1-e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{2i(c+dx)}\right)(\cos(c+dx)+1)+3i\sin(c\right.}{15ade\sqrt{e\sin(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (Sec[(c + d*x)/2]^2*(Cos[c + d*x] + I*Sin[c + d*x])*(-6 - 9*Cos[c + d*x] +
2*Sqrt[1 - E^((2*I)*(c + d*x))]*(1 + Cos[c + d*x])*Hypergeometric2F1[1/2, 3
/4, 7/4, E^((2*I)*(c + d*x))] + (3*I)*Sin[c + d*x]))/(15*a*d*e*Sqrt[e*Sin[c
 + d*x]])
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{e\sin\left(dx+c\right)}}{ae^{2}\cos\left(dx+c\right)^{2}-ae^{2}+\left(ae^{2}\cos\left(dx+c\right)^{2}-ae^{2}\right)\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*sin(d*x + c))/(a*e^2*cos(d*x + c)^2 - a*e^2 + (a*e^2*cos(d
*x + c)^2 - a*e^2)*sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(a\sec\left(dx+c\right)+a\right)\left(e\sin\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(3/2)), x)
```

maple [A] time = 3.70, size = 187, normalized size = 1.39

$$\frac{-\frac{2e}{5a(e\sin(dx+c))^{\frac{5}{2}}}+\frac{\frac{4\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\left(\sin^{\frac{7}{2}}(dx+c)\right)\text{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)}{5}-\frac{2\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\left(\sin^{\frac{7}{2}}(dx+c)\right)\text{EllipticF}\left(\sqrt{-\sin\ldots\right.}{5}}{ea\sin(dx+c)^{3}\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x)
```

```
[Out] (-2/5/a*e/(e*sin(d*x+c))^(5/2)+2/5/e*(2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)
+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-(-s
in(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(
d*x+c)+1)^(1/2),1/2*2^(1/2))+2*sin(d*x+c)^5-3*sin(d*x+c)^3+sin(d*x+c))/a/si
n(d*x+c)^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\cos\left(c+d\,x\right)}{a\,{\left(e\,\sin\left(c+d\,x\right)\right)}^{3/2}\,\left(\cos\left(c+d\,x\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x))),x)
```

```
[Out] int(cos(c + d*x)/(a*(e*sin(c + d*x))^(3/2)*(cos(c + d*x) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{(e\sin(c+dx))^{\frac{3}{2}}\sec(c+dx)+(e\sin(c+dx))^{\frac{3}{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Integral(1/((e*sin(c + d*x))**(3/2)*sec(c + d*x) + (e*sin(c + d*x))**(3/2))
, x)/a
```

3.126 $\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=135

$$\frac{4\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{21ade^2\sqrt{e\sin(c+dx)}}-\frac{2e}{7ad(e\sin(c+dx))^{7/2}}+\frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}}-\frac{4\cos(c+dx)}{21ade(e\sin(c+dx))^{3/2}}$$

```
[Out] -2/7*e/a/d/(e*sin(d*x+c))^(7/2)+2/7*e*cos(d*x+c)/a/d/(e*sin(d*x+c))^(7/2)-4
/21*cos(d*x+c)/a/d/e/(e*sin(d*x+c))^(3/2)-4/21*(sin(1/2*c+1/4*Pi+1/2*d*x)^2
)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/
2))*sin(d*x+c)^(1/2)/a/d/e^2/(e*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3872, 2839, 2564, 30, 2567, 2636, 2642, 2641}

$$\frac{4\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{21ade^2\sqrt{e\sin(c+dx)}}-\frac{2e}{7ad(e\sin(c+dx))^{7/2}}+\frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}}-\frac{4\cos(c+dx)}{21ade(e\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-2*e)/(7*a*d*(e*Sin[c + d*x])^(7/2)) + (2*e*Cos[c + d*x])/(7*a*d*(e*Sin[c
+ d*x])^(7/2)) - (4*Cos[c + d*x])/(21*a*d*e*(e*Sin[c + d*x])^(3/2)) + (4*El
lipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a*d*e^2*Sqrt[e*Sin[c
 + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx &= -\int \frac{\cos(c+dx)}{(-a-a\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx \\
&= \frac{e^2\int \frac{\cos(c+dx)}{(e\sin(c+dx))^{9/2}}\,dx}{a} - \frac{e^2\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{9/2}}\,dx}{a} \\
&= \frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}} + \frac{2\int \frac{1}{(e\sin(c+dx))^{5/2}}\,dx}{7a} + \frac{e\,\text{Subst}\left(\int \frac{1}{x^{9/2}}\,dx, x\right.}{ad} \\
&= -\frac{2e}{7ad(e\sin(c+dx))^{7/2}} + \frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}} - \frac{4\cos(c+dx)}{21ade(e\sin(c+} \\
&= -\frac{2e}{7ad(e\sin(c+dx))^{7/2}} + \frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}} - \frac{4\cos(c+dx)}{21ade(e\sin(c+} \\
&= -\frac{2e}{7ad(e\sin(c+dx))^{7/2}} + \frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}} - \frac{4\cos(c+dx)}{21ade(e\sin(c+}
\end{aligned}$$

Mathematica [A] time = 1.27, size = 91, normalized size = 0.67

$$-\frac{2\left(2\cos(c+dx)+\cos(2(c+dx))+\sin^{\frac{7}{2}}(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)F\left(\frac{1}{4}(-2c-2dx+\pi)\middle|2\right)+4\right)}{21ade(\cos(c+dx)+1)(e\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-2*(4 + 2*Cos[c + d*x] + Cos[2*(c + d*x)] + Csc[(c + d*x)/2]^2*EllipticF[(
-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(7/2)))/(21*a*d*e*(1 + Cos[c + d*x])*
(e*Sin[c + d*x])^(3/2))
```

fricas [F] time = 1.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{e\sin(dx+c)}}{\left(ae^3\cos(dx+c)^2-ae^3+\left(ae^3\cos(dx+c)^2-ae^3\right)\sec(dx+c)\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*sin(d*x + c))/((a*e^3*cos(d*x + c)^2 - a*e^3 + (a*e^3*cos(
d*x + c)^2 - a*e^3)*sec(d*x + c))*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{(a\sec(dx+c)+a)(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(5/2)), x)
```

maple [A] time = 3.28, size = 136, normalized size = 1.01

$$\frac{-\frac{2e}{7a(e\sin(dx+c))^{\frac{7}{2}}}-\frac{2\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{9}{2}}(dx+c)\right)\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-2\left(\sin^5(dx+c)\right)+5\left(\sin^3(dx+c)\right)-3\sin(dx+c\right)}{21e^2a\sin(dx+c)^4\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x)
```

```
[Out] (-2/7/a*e/(e*sin(d*x+c))^(7/2)-2/21/e^2*((-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(9/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*
sin(d*x+c)^5+5*sin(d*x+c)^3-3*sin(d*x+c))/a/sin(d*x+c)^4/cos(d*x+c)/(e*sin(
d*x+c))^(1/2))/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\cos(c+d\,x)}{a\,{(e\,\sin(c+d\,x))}^{5/2}\,(\cos(c+d\,x)+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x))),x)
```

```
[Out] int(cos(c + d*x)/(a*(e*sin(c + d*x))^(5/2)*(cos(c + d*x) + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.127 $\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=162

$$\frac{52e^4\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{21a^2d\sqrt{e\sin(c+dx)}}-\frac{4e^3\sqrt{e\sin(c+dx)}}{a^2d}+\frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7a^2d}+\frac{26e^3\cos(c+dx)\sqrt{}}{21a^2d}$$

```
[Out] 4/5*e*(e*sin(d*x+c))^(5/2)/a^2/d-52/21*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*
sin(d*x+c)^(1/2)/a^2/d/(e*sin(d*x+c))^(1/2)-4*e^3*(e*sin(d*x+c))^(1/2)/a^2/
d+26/21*e^3*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/a^2/d+2/7*e^3*cos(d*x+c)^3*(e*s
in(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.55, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3872, 2875, 2873, 2569, 2642, 2641, 2564, 14}

$$-\frac{4e^3\sqrt{e\sin(c+dx)}}{a^2d}+\frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7a^2d}+\frac{26e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d}+\frac{52e^4\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(\right.\right.}{21a^2d\sqrt{e\sin(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (52*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a^2*d*Sqrt
[e*Sin[c + d*x]]) - (4*e^3*Sqrt[e*Sin[c + d*x]])/(a^2*d) + (26*e^3*Cos[c +
d*x]*Sqrt[e*Sin[c + d*x]])/(21*a^2*d) + (2*e^3*Cos[c + d*x]^3*Sqrt[e*Sin[c
+ d*x]])/(7*a^2*d) + (4*e*(e*Sin[c + d*x])^(5/2))/(5*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
```

```
d}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{7/2}}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx}{a^4} \\
&= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{\sqrt{e\sin(c+dx)}}-\frac{2a^2\cos^3(c+dx)}{\sqrt{e\sin(c+dx)}}+\frac{a^2\cos^4(c+dx)}{\sqrt{e\sin(c+dx)}}\right)dx}{a^4} \\
&= \frac{e^4\int \frac{\cos^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{a^2}+\frac{e^4\int \frac{\cos^4(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{a^2}-\frac{\left(2e^4\right)\int \frac{\cos^3(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{a^2} \\
&= \frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d}+\frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7a^2d}-\frac{\left(2e^3\right)\text{Subst}\big(
\end{aligned}$$

$$\begin{aligned}
&= \frac{26e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d}+\frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7a^2d}-\frac{\left(2e^3\right)\text{Subst}}{} \\
&= \frac{4e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}}-\frac{4e^3\sqrt{e\sin(c+dx)}}{a^2d}+\frac{26e^3\cos(c+dx)\sqrt{}}{21a^2d} \\
&= \frac{52e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}}-\frac{4e^3\sqrt{e\sin(c+dx)}}{a^2d}+\frac{26e^3\cos(c+dx)}{21a^2}
\end{aligned}$$

Mathematica [A] time = 1.55, size = 94, normalized size = 0.58

$$-\frac{e^3\sqrt{e\sin(c+dx)}\left(520F\left(\frac{1}{4}(-2c-2dx+\pi)\middle|2\right)+\sqrt{\sin(c+dx)}\,(-305\cos(c+dx)+84\cos(2(c+dx))-15\,\text{co}\right.}{210a^2d\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sin[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/210*(e^3*(520*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + (756 - 305*Cos[c + d
*x] + 84*Cos[2*(c + d*x)] - 15*Cos[3*(c + d*x)])*Sqrt[Sin[c + d*x]])*Sqrt[e
*Sin[c + d*x]])/(a^2*d*Sqrt[Sin[c + d*x]])
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(e^{3} \cos \left(dx+c\right)^{2} - e^{3}\right) \sqrt{e \sin \left(dx+c\right)} \sin \left(dx+c\right)}{a^{2} \sec \left(dx+c\right)^{2} + 2 \, a^{2} \sec \left(dx+c\right) + a^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(e^3*cos(d*x + c)^2 - e^3)*sqrt(e*sin(d*x + c))*sin(d*x + c)/(a^2
*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e \sin \left(dx+c\right)\right)^{\frac{7}{2}}}{\left(a \sec \left(dx+c\right) + a\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(7/2)/(a*sec(d*x + c) + a)^2, x)
```

maple [A] time = 4.12, size = 145, normalized size = 0.90

$$-\frac{2 e^{4} \left(-15 \sin \left(dx+c\right) \left(\cos^{4}\left(dx+c\right)\right) + 65 \sqrt{-\sin \left(dx+c\right)+1}\, \sqrt{2 \sin \left(dx+c\right)+2}\, \left(\sqrt{\sin}\left(dx+c\right)\right) \mathrm{EllipticF}\left(\sqrt{\cdots}\right.\right.}{105 a^{2} \cos \left(dx \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -2/105/a^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^4*(-15*sin(d*x+c)*cos(d*x+c)^4
+65*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF
((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+42*cos(d*x+c)^3*sin(d*x+c)-65*cos(d*x+c
)^2*sin(d*x+c)+168*cos(d*x+c)*sin(d*x+c))/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cos\left(c+d\,x\right)}^2\,{\left(e\,\sin\left(c+d\,x\right)\right)}^{7/2}}{a^2\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(7/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(7/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(7/2)/(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

3.128 $\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=187

$$\frac{4e^3}{a^2d\sqrt{e\sin(c+dx)}}-\frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}}-\frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}}-\frac{44e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}}+\frac{4e(e\sin(\cdots}{3\cdots}$$

```
[Out] 4/3*e*(e*sin(d*x+c))^(3/2)/a^2/d-12/5*e*cos(d*x+c)*(e*sin(d*x+c))^(3/2)/a^2
/d+4*e^3/a^2/d/(e*sin(d*x+c))^(1/2)-2*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^(
1/2)-2*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(1/2)+44/5*e^2*(sin(1/2*c+1/4*
Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1
/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a^2/d/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.60, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3872, 2875, 2873, 2567, 2640, 2639, 2564, 14, 2569}

$$\frac{4e^3}{a^2d\sqrt{e\sin(c+dx)}}-\frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}}-\frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}}-\frac{44e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}}+\frac{4e(e\sin(\cdots}{3\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*e^3)/(a^2*d*Sqrt[e*Sin[c + d*x]]) - (2*e^3*Cos[c + d*x])/(a^2*d*Sqrt[e*S
in[c + d*x]]) - (2*e^3*Cos[c + d*x]^3)/(a^2*d*Sqrt[e*Sin[c + d*x]]) - (44*e
^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a^2*d*Sqrt[Sin
[c + d*x]]) + (4*e*(e*Sin[c + d*x])^(3/2))/(3*a^2*d) - (12*e*Cos[c + d*x]*(
e*Sin[c + d*x])^(3/2))/(5*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
```

```
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{5/2}}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}dx}{a^4}$$
$$= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{3/2}} - \frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{3/2}} + \frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{3/2}}\right)dx}{a^4}$$
$$= \frac{e^4\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{3/2}}dx}{a^2} + \frac{e^4\int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{3/2}}dx}{a^2} - \frac{(2e^4)\int \frac{\cos^3(c+dx)}{(e\sin(c+dx))^{3/2}}dx}{a^2}$$

$$= -\frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{(2e^2)\int\sqrt{e\sin(c+dx)}\,dx}{a^2} - \frac{(6e^2)\int \cos}{}$$
$$= -\frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{12e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d} - \frac{(12e}{}$$
$$= \frac{4e^3}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{4e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right.\right.}{a^2d\sqrt{\sin}}$$
$$= \frac{4e^3}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{44e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+d\right.\right.}{5a^2d\sqrt{\sin}}$$

Mathematica [C] time = 2.99, size = 249, normalized size = 1.33

$$\frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)(e\sin(c+dx))^{5/2}\left(\csc^2(c+dx)\left(20\sin(c)\cos(dx)-3\sin(2c)\cos(2dx)+20\cos(c)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sin[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(e*Sin[c + d*x])^(5/2)*(((352*I)*E^((2
*I)*(2*c + d*x))*(3*Hypergeometric2F1[-1/4, 1/2, 3/4, E^((2*I)*(c + d*x))]
+ E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))]))/((1
 + E^((2*I)*c))*(1 - E^((2*I)*(c + d*x)))^(5/2)) + Csc[c + d*x]^2*(20*Cos[d
*x]*Sin[c] - 3*Cos[2*d*x]*Sin[2*c] + Sec[c/2]*(-36*Sec[c]*Sin[(3*c)/2] + 60
*Sec[(c + d*x)/2]*Sin[(d*x)/2]) + 20*Cos[c]*Sin[d*x] - 3*Cos[2*c]*Sin[2*d*x
] - 96*Sec[c]*Tan[c/2])))/(15*a^2*d*(1 + Sec[c + d*x])^2)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(e^2\cos\left(dx+c\right)^2-e^2\right)\sqrt{e\sin\left(dx+c\right)}}{a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(e^2*cos(d*x + c)^2 - e^2)*sqrt(e*sin(d*x + c))/(a^2*sec(d*x + c)
^2 + 2*a^2*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

maple [A] time = 4.03, size = 173, normalized size = 0.93

$$-\frac{2e^3\left(33\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}\,(dx+c)\right)\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-66\sqrt{-}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -2/15/a^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^3*(33*(-sin(d*x+c)+1)^(1/2)*(2*
sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^
(1/2))-66*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ell
ipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-3*cos(d*x+c)^4+10*cos(d*x+c)^3+33
*cos(d*x+c)^2-40*cos(d*x+c))/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2\,(e\,\sin(c+d\,x))^{5/2}}{a^2\,(\cos(c+d\,x)+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(5/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(5/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.129 $\int \frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=189

$$\frac{4e^3}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{4e^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{a^2d\sqrt{e\sin(c+dx)}}+\frac{4e}{}$$

```
[Out] 4/3*e^3/a^2/d/(e*sin(d*x+c))^(3/2)-2/3*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^
(3/2)-2/3*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(3/2)+4*e^2*(sin(1/2*c+1/4*
Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1
/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(e*sin(d*x+c))^(1/2)+4*e*(e*sin(d*x
+c))^(1/2)/a^2/d-4/3*e*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 0.59, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3872, 2875, 2873, 2567, 2642, 2641, 2564, 14, 2569}

$$\frac{4e^3}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{4e^2\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{a^2d\sqrt{e\sin(c+dx)}}+\frac{4e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*e^3)/(3*a^2*d*(e*Sin[c + d*x])^(3/2)) - (2*e^3*Cos[c + d*x])/(3*a^2*d*(e
*Sin[c + d*x])^(3/2)) - (2*e^3*Cos[c + d*x]^3)/(3*a^2*d*(e*Sin[c + d*x])^(3
/2)) - (4*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^2*d*S
qrt[e*Sin[c + d*x]]) + (4*e*Sqrt[e*Sin[c + d*x]])/(a^2*d) - (4*e*Cos[c + d*
x]*Sqrt[e*Sin[c + d*x]])/(3*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
```

```
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{3/2}}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx}{a^4}$$
$$= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{5/2}} - \frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{5/2}} + \frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{5/2}}\right)dx}{a^4}$$
$$= \frac{e^4\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx}{a^2} + \frac{e^4\int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\cos^3(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx}{a^2}$$
$$= -\frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{(2e^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3a^2} - \frac{(2e^2)\int \cdots}{\cdots}$$
$$= -\frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{4e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d} - \cdots$$
$$= \frac{4e^3}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{4e^2F\left(\frac{1}{2}(c\cdots\right.}{3\cdots}$$
$$= \frac{4e^3}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{4e^2F\left(\frac{1}{2}(c\cdots\right.}{\cdots}$$

Mathematica [A] time = 1.91, size = 119, normalized size = 0.63

$$\frac{2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)(e\sin(c+dx))^{3/2}\left(\frac{24F\left(\frac{1}{4}(-2c-2dx+\pi)\middle|2\right)}{\sin^{\frac{3}{2}}(c+dx)} + (10\cos(c+dx)-\cos(2(c+dx))+15)\csc(\cdots\right.}{3a^2d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sin[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*((15 + 10*Cos[c + d*x] - Cos[2*(c + d*
x)])*Csc[c + d*x]*Sec[(c + d*x)/2]^2 + (24*EllipticF[(-2*c + Pi - 2*d*x)/4,
 2])/Sin[c + d*x]^(3/2))*(e*Sin[c + d*x])^(3/2))/(3*a^2*d*(1 + Sec[c + d*x]
)^2)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{e\sin(dx+c)}\,e\sin(dx+c)}{a^2\sec(dx+c)^2 + 2a^2\sec(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*sin(d*x + c))*e*sin(d*x + c)/(a^2*sec(d*x + c)^2 + 2*a^2*se
c(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] integrate((e*sin(d*x + c))^(3/2)/(a*sec(d*x + c) + a)^2, x)

maple [A] time = 4.30, size = 153, normalized size = 0.81

$$-\frac{2e^3\left(3\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{7}{2}}(dx+c)\right)\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-\left(\cos^6(d\right.\right.}{3a^2\left(e\sin(dx+c)\right)^{\frac{3}{2}}\cos(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x)

[Out] -2/3/a^2/(e*sin(d*x+c))^(3/2)/cos(d*x+c)/(cos(d*x+c)^2-1)*e^3*(3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-cos(d*x+c)^6+6*cos(d*x+c)^5+4*cos(d*x+c)^4-14*cos(d*x+c)^3-3*cos(d*x+c)^2+8*cos(d*x+c))/d

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(e\sin(dx+c)\right)^{\frac{3}{2}}}{\left(a\sec(dx+c)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] integrate((e*sin(d*x + c))^(3/2)/(a*sec(d*x + c) + a)^2, x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\cos(c+d\,x)^2\,\left(e\,\sin(c+d\,x)\right)^{3/2}}{a^2\,\left(\cos(c+d\,x)+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*sin(c + d*x))^(3/2)/(a + a/cos(c + d*x))^2,x)

[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(3/2))/(a^2*(cos(c + d*x) + 1)^2), x)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*sin(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)

[Out] Timed out

3.130 $\int \frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=188

$$\frac{4e^3}{5a^2d(e\sin(c+dx))^{5/2}}-\frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}}-\frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}}-\frac{4e}{a^2d\sqrt{e\sin(c+dx)}}+\frac{16e\cos(c+dx)}{5a^2d\sqrt{e\sin(c+dx)}}+$$

```
[Out] 4/5*e^3/a^2/d/(e*sin(d*x+c))^(5/2)-2/5*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^
(5/2)-2/5*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(5/2)-4*e/a^2/d/(e*sin(d*x+
c))^(1/2)+16/5*e*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^(1/2)-28/5*(sin(1/2*c+1/4*
Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1
/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a^2/d/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3872, 2875, 2873, 2567, 2636, 2640, 2639, 2564, 14}

$$\frac{4e^3}{5a^2d(e\sin(c+dx))^{5/2}}-\frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}}-\frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}}-\frac{4e}{a^2d\sqrt{e\sin(c+dx)}}+\frac{16e\cos(c+dx)}{5a^2d\sqrt{e\sin(c+dx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sin[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*e^3)/(5*a^2*d*(e*Sin[c + d*x])^(5/2)) - (2*e^3*Cos[c + d*x])/(5*a^2*d*(e
*Sin[c + d*x])^(5/2)) - (2*e^3*Cos[c + d*x]^3)/(5*a^2*d*(e*Sin[c + d*x])^(5
/2)) - (4*e)/(a^2*d*Sqrt[e*Sin[c + d*x]]) + (16*e*Cos[c + d*x])/(5*a^2*d*Sq
rt[e*Sin[c + d*x]]) + (28*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d
*x]])/(5*a^2*d*Sqrt[Sin[c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sqrt{e\sin(c+dx)}}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx}{a^4}$$
$$= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{7/2}} - \frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{7/2}} + \frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{7/2}}\right)dx}{a^4}$$
$$= \frac{e^4\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx}{a^2} + \frac{e^4\int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\cos^3(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx}{a^2}$$

$$= -\frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{(2e^2)\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx}{5a^2} - \frac{(6e^2)\int}{}$$

$$= -\frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} + \frac{16e\cos(c+dx)}{5a^2d\sqrt{e\sin(c+dx)}} + \frac{2\int\sqrt{e\sin}}{5}$$
$$= \frac{4e^3}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{4e}{a^2d\sqrt{e\sin}}$$

$$= \frac{4e^3}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{4e}{a^2d\sqrt{e\sin}}$$

Mathematica [C] time = 1.36, size = 222, normalized size = 1.18

$$\frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{e\sin(c+dx)}\left(\frac{3}{4}\sec(c)\left(49\sin\left(\frac{1}{2}(c-dx)\right)+35\sin\left(\frac{1}{2}(3c+dx)\right)-23\sin\left(\frac{1}{2}(c+\right.\right.\right.}{15a^2d(\sec(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sin[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*Sqrt[e*Sin[c + d*x]]*(((56*I)*E^((2*I)
*c)*(3*Hypergeometric2F1[-1/4, 1/2, 3/4, E^((2*I)*(c + d*x))] + E^((2*I)*d*
x)*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))]))/((1 + E^((2*I)*c
))*Sqrt[1 - E^((2*I)*(c + d*x))]) + (3*Sec[c]*Sec[(c + d*x)/2]^3*(49*Sin[(c
 - d*x)/2] + 35*Sin[(3*c + d*x)/2] - 23*Sin[(c + 3*d*x)/2] + 5*Sin[(5*c + 3
*d*x)/2]))/4))/(15*a^2*d*(1 + Sec[c + d*x])^2)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\sin(dx+c)}}{a^2\sec(dx+c)^2 + 2a^2\sec(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*sin(d*x + c))/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^
2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\sin(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*sin(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

maple [A] time = 4.83, size = 205, normalized size = 1.09

$$-\frac{-\frac{2e\left(-\frac{2e^2}{5(e\sin(dx+c))^{\frac{5}{2}}}+\frac{2}{\sqrt{e\sin(dx+c)}}\right)}{a^2}-\frac{2e\left(14\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{7}{2}}(dx+c)\right)\mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-7\sqrt{-\sin(dx+c)+1}}{5a^2\sin(dx+c)^3\,\mathrm{co}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] (-2*e/a^2*(-2/5*e^2/(e*sin(d*x+c))^(5/2)+2/(e*sin(d*x+c))^(1/2))-2/5*e*(14*
(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(7/2)*EllipticE((-s
in(d*x+c)+1)^(1/2),1/2*2^(1/2))-7*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1
/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+9*sin(d*x
+c)^5-11*sin(d*x+c)^3+2*sin(d*x+c))/a^2/sin(d*x+c)^3/cos(d*x+c)/(e*sin(d*x+
c))^(1/2))/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\sin(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*sin(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2\,\sqrt{e\,\sin(c+d\,x)}}{a^2\,(\cos(c+d\,x)+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(1/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(1/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{e\sin(c+dx)}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(e*sin(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a*
*2
```

3.131 $\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=190

$$\frac{4e^3}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{4e}{3a^2d(e\sin(c+dx))^{3/2}}+\frac{16e\cos(c+dx)}{21a^2d(e\sin(c+dx)}$$

```
[Out] 4/7*e^3/a^2/d/(e*sin(d*x+c))^(7/2)-2/7*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^
(7/2)-2/7*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(7/2)-4/3*e/a^2/d/(e*sin(d*
x+c))^(3/2)+16/21*e*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^(3/2)-20/21*(sin(1/2*c+
1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*
Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(e*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3872, 2875, 2873, 2567, 2636, 2642, 2641, 2564, 14}

$$\frac{4e^3}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{4e}{3a^2d(e\sin(c+dx))^{3/2}}+\frac{16e\cos(c+dx)}{21a^2d(e\sin(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (4*e^3)/(7*a^2*d*(e*Sin[c + d*x])^(7/2)) - (2*e^3*Cos[c + d*x])/(7*a^2*d*(e
*Sin[c + d*x])^(7/2)) - (2*e^3*Cos[c + d*x]^3)/(7*a^2*d*(e*Sin[c + d*x])^(7
/2)) - (4*e)/(3*a^2*d*(e*Sin[c + d*x])^(3/2)) + (16*e*Cos[c + d*x])/(21*a^2
*d*(e*Sin[c + d*x])^(3/2)) + (20*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[
c + d*x]])/(21*a^2*d*Sqrt[e*Sin[c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx &= \int \frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx \\
&= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{9/2}}\,dx}{a^4} \\
&= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{9/2}}-\frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{9/2}}+\frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{9/2}}\right)dx}{a^4} \\
&= \frac{e^4\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{9/2}}\,dx}{a^2}+\frac{e^4\int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{9/2}}\,dx}{a^2}-\frac{\left(2e^4\right)\int \frac{\cos^3(c+dx)}{(e\sin(c+dx))^{9/2}}\,dx}{a^2} \\
&= -\frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{\left(2e^2\right)\int \frac{1}{(e\sin(c+dx))^5}}{7a^2} \\
&= -\frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}}+\frac{16e\cos(c+dx}{21a^2d(e\sin(c+dx} \\
&= \frac{4e^3}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7}} \\
&= \frac{4e^3}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7}}
\end{aligned}
$$

Mathematica [A] time = 1.40, size = 82, normalized size = 0.43

$$
-\frac{\csc^3(c+dx)\left(16\sin^4\left(\frac{1}{2}(c+dx)\right)(11\cos(c+dx)+8)+40\sin^{\frac{7}{2}}(c+dx)F\left(\frac{1}{4}(-2c-2dx+\pi)\middle|2\right)\right)}{42a^2d\sqrt{e\sin(c+dx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] -1/42*(Csc[c + d*x]^3*(16*(8 + 11*Cos[c + d*x])*Sin[(c + d*x)/2]^4 + 40*Ell
ipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(7/2)))/(a^2*d*Sqrt[e*Sin[c +
 d*x]])
```

fricas [F] time = 0.78, size = 0, normalized size = 0.00

$$
\mathrm{integral}\left(\frac{\sqrt{e\sin(dx+c)}}{\left(a^2e\sec(dx+c)^2+2\,a^2e\sec(dx+c)+a^2e\right)\sin(dx+c)},x\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*sin(d*x + c))/((a^2*e*sec(d*x + c)^2 + 2*a^2*e*sec(d*x + c)
 + a^2*e)*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$
\int \frac{1}{(a\sec(dx+c)+a)^2\sqrt{e\sin(dx+c)}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(e*sin(d*x + c))), x)
```

maple [A] time = 4.18, size = 148, normalized size = 0.78

$$\frac{\frac{4e^3\left(7\left(\cos^2(dx+c)\right)-4\right)}{21a^2(e\sin(dx+c))^{\frac{7}{2}}}-\frac{2\left(5\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{9}{2}}(dx+c)\right)\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)+11\left(\sin^5(dx+c)\right)-17\left(\sin^3(dx+c)\right)+\cdots\right)}{21a^2\sin(dx+c)^4\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x)
[Out] (4/21/a^2*e^3/(e*sin(d*x+c))^(7/2)*(7*cos(d*x+c)^2-4)-2/21*(5*(-sin(d*x+c)+
1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(9/2)*EllipticF((-sin(d*x+c)+1)^
(1/2),1/2*2^(1/2))+11*sin(d*x+c)^5-17*sin(d*x+c)^3+6*sin(d*x+c))/a^2/sin(d*
x+c)^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2}{a^2\,\sqrt{e\,\sin(c+d\,x)}\,\left(\cos(c+d\,x)+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(1/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/(a^2*(e*sin(c + d*x))^(1/2)*(cos(c + d*x) + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{e\sin(c+dx)}\sec^2(c+dx)+2\sqrt{e\sin(c+dx)}\sec(c+dx)+\sqrt{e\sin(c+dx)}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(e*sin(c + d*x))*sec(c + d*x)**2 + 2*sqrt(e*sin(c + d*x))*s
ec(c + d*x) + sqrt(e*sin(c + d*x))), x)/a**2
```

3.132 $\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=224

$$\frac{4e^3}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15a^2de^2\sqrt{\sin(c+dx)}}-\frac{}{5a^2}$$

```
[Out] 4/9*e^3/a^2/d/(e*sin(d*x+c))^(9/2)-2/9*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^
(9/2)-2/9*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(9/2)-4/5*e/a^2/d/(e*sin(d*
x+c))^(5/2)+16/45*e*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^(5/2)-4/15*cos(d*x+c)/a
^2/d/e/(e*sin(d*x+c))^(1/2)+4/15*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/
2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x
+c))^(1/2)/a^2/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3872, 2875, 2873, 2567, 2636, 2640, 2639, 2564, 14}

$$\frac{4e^3}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15a^2de^2\sqrt{\sin(c+dx)}}-\frac{}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (4*e^3)/(9*a^2*d*(e*Sin[c + d*x])^(9/2)) - (2*e^3*Cos[c + d*x])/(9*a^2*d*(e
*Sin[c + d*x])^(9/2)) - (2*e^3*Cos[c + d*x]^3)/(9*a^2*d*(e*Sin[c + d*x])^(9
/2)) - (4*e)/(5*a^2*d*(e*Sin[c + d*x])^(5/2)) + (16*e*Cos[c + d*x])/(45*a^2
*d*(e*Sin[c + d*x])^(5/2)) - (4*Cos[c + d*x])/(15*a^2*d*e*Sqrt[e*Sin[c + d*
x]]) - (4*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(15*a^2*d*
e^2*Sqrt[Sin[c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
```

```
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \int \frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$$

$$= \frac{e^4 \int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{11/2}}\,dx}{a^4}$$

$$= \frac{e^4 \int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{11/2}} - \frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{11/2}} + \frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{11/2}}\right)dx}{a^4}$$

$$= \frac{e^4 \int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{11/2}}\,dx}{a^2} + \frac{e^4 \int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{11/2}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\cos^3(c+dx)}{(e\sin(c+dx))^{11/2}}}{a^2}$$

$$= -\frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{(2e^2)\int \frac{1}{(e\sin(c+dx}}{9a^2}$$

$$= -\frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} + \frac{16e\cos(c+}{45a^2d(e\sin(c+}$$

$$= \frac{4e^3}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos^3(c+dx}{9a^2d(e\sin(c+dx)}$$

$$= \frac{4e^3}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos^3(c+dx}{9a^2d(e\sin(c+dx)}$$

$$= \frac{4e^3}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos^3(c+dx}{9a^2d(e\sin(c+dx)}$$

Mathematica [C] time = 1.44, size = 163, normalized size = 0.73

$$\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)(\cos(c+dx)+i\sin(c+dx))\left(e^{-2i(c+dx)}\sqrt{1-e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^4 {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{2i(c+dx)}\right)+16i\sin\right.}{180a^2de\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (Sec[(c + d*x)/2]^4*(Cos[c + d*x] + I*Sin[c + d*x])*(-31 - 40*Cos[c + d*x]
- 19*Cos[2*(c + d*x)] + ((1 + E^(I*(c + d*x)))^4*Sqrt[1 - E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*
x)) + (16*I)*Sin[c + d*x] + (13*I)*Sin[2*(c + d*x)]))/(180*a^2*d*e*Sqrt[e*S
in[c + d*x]])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{e\sin(dx+c)}}{a^2e^2\cos(dx+c)^2 - a^2e^2 + \left(a^2e^2\cos(dx+c)^2 - a^2e^2\right)\sec(dx+c)^2 + 2\left(a^2e^2\cos(dx+c)^2 - a^2e^2\right)\sec}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*sin(d*x + c))/(a^2*e^2*cos(d*x + c)^2 - a^2*e^2 + (a^2*e^2
*cos(d*x + c)^2 - a^2*e^2)*sec(d*x + c)^2 + 2*(a^2*e^2*cos(d*x + c)^2 - a^2
*e^2)*sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^2 (e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2)), x)
```

maple [A] time = 4.42, size = 213, normalized size = 0.95

$$\frac{\frac{4e^3\left(9\left(\cos^2(dx+c)\right)-4\right)}{45a^2(e\sin(dx+c))^{\frac{9}{2}}}+\frac{\frac{4\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{11}{2}}(dx+c)\right)\mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)}{15}-\frac{2\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{11}{2}}(dx+c)\right)\mathrm{Ellip}}{15}}{e\,a^2\sin(dx+c)^5\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x)
```

```
[Out] (4/45*e^3/a^2/(e*sin(d*x+c))^(9/2)*(9*cos(d*x+c)^2-4)+2/45/e*(6*(-sin(d*x+c
)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(11/2)*EllipticE((-sin(d*x+c)+
1)^(1/2),1/2*2^(1/2))-3*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*
x+c)^(11/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+6*sin(d*x+c)^7-19*
sin(d*x+c)^5+23*sin(d*x+c)^3-10*sin(d*x+c))/a^2/sin(d*x+c)^5/cos(d*x+c)/(e*
sin(d*x+c))^(1/2))/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2}{a^2\,(e\,\sin(c+d\,x))^{3/2}\,(\cos(c+d\,x)+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(3/2)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] int(cos(c + d*x)^2/(a^2*(e*sin(c + d*x))^(3/2)*(cos(c + d*x) + 1)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

3.133 $\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=224

$$\frac{4e^3}{11a^2d(e\sin(c+dx))^{11/2}}-\frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}}-\frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}}+\frac{4\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{231a^2de^2\sqrt{e\sin(c+dx)}}-$$

```
[Out] 4/11*e^3/a^2/d/(e*sin(d*x+c))^(11/2)-2/11*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c
))^(11/2)-2/11*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(11/2)-4/7*e/a^2/d/(e*
sin(d*x+c))^(7/2)+16/77*e*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^(7/2)-4/231*cos(d
*x+c)/a^2/d/e/(e*sin(d*x+c))^(3/2)-4/231*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2
)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*si
n(d*x+c)^(1/2)/a^2/d/e^2/(e*sin(d*x+c))^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3872, 2875, 2873, 2567, 2636, 2642, 2641, 2564, 14}

$$\frac{4e^3}{11a^2d(e\sin(c+dx))^{11/2}}-\frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}}-\frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}}+\frac{4\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{231a^2de^2\sqrt{e\sin(c+dx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (4*e^3)/(11*a^2*d*(e*Sin[c + d*x])^(11/2)) - (2*e^3*Cos[c + d*x])/(11*a^2*d
*(e*Sin[c + d*x])^(11/2)) - (2*e^3*Cos[c + d*x]^3)/(11*a^2*d*(e*Sin[c + d*x
])^(11/2)) - (4*e)/(7*a^2*d*(e*Sin[c + d*x])^(7/2)) + (16*e*Cos[c + d*x])/(
77*a^2*d*(e*Sin[c + d*x])^(7/2)) - (4*Cos[c + d*x])/(231*a^2*d*e*(e*Sin[c +
 d*x])^(3/2)) + (4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(23
1*a^2*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
```

```
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \int \frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$$

$$= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{13/2}}\,dx}{a^4}$$

$$= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{13/2}} - \frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{13/2}} + \frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{13/2}}\right)dx}{a^4}$$

$$= \frac{e^4\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{13/2}}\,dx}{a^2} + \frac{e^4\int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{13/2}}\,dx}{a^2} - \frac{\left(2e^4\right)\int \frac{\cos^3(c+dx)}{(e\sin(c+dx))^{13/2}}}{a^2}$$

$$= -\frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{\left(2e^2\right)\int \frac{}{(e\sin(}}{11a}$$

$$= -\frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} + \frac{16e\cos(c}{77a^2d(e\sin(}$$

$$= \frac{4e^3}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos^3(c}{11a^2d(e\sin(c-}$$

$$= \frac{4e^3}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos^3(c}{11a^2d(e\sin(c-}$$

$$= \frac{4e^3}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos^3(c}{11a^2d(e\sin(c-}$$

Mathematica [A] time = 0.99, size = 113, normalized size = 0.50

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(97\cos(c+dx)+4\cos(2(c+dx))+\cos(3(c+dx))+\sin^{\frac{11}{2}}(c+dx)\csc^4\left(\frac{1}{2}(c+\right.\right.}{1848a^2de^2\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] -1/1848*(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]^5*(52 + 97*Cos[c + d*x] + 4*Cos[
2*(c + d*x)] + Cos[3*(c + d*x)] + Csc[(c + d*x)/2]^4*EllipticF[(-2*c + Pi -
 2*d*x)/4, 2]*Sin[c + d*x]^(11/2)))/(a^2*d*e^2*Sqrt[e*Sin[c + d*x]])
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{e\sin(dx+c)}}{\left(a^2e^3\cos(dx+c)^2 - a^2e^3 + \left(a^2e^3\cos(dx+c)^2 - a^2e^3\right)\sec(dx+c)^2 + 2\left(a^2e^3\cos(dx+c)^2 - a^2e^3\right)\text{se}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*sin(d*x + c))/((a^2*e^3*cos(d*x + c)^2 - a^2*e^3 + (a^2*e^
3*cos(d*x + c)^2 - a^2*e^3)*sec(d*x + c)^2 + 2*(a^2*e^3*cos(d*x + c)^2 - a^
2*e^3)*sec(d*x + c))*sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a\sec(dx+c)+a)^2(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2)), x)
```

maple [A] time = 4.84, size = 160, normalized size = 0.71

$$\frac{\frac{4e^3\left(11\left(\cos^2(dx+c)\right)-4\right)}{77a^2(e\sin(dx+c))^{\frac{11}{2}}}-\frac{2\left(\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sin^{\frac{13}{2}}(dx+c)\right)\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-2\left(\sin^7(dx+c)\right)+47\left(\sin^5(dx+c)\right)-\right.}{231e^2a^2\sin(dx+c)^6\cos(dx+c)\sqrt{e\sin(dx+c)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x)
[Out] (4/77*e^3/a^2/(e*sin(d*x+c))^(11/2)*(11*cos(d*x+c)^2-4)-2/231/e^2*((-sin(d*
x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(13/2)*EllipticF((-sin(d*x+
c)+1)^(1/2),1/2*2^(1/2))-2*sin(d*x+c)^7+47*sin(d*x+c)^5-87*sin(d*x+c)^3+42*
sin(d*x+c))/a^2/sin(d*x+c)^6/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2}{a^2\,{\left(e\,\sin\left(c+d\,x\right)\right)}^{5/2}\,{\left(\cos\left(c+d\,x\right)+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(5/2)*(a + a/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/(a^2*(e*sin(c + d*x))^(5/2)*(cos(c + d*x) + 1)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)
[Out] Timed out
```

3.134 $\int (a+a\sec(c+dx))^3(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=247

$$\frac{3a^3(e\sin(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{a^3(e\sin(c+dx))^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{a^3\cos}{}$$

```
[Out] 3*a^3*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+
m)/d/e/(1+m)+a^3*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(
d*x+c))^(1+m)/d/e/(1+m)+a^3*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*
m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+m)/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)+3*a^3*
hypergeom([3/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*(e*sin(d*x+
c))^(1+m)*(cos(d*x+c)^2)^(1/2)/d/e/(1+m)
```

Rubi [A] time = 0.35, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3872, 2873, 2643, 2564, 364, 2577}

$$\frac{3a^3(e\sin(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{a^3(e\sin(c+dx))^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{a^3\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a^3*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (3*a^3*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^
(1 + m))/(d*e*(1 + m)) + (a^3*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, Si
n[c + d*x]^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)) + (3*a^3*Sqrt[Cos[c +
 d*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c
 + d*x]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^3(e\sin(c+dx))^m\,dx &= -\int(-a-a\cos(c+dx))^3\sec^3(c+dx)(e\sin(c+dx))^m\,dx\\ &= -\int\left(-a^3(e\sin(c+dx))^m-3a^3\sec(c+dx)(e\sin(c+dx))^m-3a^3 \mathrm{s}\right.\\ &= a^3\int(e\sin(c+dx))^m\,dx+a^3\int\sec^3(c+dx)(e\sin(c+dx))^m\,dx+\\ &= \frac{a^3\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}}+\\ &= \frac{a^3\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}}+\end{aligned}$$

Mathematica [C] time = 7.35, size = 287, normalized size = 1.16

$$\frac{a^3(\cos(c+dx)+1)^3\tan(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(e\sin(c+dx))^m\left(3\sqrt{\cos^2(c+dx)}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+\right.\right.}{8d(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*(e*Sin[c + d*x])^m,x]
```

```
[Out] (2^(-3 - m)*a^3*E^(I*(c + d*x))*(((-I)*(-1 + E^((2*I)*(c + d*x))))/E^(I*(c
+ d*x)))^(1 + m)*(1 + Cos[c + d*x])^3*Hypergeometric2F1[1, (2 + m)/2, 1 - m
/2, E^((2*I)*(c + d*x))]*Sec[(c + d*x)/2]^6*(e*Sin[c + d*x])^m)/(d*m*Sin[c
+ d*x]^m) + (a^3*(1 + Cos[c + d*x])^3*(3*Cos[c + d*x]*Hypergeometric2F1[1,
(1 + m)/2, (3 + m)/2, Sin[c + d*x]^2] + 3*Sqrt[Cos[c + d*x]^2]*Hypergeometr
ic2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2] + Cos[c + d*x]*Hypergeomet
ric2F1[2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2])*Sec[(c + d*x)/2]^6*(e*Sin[
c + d*x])^m*Tan[c + d*x])/(8*d*(1 + m))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^3\sec(dx+c)^3+3\,a^3\sec(dx+c)^2+3\,a^3\sec(dx+c)+a^3\right)(e\sin(dx+c))^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)*(e*sin(d*x + c))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec (dx+c)+a)^3 (e\sin (dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*(e*sin(d*x + c))^m, x)
```

maple [F] time = 3.50, size = 0, normalized size = 0.00

$$\int (a+a\sec (dx+c))^3 (e\sin (dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec (dx+c)+a)^3 (e\sin (dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*(e*sin(d*x + c))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(e\,\sin \left(c+d\,x\right)\right)}^m\,{\left(a+\frac{a}{\cos \left(c+d\,x\right)}\right)}^3 \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^3,x)
```

```
[Out] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^3 \left(\int (e\sin (c+dx))^m \, dx + \int 3 (e\sin (c+dx))^m \sec (c+dx)\, dx + \int 3 (e\sin (c+dx))^m \sec^2 (c+dx)\, dx + \int (e\text{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(e*sin(d*x+c))**m,x)
```

```
[Out] a**3*(Integral((e*sin(c + d*x))**m, x) + Integral(3*(e*sin(c + d*x))**m*sec
(c + d*x), x) + Integral(3*(e*sin(c + d*x))**m*sec(c + d*x)**2, x) + Integr
al((e*sin(c + d*x))**m*sec(c + d*x)**3, x))
```

3.135 $\int (a+a\sec(c+dx))^2 (e\sin(c+dx))^m \, dx$

Optimal. Leaf size=195

$$\frac{2a^2(e\sin(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{a^2\cos(c+dx)(e\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(\right.}{de(m+1)\sqrt{\cos^2(c+dx)}}$$

```
[Out] 2*a^2*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+
m)/d/e/(1+m)+a^2*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+
c)^2)*(e*sin(d*x+c))^(1+m)/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)+a^2*hypergeom([3/
2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*(e*sin(d*x+c))^(1+m)*(co
s(d*x+c)^2)^(1/2)/d/e/(1+m)
```

Rubi [A] time = 0.29, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3872, 2873, 2643, 2564, 364, 2577}

$$\frac{2a^2(e\sin(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{a^2\cos(c+dx)(e\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(\right.}{de(m+1)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (2*a^2*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^
(1 + m))/(d*e*(1 + m)) + (a^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (
1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Sin[c + d*x])^(1 + m))
/(d*e*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^m\,dx = \int (-a-a\cos(c+dx))^2\sec^2(c+dx)(e\sin(c+dx))^m\,dx$$
$$= \int \left(a^2(e\sin(c+dx))^m + 2a^2\sec(c+dx)(e\sin(c+dx))^m + a^2\sec^2(c+$$
$$= a^2\int (e\sin(c+dx))^m\,dx + a^2\int \sec^2(c+dx)(e\sin(c+dx))^m\,dx + \left(2a\right.$$
$$= \frac{a^2\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \frac{a^2}{}$$
$$= \frac{a^2\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \frac{2a}{}$$

Mathematica [C] time = 4.61, size = 230, normalized size = 1.18

$$\frac{a^2\tan(c+dx)(e\sin(c+dx))^m\left(\sqrt{\cos^2(c+dx)}\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)+2\cos(c+dx)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin\right.\right.}{d(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^m,x]
```

```
[Out] (2^(-2 - m)*a^2*E^(I*(c + d*x))*(((-I)*(-1 + E^((2*I)*(c + d*x))))/E^(I*(c
+ d*x)))^(1 + m)*(1 + Cos[c + d*x])^2*Hypergeometric2F1[1, (2 + m)/2, 1 - m
/2, E^((2*I)*(c + d*x))]*Sec[(c + d*x)/2]^4*(e*Sin[c + d*x])^m)/(d*m*Sin[c
+ d*x]^m) + (a^2*(2*Cos[c + d*x]*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2,
 Sin[c + d*x]^2] + Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (
3 + m)/2, Sin[c + d*x]^2])*(e*Sin[c + d*x])^m*Tan[c + d*x])/(d*(1 + m))
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2\right)\left(e\sin\left(dx+c\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*(e*sin(d*x + c))^m
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^2(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^m, x)
```

maple [F] time = 3.00, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))^2(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x)
[Out] int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^2(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\sin(c+d\,x))^m\left(a+\frac{a}{\cos(c+d\,x)}\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^2,x)
[Out] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int (e\sin(c+dx))^m\,dx+\int 2(e\sin(c+dx))^m\sec(c+dx)\,dx+\int (e\sin(c+dx))^m\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*sin(d*x+c))**m,x)
[Out] a**2*(Integral((e*sin(c + d*x))**m, x) + Integral(2*(e*sin(c + d*x))**m*sec
(c + d*x), x) + Integral((e*sin(c + d*x))**m*sec(c + d*x)**2, x))
```

3.136 $\int (a + a\sec(c + dx))(e\sin(c + dx))^m\,dx$

Optimal. Leaf size=119

$$\frac{a(e\sin(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{a\cos(c+dx)(e\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx\right.}{de(m+1)\sqrt{\cos^2(c+dx)}}$$

```
[Out] a*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+m)/d
/e/(1+m)+a*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*
(e*sin(d*x+c))^(1+m)/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2838, 2564, 364, 2643}

$$\frac{a(e\sin(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{a\cos(c+dx)(e\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx\right.}{de(m+1)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2
]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (a*Hyperge
ometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1 + m
))/(d*e*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
```

```
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^m\,dx = -\int (-a-a\cos(c+dx))\sec(c+dx)(e\sin(c+dx))^m\,dx$$
$$= a\int (e\sin(c+dx))^m\,dx + a\int \sec(c+dx)(e\sin(c+dx))^m\,dx$$
$$= \frac{a\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \ldots$$
$$= \frac{a\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \ldots$$

Mathematica [A] time = 0.15, size = 97, normalized size = 0.82

$$\frac{a(e\sin(c+dx))^m\left(\sin(c+dx)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)+\sqrt{\cos^2(c+dx)}\,\tan(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin\right.\right.}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^m,x]
[Out] (a*(e*Sin[c + d*x])^m*(Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d
*x]^2]*Sin[c + d*x] + Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2
, (3 + m)/2, Sin[c + d*x]^2]*Tan[c + d*x]))/(d*(1 + m))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)(e\sin(dx+c))^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

maple [F] time = 2.50, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\sin(c+d\,x))^m\,\left(a+\frac{a}{\cos(c+d\,x)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int (e\sin(c+dx))^m\,dx+\int (e\sin(c+dx))^m\sec(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))**m,x)
[Out] a*(Integral((e*sin(c + d*x))**m, x) + Integral((e*sin(c + d*x))**m*sec(c +
d*x), x))
```

3.137 $\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=100

$$\frac{e\cos(c+dx)(e\sin(c+dx))^{m-1}\,{}_2F_1\left(-\frac{1}{2},\frac{m-1}{2};\frac{m+1}{2};\sin^2(c+dx)\right)}{ad(1-m)\sqrt{\cos^2(c+dx)}}-\frac{e(e\sin(c+dx))^{m-1}}{ad(1-m)}$$

```
[Out] -e*(e*sin(d*x+c))^(-1+m)/a/d/(1-m)+e*cos(d*x+c)*hypergeom([-1/2, -1/2+1/2*m
],[1/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(-1+m)/a/d/(1-m)/(cos(d*x+c)^2)^
(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3872, 2839, 2564, 30, 2577}

$$\frac{e\cos(c+dx)(e\sin(c+dx))^{m-1}\,{}_2F_1\left(-\frac{1}{2},\frac{m-1}{2};\frac{m+1}{2};\sin^2(c+dx)\right)}{ad(1-m)\sqrt{\cos^2(c+dx)}}-\frac{e(e\sin(c+dx))^{m-1}}{ad(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((e*(e*Sin[c + d*x])^(-1 + m))/(a*d*(1 - m))) + (e*Cos[c + d*x]*Hypergeome
tric2F1[-1/2, (-1 + m)/2, (1 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(-1 +
 m))/(a*d*(1 - m)*Sqrt[Cos[c + d*x]^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
```

```
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)(e\sin(c+dx))^m}{-a-a\cos(c+dx)}\,dx\\ &= \frac{e^2\int \cos(c+dx)(e\sin(c+dx))^{-2+m}\,dx}{a} - \frac{e^2\int \cos^2(c+dx)(e\sin(c+dx))^{-2+m}\,dx}{a}\\ &= \frac{e\cos(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{-1+m}}{ad(1-m)\sqrt{\cos^2(c+dx)}} + \frac{e\,\mathrm{Subst}\left(\int x\right.}{}\\ &= -\frac{e(e\sin(c+dx))^{-1+m}}{ad(1-m)} + \frac{e\cos(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};\sin^2(c+dx)\right)(e\sin(c+}{ad(1-m)\sqrt{\cos^2(c+dx)}}\end{aligned}$$

Mathematica [F] time = 31.64, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x]),x]
[Out] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x]), x]
```

fricas [F] time = 0.86, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(e\sin(dx+c))^m}{a\sec(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="fricas")
[Out] integral((e*sin(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

maple [F] time = 2.11, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{a+a\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x)
[Out] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)\,\left(e\,\sin(c+d\,x)\right)^m}{a\,\left(\cos(c+d\,x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^m)/(a*(cos(c + d*x) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\sin(c+dx))^m}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c)),x)
[Out] Integral((e*sin(c + d*x))**m/(sec(c + d*x) + 1), x)/a
```

3.138 $\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=207

$$-\frac{e^3\cos(c+dx)(e\sin(c+dx))^{m-3}\,{}_2F_1\left(-\frac{3}{2},\frac{m-3}{2};\frac{m-1}{2};\sin^2(c+dx)\right)}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}-\frac{e^3\cos(c+dx)(e\sin(c+dx))^{m-3}\,{}_2F_1\left(-\frac{1}{2},\frac{m-3}{2}\right.}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}$$

```
[Out] 2*e^3*(e*sin(d*x+c))^(-3+m)/a^2/d/(3-m)-2*e*(e*sin(d*x+c))^(-1+m)/a^2/d/(1-
m)-e^3*cos(d*x+c)*hypergeom([-3/2, -3/2+1/2*m],[-1/2+1/2*m],sin(d*x+c)^2)*(
e*sin(d*x+c))^(-3+m)/a^2/d/(3-m)/(cos(d*x+c)^2)^(1/2)-e^3*cos(d*x+c)*hyperg
eom([-1/2, -3/2+1/2*m],[-1/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(-3+m)/a^2
/d/(3-m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3872, 2875, 2873, 2577, 2564, 14}

$$-\frac{e^3\cos(c+dx)(e\sin(c+dx))^{m-3}\,{}_2F_1\left(-\frac{3}{2},\frac{m-3}{2};\frac{m-1}{2};\sin^2(c+dx)\right)}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}-\frac{e^3\cos(c+dx)(e\sin(c+dx))^{m-3}\,{}_2F_1\left(-\frac{1}{2},\frac{m-3}{2}\right.}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*e^3*(e*Sin[c + d*x])^(-3 + m))/(a^2*d*(3 - m)) - (e^3*Cos[c + d*x]*Hyper
geometric2F1[-3/2, (-3 + m)/2, (-1 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])
^(-3 + m))/(a^2*d*(3 - m)*Sqrt[Cos[c + d*x]^2]) - (e^3*Cos[c + d*x]*Hyperge
ometric2F1[-1/2, (-3 + m)/2, (-1 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(
-3 + m))/(a^2*d*(3 - m)*Sqrt[Cos[c + d*x]^2]) - (2*e*(e*Sin[c + d*x])^(-1 +
 m))/(a^2*d*(1 - m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
```

```
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)(e\sin(c+dx))^m}{(-a-a\cos(c+dx))^2}\,dx\\
&= \frac{e^4\int \cos^2(c+dx)(-a+a\cos(c+dx))^2(e\sin(c+dx))^{-4+m}\,dx}{a^4}\\
&= \frac{e^4\int \left(a^2\cos^2(c+dx)(e\sin(c+dx))^{-4+m}-2a^2\cos^3(c+dx)(e\sin(c+dx))^{-4+m}+\right.}{a^4}\\
&= \frac{e^4\int \cos^2(c+dx)(e\sin(c+dx))^{-4+m}\,dx}{a^2}+\frac{e^4\int \cos^4(c+dx)(e\sin(c+dx))^{-4+m}\,dx}{a^2}\\
&= -\frac{e^3\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);\sin^2(c+dx)\right)(e\sin(c+dx))^{-3+m}}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}\\
&= -\frac{e^3\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);\sin^2(c+dx)\right)(e\sin(c+dx))^{-3+m}}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}\\
&= \frac{2e^3(e\sin(c+dx))^{-3+m}}{a^2d(3-m)}-\frac{e^3\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);\sin^2(c+\right.}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}
\end{aligned}
$$

Mathematica [C] time = 22.70, size = 2833, normalized size = 13.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2^(2 - m)*E^(I*(c + d*x))*(((-I)*(-1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x
)))^(1 + m)*Cos[c/2 + (d*x)/2]^4*Hypergeometric2F1[1, (2 + m)/2, 1 - m/2, E
^((2*I)*(c + d*x))]*Sec[c + d*x]^2*(e*Sin[c + d*x])^m)/(d*m*(a + a*Sec[c +
d*x])^2*Sin[c + d*x]^m) + ((AppellF1[(1 + m)/2, 1 - m, 2*m, (3 + m)/2, Tan[
(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 6*AppellF1[(1 + m)/2, 2 - m, 2*m, (3
 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] + 12*AppellF1[(1 + m)/2,
3 - m, 2*m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 8*AppellF
1[(1 + m)/2, 4 - m, 2*m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2
])*(Sec[(c + d*x)/4]^2)^(2*m)*Sec[(c + d*x)/2]^4*Sin[c + d*x]^m*(e*Sin[c +
d*x])^m*Tan[(c + d*x)/4])/(4*a^2*d*(1 + m)*(1 - Tan[(c + d*x)/4]^2)^m*((m*(
AppellF1[(1 + m)/2, 1 - m, 2*m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*
```

```
x)/4]^2] - 6*AppellF1[(1 + m)/2, 2 - m, 2*m, (3 + m)/2, Tan[(c + d*x)/4]^2,
 -Tan[(c + d*x)/4]^2] + 12*AppellF1[(1 + m)/2, 3 - m, 2*m, (3 + m)/2, Tan[(
c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 8*AppellF1[(1 + m)/2, 4 - m, 2*m, (3
+ m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2])*(Sec[(c + d*x)/4]^2)^(1 +
 2*m)*Sin[c + d*x]^m*Tan[(c + d*x)/4]^2*(1 - Tan[(c + d*x)/4]^2)^(-1 - m))/
(2*(1 + m)) + ((AppellF1[(1 + m)/2, 1 - m, 2*m, (3 + m)/2, Tan[(c + d*x)/4]
^2, -Tan[(c + d*x)/4]^2] - 6*AppellF1[(1 + m)/2, 2 - m, 2*m, (3 + m)/2, Tan
[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] + 12*AppellF1[(1 + m)/2, 3 - m, 2*m,
(3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 8*AppellF1[(1 + m)/2,
 4 - m, 2*m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2])*(Sec[(c +
 d*x)/4]^2)^(1 + 2*m)*Sin[c + d*x]^m)/(4*(1 + m)*(1 - Tan[(c + d*x)/4]^2)^m
) + (m*(AppellF1[(1 + m)/2, 1 - m, 2*m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan
[(c + d*x)/4]^2] - 6*AppellF1[(1 + m)/2, 2 - m, 2*m, (3 + m)/2, Tan[(c + d*
x)/4]^2, -Tan[(c + d*x)/4]^2] + 12*AppellF1[(1 + m)/2, 3 - m, 2*m, (3 + m)/
2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 8*AppellF1[(1 + m)/2, 4 - m,
2*m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2])*Cos[c + d*x]*(Sec
[(c + d*x)/4]^2)^(2*m)*Sin[c + d*x]^(-1 + m)*Tan[(c + d*x)/4])/((1 + m)*(1
- Tan[(c + d*x)/4]^2)^m) + (m*(AppellF1[(1 + m)/2, 1 - m, 2*m, (3 + m)/2, T
an[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 6*AppellF1[(1 + m)/2, 2 - m, 2*m,
 (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] + 12*AppellF1[(1 + m)/
2, 3 - m, 2*m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 8*Appe
llF1[(1 + m)/2, 4 - m, 2*m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4
]^2])*(Sec[(c + d*x)/4]^2)^(2*m)*Sin[c + d*x]^m*Tan[(c + d*x)/4]^2)/((1 + m
)*(1 - Tan[(c + d*x)/4]^2)^m) + ((Sec[(c + d*x)/4]^2)^(2*m)*Sin[c + d*x]^m*
Tan[(c + d*x)/4]*(-((m*(1 + m)*AppellF1[1 + (1 + m)/2, 1 - m, 1 + 2*m, 1 +
(3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2]*Sec[(c + d*x)/4]^2*Tan[
(c + d*x)/4])/(3 + m)) + ((1 - m)*(1 + m)*AppellF1[1 + (1 + m)/2, 2 - m, 2*
m, 1 + (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2]*Sec[(c + d*x)/4]
^2*Tan[(c + d*x)/4])/(2*(3 + m)) - 6*(-((m*(1 + m)*AppellF1[1 + (1 + m)/2,
2 - m, 1 + 2*m, 1 + (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2]*Sec
[(c + d*x)/4]^2*Tan[(c + d*x)/4])/(3 + m)) + ((2 - m)*(1 + m)*AppellF1[1 +
(1 + m)/2, 3 - m, 2*m, 1 + (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]
^2]*Sec[(c + d*x)/4]^2*Tan[(c + d*x)/4])/(2*(3 + m))) + 12*(-((m*(1 + m)*Ap
pellF1[1 + (1 + m)/2, 3 - m, 1 + 2*m, 1 + (3 + m)/2, Tan[(c + d*x)/4]^2, -T
an[(c + d*x)/4]^2]*Sec[(c + d*x)/4]^2*Tan[(c + d*x)/4])/(3 + m)) + ((3 - m)
*(1 + m)*AppellF1[1 + (1 + m)/2, 4 - m, 2*m, 1 + (3 + m)/2, Tan[(c + d*x)/4
]^2, -Tan[(c + d*x)/4]^2]*Sec[(c + d*x)/4]^2*Tan[(c + d*x)/4])/(2*(3 + m)))
 - 8*(-((m*(1 + m)*AppellF1[1 + (1 + m)/2, 4 - m, 1 + 2*m, 1 + (3 + m)/2, T
an[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2]*Sec[(c + d*x)/4]^2*Tan[(c + d*x)/4]
)/(3 + m)) + ((4 - m)*(1 + m)*AppellF1[1 + (1 + m)/2, 5 - m, 2*m, 1 + (3 +
m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2]*Sec[(c + d*x)/4]^2*Tan[(c +
d*x)/4])/(2*(3 + m)))))/((1 + m)*(1 - Tan[(c + d*x)/4]^2)^m))) - (2*(3 + m)
*(AppellF1[(1 + m)/2, 1 - m, 2*m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c +
d*x)/4]^2] - 2*AppellF1[(1 + m)/2, 2 - m, 2*m, (3 + m)/2, Tan[(c + d*x)/4]^
2, -Tan[(c + d*x)/4]^2])*Sec[(c + d*x)/2]*(e*Sin[c + d*x])^m*Tan[(c + d*x)/
2])/(a^2*d*(1 + m)*((3 + m)*AppellF1[(1 + m)/2, 1 - m, 2*m, (3 + m)/2, Tan[
(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 2*((3 + m)*AppellF1[(1 + m)/2, 2 - m
, 2*m, (3 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] + (2*m*AppellF1[
(3 + m)/2, 1 - m, 1 + 2*m, (5 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]
^2] + (-1 + m)*AppellF1[(3 + m)/2, 2 - m, 2*m, (5 + m)/2, Tan[(c + d*x)/4]^
2, -Tan[(c + d*x)/4]^2] - 4*m*AppellF1[(3 + m)/2, 2 - m, 1 + 2*m, (5 + m)/2
, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] + 4*AppellF1[(3 + m)/2, 3 - m, 2
*m, (5 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2] - 2*m*AppellF1[(3 +
 m)/2, 3 - m, 2*m, (5 + m)/2, Tan[(c + d*x)/4]^2, -Tan[(c + d*x)/4]^2])*Tan
[(c + d*x)/4]^2)))
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(e\sin(dx+c))^m}{a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((e*sin(d*x + c))^m/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^2, x)
```

maple [F] time = 1.46, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{(a+a\sec(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x)
```

```
[Out] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2\,(e\,\sin(c+d\,x))^m}{a^2\,(\cos(c+d\,x)+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^m)/(a^2*(cos(c + d*x) + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\sin(c+dx))^m}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral((e*sin(c + d*x))**m/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**
2
```

3.139 $\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=236

$$\frac{e^5\cos(c+dx)(e\sin(c+dx))^{m-5}\,{}_2F_1\left(-\frac{5}{2},\frac{m-5}{2};\frac{m-3}{2};\sin^2(c+dx)\right)}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}+\frac{3e^5\cos(c+dx)(e\sin(c+dx))^{m-5}\,{}_2F_1\left(-\frac{3}{2},\frac{m-5}{2},\ldots\right.}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}$$

```
[Out] -4*e^5*(e*sin(d*x+c))^(-5+m)/a^3/d/(5-m)+7*e^3*(e*sin(d*x+c))^(-3+m)/a^3/d/
(3-m)-3*e*(e*sin(d*x+c))^(-1+m)/a^3/d/(1-m)+e^5*cos(d*x+c)*hypergeom([-5/2,
 -5/2+1/2*m],[-3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(-5+m)/a^3/d/(5-m)/(
cos(d*x+c)^2)^(1/2)+3*e^5*cos(d*x+c)*hypergeom([-3/2, -5/2+1/2*m],[-3/2+1/2
*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(-5+m)/a^3/d/(5-m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.64, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3872, 2875, 2873, 2564, 14, 2577, 270}

$$\frac{e^5\cos(c+dx)(e\sin(c+dx))^{m-5}\,{}_2F_1\left(-\frac{5}{2},\frac{m-5}{2};\frac{m-3}{2};\sin^2(c+dx)\right)}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}+\frac{3e^5\cos(c+dx)(e\sin(c+dx))^{m-5}\,{}_2F_1\left(-\frac{3}{2},\frac{m-5}{2},\ldots\right.}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-4*e^5*(e*Sin[c + d*x])^(-5 + m))/(a^3*d*(5 - m)) + (e^5*Cos[c + d*x]*Hype
rgeometric2F1[-5/2, (-5 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x]
)^(-5 + m))/(a^3*d*(5 - m)*Sqrt[Cos[c + d*x]^2]) + (3*e^5*Cos[c + d*x]*Hype
rgeometric2F1[-3/2, (-5 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x]
)^(-5 + m))/(a^3*d*(5 - m)*Sqrt[Cos[c + d*x]^2]) + (7*e^3*(e*Sin[c + d*x])^
(-3 + m))/(a^3*d*(3 - m)) - (3*e*(e*Sin[c + d*x])^(-1 + m))/(a^3*d*(1 - m))
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cos^3(c+dx)(e\sin(c+dx))^m}{(-a-a\cos(c+dx))^3}\,dx \\
&= -\frac{e^6\int \cos^3(c+dx)(-a+a\cos(c+dx))^3(e\sin(c+dx))^{-6+m}\,dx}{a^6} \\
&= -\frac{e^6\int \left(-a^3\cos^3(c+dx)(e\sin(c+dx))^{-6+m}+3a^3\cos^4(c+dx)(e\sin(c+dx))^{-6+m}\right.}{a} \\
&= \frac{e^6\int \cos^3(c+dx)(e\sin(c+dx))^{-6+m}\,dx}{a^3}-\frac{e^6\int \cos^6(c+dx)(e\sin(c+dx))^{-6+m}\,dx}{a^3} \\
&= \frac{e^5\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}(-5+m);\frac{1}{2}(-3+m);\sin^2(c+dx)\right)(e\sin(c+dx))^{-5+m}}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}+ \\
&= \frac{e^5\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}(-5+m);\frac{1}{2}(-3+m);\sin^2(c+dx)\right)(e\sin(c+dx))^{-5+m}}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}+ \\
&= -\frac{4e^5(e\sin(c+dx))^{-5+m}}{a^3d(5-m)}+\frac{e^5\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}(-5+m);\frac{1}{2}(-3+m);\sin^2(c\right.}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}
\end{aligned}
$$

Mathematica [F] time = 10.74, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^3, x]
```

fricas [F] time = 0.88, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(e\sin(dx+c))^m}{a^3\sec(dx+c)^3+3\,a^3\sec(dx+c)^2+3\,a^3\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((e*sin(d*x + c))^m/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \sin(dx+c))^m}{(a \sec(dx+c)+a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^3, x)
```

maple [F] time = 1.34, size = 0, normalized size = 0.00

$$\int \frac{(e \sin(dx+c))^m}{(a+a \sec(dx+c))^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x)
```

```
[Out] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \sin(dx+c))^m}{(a \sec(dx+c)+a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^3\,(e\,\sin(c+d\,x))^m}{a^3\,(\cos(c+d\,x)+1)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)^3*(e*sin(c + d*x))^m)/(a^3*(cos(c + d*x) + 1)^3), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e \sin(c+dx))^m}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1} \, dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral((e*sin(c + d*x))**m/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
 + d*x) + 1), x)/a**3
```

3.140 $\int (a + a\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=106

$$\frac{2ae\sqrt{a\sec(c+dx)+a}\,(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{-m/2}F_1\left(-\frac{1}{2};\frac{1-m}{2},\frac{1}{2}(-m-2);\frac{1}{2};\cos(c+dx),-\cos(\right.}{d}$$

```
[Out] 2*a*e*AppellF1(-1/2,-1-1/2*m,1/2-1/2*m,1/2,-cos(d*x+c),cos(d*x+c))*(1-cos(d
*x+c))^(1/2-1/2*m)*(e*sin(d*x+c))^(-1+m)*(a+a*sec(d*x+c))^(1/2)/d/((1+cos(d
*x+c))^(1/2*m))
```

Rubi [A] time = 0.37, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3876, 2886, 135, 133}

$$\frac{2ae\sqrt{a\sec(c+dx)+a}\,(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{-m/2}F_1\left(-\frac{1}{2};\frac{1-m}{2},\frac{1}{2}(-m-2);\frac{1}{2};\cos(c+dx),-\cos(\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m,x]
```

```
[Out] (2*a*e*AppellF1[-1/2, (1 - m)/2, (-2 - m)/2, 1/2, Cos[c + d*x], -Cos[c + d*
x]]*(1 - Cos[c + d*x])^((1 - m)/2)*Sqrt[a + a*Sec[c + d*x]]*(e*Sin[c + d*x]
)^(-1 + m))/(d*(1 + Cos[c + d*x])^(m/2))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2886

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos
[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\int (a + a\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx = \frac{\left(\sqrt{-\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))^{3/2}(e\sin(c+dx))^m}{(-\cos(c+dx))^{3/2}} d}{\sqrt{-a-a\cos(c+dx)}}$$

$$= -\frac{\left(e\sqrt{-\cos(c+dx)}\,(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}}(-a+a\cos(c+dx))^{\frac{1}{}}\right.}{}$$

$$= \frac{\left(ae\sqrt{-\cos(c+dx)}\,(1+\cos(c+dx))^{-m/2}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}}\right.}{}$$

$$= \frac{\left(ae(1-\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}}\sqrt{-\cos(c+dx)}\,(1+\cos(c+dx))^{-m/2}(-a-\right.}{}$$

$$= \frac{2aeF_1\left(-\frac{1}{2};\frac{1-m}{2},\frac{1}{2}(-2-m);\frac{1}{2};\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c}{}$$

Mathematica [B] time = 9.75, size = 1243, normalized size = 11.73

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m,x]
```

```
[Out] (4*(3 + m)*(AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + AppellF1[(1 + m)/2, 1/2, m, (3 + m)/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[(1 + m)/2, 3/2, m, (3 + m)/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[(c + d*x)/2]^3*(a*(1 + Sec[c + d
*x]))^(3/2)*Sin[(c + d*x)/2]*(e*Sin[c + d*x])^m)/(d*(1 + m)*(6*AppellF1[(1
+ m)/2, 3/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*m*A
ppellF1[(1 + m)/2, 3/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2] - 2*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] - 2*m*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[(3 + m)/2, 1/2, 1 + m, (5 + m)
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*m*AppellF1[(3 + m)/2, 1/2,
 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[(3 +
 m)/2, 3/2, m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 4*m*Ap
pellF1[(3 + m)/2, 3/2, 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2] + 6*AppellF1[(3 + m)/2, 5/2, m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2] + 6*AppellF1[(1 + m)/2, 3/2, m, (3 + m)/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] + 2*m*AppellF1[(1 + m)/2, 3/2, m, (3
+ m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] + 2*AppellF1[
(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
*Cos[c + d*x] + 2*m*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] + AppellF1[(3 + m)/2, 1/2, 1 + m
, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] + 2*m*Ap
pellF1[(3 + m)/2, 1/2, 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2]*Cos[c + d*x] - AppellF1[(3 + m)/2, 3/2, m, (5 + m)/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] + 4*m*AppellF1[(3 + m)/2, 3/2, 1 +
m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] - 6*App
ellF1[(3 + m)/2, 5/2, m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
]*Cos[c + d*x] + (3 + m)*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x]) + (3 + m)*AppellF1[(1
 + m)/2, 1/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + C
os[c + d*x])))
```

fricas [F] time = 1.50, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^{\frac{3}{2}}(e\sin(dx+c))^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{3}{2}}(e\sin(dx+c))^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

maple [F] time = 1.17, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))^{\frac{3}{2}}(e\sin(dx+c))^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)
[Out] int((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^{\frac{3}{2}}(e\sin(dx+c))^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\sin(c+d\,x))^m\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^(3/2),x)
[Out] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(e*sin(d*x+c))**m,x)
[Out] Timed out
```

3.141 $\int \sqrt{a+a\sec(c+dx)}\,(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=107

$$-\frac{2e\cos(c+dx)\sqrt{a\sec(c+dx)+a}\,(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{-m/2}F_1\left(\frac{1}{2};\frac{1-m}{2},-\frac{m}{2};\frac{3}{2};\cos(c+dx),-\cos\right.}{d}$$

```
[Out] -2*e*AppellF1(1/2,-1/2*m,1/2-1/2*m,3/2,-cos(d*x+c),cos(d*x+c))*(1-cos(d*x+c
))^(1/2-1/2*m)*cos(d*x+c)*(e*sin(d*x+c))^(-1+m)*(a+a*sec(d*x+c))^(1/2)/d/((
1+cos(d*x+c))^(1/2*m))
```

Rubi [A] time = 0.31, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3876, 2886, 135, 133}

$$-\frac{2e\cos(c+dx)\sqrt{a\sec(c+dx)+a}\,(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{-m/2}F_1\left(\frac{1}{2};\frac{1-m}{2},-\frac{m}{2};\frac{3}{2};\cos(c+dx),-\cos\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*(e*Sin[c + d*x])^m,x]
```

```
[Out] (-2*e*AppellF1[1/2, (1 - m)/2, -m/2, 3/2, Cos[c + d*x], -Cos[c + d*x]]*(1 -
 Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(e*Sin[c +
 d*x])^(-1 + m))/(d*(1 + Cos[c + d*x])^(m/2))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2886

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos
[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\,(e\sin(c+dx))^m\,dx = \frac{\left(\sqrt{-\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}\right)\int \frac{\sqrt{-a-a\cos(c+dx)}\,(e\sin(c+dx))^m}{\sqrt{-\cos(c+dx)}}\,d}{\sqrt{-a-a\cos(c+dx)}}$$

$$= -\frac{\left(e\sqrt{-\cos(c+dx)}\,(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}}(-a+a\cos(c+dx)\right.}{}$$

$$= -\frac{\left(e\sqrt{-\cos(c+dx)}\,(1+\cos(c+dx))^{-m/2}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-}{2}}\right.}{}$$

$$= -\frac{\left(e(1-\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}}\sqrt{-\cos(c+dx)}\,(1+\cos(c+dx))^{-m/2}(-a\right.}{}$$

$$= -\frac{2eF_1\left(\frac{1}{2};\frac{1-m}{2},-\frac{m}{2};\frac{3}{2};\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx)}{}$$

Mathematica [B] time = 2.90, size = 433, normalized size = 4.05

$$\frac{4(m+3}{d(m+1)\left((\cos(c+dx)-1)\left(2(m+1)F_1\left(\frac{m+3}{2};-\frac{1}{2},m+2;\frac{m+5}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+(2m+\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*(e*Sin[c + d*x])^m,x]
```

```
[Out] (4*(3 + m)*(AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + AppellF1[(1 + m)/2, 1/2, m, (3 + m)/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[(c + d*x)/2]^3*Sqrt[a*(1 + Sec[c + d*x])]
*Sin[(c + d*x)/2]*(e*Sin[c + d*x])^m)/(d*(1 + m)*((2*(1 + m)*AppellF1[(3 +
m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (1
 + 2*m)*AppellF1[(3 + m)/2, 1/2, 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2] - AppellF1[(3 + m)/2, 3/2, m, (5 + m)/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + (3 + m)*AppellF1[(1 + m)/2,
-1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c
 + d*x]) + (3 + m)*AppellF1[(1 + m)/2, 1/2, m, (3 + m)/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x])))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{a\sec(dx+c)+a}\,(e\sin(dx+c))^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sec(dx+c)+a}\,(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

maple [F] time = 1.24, size = 0, normalized size = 0.00

$$\int \sqrt{a + a \sec (dx + c)} \; (e \sin (dx + c))^{m} \; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x)
[Out] int((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sec (dx + c) + a} \, (e \sin (dx + c))^{m} \; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e \; \sin (c + d\,x))^{m} \sqrt{a + \frac{a}{\cos (c + d\,x)}} \; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^(1/2),x)
[Out] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a (\sec (c + dx) + 1)} \, (e \sin (c + dx))^{m} \; dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*(e*sin(d*x+c))**m,x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(e*sin(c + d*x))**m, x)
```

3.142 $\int \frac{(e\sin(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=115

$$-\frac{2e\cos(c+dx)(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{1-\frac{m}{2}}F_1\left(\frac{3}{2};\frac{1-m}{2},\frac{2-m}{2};\frac{5}{2};\cos(c+dx),-\cos(c+dx)\right)(e\sin(c+dx))^{m-1}}{3d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] -2/3*e*AppellF1(3/2,1-1/2*m,1/2-1/2*m,5/2,-cos(d*x+c),cos(d*x+c))*(1-cos(d*
x+c))^(1/2-1/2*m)*cos(d*x+c)*(1+cos(d*x+c))^(1-1/2*m)*(e*sin(d*x+c))^(-1+m)
/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3876, 2886, 135, 133}

$$-\frac{2e\cos(c+dx)(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{1-\frac{m}{2}}F_1\left(\frac{3}{2};\frac{1-m}{2},\frac{2-m}{2};\frac{5}{2};\cos(c+dx),-\cos(c+dx)\right)(e\sin(c+dx))^{m-1}}{3d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*e*AppellF1[3/2, (1 - m)/2, (2 - m)/2, 5/2, Cos[c + d*x], -Cos[c + d*x]]
*(1 - Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]*(1 + Cos[c + d*x])^(1 - m/2)*(
e*Sin[c + d*x])^(-1 + m))/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2886

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos
[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{-a-a\cos(c+dx)}\int \frac{\sqrt{-\cos(c+dx)}\,(e\sin(c+dx))^m}{\sqrt{-a-a\cos(c+dx)}}\,dx}{\sqrt{-\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{\left(e(-a-a\cos(c+dx))^{\frac{1}{2}+\frac{1-m}{2}}(-a+a\cos(c+dx))^{\frac{1-m}{2}}(e\sin(c+dx))^{-1+m}\right)\text{Subst}\left(\int \cdots\right.}{d\sqrt{-\cos(c+dx)}\,\sqrt{a+a\sec(c\cdots}}$$

$$= -\frac{\left(e(1+\cos(c+dx))^{1-\frac{m}{2}}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\frac{m}{2}}(-a+a\cos(c+dx))^{\frac{1-m}{2}}(e\sin(\cdots\right.}{d\sqrt{-\cos(c+dx)}\cdots}$$

$$= -\frac{\left(e(1-\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}}(1+\cos(c+dx))^{1-\frac{m}{2}}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\frac{m}{2}}(-a+a\,c\cdots\right.}{d\sqrt{-c\cdots}}$$

$$= -\frac{2eF_1\left(\frac{3}{2};\frac{1-m}{2},\frac{2-m}{2};\frac{5}{2};\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}\cos(c+dx)(1\cdots}{3d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [B] time = 2.07, size = 277, normalized size = 2.41

$$\frac{4(m+3)\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c\cdots\right.}{d(m+1)\sqrt{a(\sec(c+dx)+1)}\left((\cos(c+dx)-1)\left(2(m+1)F_1\left(\frac{m+3}{2};-\frac{1}{2},m+2;\frac{m+5}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c\cdots\right.\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (4*(3 + m)*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^3*Sin[(c + d*x)/2]*(e*Sin[c + d*x])^m
)/(d*(1 + m)*((2*(1 + m)*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[(3 + m)/2, 1/2, 1 + m, (5 + m
)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + (3 + m
)*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*(1 + Cos[c + d*x]))*Sqrt[a*(1 + Sec[c + d*x])])
```

fricas [F] time = 0.68, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(e\sin(dx+c))^m}{\sqrt{a\sec(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*sin(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

maple [F] time = 1.16, size = 0, normalized size = 0.00

$$\int \frac{\left(e\sin\left(dx+c\right)\right)^{m}}{\sqrt{a+a\sec\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x)
[Out] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\sin\left(dx+c\right)\right)^{m}}{\sqrt{a\sec\left(dx+c\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(e\,\sin\left(c+d\,x\right)\right)^{m}}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^(1/2),x)
[Out] int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\sin\left(c+dx\right)\right)^{m}}{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral((e*sin(c + d*x))**m/sqrt(a*(sec(c + d*x) + 1)), x)
```

3.143 $\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=120

$$-\frac{2e\cos^2(c+dx)(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{1-\frac{m}{2}}F_1\left(\frac{5}{2};\frac{1-m}{2},\frac{4-m}{2};\frac{7}{2};\cos(c+dx),-\cos(c+dx)\right)(e\sin(c+}{5ad\sqrt{a\sec(c+dx)+a}}$$

```
[Out] -2/5*e*AppellF1(5/2,2-1/2*m,1/2-1/2*m,7/2,-cos(d*x+c),cos(d*x+c))*(1-cos(d*
x+c))^(1/2-1/2*m)*cos(d*x+c)^2*(1+cos(d*x+c))^(1-1/2*m)*(e*sin(d*x+c))^(-1+
m)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3876, 2886, 135, 133}

$$-\frac{2e\cos^2(c+dx)(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{1-\frac{m}{2}}F_1\left(\frac{5}{2};\frac{1-m}{2},\frac{4-m}{2};\frac{7}{2};\cos(c+dx),-\cos(c+dx)\right)(e\sin(c+}{5ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*e*AppellF1[5/2, (1 - m)/2, (4 - m)/2, 7/2, Cos[c + d*x], -Cos[c + d*x]]
*(1 - Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]^2*(1 + Cos[c + d*x])^(1 - m/2)
*(e*Sin[c + d*x])^(-1 + m))/(5*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2886

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos
[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{-a-a\cos(c+dx)}\int \frac{(-\cos(c+dx))^{3/2}(e\sin(c+dx))^m}{(-a-a\cos(c+dx))^{3/2}}\,dx}{\sqrt{-\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{\left(e(-a-a\cos(c+dx))^{\frac{1}{2}+\frac{1-m}{2}}(-a+a\cos(c+dx))^{\frac{1-m}{2}}(e\sin(c+dx))^{-1+m}\right)\text{Subst}}{d\sqrt{-\cos(c+dx)}\sqrt{a+a}}$$

$$= \frac{\left(e(1+\cos(c+dx))^{1-\frac{m}{2}}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\frac{m}{2}}(-a+a\cos(c+dx))^{\frac{1-m}{2}}(e\,s\right.}{ad\sqrt{-\cos(c+}}$$

$$= \frac{\left(e(1-\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}}(1+\cos(c+dx))^{1-\frac{m}{2}}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\frac{m}{2}}(-a+\right.}{ad}$$

$$= -\frac{2eF_1\left(\frac{5}{2};\frac{1-m}{2},\frac{4-m}{2};\frac{7}{2};\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}\cos^2(c+}{5ad\sqrt{a+a\sec(c+dx)}}$$

Mathematica [B] time = 3.01, size = 484, normalized size = 4.03

$$\frac{}{d(m+1)(a(\sec(c+dx)+1))^{3/2}\left(-4(m+3)\cos^2\left(\frac{1}{2}(c+dx)\right)F_1\left(\frac{m+1}{2};-\frac{1}{2},m+1;\frac{m+3}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (4*(3 + m)*(AppellF1[(1 + m)/2, -1/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] - 2*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[(c + d*x)/2]^3*Sin[(c + d*x)/2]*(e*Sin
[c + d*x])^m)/(d*(1 + m)*(-4*(3 + m)*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 +
m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2 + (2*m*Ap
pellF1[(3 + m)/2, -1/2, 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2] - 4*(1 + m)*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[(3 + m)/2, 1/2, m, (5 + m)/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[(3 + m)/2, 1/2, 1 + m, (5
 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + (3
 + m)*AppellF1[(1 + m)/2, -1/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*(1 + Cos[c + d*x]))*(a*(1 + Sec[c + d*x]))^(3/2))
```

fricas [F] time = 1.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{a\sec(dx+c)+a}\,(e\sin(dx+c))^m}{a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sin(d*x + c))^m/(a^2*sec(d*x + c)^2 +
2*a^2*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^(3/2), x)
```

maple [F] time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{(a+a\sec(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x)
[Out] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\ \sin(c+d\,x))^m}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^(3/2),x)
[Out] int((e*sin(c + d*x))^m/(a + a/cos(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(c+dx))^m}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c))**(3/2),x)
[Out] Integral((e*sin(c + d*x))**m/(a*(sec(c + d*x) + 1))**(3/2), x)
```

3.144 $\int (a+a\sec(c+dx))^n(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=130

$$-\frac{e\cos(c+dx)(1-\cos(c+dx))^{\frac{1-m}{2}}(a\sec(c+dx)+a)^n(e\sin(c+dx))^{m-1}(\cos(c+dx)+1)^{\frac{1}{2}(-m-2n+1)}F_1\Big(1-n;}{d(1-n)}$$

[Out] -e*AppellF1(1-n,1/2-1/2*m-n,1/2-1/2*m,2-n,-cos(d*x+c),cos(d*x+c))*(1-cos(d*x+c))^(1/2-1/2*m)*cos(d*x+c)*(1+cos(d*x+c))^(1/2-1/2*m-n)*(a+a*sec(d*x+c))^n*(e*sin(d*x+c))^(-1+m)/d/(1-n)

Rubi [A] time = 0.28, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3876, 2886, 135, 133}

$$-\frac{e\cos(c+dx)(1-\cos(c+dx))^{\frac{1-m}{2}}(a\sec(c+dx)+a)^n(e\sin(c+dx))^{m-1}(\cos(c+dx)+1)^{\frac{1}{2}(-m-2n+1)}F_1\Big(1-n;}{d(1-n)}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Sec[c + d*x])^n*(e*Sin[c + d*x])^m,x]

[Out] -((e*AppellF1[1 - n, (1 - m)/2, (1 - m - 2*n)/2, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 - Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]*(1 + Cos[c + d*x])^((1 - m - 2*n)/2)*(a + a*Sec[c + d*x])^n*(e*Sin[c + d*x])^(-1 + m))/(d*(1 - n)))

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 2886

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x])^((p - 1)/2)), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p - 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x])^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\int (a + a\sec(c+dx))^n (e\sin(c+dx))^m\,dx = ((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n)\int(-\cos$$

$$= -\frac{\left(e(-\cos(c+dx))^n(-a-a\cos(c+dx))^{\frac{1-m}{2}-n}(-a+a\cos(c+dx))^{\frac{1-m}{2}}\right.}{}$$

$$= -\frac{\left(e(-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}-n}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-}{2}}\right.}{}$$

$$= -\frac{\left(e(1-\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}}(-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}-n}(-a\right.}{}$$

$$= -\frac{eF_1\left(1-n;\frac{1-m}{2},\frac{1}{2}(1-m-2n);2-n;\cos(c+dx),-\cos(c+dx)\right)(1}{}$$

Mathematica [B] time = 1.92, size = 276, normalized size = 2.12

$$\frac{4(m+3)\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)(a(\sec(c+d}{d(m+1)\left((m+3)(\cos(c+dx)+1)F_1\left(\frac{m+1}{2};n,m+1;\frac{m+3}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-4\sin^2\left(\frac{1}{2}(c+d\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*(e*Sin[c + d*x])^m,x]
```

```
[Out] (4*(3 + m)*AppellF1[(1 + m)/2, n, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^3*(a*(1 + Sec[c + d*x]))^n*Sin[(c + d*x)
/2]*(e*Sin[c + d*x])^m)/(d*(1 + m)*((3 + m)*AppellF1[(1 + m)/2, n, 1 + m, (
3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x]) - 4*(
(1 + m)*AppellF1[(3 + m)/2, n, 2 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] - n*AppellF1[(3 + m)/2, 1 + n, 1 + m, (5 + m)/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2])*Sin[(c + d*x)/2]^2))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^n(e\sin(dx+c))^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n (e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

maple [F] time = 2.79, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))^n (e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
[Out] int((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n (e\sin(dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\sin(c+d\,x))^m \left(a+\frac{a}{\cos(c+d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^n,x)
[Out] int((e*sin(c + d*x))^m*(a + a/cos(c + d*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*(e*sin(d*x+c))**m,x)
[Out] Timed out
```

3.145 $\int (a+a\sec(c+dx))^n \sin^7(c+dx)\,dx$

Optimal. Leaf size=180

$$-\frac{(3-n)(8-n)(16-n)(a\sec(c+dx)+a)^{n+4}\,{}_2F_1(6,n+4;n+5;\sec(c+dx)+1)}{42a^4d(1-n)(n+4)}+\frac{\cos^7(c+dx)\left(6(8-n)-\left(n^2-\right.\right.}{}$$

```
[Out] -1/42*(3-n)*(8-n)*(16-n)*hypergeom([6, 4+n],[5+n],1+sec(d*x+c))*(a+a*sec(d*
x+c))^(4+n)/a^4/d/(-n^2-3*n+4)-cos(d*x+c)^7*(1-sec(d*x+c))^2*(a+a*sec(d*x+c
))^(4+n)/a^4/d/(1-n)+1/42*cos(d*x+c)^7*(a+a*sec(d*x+c))^(4+n)*(48-6*n-(n^2-
25*n+108)*sec(d*x+c))/a^4/d/(1-n)
```

Rubi [A] time = 0.17, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3873, 100, 145, 65}

$$-\frac{(3-n)(8-n)(16-n)(a\sec(c+dx)+a)^{n+4}\,{}_2F_1(6,n+4;n+5;\sec(c+dx)+1)}{42a^4d(1-n)(n+4)}+\frac{\cos^7(c+dx)\left(6(8-n)-\left(n^2-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^7,x]
```

```
[Out] -((3 - n)*(8 - n)*(16 - n)*Hypergeometric2F1[6, 4 + n, 5 + n, 1 + Sec[c + d
*x]]*(a + a*Sec[c + d*x])^(4 + n))/(42*a^4*d*(1 - n)*(4 + n)) - (Cos[c + d*
x]^7*(1 - Sec[c + d*x])^2*(a + a*Sec[c + d*x])^(4 + n))/(a^4*d*(1 - n)) + (
Cos[c + d*x]^7*(a + a*Sec[c + d*x])^(4 + n)*(6*(8 - n) - (108 - 25*n + n^2)
*Sec[c + d*x]))/(42*a^4*d*(1 - n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 145

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)*(
a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x]
 + Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] && !LtQ[n, -2]))
```

Rule 3873

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[(f*b^(p - 1))^(-1), Subst[Int[((-a + b*x)^((p - 1)/2
)*(a + b*x)^(m + (p - 1)/2))/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n \sin^7(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(-a-ax)^3(a-ax)^{3+n}}{x^8}\,dx, x, -\sec(c+dx)\right)}{a^6 d}$$

$$= -\frac{\cos^7(c+dx)(1-\sec(c+dx))^2(a+a\sec(c+dx))^{4+n}}{a^4 d(1-n)} - \frac{\text{Subst}\Big(\int \frac{(-a}{}}{}$$

$$= -\frac{\cos^7(c+dx)(1-\sec(c+dx))^2(a+a\sec(c+dx))^{4+n}}{a^4 d(1-n)} + \frac{\cos^7(c+dx)}{}$$

$$= -\frac{(3-n)(8-n)(16-n)\,{}_2F_1(6,4+n;5+n;1+\sec(c+dx))(a+a\sec}{42a^4 d(1-n)(4+n)}$$

Mathematica [A] time = 1.51, size = 113, normalized size = 0.63

$$\frac{(\sec(c+dx)+1)^4(a(\sec(c+dx)+1))^n\left((n+4)\cos^5(c+dx)\left(\left(n^2-25n+24\right)\cos(c+dx)+6(n-1)\cos^2(c+\right.\right.}{42d(n-1)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^7,x]
```

```
[Out] (((4 + n)*Cos[c + d*x]^5*(42 + (24 - 25*n + n^2)*Cos[c + d*x] + 6*(-1 + n)*
Cos[c + d*x]^2) - (-384 + 200*n - 27*n^2 + n^3)*Hypergeometric2F1[6, 4 + n,
 5 + n, 1 + Sec[c + d*x]])*(1 + Sec[c + d*x])^4*(a*(1 + Sec[c + d*x]))^n)/(
42*d*(-1 + n)*(4 + n))
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(dx+c\right)^6 - 3\cos\left(dx+c\right)^4 + 3\cos\left(dx+c\right)^2 - 1\right)\left(a\sec\left(dx+c\right)+a\right)^n \sin\left(dx+c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*(a*sec
(d*x + c) + a)^n*sin(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sec\left(dx+c\right)+a\right)^n \sin\left(dx+c\right)^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^7, x)
```

maple [F] time = 3.01, size = 0, normalized size = 0.00

$$\int \left(a+a\sec\left(dx+c\right)\right)^n \left(\sin^7\left(dx+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x)
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n \sin(dx+c)^7 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+d\,x)^7 \left(a+\frac{a}{\cos(c+d\,x)}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7*(a + a/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^7*(a + a/cos(c + d*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**7,x)
[Out] Timed out
```

3.146 $\int (a + a\sec(c + dx))^n \sin^5(c + dx)\, dx$

Optimal. Leaf size=123

$$\frac{(n^2 - 13n + 32)\,(a\sec(c + dx) + a)^{n+3}\,{}_2F_1(4, n + 3; n + 4; \sec(c + dx) + 1)}{20a^3d(n + 3)} - \frac{\cos^5(c + dx)(a\sec(c + dx) + a)^{n+3}}{5a^3d}$$

```
[Out] 1/20*(12-n)*cos(d*x+c)^4*(a+a*sec(d*x+c))^(3+n)/a^3/d-1/5*cos(d*x+c)^5*(a+a
*sec(d*x+c))^(3+n)/a^3/d+1/20*(n^2-13*n+32)*hypergeom([4, 3+n],[4+n],1+sec(
d*x+c))*(a+a*sec(d*x+c))^(3+n)/a^3/d/(3+n)
```

Rubi [A] time = 0.11, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3873, 89, 78, 65}

$$\frac{(n^2 - 13n + 32)\,(a\sec(c + dx) + a)^{n+3}\,{}_2F_1(4, n + 3; n + 4; \sec(c + dx) + 1)}{20a^3d(n + 3)} - \frac{\cos^5(c + dx)(a\sec(c + dx) + a)^{n+3}}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^5,x]
```

```
[Out] ((12 - n)*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(3 + n))/(20*a^3*d) - (Cos[c
+ d*x]^5*(a + a*Sec[c + d*x])^(3 + n))/(5*a^3*d) + ((32 - 13*n + n^2)*Hyper
geometric2F1[4, 3 + n, 4 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3 + n
))/(20*a^3*d*(3 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 3873

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[(f*b^(p - 1))^(-1), Subst[Int[((-a + b*x)^((p - 1)/2
)*(a + b*x)^(m + (p - 1)/2))/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n \sin^5(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(-a-ax)^2(a-ax)^{2+n}}{x^6}\,dx, x, -\sec(c+dx)\right)}{a^4 d}$$

$$= -\frac{\cos^5(c+dx)(a+a\sec(c+dx))^{3+n}}{5a^3 d} - \frac{\text{Subst}\left(\int \frac{(a-ax)^{2+n}\left(a^3(12-n)+5a^3x\right)}{x^5}\,d\right.}{5a^5 d}$$

$$= \frac{(12-n)\cos^4(c+dx)(a+a\sec(c+dx))^{3+n}}{20a^3 d} - \frac{\cos^5(c+dx)(a+a\sec(c+}{5a^3 d}$$

$$= \frac{(12-n)\cos^4(c+dx)(a+a\sec(c+dx))^{3+n}}{20a^3 d} - \frac{\cos^5(c+dx)(a+a\sec(c+}{5a^3 d}$$

Mathematica [A] time = 0.52, size = 84, normalized size = 0.68

$$-\frac{(\sec(c+dx)+1)^3(a(\sec(c+dx)+1))^n\left((n+3)\cos^4(c+dx)(4\cos(c+dx)+n-12)-\left(n^2-13n+32\right){}_2F_1(4,n\right.}{20d(n+3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^5,x]

[Out] -1/20*(((3 + n)*Cos[c + d*x]^4*(-12 + n + 4*Cos[c + d*x]) - (32 - 13*n + n^2)*Hypergeometric2F1[4, 3 + n, 4 + n, 1 + Sec[c + d*x]])*(1 + Sec[c + d*x])^3*(a*(1 + Sec[c + d*x]))^n)/(d*(3 + n))

fricas [F] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1\right)(a\sec(dx+c)+a)^n \sin(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="fricas")

[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(a*sec(d*x + c) + a)^n*sin(d*x + c), x)

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n \sin(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^5, x)

maple [F] time = 2.79, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))^n \left(\sin^5(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x)

[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec (dx + c) + a)^n \sin (dx + c)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin (c + d\,x)^5 \left(a + \frac{a}{\cos (c + d\,x)}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5*(a + a/cos(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)^5*(a + a/cos(c + d*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.147 $\int (a+a\sec(c+dx))^n \sin^3(c+dx)\,dx$

Optimal. Leaf size=83

$$\frac{\cos^3(c+dx)(a\sec(c+dx)+a)^{n+2}}{3a^2d}-\frac{(4-n)(a\sec(c+dx)+a)^{n+2}\,{}_2F_1(3,n+2;n+3;\sec(c+dx)+1)}{3a^2d(n+2)}$$

```
[Out] 1/3*cos(d*x+c)^3*(a+a*sec(d*x+c))^(2+n)/a^2/d-1/3*(4-n)*hypergeom([3, 2+n],
[3+n],1+sec(d*x+c))*(a+a*sec(d*x+c))^(2+n)/a^2/d/(2+n)
```

Rubi [A] time = 0.07, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3873, 78, 65}

$$\frac{\cos^3(c+dx)(a\sec(c+dx)+a)^{n+2}}{3a^2d}-\frac{(4-n)(a\sec(c+dx)+a)^{n+2}\,{}_2F_1(3,n+2;n+3;\sec(c+dx)+1)}{3a^2d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^3,x]
```

```
[Out] (Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(2 + n))/(3*a^2*d) - ((4 - n)*Hypergeo
metric2F1[3, 2 + n, 3 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2 + n))/
(3*a^2*d*(2 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 3873

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[(f*b^(p - 1))^(-1), Subst[Int[((-a + b*x)^((p - 1)/2
)*(a + b*x)^(m + (p - 1)/2))/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n\sin^3(c+dx)\,dx=-\frac{\operatorname{Subst}\left(\int \frac{(-a-ax)(a-ax)^{1+n}}{x^4}\,dx,x,-\sec(c+dx)\right)}{a^2d}$$

$$=\frac{\cos^3(c+dx)(a+a\sec(c+dx))^{2+n}}{3a^2d}+\frac{(4-n)\operatorname{Subst}\left(\int \frac{(a-ax)^{1+n}}{x^3}\,dx,x,-\mathrm{s}\right.}{3ad}$$

$$=\frac{\cos^3(c+dx)(a+a\sec(c+dx))^{2+n}}{3a^2d}-\frac{(4-n)\,{}_2F_1(3,2+n;3+n;1+\sec(}{3a^2d(2-}$$

Mathematica [A] time = 0.16, size = 67, normalized size = 0.81

$$\frac{(\sec(c+dx)+1)^2(a(\sec(c+dx)+1))^n\left((n-4)\,{}_2F_1(3,n+2;n+3;\sec(c+dx)+1)+(n+2)\cos^3(c+dx)\right)}{3d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^3,x]
```

```
[Out] (((2 + n)*Cos[c + d*x]^3 + (-4 + n)*Hypergeometric2F1[3, 2 + n, 3 + n, 1 +
Sec[c + d*x]])*(1 + Sec[c + d*x])^2*(a*(1 + Sec[c + d*x]))^n)/(3*d*(2 + n))
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos(dx+c)^2-1\right)(a\sec(dx+c)+a)^n\sin(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(a*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 1.35sym2poly/r2sym(const gen & e,const i
ndex_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 3.02, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))^n\left(\sin^3(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\sin(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+d\,x)^3\left(a+\frac{a}{\cos(c+d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + a/cos(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)^3*(a + a/cos(c + d*x))^n, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.148 $\int (a+a\sec(c+dx))^n \sin(c+dx)\,dx$

Optimal. Leaf size=42

$$\frac{(a\sec(c+dx)+a)^{n+1}\,{}_2F_1(2,n+1;n+2;\sec(c+dx)+1)}{ad(n+1)}$$

```
[Out] hypergeom([2, 1+n],[2+n],1+sec(d*x+c))*(a+a*sec(d*x+c))^(1+n)/a/d/(1+n)
```

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3873, 65}

$$\frac{(a\sec(c+dx)+a)^{n+1}\,{}_2F_1(2,n+1;n+2;\sec(c+dx)+1)}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x],x]
```

```
[Out] (Hypergeometric2F1[2, 1 + n, 2 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^
(1 + n))/(a*d*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 3873

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[(f*b^(p - 1))^(-1), Subst[Int[((-a + b*x)^((p - 1)/2
)*(a + b*x)^(m + (p - 1)/2))/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^n \sin(c+dx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^n}{x^2}\,dx, x, -\sec(c+dx)\right)}{d}\\ &= \frac{{}_2F_1(2,1+n;2+n;1+\sec(c+dx))(a+a\sec(c+dx))^{1+n}}{ad(1+n)}\end{aligned}$$

Mathematica [A] time = 0.04, size = 42, normalized size = 1.00

$$\frac{(a(\sec(c+dx)+1))^{n+1}\,{}_2F_1(2,n+1;n+2;\sec(c+dx)+1)}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x],x]
```

```
[Out] (Hypergeometric2F1[2, 1 + n, 2 + n, 1 + Sec[c + d*x]]*(a*(1 + Sec[c + d*x])
)^(1 + n))/(a*d*(1 + n))
```

fricas [F] time = 1.26, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^n\sin(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n \sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

maple [F] time = 1.21, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))^n \sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c),x)
```

```
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n \sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

mupad [B] time = 1.18, size = 64, normalized size = 1.52

$$\frac{\cos(c+d\,x)\left(a+\frac{a}{\cos(c+d\,x)}\right)^n {}_2F_1\left(1-n,-n;\ 2-n;\ -\cos(c+d\,x)\right)}{d\,(\cos(c+d\,x)+1)^n\ (n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + a/cos(c + d*x))^n,x)
```

```
[Out] (cos(c + d*x)*(a + a/cos(c + d*x))^n*hypergeom([1 - n, -n], 2 - n, -cos(c +
d*x)))/(d*(cos(c + d*x) + 1)^n*(n - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^n \sin(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*sin(c + d*x), x)
```

3.149 $\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=40

$$-\frac{(a\sec(c+dx)+a)^n\,{}_2F_1\left(1,n;n+1;\frac{1}{2}(\sec(c+dx)+1)\right)}{2dn}$$

```
[Out] -1/2*hypergeom([1, n],[1+n],1/2+1/2*sec(d*x+c))*(a+a*sec(d*x+c))^n/d/n
```

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3873, 68}

$$-\frac{(a\sec(c+dx)+a)^n\,{}_2F_1\left(1,n;n+1;\frac{1}{2}(\sec(c+dx)+1)\right)}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -(Hypergeometric2F1[1, n, 1 + n, (1 + Sec[c + d*x])/2]*(a + a*Sec[c + d*x])
^n)/(2*d*n)
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3873

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[(f*b^(p - 1))^(-1), Subst[Int[((-a + b*x)^((p - 1)/2
)*(a + b*x)^(m + (p - 1)/2))/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx &= -\frac{a^2\,\text{Subst}\left(\int \frac{(a-ax)^{-1+n}}{-a-ax}\,dx, x, -\sec(c+dx)\right)}{d}\\ &= -\frac{{}_2F_1\left(1,n;1+n;\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^n}{2dn}\end{aligned}$$

Mathematica [B] time = 0.68, size = 92, normalized size = 2.30

$$\frac{2^{n-1}(\sec(c+dx)+1)^{-n}(a(\sec(c+dx)+1))^n\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{n-1}\,{}_2F_1\left(1,1-n;2-n;\cos(c+dx\right)}{d(n-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] (2^(-1 + n)*Hypergeometric2F1[1, 1 - n, 2 - n, Cos[c + d*x]*Sec[(c + d*x)/2
]^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n)*(a*(1 + Sec[c + d*x]))^n)/(
d*(-1 + n)*(1 + Sec[c + d*x])^n)
```

fricas [F] time = 0.77, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^n\csc(dx+c)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

maple [F] time = 1.31, size = 0, normalized size = 0.00

$$\int \csc(dx+c)\,(a+a\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sec(d*x+c))^n,x)
[Out] int(csc(d*x+c)*(a+a*sec(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^n/sin(c + d*x),x)
[Out] int((a + a/cos(c + d*x))^n/sin(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^n\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))**n,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*csc(c + d*x), x)
```

3.150 $\int \csc^3(c+dx)(a+a\sec(c+dx))^n \, dx$

Optimal. Leaf size=112

$$-\frac{(n+2)(a\sec(c+dx)+a)^n\,{}_2F_1\left(1,n;n+1;\frac{1}{2}(\sec(c+dx)+1)\right)}{8dn}-\frac{a(2-n)(a\sec(c+dx)+a)^{n-1}}{4d(1-n)}+\frac{a(a\sec(c+\ldots}{2d(1-\sec\ldots}$$

```
[Out] -1/4*a*(2-n)*(a+a*sec(d*x+c))^(-1+n)/d/(1-n)+1/2*a*(a+a*sec(d*x+c))^(-1+n)/
d/(1-sec(d*x+c))-1/8*(2+n)*hypergeom([1, n],[1+n],1/2+1/2*sec(d*x+c))*(a+a*
sec(d*x+c))^n/d/n
```

Rubi [A] time = 0.10, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3873, 89, 79, 68}

$$-\frac{(n+2)(a\sec(c+dx)+a)^n\,{}_2F_1\left(1,n;n+1;\frac{1}{2}(\sec(c+dx)+1)\right)}{8dn}-\frac{a(2-n)(a\sec(c+dx)+a)^{n-1}}{4d(1-n)}+\frac{a(a\sec(c+\ldots}{2d(1-\sec\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -(a*(2 - n)*(a + a*Sec[c + d*x])^(-1 + n))/(4*d*(1 - n)) + (a*(a + a*Sec[c
+ d*x])^(-1 + n))/(2*d*(1 - Sec[c + d*x])) - ((2 + n)*Hypergeometric2F1[1,
n, 1 + n, (1 + Sec[c + d*x])/2]*(a + a*Sec[c + d*x])^n)/(8*d*n)
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 3873

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[(f*b^(p - 1))^(-1), Subst[Int[((-a + b*x)^((p - 1)/2
)*(a + b*x)^(m + (p - 1)/2))/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{a^4\,\text{Subst}\left(\int \frac{x^2(a-ax)^{-2+n}}{(-a-ax)^2}\,dx, x, -\sec(c+dx)\right)}{d}$$

$$= \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))} + \frac{\text{Subst}\left(\int \frac{(a-ax)^{-2+n}\left(-a^3n+2a^3x\right)}{-a-ax}\,dx, x, -\sec(c+\right.}{2d}$$

$$= -\frac{a(2-n)(a+a\sec(c+dx))^{-1+n}}{4d(1-n)} + \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))} - \frac{\left(a^2(2+\right.}{}$$

$$= -\frac{a(2-n)(a+a\sec(c+dx))^{-1+n}}{4d(1-n)} + \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))} - \frac{(2+n)\,_2}{}$$

Mathematica [A] time = 2.17, size = 179, normalized size = 1.60

$$\frac{\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^{-n}(a(\sec(c+dx)+1))^n\left(2^{n+1}\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^n\,_2F_1\left(1\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] (Cos[c + d*x]*Sec[(c + d*x)/2]^2*(a*(1 + Sec[c + d*x]))^n*(2^(1 + n)*Hyperg
eometric2F1[1, 1 - n, 2 - n, Cos[c + d*x]*Sec[(c + d*x)/2]^2]*(Cos[(c + d*x
)/2]^2*Sec[c + d*x])^n + 2^n*Hypergeometric2F1[2, 1 - n, 2 - n, Cos[c + d*x
]*Sec[(c + d*x)/2]^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n + (1 + Sec[c + d*
x])^n))/(8*d*(-1 + n)*(1 + Sec[c + d*x])^n)
```

fricas [F] time = 1.89, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^n\csc(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

maple [F] time = 1.40, size = 0, normalized size = 0.00

$$\int \left(\csc^3(dx+c)\right)(a+a\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n \csc(dx+c)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sin(c+d\,x)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^n/sin(c + d*x)^3,x)
[Out] int((a + a/cos(c + d*x))^n/sin(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+a*sec(d*x+c))**n,x)
[Out] Timed out
```

3.151 $\int \csc^5(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=240

$$\frac{a^2\left(n^2+9n+12\right)(a\sec(c+dx)+a)^{n-2}\,{}_2F_1\left(1,n-2;n-1;\frac{1}{2}(\sec(c+dx)+1)\right)}{16d(2-n)}-\frac{a^2\left(-2(1-n)(n+6)\sec(c+dx)\right.}{8d\left(n^2-3\right.}$$

```
[Out] 1/16*a^2*(n^2+9*n+12)*hypergeom([1, -2+n],[-1+n],1/2+1/2*sec(d*x+c))*(a+a*s
ec(d*x+c))^(-2+n)/d/(2-n)+1/4*a^2*(3+n)*sec(d*x+c)^2*(a+a*sec(d*x+c))^(-2+n
)/d/(1-n)/(1-sec(d*x+c))^2-a^2*sec(d*x+c)^3*(a+a*sec(d*x+c))^(-2+n)/d/(1-n)
/(1-sec(d*x+c))^2-1/8*a^2*(a+a*sec(d*x+c))^(-2+n)*(12+4*n-7*n^2-n^3-2*(1-n)
*(6+n)*sec(d*x+c))/d/(n^2-3*n+2)/(1-sec(d*x+c))
```

Rubi [A] time = 0.22, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3873, 100, 149, 146, 68}

$$\frac{a^2\left(n^2+9n+12\right)(a\sec(c+dx)+a)^{n-2}\,{}_2F_1\left(1,n-2;n-1;\frac{1}{2}(\sec(c+dx)+1)\right)}{16d(2-n)}-\frac{a^2\left(-2(1-n)(n+6)\sec(c+dx)\right.}{8d\left(n^2-3\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] (a^2*(12 + 9*n + n^2)*Hypergeometric2F1[1, -2 + n, -1 + n, (1 + Sec[c + d*x
])/2]*(a + a*Sec[c + d*x])^(-2 + n))/(16*d*(2 - n)) + (a^2*(3 + n)*Sec[c +
d*x]^2*(a + a*Sec[c + d*x])^(-2 + n))/(4*d*(1 - n)*(1 - Sec[c + d*x])^2) -
(a^2*Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(-2 + n))/(d*(1 - n)*(1 - Sec[c +
d*x])^2) - (a^2*(a + a*Sec[c + d*x])^(-2 + n)*(12 + 4*n - 7*n^2 - n^3 - 2*(
1 - n)*(6 + n)*Sec[c + d*x]))/(8*d*(2 - 3*n + n^2)*(1 - Sec[c + d*x]))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
```

```
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 3873

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[(f*b^(p - 1))^(-1), Subst[Int[((-a + b*x)^((p - 1)/2
)*(a + b*x)^(m + (p - 1)/2))/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{a^6\,\text{Subst}\left(\int \frac{x^4(a-ax)^{-3+n}}{(-a-ax)^3}\,dx, x, -\sec(c+dx)\right)}{d}$$

$$= -\frac{a^2\sec^3(c+dx)(a+a\sec(c+dx))^{-2+n}}{d(1-n)(1-\sec(c+dx))^2} + \frac{a^4\,\text{Subst}\left(\int \frac{x^2(a-ax)^{-3+n}(3a^2}{(-a-ax)^3}\right.}{d(1}$$

$$= \frac{a^2(3+n)\sec^2(c+dx)(a+a\sec(c+dx))^{-2+n}}{4d(1-n)(1-\sec(c+dx))^2} - \frac{a^2\sec^3(c+dx)(a+a}{d(1-n)(1-\text{se}}$$

$$= \frac{a^2(3+n)\sec^2(c+dx)(a+a\sec(c+dx))^{-2+n}}{4d(1-n)(1-\sec(c+dx))^2} - \frac{a^2\sec^3(c+dx)(a+a}{d(1-n)(1-\text{se}}$$

$$= \frac{a^2\left(12+9n+n^2\right)\,{}_2F_1\left(1,-2+n;-1+n;\frac{1}{2}(1+\sec(c+dx))\right)(a+a\,\text{se}}{16d(2-n)}$$

Mathematica [B] time = 6.53, size = 492, normalized size = 2.05

$$-\frac{\cos(c+dx)(\sec(c+dx)+1)^{-n}(a(\sec(c+dx)+1))^n\left(-2^n\left(n^2+7n-18\right)\sec(c+dx)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -1/64*(Cos[c + d*x]*(a*(1 + Sec[c + d*x]))^n*(2^(1 + n)*Cot[(c + d*x)/2]^4*
Sec[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n) - 3*2^n*n*Cot[(c +
d*x)/2]^4*Sec[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n) + 2^n*n^2
*Cot[(c + d*x)/2]^4*Sec[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n)
 - 3*2^(2 + n)*(-2 + n)*Hypergeometric2F1[1, 1 - n, 2 - n, Cos[c + d*x]*Sec
[(c + d*x)/2]^2]*Sec[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n) -
2^n*(-18 + 7*n + n^2)*Hypergeometric2F1[2, 1 - n, 2 - n, Cos[c + d*x]*Sec[(
c + d*x)/2]^2]*Sec[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n) + 32
```

```
*Sec[c + d*x]*(1 + Sec[c + d*x])^n - 12*n*Sec[c + d*x]*(1 + Sec[c + d*x])^n
 - 12*Sec[(c + d*x)/2]^2*Sec[c + d*x]*(1 + Sec[c + d*x])^n + 2*n*Sec[(c + d
*x)/2]^2*Sec[c + d*x]*(1 + Sec[c + d*x])^n - 2*Sec[(c + d*x)/2]^4*Sec[c + d
*x]*(1 + Sec[c + d*x])^n + 2*n*Sec[(c + d*x)/2]^4*Sec[c + d*x]*(1 + Sec[c +
 d*x])^n))/(d*(-2 + n)*(-1 + n)*(1 + Sec[c + d*x])^n)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left((a \sec(dx+c)+a)^n \csc(dx+c)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((a*sec(d*x + c) + a)^n*csc(d*x + c)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(dx+c)+a)^n \csc(dx+c)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^5, x)
```

maple [F] time = 1.30, size = 0, normalized size = 0.00

$$\int \left(\csc^5(dx+c)\right)(a+a\sec(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x)
[Out] int(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a \sec(dx+c)+a)^n \csc(dx+c)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sin(c+d\,x)^5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^n/sin(c + d*x)^5,x)
[Out] int((a + a/cos(c + d*x))^n/sin(c + d*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+a*sec(d*x+c))**n,x)
[Out] Timed out
```

3.152 $\int (a + a\sec(c + dx))^n \sin^4(c + dx)\, dx$

Optimal. Leaf size=230

$$\frac{2^{n+\frac{1}{2}} \sin(c+dx) \cos^n(c+dx)(\cos(c+dx)+1)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n F_1\left(\frac{1}{2}; n-4, \frac{1}{2}-n; \frac{3}{2}; 1-\cos(c+dx), \frac{1}{2}\right.}{d}$$

```
[Out] -cos(d*x+c)*(a+a*sec(d*x+c))^n*sin(d*x+c)/d+2^(1/2+n)*AppellF1(1/2,-4+n,1/2
-n,3/2,1-cos(d*x+c),1/2-1/2*cos(d*x+c))*cos(d*x+c)^n*(1+cos(d*x+c))^(-1/2-n
)*(a+a*sec(d*x+c))^n*sin(d*x+c)/d-AppellF1(1-n,1/2-n,-1/2,2-n,-cos(d*x+c),c
os(d*x+c))*(1+cos(d*x+c))^(1/2-n)*(n-n*cos(d*x+c))*cot(d*x+c)*(a+a*sec(d*x+
c))^n/d/(1-n)/(1-cos(d*x+c))^(1/2)
```

Rubi [A] time = 0.67, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3876, 2881, 2787, 2786, 2785, 133, 3046, 3008, 135}

$$\frac{2^{n+\frac{1}{2}} \sin(c+dx) \cos^n(c+dx)(\cos(c+dx)+1)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n F_1\left(\frac{1}{2}; n-4, \frac{1}{2}-n; \frac{3}{2}; 1-\cos(c+dx), \frac{1}{2}\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^4,x]
```

```
[Out] -((AppellF1[1 - n, -1/2, 1/2 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 +
Cos[c + d*x])^(1/2 - n)*(n - n*Cos[c + d*x])*Cot[c + d*x]*(a + a*Sec[c + d*
x])^n)/(d*(1 - n)*Sqrt[1 - Cos[c + d*x]])) - (Cos[c + d*x]*(a + a*Sec[c + d
*x])^n*Sin[c + d*x])/d + (2^(1/2 + n)*AppellF1[1/2, -4 + n, 1/2 - n, 3/2, 1
 - Cos[c + d*x], (1 - Cos[c + d*x])/2]*Cos[c + d*x]^n*(1 + Cos[c + d*x])^(-
1/2 - n)*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/d
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
```

```
tegerQ[m] && GtQ[a, 0] && !GtQ[d/b, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] && !IGtQ[m, 0]
```

Rule 3008

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(f*Cos[e + f*x]
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] && !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\int (a + a\sec(c+dx))^n \sin^4(c+dx)\,dx = ((-\cos(c+dx))^n(-a - a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n) \int (-\cos$$
$$= ((-\cos(c+dx))^n(-a - a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n) \int (-\cos$$
$$= -\frac{\cos(c+dx)(a+a\sec(c+dx))^n \sin(c+dx)}{d} + ((-\cos(c+dx))^n(1+$$
$$= -\frac{\cos(c+dx)(a+a\sec(c+dx))^n \sin(c+dx)}{d} + (\cos^n(c+dx)(1+\cos$$
$$= -\frac{\cos(c+dx)(a+a\sec(c+dx))^n \sin(c+dx)}{d} - \frac{\left((-\cos(c+dx))^n(1+\right.}{}$$
$$= -\frac{\cos(c+dx)(a+a\sec(c+dx))^n \sin(c+dx)}{d} + \frac{2^{\frac{1}{2}+n}F_1\left(\frac{1}{2};-4+n,\frac{1}{2}\right.}{}$$
$$= -\frac{F_1\left(1-n;-\frac{1}{2},\frac{1}{2}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)(1+\cos(c+}{d(1-n)\sqrt{1-\cos(}}$$

Mathematica [C] time = 22.87, size = 7069, normalized size = 30.73

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^4,x]
[Out] Result too large to show
```

fricas [F] time = 2.28, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1\right)(a\sec(dx+c)+a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="fricas")
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(a*sec(d*x + c) + a)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n \sin(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

maple [F] time = 3.55, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))^n \left(\sin^4(dx+c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n \sin(dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin(c+d\,x)^4 \left(a+\frac{a}{\cos(c+d\,x)}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + a/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^4*(a + a/cos(c + d*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**4,x)
[Out] Timed out
```

3.153 $\int (a + a\sec(c+dx))^n \sin^2(c+dx)\,dx$

Optimal. Leaf size=95

$$-\frac{\sqrt{1-\cos(c+dx)}\,\cot(c+dx)(\cos(c+dx)+1)^{\frac{1}{2}-n}(a\sec(c+dx)+a)^n F_1\left(1-n;-\frac{1}{2},-n-\frac{1}{2};2-n;\cos(c+dx\right.}{d(1-n)}$$

```
[Out] -AppellF1(1-n,-1/2-n,-1/2,2-n,-cos(d*x+c),cos(d*x+c))*(1+cos(d*x+c))^(1/2-n
)*cot(d*x+c)*(a+a*sec(d*x+c))^n*(1-cos(d*x+c))^(1/2)/d/(1-n)
```

Rubi [A] time = 0.35, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3876, 2874, 3008, 135, 133}

$$-\frac{\sqrt{1-\cos(c+dx)}\,\cot(c+dx)(\cos(c+dx)+1)^{\frac{1}{2}-n}(a\sec(c+dx)+a)^n F_1\left(1-n;-\frac{1}{2},-n-\frac{1}{2};2-n;\cos(c+dx\right.}{d(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^2,x]
```

```
[Out] -((AppellF1[1 - n, -1/2, -1/2 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*Sqrt
[1 - Cos[c + d*x]]*(1 + Cos[c + d*x])^(1/2 - n)*Cot[c + d*x]*(a + a*Sec[c +
 d*x])^n)/(d*(1 - n)))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3008

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(f*Cos[e + f*x]
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
```

```
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\int (a+a\sec(c+dx))^n\sin^2(c+dx)\,dx = ((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n)\int(-\cos(c$$

$$= \frac{((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n)\int(-\cos(c+}{a^2}$$

$$= -\frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{\frac{1}{2}-n}\sqrt{-a+a\cos(c+dx)}\,\csc(c+}{}$$

$$= -\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{-\frac{1}{2}-n}(-a-a\cos(c+dx))\sqrt{-a+a\,\mathrm{co}}\right.}{}$$

$$= -\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{-\frac{1}{2}-n}(-a-a\cos(c+dx))(-a+a\cos\right.}{}$$

$$= -\frac{F_1\left(1-n;-\frac{1}{2},-\frac{1}{2}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)\sqrt{1-\cos(c+}}{d(1-n)}$$

Mathematica [C] time = 17.14, size = 4297, normalized size = 45.23

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^2,x]
```

```
[Out] (2^(3 + n)*Cos[(c + d*x)/2]^5*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(a*(1 + S
ec[c + d*x]))^n*Sin[(c + d*x)/2]*(Cos[2*(c + d*x)]*(-1/4*(1 + Sec[c + d*x])
^n - ((1 + Sec[c + d*x])^n*Sin[c + d*x]^2)/2 - ((1 + Sec[c + d*x])^n*Sin[c
+ d*x]^4)/4) + (I/4)*(1 + Sec[c + d*x])^n*Sin[2*(c + d*x)] + (I/2)*(1 + Sec
[c + d*x])^n*Sin[c + d*x]^2*Sin[2*(c + d*x)] + (I/4)*(1 + Sec[c + d*x])^n*S
in[c + d*x]^4*Sin[2*(c + d*x)] + Cos[c + d*x]^4*(-1/4*(Cos[2*(c + d*x)]*(1
+ Sec[c + d*x])^n) + (I/4)*(1 + Sec[c + d*x])^n*Sin[2*(c + d*x)]) + Cos[c +
 d*x]^3*((-I)*Cos[2*(c + d*x)]*(1 + Sec[c + d*x])^n*Sin[c + d*x] - (1 + Sec
[c + d*x])^n*Sin[c + d*x]*Sin[2*(c + d*x)]) + Cos[c + d*x]^2*(Cos[2*(c + d*
x)]*((1 + Sec[c + d*x])^n/2 + (3*(1 + Sec[c + d*x])^n*Sin[c + d*x]^2)/2) -
(I/2)*(1 + Sec[c + d*x])^n*Sin[2*(c + d*x)] - ((3*I)/2)*(1 + Sec[c + d*x])^
n*Sin[c + d*x]^2*Sin[2*(c + d*x)]) + Cos[c + d*x]*(Cos[2*(c + d*x)]*(I*(1 +
 Sec[c + d*x])^n*Sin[c + d*x] + I*(1 + Sec[c + d*x])^n*Sin[c + d*x]^3) + (1
 + Sec[c + d*x])^n*Sin[c + d*x]*Sin[2*(c + d*x)] + (1 + Sec[c + d*x])^n*Sin
[c + d*x]^3*Sin[2*(c + d*x)]))*((3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2
]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2)/(3*AppellF1[1/2, n, 2, 3/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-2*AppellF1[3/2, n, 3, 5/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 2, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) - AppellF1[1/2,
n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]/(AppellF1[1/2, n, 3, 3/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/2, n, 4, 5/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/3)))/(d*(1
+ Sec[c + d*x])^n*(2^(2 + n)*Cos[(c + d*x)/2]^6*(Cos[(c + d*x)/2]^2*Sec[c +
 d*x])^n*((3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2]*Sec[(c + d*x)/2]^2)/(3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Ta
```

```
n[(c + d*x)/2]^2] + 2*(-2*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) - AppellF1[1/2, n, 3, 3/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]/(AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/3)) - 5*2^(2 + n)*Cos[(c + d*x)/2
]^4*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*Sin[(c + d*x)/2]^2*((3*AppellF1[1/2
, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2)/(
3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-2
*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*Appe
llF1[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c +
 d*x)/2]^2) - AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]/(AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (
2*(-3*AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n
*AppellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan
[(c + d*x)/2]^2)/3)) + 2^(3 + n)*Cos[(c + d*x)/2]^5*(Cos[(c + d*x)/2]^2*Sec
[c + d*x])^n*Sin[(c + d*x)/2]*((3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3*AppellF1[1
/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-2*AppellF1[3/
2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*AppellF1[3/2, 1
+ n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)
+ (3*Sec[(c + d*x)/2]^2*((-2*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (n*AppellF1[3/
2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2])/3))/(3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2] + 2*(-2*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) - (-(AppellF1[3/2, n, 4, 5/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (n
*AppellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[
(c + d*x)/2]^2*Tan[(c + d*x)/2])/3)/(AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/3) - (3*AppellF1[1/2, n, 2,
 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*(2*(-2*Ap
pellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*AppellF
1[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2] + 3*((-2*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (n*Appell
F1[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2])/3) + 2*Tan[(c + d*x)/2]^2*(-2*((-9*AppellF1[5/2,
n, 4, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2])/5 + (3*n*AppellF1[5/2, 1 + n, 3, 7/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5) + n*((-6*AppellF1
[5/2, 1 + n, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2])/5 + (3*(1 + n)*AppellF1[5/2, 2 + n, 2, 7/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5)))
)/(3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*
(-2*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*A
ppellF1[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(
c + d*x)/2]^2)^2 + (AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2]*(-(AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (n*AppellF1[3/2, 1 + n, 3, 5/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])/3 + (2*(-3*AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2] + n*AppellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (2*Tan[(c + d*x)/2]^2*(-3*((
-12*AppellF1[5/2, n, 5, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(
```

```
c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (3*n*AppellF1[5/2, 1 + n, 4, 7/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5)
 + n*((-9*AppellF1[5/2, 1 + n, 4, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (3*(1 + n)*AppellF1[5/2, 2 +
n, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2])/5)))/3))/(AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] + (2*(-3*AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/3)^2) + 2^(3 + n)*n*Cos[(c + d*x)/2]^5*(
Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n)*Sin[(c + d*x)/2]*((3*AppellF1[1/2
, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2)/(
3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-2
*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*Appe
llF1[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c +
 d*x)/2]^2) - AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]/(AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (
2*(-3*AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n
*AppellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan
[(c + d*x)/2]^2)/3))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + C
os[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x])))
```

fricas [F] time = 1.26, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(\cos\left(dx+c\right)^{2}-1\right)\left(a\sec\left(dx+c\right)+a\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^2 - 1)*(a*sec(d*x + c) + a)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(dx+c\right)+a\right)^{n}\sin\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

maple [F] time = 2.79, size = 0, normalized size = 0.00

$$\int\left(a+a\sec\left(dx+c\right)\right)^{n}\left(\sin^{2}\left(dx+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x)
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\sec\left(dx+c\right)+a\right)^{n}\sin\left(dx+c\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c+d\,x)^2\left(a+\frac{a}{\cos(c+d\,x)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + a/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^2*(a + a/cos(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^n\sin^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**2,x)
[Out] Integral((a*(sec(c + d*x) + 1))**n*sin(c + d*x)**2, x)
```

3.154 $\int \csc^2(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=98

$$\frac{2^{n-\frac{1}{2}}n\tan(c+dx)(\sec(c+dx)+1)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)}{d}-\frac{\cot(c+dx)(a\sec}{d}$$

```
[Out] -cot(d*x+c)*(a+a*sec(d*x+c))^n/d+2^(-1/2+n)*n*hypergeom([1/2, 3/2-n],[3/2],
1/2-1/2*sec(d*x+c))*(1+sec(d*x+c))^(-1/2-n)*(a+a*sec(d*x+c))^n*tan(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3875, 3828, 3827, 69}

$$\frac{2^{n-\frac{1}{2}}n\tan(c+dx)(\sec(c+dx)+1)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)}{d}-\frac{\cot(c+dx)(a\sec}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -((Cot[c + d*x]*(a + a*Sec[c + d*x])^n)/d) + (2^(-1/2 + n)*n*Hypergeometric
2F1[1/2, 3/2 - n, 3/2, (1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x])^(-1/2 - n)*
(a + a*Sec[c + d*x])^n*Tan[c + d*x])/d
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule steps

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{\cot(c+dx)(a+a\sec(c+dx))^n}{d} + (an)\int \sec(c+dx)(a+a\sec(c+$$

$$= -\frac{\cot(c+dx)(a+a\sec(c+dx))^n}{d} + (n(1+\sec(c+dx))^{-n}(a+a\sec(c$$

$$= -\frac{\cot(c+dx)(a+a\sec(c+dx))^n}{d} - \frac{\left(n(1+\sec(c+dx))^{-\frac{1}{2}-n}(a+a\sec\right.}{}$$

$$= -\frac{\cot(c+dx)(a+a\sec(c+dx))^n}{d} + \frac{2^{-\frac{1}{2}+n} n\, {}_2F_1\left(\frac{1}{2},\frac{3}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sec\right.}{}$$

Mathematica [A] time = 1.16, size = 142, normalized size = 1.45

$$-\frac{2^{n-1}\tan\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^{-n}(a(\sec(c+dx)+1))^n\left(\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)^n\left(\cos^2\left(\frac{1}{2}(c+dx\right.\right.}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -((2^(-1 + n)*(Cot[(c + d*x)/2]^2*Hypergeometric2F1[-1/2, n, 1/2, Tan[(c +
d*x)/2]^2] - Hypergeometric2F1[1/2, n, 3/2, Tan[(c + d*x)/2]^2])*(Cos[c + d
*x]*Sec[(c + d*x)/2]^2)^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(a*(1 + Sec[c
 + d*x]))^n*Tan[(c + d*x)/2])/(d*(1 + Sec[c + d*x])^n))
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^n\csc(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

maple [F] time = 1.32, size = 0, normalized size = 0.00

$$\int \left(\csc^2(dx+c)\right)(a+a\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sin\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^n/sin(c + d*x)^2,x)
```

```
[Out] int((a + a/cos(c + d*x))^n/sin(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(c+dx\right)+1\right)\right)^n \csc^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sec(d*x+c))**n,x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*csc(c + d*x)**2, x)
```

3.155 $\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=349

$$-\frac{a^4\sin(c+dx)\cos(c+dx)(a\sec(c+dx)+a)^n}{d(3-2n)(a-a\cos(c+dx))^2(a\cos(c+dx)+a)^2}-\frac{a^3(4-n)\sin(c+dx)\cos(c+dx)(a\sec(c+dx)+a)^n}{d\left(4n^2-8n+3\right)(a-a\cos(c+dx))^2(a\cos(c+dx)+a)}+$$

```
[Out] (n^2-n+2)*cos(d*x+c)*(a+a*sec(d*x+c))^n*sin(d*x+c)/d/(3-2*n)/(-4*n^2+1)/(1-
cos(d*x+c))^2-a^4*cos(d*x+c)*(a+a*sec(d*x+c))^n*sin(d*x+c)/d/(3-2*n)/(a-a*c
os(d*x+c))^2/(a+a*cos(d*x+c))^2-a^3*(4-n)*cos(d*x+c)*(a+a*sec(d*x+c))^n*sin
(d*x+c)/d/(4*n^2-8*n+3)/(a-a*cos(d*x+c))^2/(a+a*cos(d*x+c))+n*(-n^2-3*n+7)*
cos(d*x+c)*((1+cos(d*x+c))/(1-cos(d*x+c)))^(-1/2-n)*hypergeom([1-n, -1/2-n]
,[2-n],-2*cos(d*x+c)/(1-cos(d*x+c)))*(a+a*sec(d*x+c))^n*sin(d*x+c)/d/(-8*n^
4+20*n^3-10*n^2-5*n+3)/(1-cos(d*x+c))^2
```

Rubi [A] time = 0.54, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3876, 2883, 129, 155, 12, 132}

$$-\frac{a^3(4-n)\sin(c+dx)\cos(c+dx)(a\sec(c+dx)+a)^n}{d\left(4n^2-8n+3\right)(a-a\cos(c+dx))^2(a\cos(c+dx)+a)}-\frac{a^4\sin(c+dx)\cos(c+dx)(a\sec(c+dx)+a)^n}{d(3-2n)(a-a\cos(c+dx))^2(a\cos(c+dx)+a)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] ((2 - n + n^2)*Cos[c + d*x]*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*
n)*(1 - 4*n^2)*(1 - Cos[c + d*x])^2) - (a^4*Cos[c + d*x]*(a + a*Sec[c + d*x
])^n*Sin[c + d*x])/(d*(3 - 2*n)*(a - a*Cos[c + d*x])^2*(a + a*Cos[c + d*x])
^2) - (a^3*(4 - n)*Cos[c + d*x]*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3
- 8*n + 4*n^2)*(a - a*Cos[c + d*x])^2*(a + a*Cos[c + d*x])) + (n*(7 - 3*n -
 n^2)*Cos[c + d*x]*((1 + Cos[c + d*x])/(1 - Cos[c + d*x]))^(-1/2 - n)*Hyper
geometric2F1[-1/2 - n, 1 - n, 2 - n, (-2*Cos[c + d*x])/(1 - Cos[c + d*x])]*
(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*(3 - 2*n)*(1 - n)*(1 + 2*
n)*(1 - Cos[c + d*x])^2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
```

```
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) &&  !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 2883

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[Cos[e + f*x]/(a^(
p - 2)*f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(d*x
)^n*(a + b*x)^(m + p/2 - 1/2)*(a - b*x)^(p/2 - 1/2), x], x, Sin[e + f*x]],
x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[p/2]
 &&  !IntegerQ[m]
```

Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx = ((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n)\int(-\cos$$

$$= -\frac{\left(a^6(-\cos(c+dx))^n(-a-a\cos(c+dx))^{-\frac{1}{2}-n}(a+a\sec(c+dx))^n \sin\right.}{d\sqrt{-a+a\cos(c+}}$$

$$= -\frac{a^4\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(3-2n)(a-a\cos(c+dx))^2(a+a\cos(c+dx))^2} - \frac{\left(a^3(-\cos(c+d\right.}{}$$

$$= -\frac{a^4\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(3-2n)(a-a\cos(c+dx))^2(a+a\cos(c+dx))^2} - \frac{a^3(4-n)\cos(}{d(1-2n)(3-2}$$

$$= \frac{(2-n+n^2)\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1-2n)(3-2n)(1+2n)(1-\cos(c+dx))^2} - \frac{a^4\cos(c}{d(3-2n)(}$$

$$= \frac{(2-n+n^2)\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1-2n)(3-2n)(1+2n)(1-\cos(c+dx))^2} - \frac{a^4\cos(c}{d(3-2n)(}$$

$$= \frac{(2-n+n^2)\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1-2n)(3-2n)(1+2n)(1-\cos(c+dx))^2} - \frac{a^4\cos(c}{d(3-2n)(}$$

Mathematica [A] time = 7.04, size = 350, normalized size = 1.00

$$\tan\left(\frac{1}{2}(c+dx)\right)(a(\sec(c+dx)+1))^n\left(\frac{24(\sec(c+dx)+1)^{-n}\,{}_2F_1\left(\frac{1}{2},n;\frac{3}{2};\tan^2\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)^n\left(-2(\sec(c+dx)+1)^n\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] ((a*(1 + Sec[c + d*x]))^n*((-2*Cot[(c + d*x)/2]^2*Hypergeometric2F1[-1/2, n
, 1/2, Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(3*2^n*(Cos[
(c + d*x)/2]^2*Sec[c + d*x])^n + 2*(1 + Sec[c + d*x])^n + n*(1 + Sec[c + d*
x])^n))/(1 + Sec[c + d*x])^n + (-(Cos[c + d*x]*(4*n*Cos[c + d*x] + (-3 + n)
*(3 + Cos[2*(c + d*x)]))*Csc[(c + d*x)/2]^4*Sec[(c + d*x)/2]^2) + (24*Hyper
geometric2F1[1/2, n, 3/2, Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2
]^2)^n*(-3*2^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n - 2*(1 + Sec[c + d*x])^n
 + n*(2^(1 + n)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n + (1 + Sec[c + d*x])^n)
))/(1 + Sec[c + d*x])^n)/(4*(-3 + 2*n)))*Tan[(c + d*x)/2])/(24*d)
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^n\csc(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n \csc(dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

maple [F] time = 1.39, size = 0, normalized size = 0.00

$$\int \left(\csc^4(dx+c)\right)(a+a\sec(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x)
[Out] int(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n \csc(dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\sin(c+d\,x)^4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^n/sin(c + d*x)^4,x)
[Out] int((a + a/cos(c + d*x))^n/sin(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sec(d*x+c))**n,x)
[Out] Timed out
```

3.156 $\int (a+a\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=105

$$-\frac{\sqrt{\sin(c+dx)}\cos(c+dx)(\cos(c+dx)+1)^{-n-\frac{1}{4}}(a\sec(c+dx)+a)^n F_1\left(1-n;-\frac{1}{4},-n-\frac{1}{4};2-n;\cos(c+dx),-\right.}{d(1-n)\sqrt[4]{1-\cos(c+dx)}}$$

```
[Out] -AppellF1(1-n,-1/4-n,-1/4,2-n,-cos(d*x+c),cos(d*x+c))*cos(d*x+c)*(1+cos(d*x
+c))^(-1/4-n)*(a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2)/d/(1-n)/(1-cos(d*x+c))^(1
/4)
```

Rubi [A] time = 0.26, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3876, 2886, 135, 133}

$$-\frac{\sqrt{\sin(c+dx)}\cos(c+dx)(\cos(c+dx)+1)^{-n-\frac{1}{4}}(a\sec(c+dx)+a)^n F_1\left(1-n;-\frac{1}{4},-n-\frac{1}{4};2-n;\cos(c+dx),-\right.}{d(1-n)\sqrt[4]{1-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^(3/2),x]
```

```
[Out] -((AppellF1[1 - n, -1/4, -1/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*Cos[
c + d*x]*(1 + Cos[c + d*x])^(-1/4 - n)*(a + a*Sec[c + d*x])^n*Sqrt[Sin[c +
d*x]])/(d*(1 - n)*(1 - Cos[c + d*x])^(1/4)))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2886

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos
[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\int (a+a\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx = ((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n)\int(-\cos(c$$

$$= -\frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-\frac{1}{4}-n}(a+a\sec(c+dx))^n\sqrt{\sin(c+}\right.}{d\sqrt[4]{-a+a\cos}}$$

$$= -\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{-\frac{1}{4}-n}(a+a\sec(c+dx))^n\sqrt{\sin(c+d}\right.}{d\sqrt[4]{-a+a\cos(c}}$$

$$= -\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{-\frac{1}{4}-n}(a+a\sec(c+dx))^n\sqrt{\sin(c+d}\right.}{d\sqrt[4]{1-\cos(c+d}}$$

$$= -\frac{F_1\left(1-n;-\frac{1}{4},-\frac{1}{4}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)\cos(c+dx)(1}{d(1-n)\sqrt[4]{1-\cos(c+}}$$

Mathematica [B] time = 3.36, size = 382, normalized size = 3.64

$$\overline{d\left(2(\cos(c+dx)-1)\left(3F_1\left(\frac{5}{4};n,\frac{5}{2};\frac{9}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-5F_1\left(\frac{5}{4};n,\frac{7}{2};\frac{9}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\mathrm{t}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^(3/2),x]
```

```
[Out] (10*(AppellF1[1/4, n, 3/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] -
AppellF1[1/4, n, 5/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(1 + C
os[c + d*x])*(a*(1 + Sec[c + d*x]))^n*Sin[c + d*x]^(5/2))/(d*(2*(3*AppellF1
[5/4, n, 5/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 5*AppellF1[5/
4, n, 7/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*n*AppellF1[5/4
, 1 + n, 3/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*n*AppellF1[
5/4, 1 + n, 5/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c
 + d*x]) + 5*AppellF1[1/4, n, 3/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*(1 + Cos[c + d*x]) - 5*AppellF1[1/4, n, 5/2, 5/4, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x])))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\mathrm{integral}\left((a\sec(dx+c)+a)^n\sin(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\sin(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

maple [F] time = 0.82, size = 0, normalized size = 0.00

$$\int (a + a\sec(dx + c))^n \left(\sin^{\frac{3}{2}}(dx + c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx + c) + a)^n \sin(dx + c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin(c + d\,x)^{3/2} \left(a + \frac{a}{\cos(c + d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^(3/2)*(a + a/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^(3/2)*(a + a/cos(c + d*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**(3/2),x)
[Out] Timed out
```

3.157 $\int (a + a\sec(c + dx))^n \sqrt{\sin(c + dx)}\, dx$

Optimal. Leaf size=105

$$-\frac{\sqrt[4]{1-\cos(c+dx)}\cos(c+dx)(\cos(c+dx)+1)^{\frac{1}{4}-n}(a\sec(c+dx)+a)^n F_1\left(1-n;\frac{1}{4},\frac{1}{4}-n;2-n;\cos(c+dx),-\mathrm{cc}\right.}{d(1-n)\sqrt{\sin(c+dx)}}$$

```
[Out] -AppellF1(1-n,1/4-n,1/4,2-n,-cos(d*x+c),cos(d*x+c))*(1-cos(d*x+c))^(1/4)*co
s(d*x+c)*(1+cos(d*x+c))^(1/4-n)*(a+a*sec(d*x+c))^n/d/(1-n)/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3876, 2886, 135, 133}

$$-\frac{\sqrt[4]{1-\cos(c+dx)}\cos(c+dx)(\cos(c+dx)+1)^{\frac{1}{4}-n}(a\sec(c+dx)+a)^n F_1\left(1-n;\frac{1}{4},\frac{1}{4}-n;2-n;\cos(c+dx),-\mathrm{cc}\right.}{d(1-n)\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]],x]
```

```
[Out] -((AppellF1[1 - n, 1/4, 1/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 - C
os[c + d*x])^(1/4)*Cos[c + d*x]*(1 + Cos[c + d*x])^(1/4 - n)*(a + a*Sec[c +
 d*x])^n)/(d*(1 - n)*Sqrt[Sin[c + d*x]]))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2886

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos
[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\int (a+a\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx = ((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n)\int (-\cos$$

$$= -\frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{\frac{1}{4}-n}\sqrt[4]{-a+a\cos(c+dx)}\,(a+\right.}{d\sqrt{\sin(c+}}$$

$$= -\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{4}-n}\sqrt[4]{-a+a\cos(c+dx)}\,(a+a\,\mathrm{se}\right.}{d\sqrt{\sin(c+dx}}$$

$$= -\frac{\left(\sqrt[4]{1-\cos(c+dx)}\,(-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{4}-n}(a+a\sec(c\right.}{d\sqrt{\sin(c+dx)}}$$

$$= -\frac{F_1\left(1-n;\frac{1}{4},\frac{1}{4}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)\sqrt[4]{1-\cos(c+}}{d(1-n)\sqrt{\sin(c-}}$$

Mathematica [B] time = 1.44, size = 214, normalized size = 2.04

$$\frac{14\sin^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)(a(\sec(c+dx)+1))^nF_1}{d\left(6(\cos(c+dx)-1)\left(3F_1\left(\frac{7}{4};n,\frac{5}{2};\frac{11}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-2nF_1\left(\frac{7}{4};n+1,\frac{3}{2};\frac{11}{4};\tan^2\left(\frac{1}{2}(c-\right.\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]],x]
```

```
[Out] (14*AppellF1[3/4, n, 3/2, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1
+ Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^n*Sin[c + d*x]^(3/2))/(d*(6*(3*Appel
lF1[7/4, n, 5/2, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*n*Appel
lF1[7/4, 1 + n, 3/2, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 +
Cos[c + d*x]) + 21*AppellF1[3/4, n, 3/2, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*(1 + Cos[c + d*x])))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left((a\sec(dx+c)+a)^n\sqrt{\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\sqrt{\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

maple [F] time = 0.79, size = 0, normalized size = 0.00

$$\int (a+a\sec(dx+c))^n\left(\sqrt{\sin}\,(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
```

```
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a\sec(dx+c)+a)^n\sqrt{\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\sin(c+d\,x)}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^(1/2)*(a + a/cos(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)^(1/2)*(a + a/cos(c + d*x))^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^n\,\sqrt{\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**(1/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*sqrt(sin(c + d*x)), x)
```

3.158 $\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$

Optimal. Leaf size=105

$$-\frac{(1-\cos(c+dx))^{3/4}\cos(c+dx)(\cos(c+dx)+1)^{\frac{3}{4}-n}(a\sec(c+dx)+a)^n F_1\left(1-n;\frac{3}{4},\frac{3}{4}-n;2-n;\cos(c+dx)\right.}{d(1-n)\sin^{\frac{3}{2}}(c+dx)}$$

```
[Out] -AppellF1(1-n,3/4-n,3/4,2-n,-cos(d*x+c),cos(d*x+c))*(1-cos(d*x+c))^(3/4)*co
s(d*x+c)*(1+cos(d*x+c))^(3/4-n)*(a+a*sec(d*x+c))^n/d/(1-n)/sin(d*x+c)^(3/2)
```

Rubi [A] time = 0.25, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3876, 2886, 135, 133}

$$-\frac{(1-\cos(c+dx))^{3/4}\cos(c+dx)(\cos(c+dx)+1)^{\frac{3}{4}-n}(a\sec(c+dx)+a)^n F_1\left(1-n;\frac{3}{4},\frac{3}{4}-n;2-n;\cos(c+dx)\right.}{d(1-n)\sin^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]],x]
```

```
[Out] -((AppellF1[1 - n, 3/4, 3/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 - C
os[c + d*x])^(3/4)*Cos[c + d*x]*(1 + Cos[c + d*x])^(3/4 - n)*(a + a*Sec[c +
 d*x])^n)/(d*(1 - n)*Sin[c + d*x]^(3/2)))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2886

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos
[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\int \frac{(a + a\sec(c + dx))^n}{\sqrt{\sin(c + dx)}}\,dx = ((-\cos(c + dx))^n(-a - a\cos(c + dx))^{-n}(a + a\sec(c + dx))^n)\int \frac{(-\cos(c + dx))^{-n}(-a}{\sqrt{\sin(c}}$$

$$= -\frac{\left((-\cos(c + dx))^n(-a - a\cos(c + dx))^{\frac{3}{4}-n}(-a + a\cos(c + dx))^{3/4}(a + a\sec(c + dx))\right.}{d\sin^{\frac{3}{2}}(c + dx)}$$

$$= -\frac{\left((-\cos(c + dx))^n(1 + \cos(c + dx))^{\frac{3}{4}-n}(-a + a\cos(c + dx))^{3/4}(a + a\sec(c + dx))^n\right)}{d\sin^{\frac{3}{2}}(c + dx)}$$

$$= -\frac{\left((1 - \cos(c + dx))^{3/4}(-\cos(c + dx))^n(1 + \cos(c + dx))^{\frac{3}{4}-n}(a + a\sec(c + dx))^n\right)\text{Sub}}{d\sin^{\frac{3}{2}}(c + dx)}$$

$$= -\frac{F_1\left(1 - n; \frac{3}{4}, \frac{3}{4} - n; 2 - n; \cos(c + dx), -\cos(c + dx)\right)(1 - \cos(c + dx))^{3/4}\cos(c + d}{d(1 - n)\sin^{\frac{3}{2}}(c + dx)}$$

Mathematica [B] time = 1.04, size = 212, normalized size = 2.02

$$\frac{10\sqrt{\sin(c + dx)}(\cos(c + dx) + 1)(a(\sec(c + dx) + 1))^n F_1\left(\frac{1}{4}; n\right.}{d\left(2(\cos(c + dx) - 1)\left(F_1\left(\frac{5}{4}; n, \frac{3}{2}; \frac{9}{4}; \tan^2\left(\frac{1}{2}(c + dx)\right), -\tan^2\left(\frac{1}{2}(c + dx)\right)\right) - 2nF_1\left(\frac{5}{4}; n + 1, \frac{1}{2}; \frac{9}{4}; \tan^2\left(\frac{1}{2}(c + dx)\right)\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]],x]
```

```
[Out] (10*AppellF1[1/4, n, 1/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1
+ Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^n*Sqrt[Sin[c + d*x]])/(d*(2*(AppellF
1[5/4, n, 3/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*n*AppellF1
[5/4, 1 + n, 1/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[
c + d*x]) + 5*AppellF1[1/4, n, 1/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2]*(1 + Cos[c + d*x])))
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(a\sec(dx + c) + a)^n}{\sqrt{\sin(dx + c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx + c) + a)^n}{\sqrt{\sin(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

maple [F] time = 0.84, size = 0, normalized size = 0.00

$$\int \frac{(a + a\sec(dx + c))^n}{\sqrt{\sin(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
[Out] int((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx + c) + a)^n}{\sqrt{\sin(dx + c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\cos(c+d\,x)}\right)^n}{\sqrt{\sin(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^n/sin(c + d*x)^(1/2),x)
[Out] int((a + a/cos(c + d*x))^n/sin(c + d*x)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec(c + dx) + 1))^n}{\sqrt{\sin(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n/sin(d*x+c)**(1/2),x)
[Out] Integral((a*(sec(c + d*x) + 1))**n/sqrt(sin(c + d*x)), x)
```

3.159 $\int \frac{(a+a\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=105

$$-\frac{(1-\cos(c+dx))^{5/4}\cos(c+dx)(\cos(c+dx)+1)^{\frac{5}{4}-n}(a\sec(c+dx)+a)^n F_1\left(1-n;\frac{5}{4},\frac{5}{4}-n;2-n;\cos(c+dx),-\right.}{d(1-n)\sin^{\frac{5}{2}}(c+dx)}$$

```
[Out] -AppellF1(1-n,5/4-n,5/4,2-n,-cos(d*x+c),cos(d*x+c))*(1-cos(d*x+c))^(5/4)*co
s(d*x+c)*(1+cos(d*x+c))^(5/4-n)*(a+a*sec(d*x+c))^n/d/(1-n)/sin(d*x+c)^(5/2)
```

Rubi [A] time = 0.27, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3876, 2886, 135, 133}

$$-\frac{(1-\cos(c+dx))^{5/4}\cos(c+dx)(\cos(c+dx)+1)^{\frac{5}{4}-n}(a\sec(c+dx)+a)^n F_1\left(1-n;\frac{5}{4},\frac{5}{4}-n;2-n;\cos(c+dx),-\right.}{d(1-n)\sin^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n/Sin[c + d*x]^(3/2),x]
```

```
[Out] -((AppellF1[1 - n, 5/4, 5/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 - C
os[c + d*x])^(5/4)*Cos[c + d*x]*(1 + Cos[c + d*x])^(5/4 - n)*(a + a*Sec[c +
 d*x])^n)/(d*(1 - n)*Sin[c + d*x]^(5/2)))
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 2886

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos
[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx = \left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int \frac{(-\cos(c+dx))^{-n}}{\sin^{\frac{3}{2}}}$$

$$= -\frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{\frac{5}{4}-n}(-a+a\cos(c+dx))^{5/4}(a+a\sec(c+d\right.}{d\sin^{\frac{5}{2}}(c+dx)}$$

$$= -\left(-\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{4}-n}(-a-a\cos(c+dx))(-a+a\cos(c+dx\right.}{ad\sin^{\frac{5}{2}}(c+}\right.$$

$$= -\frac{\left(\sqrt[4]{1-\cos(c+dx)}\,(-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{4}-n}(-a-a\cos(c+dx))(-a\right.}{a^2d\sin^{\frac{5}{2}}}$$

$$= -\frac{F_1\left(1-n;\frac{5}{4},\frac{5}{4}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{5/4}\cos(c}{d(1-n)\sin^{\frac{5}{2}}(c+dx)}$$

Mathematica [B] time = 1.24, size = 212, normalized size = 2.02

$$-\frac{6(\cos(c+dx)+1)(a(\sec(c+dx)+1))^nF_1}{d\sqrt{\sin(c+dx)}\left(3(\cos(c+dx)+1)F_1\left(-\frac{1}{4};n,-\frac{1}{2};\frac{3}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-2(\cos(c+dx)-1\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n/Sin[c + d*x]^(3/2),x]
```

```
[Out] (-6*AppellF1[-1/4, n, -1/2, 3/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(
1 + Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^n)/(d*(-2*(AppellF1[3/4, n, 1/2, 7
/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*n*AppellF1[3/4, 1 + n, -1/
2, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + 3*A
ppellF1[-1/4, n, -1/2, 3/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + C
os[c + d*x]))*Sqrt[Sin[c + d*x]])
```

fricas [F] time = 0.64, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{(a\sec(dx+c)+a)^n\sqrt{\sin(dx+c)}}{\cos(dx+c)^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*sec(d*x + c) + a)^n*sqrt(sin(d*x + c))/(cos(d*x + c)^2 - 1), x
)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^n}{\sin(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^n/sin(d*x + c)^(3/2), x)
```

maple [F] time = 0.81, size = 0, normalized size = 0.00

$$\int \frac{(a + a\sec(dx + c))^n}{\sin(dx + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)
[Out] int((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx + c) + a)^n}{\sin(dx + c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((a*sec(d*x + c) + a)^n/sin(d*x + c)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a + \frac{a}{\cos(c+d\,x)}\right)^n}{\sin(c + d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^n/sin(c + d*x)^(3/2),x)
[Out] int((a + a/cos(c + d*x))^n/sin(c + d*x)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec(c + dx) + 1))^n}{\sin^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n/sin(d*x+c)**(3/2),x)
[Out] Integral((a*(sec(c + d*x) + 1))**n/sin(c + d*x)**(3/2), x)
```

3.160 $\int (a+b\sec(c+dx))\sin^7(c+dx)\,dx$

Optimal. Leaf size=119

$$\frac{a\cos^7(c+dx)}{7d}-\frac{3a\cos^5(c+dx)}{5d}+\frac{a\cos^3(c+dx)}{d}-\frac{a\cos(c+dx)}{d}+\frac{b\cos^6(c+dx)}{6d}-\frac{3b\cos^4(c+dx)}{4d}+\frac{3b\cos^2(c+}{2d}$$

```
[Out] -a*cos(d*x+c)/d+3/2*b*cos(d*x+c)^2/d+a*cos(d*x+c)^3/d-3/4*b*cos(d*x+c)^4/d-
3/5*a*cos(d*x+c)^5/d+1/6*b*cos(d*x+c)^6/d+1/7*a*cos(d*x+c)^7/d-b*ln(cos(d*x
+c))/d
```

Rubi [A] time = 0.11, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 2837, 12, 766}

$$\frac{a\cos^7(c+dx)}{7d}-\frac{3a\cos^5(c+dx)}{5d}+\frac{a\cos^3(c+dx)}{d}-\frac{a\cos(c+dx)}{d}+\frac{b\cos^6(c+dx)}{6d}-\frac{3b\cos^4(c+dx)}{4d}+\frac{3b\cos^2(c+}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^7,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (3*b*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]^3)/d -
 (3*b*Cos[c + d*x]^4)/(4*d) - (3*a*Cos[c + d*x]^5)/(5*d) + (b*Cos[c + d*x]^
6)/(6*d) + (a*Cos[c + d*x]^7)/(7*d) - (b*Log[Cos[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))\sin^7(c+dx)\,dx = -\int (-b-a\cos(c+dx))\sin^6(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a(-b+x)\left(a^2-x^2\right)^3}{x}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-b+x)\left(a^2-x^2\right)^3}{x}\,dx, x, -a\cos(c+dx)\right)}{a^6 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^6-\frac{a^6 b}{x}+3a^4bx-3a^4x^2-3a^2bx^3+3a^2x^4+bx^5-x^6\right)dx, x, -\right.}{a^6 d}$$
$$= -\frac{a\cos(c+dx)}{d}+\frac{3b\cos^2(c+dx)}{2d}+\frac{a\cos^3(c+dx)}{d}-\frac{3b\cos^4(c+dx)}{4d}-\frac{3}{}$$

Mathematica [A] time = 0.15, size = 115, normalized size = 0.97

$$-\frac{35a\cos(c+dx)}{64d}+\frac{7a\cos(3(c+dx))}{64d}-\frac{7a\cos(5(c+dx))}{320d}+\frac{a\cos(7(c+dx))}{448d}-\frac{b\left(-\frac{1}{3}\cos^6(c+dx)+\frac{3}{2}\cos^4(c+dx)-\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^7,x]
```

```
[Out] (-35*a*Cos[c + d*x])/(64*d) + (7*a*Cos[3*(c + d*x)])/(64*d) - (7*a*Cos[5*(c
 + d*x)])/(320*d) + (a*Cos[7*(c + d*x)])/(448*d) - (b*(-3*Cos[c + d*x]^2 +
(3*Cos[c + d*x]^4)/2 - Cos[c + d*x]^6/3 + 2*Log[Cos[c + d*x]]))/(2*d)
```

fricas [A] time = 0.73, size = 93, normalized size = 0.78

$$\frac{60\,a\cos(dx+c)^7+70\,b\cos(dx+c)^6-252\,a\cos(dx+c)^5-315\,b\cos(dx+c)^4+420\,a\cos(dx+c)^3+630\,b\cos}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/420*(60*a*cos(d*x + c)^7 + 70*b*cos(d*x + c)^6 - 252*a*cos(d*x + c)^5 - 3
15*b*cos(d*x + c)^4 + 420*a*cos(d*x + c)^3 + 630*b*cos(d*x + c)^2 - 420*a*c
os(d*x + c) - 420*b*log(-cos(d*x + c)))/d
```

giac [B] time = 0.26, size = 317, normalized size = 2.66

$$\frac{420\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-420\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{384\,a+1089\,b-\frac{2688\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{8463\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8064\,a(\cos(dx+}{(\cos(dx+c)+}}{}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/420*(420*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*b*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (384*a + 1089*b - 268
8*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 8463*b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 8064*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 28749*b*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 13440*a*(cos(d*x + c) - 1)^3/(c
os(d*x + c) + 1)^3 - 56035*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56
```

```
035*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 28749*b*(cos(d*x + c) - 1
)^5/(cos(d*x + c) + 1)^5 + 8463*b*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6
 - 1089*b*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 1)^7)/d
```

maple [A] time = 0.63, size = 129, normalized size = 1.08

$$-\frac{16a\cos(dx+c)}{35d}-\frac{a\cos(dx+c)\left(\sin^{6}(dx+c)\right)}{7d}-\frac{6a\cos(dx+c)\left(\sin^{4}(dx+c)\right)}{35d}-\frac{8a\cos(dx+c)\left(\sin^{2}(dx+c\right.}{35d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c)^7,x)
```

```
[Out] -16/35*a*cos(d*x+c)/d-1/7/d*a*cos(d*x+c)*sin(d*x+c)^6-6/35/d*a*cos(d*x+c)*s
in(d*x+c)^4-8/35/d*a*cos(d*x+c)*sin(d*x+c)^2-1/6/d*b*sin(d*x+c)^6-1/4/d*b*s
in(d*x+c)^4-1/2/d*b*sin(d*x+c)^2-b*ln(cos(d*x+c))/d
```

maxima [A] time = 0.34, size = 91, normalized size = 0.76

$$\frac{60\,a\cos(dx+c)^{7}+70\,b\cos(dx+c)^{6}-252\,a\cos(dx+c)^{5}-315\,b\cos(dx+c)^{4}+420\,a\cos(dx+c)^{3}+630\,b}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/420*(60*a*cos(d*x + c)^7 + 70*b*cos(d*x + c)^6 - 252*a*cos(d*x + c)^5 - 3
15*b*cos(d*x + c)^4 + 420*a*cos(d*x + c)^3 + 630*b*cos(d*x + c)^2 - 420*a*c
os(d*x + c) - 420*b*log(cos(d*x + c)))/d
```

mupad [B] time = 0.93, size = 89, normalized size = 0.75

$$-\frac{a\,\cos(c+d\,x)-a\cos(c+d\,x)^{3}+\frac{3\,a\cos(c+d\,x)^{5}}{5}-\frac{a\cos(c+d\,x)^{7}}{7}-\frac{3\,b\cos(c+d\,x)^{2}}{2}+\frac{3\,b\cos(c+d\,x)^{4}}{4}-\frac{b\cos(c+d\,x)^{6}}{6}+b\,\ln}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7*(a + b/cos(c + d*x)),x)
```

```
[Out] -(a*cos(c + d*x) - a*cos(c + d*x)^3 + (3*a*cos(c + d*x)^5)/5 - (a*cos(c + d
*x)^7)/7 - (3*b*cos(c + d*x)^2)/2 + (3*b*cos(c + d*x)^4)/4 - (b*cos(c + d*x
)^6)/6 + b*log(cos(c + d*x)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**7,x)
```

```
[Out] Timed out
```

3.161 $\int (a+b\sec(c+dx))\sin^5(c+dx)\,dx$

Optimal. Leaf size=87

$$-\frac{a\cos^5(c+dx)}{5d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}-\frac{b\cos^4(c+dx)}{4d}+\frac{b\cos^2(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] -a*cos(d*x+c)/d+b*cos(d*x+c)^2/d+2/3*a*cos(d*x+c)^3/d-1/4*b*cos(d*x+c)^4/d-
1/5*a*cos(d*x+c)^5/d-b*ln(cos(d*x+c))/d
```

Rubi [A] time = 0.10, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 2837, 12, 766}

$$-\frac{a\cos^5(c+dx)}{5d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}-\frac{b\cos^4(c+dx)}{4d}+\frac{b\cos^2(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^5,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (b*Cos[c + d*x]^2)/d + (2*a*Cos[c + d*x]^3)/(3*d) -
 (b*Cos[c + d*x]^4)/(4*d) - (a*Cos[c + d*x]^5)/(5*d) - (b*Log[Cos[c + d*x]]
)/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))\sin^5(c+dx)\,dx = -\int (-b-a\cos(c+dx))\sin^4(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a(-b+x)(a^2-x^2)^2}{x}\,dx, x, -a\cos(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-b+x)(a^2-x^2)^2}{x}\,dx, x, -a\cos(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^4 - \frac{a^4 b}{x} + 2a^2bx - 2a^2x^2 - bx^3 + x^4\right)dx, x, -a\cos(c+dx)\right)}{a^4 d}$$
$$= -\frac{a\cos(c+dx)}{d} + \frac{b\cos^2(c+dx)}{d} + \frac{2a\cos^3(c+dx)}{3d} - \frac{b\cos^4(c+dx)}{4d} -$$

Mathematica [**A**] time = 0.08, size = 83, normalized size = 0.95

$$-\frac{5a\cos(c+dx)}{8d} + \frac{5a\cos(3(c+dx))}{48d} - \frac{a\cos(5(c+dx))}{80d} - \frac{b\left(\frac{1}{4}\cos^4(c+dx) - \cos^2(c+dx) + \log(\cos(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^5,x]
[Out] (-5*a*Cos[c + d*x])/(8*d) + (5*a*Cos[3*(c + d*x)])/(48*d) - (a*Cos[5*(c + d
*x)])/(80*d) - (b*(-Cos[c + d*x]^2 + Cos[c + d*x]^4/4 + Log[Cos[c + d*x]]))
/d
```

fricas [**A**] time = 0.78, size = 71, normalized size = 0.82

$$-\frac{12\,a\cos(dx+c)^5 + 15\,b\cos(dx+c)^4 - 40\,a\cos(dx+c)^3 - 60\,b\cos(dx+c)^2 + 60\,a\cos(dx+c) + 60\,b\log(}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="fricas")
[Out] -1/60*(12*a*cos(d*x + c)^5 + 15*b*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 60
*b*cos(d*x + c)^2 + 60*a*cos(d*x + c) + 60*b*log(-cos(d*x + c)))/d
```

giac [**B**] time = 0.24, size = 248, normalized size = 2.85

$$\frac{60\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 60\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{64\,a+137\,b-\frac{320\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{805\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{640\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{(}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="giac")
[Out] 1/60*(60*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*b*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (64*a + 137*b - 320*a*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) - 805*b*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 640*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970*b*(cos(d*x
+ c) - 1)^2/(cos(d*x + c) + 1)^2 - 1970*b*(cos(d*x + c) - 1)^3/(cos(d*x + c
) + 1)^3 + 805*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 137*b*(cos(d*x
```

```
+ c) - 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
1)^5)/d
```

maple [A] time = 0.59, size = 95, normalized size = 1.09

$$-\frac{8a\cos(dx+c)}{15d}-\frac{a\cos(dx+c)\left(\sin^4(dx+c)\right)}{5d}-\frac{4a\cos(dx+c)\left(\sin^2(dx+c)\right)}{15d}-\frac{b\left(\sin^4(dx+c)\right)}{4d}-\frac{b\left(\sin^2(dx+c\right.}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c)^5,x)
```

```
[Out] -8/15*a*cos(d*x+c)/d-1/5/d*a*cos(d*x+c)*sin(d*x+c)^4-4/15/d*a*cos(d*x+c)*si
n(d*x+c)^2-1/4/d*b*sin(d*x+c)^4-1/2/d*b*sin(d*x+c)^2-b*ln(cos(d*x+c))/d
```

maxima [A] time = 0.45, size = 69, normalized size = 0.79

$$-\frac{12\,a\cos(dx+c)^5+15\,b\cos(dx+c)^4-40\,a\cos(dx+c)^3-60\,b\cos(dx+c)^2+60\,a\cos(dx+c)+60\,b\log(\mathrm{co}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(12*a*cos(d*x + c)^5 + 15*b*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 60
*b*cos(d*x + c)^2 + 60*a*cos(d*x + c) + 60*b*log(cos(d*x + c)))/d
```

mupad [B] time = 0.91, size = 67, normalized size = 0.77

$$-\frac{a\,\cos(c+d\,x)-\frac{2\,a\cos(c+d\,x)^3}{3}+\frac{a\cos(c+d\,x)^5}{5}-b\cos(c+d\,x)^2+\frac{b\cos(c+d\,x)^4}{4}+b\,\ln(\cos(c+d\,x))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5*(a + b/cos(c + d*x)),x)
```

```
[Out] -(a*cos(c + d*x) - (2*a*cos(c + d*x)^3)/3 + (a*cos(c + d*x)^5)/5 - b*cos(c
+ d*x)^2 + (b*cos(c + d*x)^4)/4 + b*log(cos(c + d*x)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))\sin^5(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**5,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sin(c + d*x)**5, x)
```

3.162 $\int (a + b\sec(c + dx))\sin^3(c + dx)\,dx$

Optimal. Leaf size=58

$$\frac{a\cos^3(c+dx)}{3d} - \frac{a\cos(c+dx)}{d} + \frac{b\cos^2(c+dx)}{2d} - \frac{b\log(\cos(c+dx))}{d}$$

```
[Out] -a*cos(d*x+c)/d+1/2*b*cos(d*x+c)^2/d+1/3*a*cos(d*x+c)^3/d-b*ln(cos(d*x+c))/
d
```

Rubi [A] time = 0.09, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 2837, 12, 766}

$$\frac{a\cos^3(c+dx)}{3d} - \frac{a\cos(c+dx)}{d} + \frac{b\cos^2(c+dx)}{2d} - \frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^3,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (b*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]^3)/(3*d)
 - (b*Log[Cos[c + d*x]])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))\sin^3(c+dx)\,dx = -\int (-b-a\cos(c+dx))\sin^2(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a(-b+x)(a^2-x^2)}{x}\,dx, x, -a\cos(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-b+x)(a^2-x^2)}{x}\,dx, x, -a\cos(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2-\frac{a^2 b}{x}+bx-x^2\right)dx, x, -a\cos(c+dx)\right)}{a^2 d}$$
$$= -\frac{a\cos(c+dx)}{d}+\frac{b\cos^2(c+dx)}{2d}+\frac{a\cos^3(c+dx)}{3d}-\frac{b\log(\cos(c+dx))}{d}$$

Mathematica [A] time = 0.05, size = 57, normalized size = 0.98

$$-\frac{3a\cos(c+dx)}{4d}+\frac{a\cos(3(c+dx))}{12d}-\frac{b\left(\log(\cos(c+dx))-\frac{1}{2}\cos^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^3,x]
```

```
[Out] (-3*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)])/(12*d) - (b*(-1/2*Cos[c +
d*x]^2 + Log[Cos[c + d*x]]))/d
```

fricas [A] time = 0.62, size = 49, normalized size = 0.84

$$\frac{2\,a\cos(dx+c)^3+3\,b\cos(dx+c)^2-6\,a\cos(dx+c)-6\,b\log(-\cos(dx+c))}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*cos(d*x + c)^3 + 3*b*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 6*b*log(-
cos(d*x + c)))/d
```

giac [A] time = 0.34, size = 66, normalized size = 1.14

$$-\frac{b\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{2\,ad^2\cos(dx+c)^3+3\,bd^2\cos(dx+c)^2-6\,ad^2\cos(dx+c)}{6\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -b*log(abs(cos(d*x + c))/abs(d))/d + 1/6*(2*a*d^2*cos(d*x + c)^3 + 3*b*d^2*
cos(d*x + c)^2 - 6*a*d^2*cos(d*x + c))/d^3
```

maple [A] time = 0.60, size = 61, normalized size = 1.05

$$-\frac{a\cos(dx+c)\left(\sin^2(dx+c)\right)}{3d}-\frac{2a\cos(dx+c)}{3d}-\frac{b\left(\sin^2(dx+c)\right)}{2d}-\frac{b\ln(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c)^3,x)
```

```
[Out] -1/3/d*a*cos(d*x+c)*sin(d*x+c)^2-2/3*a*cos(d*x+c)/d-1/2/d*b*sin(d*x+c)^2-b*
ln(cos(d*x+c))/d
```

maxima [A] time = 0.51, size = 47, normalized size = 0.81

$$\frac{2\,a\cos\left(dx+c\right)^{3}+3\,b\cos\left(dx+c\right)^{2}-6\,a\cos\left(dx+c\right)-6\,b\log\left(\cos\left(dx+c\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a*cos(d*x + c)^3 + 3*b*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 6*b*log(c
os(d*x + c)))/d
```

mupad [B] time = 0.06, size = 45, normalized size = 0.78

$$-\frac{a\,\cos\left(c+d\,x\right)-\frac{a\cos(c+d\,x)^3}{3}-\frac{b\cos(c+d\,x)^2}{2}+b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + b/cos(c + d*x)),x)
```

```
[Out] -(a*cos(c + d*x) - (a*cos(c + d*x)^3)/3 - (b*cos(c + d*x)^2)/2 + b*log(cos(
c + d*x)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)\sin^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sin(c + d*x)**3, x)
```

3.163 $\int (a+b\sec(c+dx))\sin(c+dx)\,dx$

Optimal. Leaf size=26

$$-\frac{a\cos(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] -a*cos(d*x+c)/d-b*ln(cos(d*x+c))/d
```

Rubi [A] time = 0.03, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3872, 2721, 43}

$$-\frac{a\cos(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x],x]
```

```
[Out] -((a*Cos[c + d*x])/d) - (b*Log[Cos[c + d*x]])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (a+b\sec(c+dx))\sin(c+dx)\,dx &= -\int (-b-a\cos(c+dx))\tan(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{-b+x}{x}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(1-\frac{b}{x}\right)dx, x, -a\cos(c+dx)\right)}{d}\\
&= -\frac{a\cos(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 37, normalized size = 1.42

$$\frac{a\sin(c)\sin(dx)}{d}-\frac{a\cos(c)\cos(dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x],x]
[Out] -((a*Cos[c]*Cos[d*x])/d) - (b*Log[Cos[c + d*x]])/d + (a*Sin[c]*Sin[d*x])/d
```

fricas [A] time = 0.70, size = 25, normalized size = 0.96

$$-\frac{a\cos\left(dx+c\right)+b\log\left(-\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c),x, algorithm="fricas")
[Out] -(a*cos(d*x + c) + b*log(-cos(d*x + c)))/d
```

giac [A] time = 0.24, size = 32, normalized size = 1.23

$$-\frac{a\cos\left(dx+c\right)}{d}-\frac{b\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c),x, algorithm="giac")
[Out] -a*cos(d*x + c)/d - b*log(abs(cos(d*x + c))/abs(d))/d
```

maple [A] time = 0.19, size = 28, normalized size = 1.08

$$\frac{b\ln\left(\sec\left(dx+c\right)\right)}{d}-\frac{a}{d\sec\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c),x)
[Out] 1/d*b*ln(sec(d*x+c))-1/d*a/sec(d*x+c)
```

maxima [A] time = 0.44, size = 23, normalized size = 0.88

$$-\frac{a\cos\left(dx+c\right)+b\log\left(\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c),x, algorithm="maxima")
[Out] -(a*cos(d*x + c) + b*log(cos(d*x + c)))/d
```

mupad [B] time = 0.04, size = 23, normalized size = 0.88

$$-\frac{a\,\cos\left(c+d\,x\right)+b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b/cos(c + d*x)),x)
[Out] -(a*cos(c + d*x) + b*log(cos(c + d*x)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)\sin\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sin(c + d*x), x)
```

3.164 $\int \csc(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=26

$$\frac{b\log(\tan(c+dx))}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -a*arctanh(cos(d*x+c))/d+b*ln(tan(d*x+c))/d
```

Rubi [A] time = 0.07, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3872, 2834, 2620, 29, 3770}

$$\frac{b\log(\tan(c+dx))}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Sec[c + d*x]),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) + (b*Log[Tan[c + d*x]])/d
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)(a+b\sec(c+dx))\,dx &= -\int(-b-a\cos(c+dx))\csc(c+dx)\sec(c+dx)\,dx\\ &= a\int\csc(c+dx)\,dx+b\int\csc(c+dx)\sec(c+dx)\,dx\\ &= -\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\,\text{Subst}\left(\int\frac{1}{x}\,dx,x,\tan(c+dx)\right)}{d}\\ &= -\frac{a\tanh^{-1}(\cos(c+dx))}{d}+\frac{b\log(\tan(c+dx))}{d}\end{aligned}$$

Mathematica [B] time = 0.04, size = 63, normalized size = 2.42

$$\frac{a\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{b(\log(\cos(c+dx))-\log(\sin(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Sec[c + d*x]),x]
```

```
[Out] -((a*Log[Cos[c/2 + (d*x)/2]])/d) + (a*Log[Sin[c/2 + (d*x)/2]])/d - (b*(Log[
Cos[c + d*x]] - Log[Sin[c + d*x]]))/d
```

fricas [A] time = 0.48, size = 51, normalized size = 1.96

$$-\frac{2\,b\log\left(-\cos\left(dx+c\right)\right)+(a-b)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)-(a+b)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*log(-cos(d*x + c)) + (a - b)*log(1/2*cos(d*x + c) + 1/2) - (a + b
)*log(-1/2*cos(d*x + c) + 1/2))/d
```

giac [B] time = 0.21, size = 61, normalized size = 2.35

$$\frac{(a+b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-2\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((a + b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 2*b*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)))/d
```

maple [A] time = 0.34, size = 35, normalized size = 1.35

$$\frac{b\ln\left(\tan\left(dx+c\right)\right)}{d}+\frac{a\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*sec(d*x+c)),x)
```

```
[Out] b*ln(tan(d*x+c))/d+1/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.55, size = 45, normalized size = 1.73

$$-\frac{(a-b)\log(\cos(dx+c)+1)-(a+b)\log(\cos(dx+c)-1)+2\,b\log(\cos(dx+c))}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*((a - b)*log(cos(d*x + c) + 1) - (a + b)*log(cos(d*x + c) - 1) + 2*b*l
og(cos(d*x + c)))/d
```

mupad [B] time = 0.11, size = 63, normalized size = 2.42

$$\frac{\frac{a\,\ln(\cos(c+d\,x)-1)}{2}-b\,\ln(\cos(c+d\,x))-\frac{a\,\ln(\cos(c+d\,x)+1)}{2}+\frac{b\,\ln(\cos(c+d\,x)-1)}{2}+\frac{b\,\ln(\cos(c+d\,x)+1)}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/sin(c + d*x),x)
```

```
[Out] ((a*log(cos(c + d*x) - 1))/2 - b*log(cos(c + d*x)) - (a*log(cos(c + d*x) +
1))/2 + (b*log(cos(c + d*x) - 1))/2 + (b*log(cos(c + d*x) + 1))/2)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))\csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*csc(c + d*x), x)
```

3.165 $\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=64

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}-\frac{b\cot^2(c+dx)}{2d}+\frac{b\log(\tan(c+dx))}{d}$$

```
[Out] -1/2*a*arctanh(cos(d*x+c))/d-1/2*b*cot(d*x+c)^2/d-1/2*a*cot(d*x+c)*csc(d*x+
c)/d+b*ln(tan(d*x+c))/d
```

Rubi [A] time = 0.10, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3872, 2834, 2620, 14, 3768, 3770}

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}-\frac{b\cot^2(c+dx)}{2d}+\frac{b\log(\tan(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] -(a*ArcTanh[Cos[c + d*x]])/(2*d) - (b*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*
x]*Csc[c + d*x])/(2*d) + (b*Log[Tan[c + d*x]])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
```

```
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx = -\int (-b-a\cos(c+dx))\csc^3(c+dx)\sec(c+dx)\,dx$$
$$= a\int \csc^3(c+dx)\,dx + b\int \csc^3(c+dx)\sec(c+dx)\,dx$$
$$= -\frac{a\cot(c+dx)\csc(c+dx)}{2d} + \frac{1}{2}a\int \csc(c+dx)\,dx + \frac{b\,\text{Subst}\left(\int \frac{1+x^2}{x^3}\,d\right.}{d}$$
$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{a\cot(c+dx)\csc(c+dx)}{2d} + \frac{b\,\text{Subst}\left(\int\left(\frac{1}{x^3}\right.\right.}{}$$
$$= -\frac{a\tanh^{-1}(\cos(c+dx))}{2d} - \frac{b\cot^2(c+dx)}{2d} - \frac{a\cot(c+dx)\csc(c+dx)}{2d} +$$

Mathematica [A] time = 0.51, size = 114, normalized size = 1.78

$$-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{b\left(\csc^2(c+dx)-2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/8*(a*Csc[(c + d*x)/2]^2)/d - (a*Log[Cos[(c + d*x)/2]])/(2*d) + (a*Log[Si
n[(c + d*x)/2]])/(2*d) - (b*(Csc[c + d*x]^2 + 2*Log[Cos[c + d*x]] - 2*Log[S
in[c + d*x]]))/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

fricas [B] time = 0.49, size = 123, normalized size = 1.92

$$\frac{2a\cos(dx+c) - 4\left(b\cos(dx+c)^2 - b\right)\log(-\cos(dx+c)) - \left((a-2b)\cos(dx+c)^2 - a + 2b\right)\log\left(\frac{1}{2}\cos(dx\right.}{4\left(d\cos(dx+c)^2 - d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*cos(d*x + c) - 4*(b*cos(d*x + c)^2 - b)*log(-cos(d*x + c)) - ((a -
 2*b)*cos(d*x + c)^2 - a + 2*b)*log(1/2*cos(d*x + c) + 1/2) + ((a + 2*b)*co
s(d*x + c)^2 - a - 2*b)*log(-1/2*cos(d*x + c) + 1/2) + 2*b)/(d*cos(d*x + c)
^2 - d)
```

giac [B] time = 0.25, size = 169, normalized size = 2.64

$$\frac{2(a+2b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 8b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) + \frac{\left(a+b-\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{4b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1} - \frac{a(\cos(dx}{\cos(dx}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(2*(a + 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 8*b*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a + b - 2*a*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*(
```

```
cos(d*x + c) + 1)/(cos(d*x + c) - 1) - a*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/d
```

maple [A] time = 0.78, size = 68, normalized size = 1.06

$$-\frac{a\cot(dx+c)\csc(dx+c)}{2d}+\frac{a\ln(\csc(dx+c)-\cot(dx+c))}{2d}-\frac{b}{2d\sin(dx+c)^2}+\frac{b\ln(\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*sec(d*x+c)),x)
```

```
[Out] -1/2*a*cot(d*x+c)*csc(d*x+c)/d+1/2/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/2/d*b/si
n(d*x+c)^2+b*ln(tan(d*x+c))/d
```

maxima [A] time = 0.40, size = 71, normalized size = 1.11

$$-\frac{(a-2b)\log(\cos(dx+c)+1)-(a+2b)\log(\cos(dx+c)-1)+4b\log(\cos(dx+c))-\frac{2(a\cos(dx+c)+b)}{\cos(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*((a - 2*b)*log(cos(d*x + c) + 1) - (a + 2*b)*log(cos(d*x + c) - 1) + 4
*b*log(cos(d*x + c)) - 2*(a*cos(d*x + c) + b)/(cos(d*x + c)^2 - 1))/d
```

mupad [B] time = 0.10, size = 76, normalized size = 1.19

$$\frac{\frac{\frac{b}{2}+\frac{a\cos(c+dx)}{2}}{\cos(c+dx)^2-1}+\ln(\cos(c+dx)-1)\left(\frac{a}{4}+\frac{b}{2}\right)-\ln(\cos(c+dx)+1)\left(\frac{a}{4}-\frac{b}{2}\right)-b\ln(\cos(c+dx))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/sin(c + d*x)^3,x)
```

```
[Out] ((b/2 + (a*cos(c + d*x))/2)/(cos(c + d*x)^2 - 1) + log(cos(c + d*x) - 1)*(a
/4 + b/2) - log(cos(c + d*x) + 1)*(a/4 - b/2) - b*log(cos(c + d*x)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sec(c+dx))\csc^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*csc(c + d*x)**3, x)
```

3.166 $\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=100

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\cot^4(c+dx)}{4d}-\frac{b\cot^2(c+dx)}{d}$$

```
[Out] -3/8*a*arctanh(cos(d*x+c))/d-b*cot(d*x+c)^2/d-1/4*b*cot(d*x+c)^4/d-3/8*a*co
t(d*x+c)*csc(d*x+c)/d-1/4*a*cot(d*x+c)*csc(d*x+c)^3/d+b*ln(tan(d*x+c))/d
```

Rubi [A] time = 0.12, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3872, 2834, 2620, 266, 43, 3768, 3770}

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\cot^4(c+dx)}{4d}-\frac{b\cot^2(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(8*d) - (b*Cot[c + d*x]^2)/d - (b*Cot[c + d*x]
^4)/(4*d) - (3*a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]*Csc[c +
 d*x]^3)/(4*d) + (b*Log[Tan[c + d*x]])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx = -\int(-b-a\cos(c+dx))\csc^5(c+dx)\sec(c+dx)\,dx$$
$$= a\int \csc^5(c+dx)\,dx + b\int \csc^5(c+dx)\sec(c+dx)\,dx$$
$$= -\frac{a\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{1}{4}(3a)\int \csc^3(c+dx)\,dx + \frac{b\,\mathrm{Subst}\left(\int \frac{(1+x^2}{x^5}\right.}{}$$
$$= -\frac{3a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot(c+dx)\csc^3(c+dx)}{4d} + \frac{1}{8}(3a)\int \csc(c$$
$$= -\frac{3a\tanh^{-1}(\cos(c+dx))}{8d} - \frac{3a\cot(c+dx)\csc(c+dx)}{8d} - \frac{a\cot(c+dx)\csc}{4d}$$
$$= -\frac{3a\tanh^{-1}(\cos(c+dx))}{8d} - \frac{b\cot^2(c+dx)}{d} - \frac{b\cot^4(c+dx)}{4d} - \frac{3a\cot(c+a}{}$$

Mathematica [A] time = 0.65, size = 164, normalized size = 1.64

$$-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{3a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d} - \frac{3a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-3*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (3*a*Log
[Cos[(c + d*x)/2]])/(8*d) + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) - (b*(2*Csc[c
 + d*x]^2 + Csc[c + d*x]^4 + 4*Log[Cos[c + d*x]] - 4*Log[Sin[c + d*x]]))/(4
*d) + (3*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

fricas [B] time = 0.51, size = 201, normalized size = 2.01

$$\frac{6\,a\cos(dx+c)^3 + 8\,b\cos(dx+c)^2 - 10\,a\cos(dx+c) - 16\left(b\cos(dx+c)^4 - 2\,b\cos(dx+c)^2 + b\right)\log(-\cos(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(6*a*cos(d*x + c)^3 + 8*b*cos(d*x + c)^2 - 10*a*cos(d*x + c) - 16*(b*c
os(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)*log(-cos(d*x + c)) - ((3*a - 8*b)*c
os(d*x + c)^4 - 2*(3*a - 8*b)*cos(d*x + c)^2 + 3*a - 8*b)*log(1/2*cos(d*x +
 c) + 1/2) + ((3*a + 8*b)*cos(d*x + c)^4 - 2*(3*a + 8*b)*cos(d*x + c)^2 + 3
```

```
*a + 8*b)*log(-1/2*cos(d*x + c) + 1/2) - 12*b)/(d*cos(d*x + c)^4 - 2*d*cos(
d*x + c)^2 + d)
```

giac [B] time = 0.24, size = 266, normalized size = 2.66

$$\frac{4\,(3\,a+8\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-64\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\frac{\left(a+b-\frac{8\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{12\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{18\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{48\,b}{(}\right.}{(\cos(dx+c)-1)^2}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/64*(4*(3*a + 8*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 64*
b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a + b - 8*a*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) - 12*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 18*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 48*b*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 - 8*a*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12*b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2)/d
```

maple [A] time = 0.73, size = 102, normalized size = 1.02

$$-\frac{a\cot(dx+c)\left(\csc^3(dx+c)\right)}{4d}-\frac{3a\cot(dx+c)\csc(dx+c)}{8d}+\frac{3a\ln(\csc(dx+c)-\cot(dx+c))}{8d}-\frac{b}{4d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+b*sec(d*x+c)),x)
```

```
[Out] -1/4*a*cot(d*x+c)*csc(d*x+c)^3/d-3/8*a*cot(d*x+c)*csc(d*x+c)/d+3/8/d*a*ln(c
sc(d*x+c)-cot(d*x+c))-1/4/d*b/sin(d*x+c)^4-1/2/d*b/sin(d*x+c)^2+b*ln(tan(d*
x+c))/d
```

maxima [A] time = 0.67, size = 110, normalized size = 1.10

$$-\frac{(3\,a-8\,b)\log(\cos(dx+c)+1)-(3\,a+8\,b)\log(\cos(dx+c)-1)+16\,b\log(\cos(dx+c))-\frac{2\left(3\,a\cos(dx+c)^3+4\right.}{\cos(dx-}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*((3*a - 8*b)*log(cos(d*x + c) + 1) - (3*a + 8*b)*log(cos(d*x + c) - 1
) + 16*b*log(cos(d*x + c)) - 2*(3*a*cos(d*x + c)^3 + 4*b*cos(d*x + c)^2 - 5
*a*cos(d*x + c) - 6*b)/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1))/d
```

mupad [B] time = 0.98, size = 117, normalized size = 1.17

$$\frac{\ln(\cos(c+d\,x)-1)\left(\frac{3\,a}{16}+\frac{b}{2}\right)}{d}-\frac{-\frac{3\,a\cos(c+d\,x)^3}{8}-\frac{b\cos(c+d\,x)^2}{2}+\frac{5\,a\,\cos(c+d\,x)}{8}+\frac{3\,b}{4}}{d\,\left(\cos(c+d\,x)^4-2\cos(c+d\,x)^2+1\right)}-\frac{\ln(\cos(c+d\,x)+1)\left(\frac{3\,a}{16}-\frac{b}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/sin(c + d*x)^5,x)
```

```
[Out] (log(cos(c + d*x) - 1)*((3*a)/16 + b/2))/d - ((3*b)/4 + (5*a*cos(c + d*x))/
8 - (3*a*cos(c + d*x)^3)/8 - (b*cos(c + d*x)^2)/2)/(d*(cos(c + d*x)^4 - 2*c
```

```
os(c + d*x)^2 + 1)) - (log(cos(c + d*x) + 1)*((3*a)/16 - b/2))/d - (b*log(c
os(c + d*x)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec (c + dx))\csc^5 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*csc(c + d*x)**5, x)
```

3.167 $\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=140

$$-\frac{5a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{5a\cot(c+dx)\csc^3(c+dx)}{24d}-\frac{5a\cot(c+dx)\csc(c+dx)}{16d}-$$

```
[Out] -5/16*a*arctanh(cos(d*x+c))/d-3/2*b*cot(d*x+c)^2/d-3/4*b*cot(d*x+c)^4/d-1/6
*b*cot(d*x+c)^6/d-5/16*a*cot(d*x+c)*csc(d*x+c)/d-5/24*a*cot(d*x+c)*csc(d*x+
c)^3/d-1/6*a*cot(d*x+c)*csc(d*x+c)^5/d+b*ln(tan(d*x+c))/d
```

Rubi [A] time = 0.14, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3872, 2834, 2620, 266, 43, 3768, 3770}

$$-\frac{5a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{5a\cot(c+dx)\csc^3(c+dx)}{24d}-\frac{5a\cot(c+dx)\csc(c+dx)}{16d}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^7*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-5*a*ArcTanh[Cos[c + d*x]])/(16*d) - (3*b*Cot[c + d*x]^2)/(2*d) - (3*b*Cot
[c + d*x]^4)/(4*d) - (b*Cot[c + d*x]^6)/(6*d) - (5*a*Cot[c + d*x]*Csc[c + d
*x])/(16*d) - (5*a*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a*Cot[c + d*x]*Cs
c[c + d*x]^5)/(6*d) + (b*Log[Tan[c + d*x]])/d
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx = -\int (-b-a\cos(c+dx))\csc^7(c+dx)\sec(c+dx)\,dx$$
$$= a\int \csc^7(c+dx)\,dx + b\int \csc^7(c+dx)\sec(c+dx)\,dx$$
$$= -\frac{a\cot(c+dx)\csc^5(c+dx)}{6d} + \frac{1}{6}(5a)\int \csc^5(c+dx)\,dx + \frac{b\,\mathrm{Subst}\left(\int \frac{(1+x^2}{x^7}\right.}{}$$
$$= -\frac{5a\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{a\cot(c+dx)\csc^5(c+dx)}{6d} + \frac{1}{8}(5a)\int \csc^3$$
$$= -\frac{5a\cot(c+dx)\csc(c+dx)}{16d} - \frac{5a\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{a\cot(c+dx)}{6}$$
$$= -\frac{5a\tanh^{-1}(\cos(c+dx))}{16d} - \frac{3b\cot^2(c+dx)}{2d} - \frac{3b\cot^4(c+dx)}{4d} - \frac{b\cot^6(c-}{6d}$$

Mathematica [A] time = 0.62, size = 216, normalized size = 1.54

$$-\frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{5a\sec^2\left(\frac{1}{2}\right.}{64}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^7*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-5*a*Csc[(c + d*x)/2]^2)/(64*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (a*Csc[(
c + d*x)/2]^6)/(384*d) - (5*a*Log[Cos[(c + d*x)/2]])/(16*d) + (5*a*Log[Sin[
(c + d*x)/2]])/(16*d) - (b*(6*Csc[c + d*x]^2 + 3*Csc[c + d*x]^4 + 2*Csc[c +
 d*x]^6 + 12*Log[Cos[c + d*x]] - 12*Log[Sin[c + d*x]]))/(12*d) + (5*a*Sec[(
c + d*x)/2]^2)/(64*d) + (a*Sec[(c + d*x)/2]^4)/(64*d) + (a*Sec[(c + d*x)/2]
^6)/(384*d)
```

fricas [B] time = 0.46, size = 284, normalized size = 2.03

$$\frac{30\,a\cos(dx+c)^5 + 48\,b\cos(dx+c)^4 - 80\,a\cos(dx+c)^3 - 120\,b\cos(dx+c)^2 + 66\,a\cos(dx+c) - 96\left(b\cos(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(30*a*cos(d*x + c)^5 + 48*b*cos(d*x + c)^4 - 80*a*cos(d*x + c)^3 - 120
*b*cos(d*x + c)^2 + 66*a*cos(d*x + c) - 96*(b*cos(d*x + c)^6 - 3*b*cos(d*x
+ c)^4 + 3*b*cos(d*x + c)^2 - b)*log(-cos(d*x + c)) - 3*((5*a - 16*b)*cos(d
*x + c)^6 - 3*(5*a - 16*b)*cos(d*x + c)^4 + 3*(5*a - 16*b)*cos(d*x + c)^2 -
 5*a + 16*b)*log(1/2*cos(d*x + c) + 1/2) + 3*((5*a + 16*b)*cos(d*x + c)^6 -
 3*(5*a + 16*b)*cos(d*x + c)^4 + 3*(5*a + 16*b)*cos(d*x + c)^2 - 5*a - 16*b
)*log(-1/2*cos(d*x + c) + 1/2) + 88*b)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)
^4 + 3*d*cos(d*x + c)^2 - d)
```

giac [B] time = 0.45, size = 357, normalized size = 2.55

$$\frac{12\,(5\,a+16\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-384\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{\left(a+b-\frac{9\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{12\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{45\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/384*(12*(5*a + 16*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) -
384*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a + b - 9*a*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*b*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 45*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 87*b*(cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2 - 110*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
1)^3 - 352*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^
3/(cos(d*x + c) - 1)^3 - 45*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 87*b*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 - 12*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - a*(cos(d*x +
c) - 1)^3/(cos(d*x + c) + 1)^3 + b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^
3)/d
```

maple [A] time = 0.75, size = 136, normalized size = 0.97

$$-\frac{a\cot(dx+c)\left(\csc^5(dx+c)\right)}{6d}-\frac{5a\cot(dx+c)\left(\csc^3(dx+c)\right)}{24d}-\frac{5a\cot(dx+c)\csc(dx+c)}{16d}+\frac{5a\ln(\csc(dx+c\cdots}{16\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^7*(a+b*sec(d*x+c)),x)
```

```
[Out] -1/6*a*cot(d*x+c)*csc(d*x+c)^5/d-5/24*a*cot(d*x+c)*csc(d*x+c)^3/d-5/16*a*co
t(d*x+c)*csc(d*x+c)/d+5/16/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/6/d*b/sin(d*x+c)
^6-1/4/d*b/sin(d*x+c)^4-1/2/d*b/sin(d*x+c)^2+b*ln(tan(d*x+c))/d
```

maxima [A] time = 0.35, size = 143, normalized size = 1.02

$$-\frac{3\,(5\,a-16\,b)\log(\cos(dx+c)+1)-3\,(5\,a+16\,b)\log(\cos(dx+c)-1)+96\,b\log(\cos(dx+c))-\frac{2\left(15\,a\cos(\cdots\right.}{\cdots}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(3*(5*a - 16*b)*log(cos(d*x + c) + 1) - 3*(5*a + 16*b)*log(cos(d*x +
c) - 1) + 96*b*log(cos(d*x + c)) - 2*(15*a*cos(d*x + c)^5 + 24*b*cos(d*x +
c)^4 - 40*a*cos(d*x + c)^3 - 60*b*cos(d*x + c)^2 + 33*a*cos(d*x + c) + 44*b
)/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1))/d
```

mupad [B] time = 1.02, size = 148, normalized size = 1.06

$$\frac{\frac{5\,a\cos(c+d\,x)^5}{16}+\frac{b\cos(c+d\,x)^4}{2}-\frac{5\,a\cos(c+d\,x)^3}{6}-\frac{5\,b\cos(c+d\,x)^2}{4}+\frac{11\,a\,\cos(c+d\,x)}{16}+\frac{11\,b}{12}}{d\,\left(\cos(c+d\,x)^6-3\cos(c+d\,x)^4+3\cos(c+d\,x)^2-1\right)}+\frac{\ln(\cos(c+d\,x)-1)\left(\frac{5\,a}{32}+\frac{b}{2}\right)}{d}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/sin(c + d*x)^7,x)
```

```
[Out] ((11*b)/12 + (11*a*cos(c + d*x))/16 - (5*a*cos(c + d*x)^3)/6 + (5*a*cos(c +
 d*x)^5)/16 - (5*b*cos(c + d*x)^2)/4 + (b*cos(c + d*x)^4)/2)/(d*(3*cos(c +
d*x)^2 - 3*cos(c + d*x)^4 + cos(c + d*x)^6 - 1)) + (log(cos(c + d*x) - 1)*(
(5*a)/32 + b/2))/d - (log(cos(c + d*x) + 1)*((5*a)/32 - b/2))/d - (b*log(co
s(c + d*x)))/d
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**7*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.168 $\int (a+b\sec(c+dx))\sin^6(c+dx)\,dx$

Optimal. Leaf size=127

$$-\frac{a\sin^5(c+dx)\cos(c+dx)}{6d}-\frac{5a\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}-\frac{b\sin^5(c+dx)}{5d}-\frac{b}{}$$

```
[Out] 5/16*a*x+b*arctanh(sin(d*x+c))/d-b*sin(d*x+c)/d-5/16*a*cos(d*x+c)*sin(d*x+c
)/d-1/3*b*sin(d*x+c)^3/d-5/24*a*cos(d*x+c)*sin(d*x+c)^3/d-1/5*b*sin(d*x+c)^
5/d-1/6*a*cos(d*x+c)*sin(d*x+c)^5/d
```

Rubi [A] time = 0.13, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3872, 2838, 2592, 302, 206, 2635, 8}

$$-\frac{a\sin^5(c+dx)\cos(c+dx)}{6d}-\frac{5a\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}-\frac{b\sin^5(c+dx)}{5d}-\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^6,x]
```

```
[Out] (5*a*x)/16 + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d - (5*a*Cos[c
+ d*x]*Sin[c + d*x])/(16*d) - (b*Sin[c + d*x]^3)/(3*d) - (5*a*Cos[c + d*x]*
Sin[c + d*x]^3)/(24*d) - (b*Sin[c + d*x]^5)/(5*d) - (a*Cos[c + d*x]*Sin[c +
 d*x]^5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
```

```
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))\sin^6(c+dx)\,dx &= -\int (-b-a\cos(c+dx))\sin^5(c+dx)\tan(c+dx)\,dx\\ &= a\int \sin^6(c+dx)\,dx + b\int \sin^5(c+dx)\tan(c+dx)\,dx\\ &= -\frac{a\cos(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{6}(5a)\int \sin^4(c+dx)\,dx + \frac{b\,\mathrm{Subst}\left(\int \frac{x^6}{1-x^2}\right.}{}\\ &= -\frac{5a\cos(c+dx)\sin^3(c+dx)}{24d} - \frac{a\cos(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{8}(5a)\int \sin^2\\ &= -\frac{b\sin(c+dx)}{d} - \frac{5a\cos(c+dx)\sin(c+dx)}{16d} - \frac{b\sin^3(c+dx)}{3d} - \frac{5a\cos(c+}{}\\ &= \frac{5ax}{16} + \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{b\sin(c+dx)}{d} - \frac{5a\cos(c+dx)\sin(c+dx)}{16d}\end{aligned}$$

Mathematica [A] time = 0.21, size = 118, normalized size = 0.93

$$\frac{5a(c+dx)}{16d} - \frac{15a\sin(2(c+dx))}{64d} + \frac{3a\sin(4(c+dx))}{64d} - \frac{a\sin(6(c+dx))}{192d} - \frac{b\sin^5(c+dx)}{5d} - \frac{b\sin^3(c+dx)}{3d} - \frac{b\sin(c+dx}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^6,x]
```

```
[Out] (5*a*(c + d*x))/(16*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d -
 (b*Sin[c + d*x]^3)/(3*d) - (b*Sin[c + d*x]^5)/(5*d) - (15*a*Sin[2*(c + d*x
)])/(64*d) + (3*a*Sin[4*(c + d*x)])/(64*d) - (a*Sin[6*(c + d*x)])/(192*d)
```

fricas [A] time = 0.49, size = 102, normalized size = 0.80

$$\frac{75\,adx + 120\,b\log(\sin(dx+c)+1) - 120\,b\log(-\sin(dx+c)+1) - \left(40\,a\cos(dx+c)^5 + 48\,b\cos(dx+c)^4 - 13\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(75*a*d*x + 120*b*log(sin(d*x + c) + 1) - 120*b*log(-sin(d*x + c) + 1
) - (40*a*cos(d*x + c)^5 + 48*b*cos(d*x + c)^4 - 130*a*cos(d*x + c)^3 - 176
*b*cos(d*x + c)^2 + 165*a*cos(d*x + c) + 368*b)*sin(d*x + c))/d
```

giac [A] time = 0.45, size = 228, normalized size = 1.80

$$\frac{75(dx+c)a + 240\,b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - 240\,b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) + \frac{2\left(75\,a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11} - 240\,b\tan\left(\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/240*(75*(d*x + c)*a + 240*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 240*b*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(75*a*tan(1/2*d*x + 1/2*c)^11 - 240*b*
tan(1/2*d*x + 1/2*c)^11 + 425*a*tan(1/2*d*x + 1/2*c)^9 - 1520*b*tan(1/2*d*x
 + 1/2*c)^9 + 990*a*tan(1/2*d*x + 1/2*c)^7 - 4128*b*tan(1/2*d*x + 1/2*c)^7
- 990*a*tan(1/2*d*x + 1/2*c)^5 - 4128*b*tan(1/2*d*x + 1/2*c)^5 - 425*a*tan(
1/2*d*x + 1/2*c)^3 - 1520*b*tan(1/2*d*x + 1/2*c)^3 - 75*a*tan(1/2*d*x + 1/2
*c) - 240*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

maple [A] time = 0.65, size = 130, normalized size = 1.02

$$-\frac{a\cos\left(dx+c\right)\left(\sin^{5}\left(dx+c\right)\right)}{6d}-\frac{5a\cos\left(dx+c\right)\left(\sin^{3}\left(dx+c\right)\right)}{24d}-\frac{5a\cos\left(dx+c\right)\sin\left(dx+c\right)}{16d}+\frac{5ax}{16}+\frac{5ca}{16d}-\frac{b\left(\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c)^6,x)
```

```
[Out] -1/6*a*cos(d*x+c)*sin(d*x+c)^5/d-5/24*a*cos(d*x+c)*sin(d*x+c)^3/d-5/16*a*co
s(d*x+c)*sin(d*x+c)/d+5/16*a*x+5/16/d*c*a-1/5*b*sin(d*x+c)^5/d-1/3*b*sin(d*
x+c)^3/d-b*sin(d*x+c)/d+1/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.33, size = 106, normalized size = 0.83

$$\frac{5\left(4\sin\left(2dx+2c\right)^{3}+60dx+60c+9\sin\left(4dx+4c\right)-48\sin\left(2dx+2c\right)\right)a-32\left(6\sin\left(dx+c\right)^{5}+10\sin\right.}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/960*(5*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*si
n(2*d*x + 2*c))*a - 32*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d
*x + c) + 1) + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*b)/d
```

mupad [B] time = 2.17, size = 332, normalized size = 2.61

$$\frac{5a\,\mathrm{atan}\left(\frac{125a^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{64\left(\frac{125a^{3}}{64}+20ab^{2}\right)}+\frac{20ab^{2}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{\frac{125a^{3}}{64}+20ab^{2}}\right)}{8d}+\frac{2b\,\mathrm{atanh}\left(\frac{64b^{3}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{\frac{25a^{2}b}{4}+64b^{3}}+\frac{25a^{2}b\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{4\left(\frac{25a^{2}b}{4}+64b^{3}\right)}\right)}{d}-\frac{\left(2b-\frac{5a}{8}\right)\tan\left(\frac{c}{2}+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6*(a + b/cos(c + d*x)),x)
```

```
[Out] (5*a*atan((125*a^3*tan(c/2 + (d*x)/2))/(64*(20*a*b^2 + (125*a^3)/64)) + (20
*a*b^2*tan(c/2 + (d*x)/2))/(20*a*b^2 + (125*a^3)/64)))/(8*d) + (2*b*atanh((
64*b^3*tan(c/2 + (d*x)/2))/((25*a^2*b)/4 + 64*b^3) + (25*a^2*b*tan(c/2 + (d
*x)/2))/(4*((25*a^2*b)/4 + 64*b^3))))/d - (tan(c/2 + (d*x)/2)*((5*a)/8 + 2*
b) - tan(c/2 + (d*x)/2)^11*((5*a)/8 - 2*b) + tan(c/2 + (d*x)/2)^3*((85*a)/2
4 + (38*b)/3) - tan(c/2 + (d*x)/2)^9*((85*a)/24 - (38*b)/3) + tan(c/2 + (d*
x)/2)^5*((33*a)/4 + (172*b)/5) - tan(c/2 + (d*x)/2)^7*((33*a)/4 - (172*b)/5
))/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x
)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x
)/2)^12 + 1))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**6,x)
```

```
[Out] Timed out
```

3.169 $\int (a + b\sec(c + dx))\sin^4(c + dx)\,dx$

Optimal. Leaf size=89

$$-\frac{a\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+}{d}$$

```
[Out] 3/8*a*x+b*arctanh(sin(d*x+c))/d-b*sin(d*x+c)/d-3/8*a*cos(d*x+c)*sin(d*x+c)/
d-1/3*b*sin(d*x+c)^3/d-1/4*a*cos(d*x+c)*sin(d*x+c)^3/d
```

Rubi [A] time = 0.11, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3872, 2838, 2592, 302, 206, 2635, 8}

$$-\frac{a\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^4,x]
```

```
[Out] (3*a*x)/8 + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d - (3*a*Cos[c +
 d*x]*Sin[c + d*x])/(8*d) - (b*Sin[c + d*x]^3)/(3*d) - (a*Cos[c + d*x]*Sin[
c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
```

```
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))\sin^4(c+dx)\,dx = -\int (-b-a\cos(c+dx))\sin^3(c+dx)\tan(c+dx)\,dx$$
$$= a\int \sin^4(c+dx)\,dx + b\int \sin^3(c+dx)\tan(c+dx)\,dx$$
$$= -\frac{a\cos(c+dx)\sin^3(c+dx)}{4d} + \frac{1}{4}(3a)\int \sin^2(c+dx)\,dx + \frac{b\,\text{Subst}\left(\int \frac{x^4}{1-x^2}\right.}{}$$
$$= -\frac{3a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\cos(c+dx)\sin^3(c+dx)}{4d} + \frac{1}{8}(3a)\int 1\,dx$$
$$= \frac{3ax}{8} - \frac{b\sin(c+dx)}{d} - \frac{3a\cos(c+dx)\sin(c+dx)}{8d} - \frac{b\sin^3(c+dx)}{3d} - \frac{a\cos}{}$$
$$= \frac{3ax}{8} + \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{b\sin(c+dx)}{d} - \frac{3a\cos(c+dx)\sin(c+dx)}{8d}$$

Mathematica [A] time = 0.15, size = 86, normalized size = 0.97

$$\frac{3a(c+dx)}{8d}-\frac{a\sin(2(c+dx))}{4d}+\frac{a\sin(4(c+dx))}{32d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^4,x]
```

```
[Out] (3*a*(c + d*x))/(8*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d -
(b*Sin[c + d*x]^3)/(3*d) - (a*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)]
)/(32*d)
```

fricas [A] time = 0.48, size = 79, normalized size = 0.89

$$\frac{9\,adx + 12\,b\log(\sin(dx+c)+1) - 12\,b\log(-\sin(dx+c)+1) + \left(6\,a\cos(dx+c)^3 + 8\,b\cos(dx+c)^2 - 15\,a\cos\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(9*a*d*x + 12*b*log(sin(d*x + c) + 1) - 12*b*log(-sin(d*x + c) + 1) +
(6*a*cos(d*x + c)^3 + 8*b*cos(d*x + c)^2 - 15*a*cos(d*x + c) - 32*b)*sin(d*
x + c))/d
```

giac [B] time = 0.24, size = 172, normalized size = 1.93

$$\frac{9\,(dx+c)a + 24\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 24\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{2\left(9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 24\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/24*(9*(d*x + c)*a + 24*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*b*log(ab
s(tan(1/2*d*x + 1/2*c) - 1)) + 2*(9*a*tan(1/2*d*x + 1/2*c)^7 - 24*b*tan(1/2
*d*x + 1/2*c)^7 + 33*a*tan(1/2*d*x + 1/2*c)^5 - 104*b*tan(1/2*d*x + 1/2*c)^
5 - 33*a*tan(1/2*d*x + 1/2*c)^3 - 104*b*tan(1/2*d*x + 1/2*c)^3 - 9*a*tan(1/
2*d*x + 1/2*c) - 24*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)
/d
```

maple [A] time = 0.65, size = 96, normalized size = 1.08

$$-\frac{a\cos(dx+c)\left(\sin^3(dx+c)\right)}{4d}-\frac{3a\cos(dx+c)\sin(dx+c)}{8d}+\frac{3ax}{8}+\frac{3ca}{8d}-\frac{b\left(\sin^3(dx+c)\right)}{3d}-\frac{b\sin(dx+c)}{d}+\frac{b\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c)^4,x)
```

```
[Out] -1/4*a*cos(d*x+c)*sin(d*x+c)^3/d-3/8*a*cos(d*x+c)*sin(d*x+c)/d+3/8*a*x+3/8/
d*c*a-1/3*b*sin(d*x+c)^3/d-b*sin(d*x+c)/d+1/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.47, size = 81, normalized size = 0.91

$$\frac{3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)-8\,\sin(2\,dx+2\,c))a-16\left(2\,\sin(dx+c)^3-3\,\log(\sin(dx+c)+1)+3\,\log(\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a - 16*(2*s
in(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d
*x + c))*b)/d
```

mupad [B] time = 1.88, size = 267, normalized size = 3.00

$$\frac{3\,a\,\mathrm{atan}\left(\frac{27\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8\left(\frac{27\,a^3}{8}+24\,a\,b^2\right)}+\frac{24\,a\,b^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\frac{27\,a^3}{8}+24\,a\,b^2}\right)}{4\,d}+\frac{2\,b\,\mathrm{atanh}\left(\frac{64\,b^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{9\,a^2\,b+64\,b^3}+\frac{9\,a^2\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{9\,a^2\,b+64\,b^3}\right)}{d}-\frac{\left(2\,b-\frac{3\,a}{4}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right.}{d\left(\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + b/cos(c + d*x)),x)
```

```
[Out] (3*a*atan((27*a^3*tan(c/2 + (d*x)/2))/(8*(24*a*b^2 + (27*a^3)/8)) + (24*a*b
^2*tan(c/2 + (d*x)/2))/(24*a*b^2 + (27*a^3)/8)))/(4*d) + (2*b*atanh((64*b^3
*tan(c/2 + (d*x)/2))/(9*a^2*b + 64*b^3) + (9*a^2*b*tan(c/2 + (d*x)/2))/(9*a
^2*b + 64*b^3)))/d - (tan(c/2 + (d*x)/2)*((3*a)/4 + 2*b) - tan(c/2 + (d*x)/
2)^7*((3*a)/4 - 2*b) + tan(c/2 + (d*x)/2)^3*((11*a)/4 + (26*b)/3) - tan(c/2
 + (d*x)/2)^5*((11*a)/4 - (26*b)/3))/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2
 + (d*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))\sin^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sin(c + d*x)**4, x)
```

3.170 $\int (a+b\sec(c+dx))\sin^2(c+dx)\,dx$

Optimal. Leaf size=51

$$-\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] 1/2*a*x+b*arctanh(sin(d*x+c))/d-b*sin(d*x+c)/d-1/2*a*cos(d*x+c)*sin(d*x+c)/
d
```

Rubi [A] time = 0.08, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3872, 2838, 2592, 321, 206, 2635, 8}

$$-\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^2,x]
```

```
[Out] (a*x)/2 + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d - (a*Cos[c + d*x
]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
```

```
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (a+b\sec(c+dx))\sin^2(c+dx)\,dx &= -\int (-b-a\cos(c+dx))\sin(c+dx)\tan(c+dx)\,dx \\
&= a\int \sin^2(c+dx)\,dx + b\int \sin(c+dx)\tan(c+dx)\,dx \\
&= -\frac{a\cos(c+dx)\sin(c+dx)}{2d} + \frac{1}{2}a\int 1\,dx + \frac{b\,\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx, x, \sin(c\right.}{d} \\
&= \frac{ax}{2} - \frac{b\sin(c+dx)}{d} - \frac{a\cos(c+dx)\sin(c+dx)}{2d} + \frac{b\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x\right.}{d} \\
&= \frac{ax}{2} + \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{b\sin(c+dx)}{d} - \frac{a\cos(c+dx)\sin(c+dx}{2d}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 54, normalized size = 1.06

$$\frac{a(c+dx)}{2d} - \frac{a\sin(2(c+dx))}{4d} - \frac{b\sin(c+dx)}{d} + \frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^2,x]
```

```
[Out] (a*(c + d*x))/(2*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d - (a
*Sin[2*(c + d*x)])/(4*d)
```

fricas [A] time = 0.50, size = 55, normalized size = 1.08

$$\frac{adx + b\log(\sin(dx+c)+1) - b\log(-\sin(dx+c)+1) - (a\cos(dx+c)+2b)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*x + b*log(sin(d*x + c) + 1) - b*log(-sin(d*x + c) + 1) - (a*cos(d*
x + c) + 2*b)*sin(d*x + c))/d
```

giac [B] time = 0.20, size = 114, normalized size = 2.24

$$\frac{(dx+c)a + 2b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - 2b\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) + \frac{2\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 -\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}\right.\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)*a + 2*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*b*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 2*(a*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1
/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - 2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.31, size = 62, normalized size = 1.22

$$-\frac{a\cos\left(dx+c\right)\sin\left(dx+c\right)}{2d}+\frac{ax}{2}+\frac{ca}{2d}-\frac{b\sin\left(dx+c\right)}{d}+\frac{b\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c)^2,x)
```

```
[Out] -1/2*a*cos(d*x+c)*sin(d*x+c)/d+1/2*a*x+1/2/d*c*a-b*sin(d*x+c)/d+1/d*b*ln(se
c(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.69, size = 59, normalized size = 1.16

$$\frac{(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right))a+2\,b\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\,\sin\left(dx+c\right)\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a + 2*b*(log(sin(d*x + c) + 1) - log(
sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

mupad [B] time = 1.09, size = 83, normalized size = 1.63

$$\frac{a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}-\frac{b\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + b/cos(c + d*x)),x)
```

```
[Out] (a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*b*atanh(sin(c/2 + (d
*x)/2)/cos(c/2 + (d*x)/2)))/d - (a*sin(2*c + 2*d*x))/(4*d) - (b*sin(c + d*x
))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)\sin^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sin(c + d*x)**2, x)
```

3.171 $\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=37

$$-\frac{a\cot(c+dx)}{d}-\frac{b\csc(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] b*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-b*csc(d*x+c)/d
```

Rubi [A] time = 0.10, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = $\{3872, 2838, 2621, 321, 207, 3767, 8\}$

$$-\frac{a\cot(c+dx)}{d}-\frac{b\csc(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Sec[c + d*x]),x]
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (b*Csc[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx &= -\int(-b-a\cos(c+dx))\csc^2(c+dx)\sec(c+dx)\,dx\\ &= a\int\csc^2(c+dx)\,dx+b\int\csc^2(c+dx)\sec(c+dx)\,dx\\ &= -\frac{a\,\mathrm{Subst}(\int 1\,dx,x,\cot(c+dx))}{d}-\frac{b\,\mathrm{Subst}\left(\int\frac{x^2}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d}\\ &= -\frac{a\cot(c+dx)}{d}-\frac{b\csc(c+dx)}{d}-\frac{b\,\mathrm{Subst}\left(\int\frac{1}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d}\\ &= \frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\cot(c+dx)}{d}-\frac{b\csc(c+dx)}{d}\end{aligned}$$

Mathematica [C] time = 0.03, size = 41, normalized size = 1.11

$$-\frac{a\cot(c+dx)}{d}-\frac{b\csc(c+dx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\sin^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (b*Csc[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin
[c + d*x]^2])/d
```

fricas [A] time = 0.60, size = 63, normalized size = 1.70

$$\frac{b\log(\sin(dx+c)+1)\sin(dx+c)-b\log(-\sin(dx+c)+1)\sin(dx+c)-2\,a\cos(dx+c)-2\,b}{2\,d\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*log(sin(d*x + c) + 1)*sin(d*x + c) - b*log(-sin(d*x + c) + 1)*sin(d*
x + c) - 2*a*cos(d*x + c) - 2*b)/(d*sin(d*x + c))
```

giac [B] time = 1.28, size = 77, normalized size = 2.08

$$\frac{2\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{a+}{\tan\left(\frac{1}{2}\,d\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*b*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c) - (a + b)/tan(1
/2*d*x + 1/2*c))/d
```

maple [A] time = 0.73, size = 47, normalized size = 1.27

$$-\frac{a\cot(dx+c)}{d}-\frac{b}{d\sin(dx+c)}+\frac{b\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*sec(d*x+c)),x)
```

```
[Out] -a*cot(d*x+c)/d-1/d*b/sin(d*x+c)+1/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.54, size = 50, normalized size = 1.35

$$-\frac{b\left(\frac{2}{\sin(dx+c)}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+\frac{2a}{\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(b*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) +
2*a/tan(d*x + c))/d
```

mupad [B] time = 1.02, size = 60, normalized size = 1.62

$$\frac{2b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{\frac{a}{2}+\frac{b}{2}}{d\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{a}{2}-\frac{b}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/sin(c + d*x)^2,x)
```

```
[Out] (2*b*atanh(tan(c/2 + (d*x)/2)))/d - (a/2 + b/2)/(d*tan(c/2 + (d*x)/2)) + (t
an(c/2 + (d*x)/2)*(a/2 - b/2))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sec(c+dx))\csc^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*csc(c + d*x)**2, x)
```

3.172 $\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=69

$$-\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] b*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-1/3*a*cot(d*x+c)^3/d-b*csc(d*x+c)/d-
1/3*b*csc(d*x+c)^3/d
```

Rubi [A] time = 0.10, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {3872, 2838, 2621, 302, 207, 3767}

$$-\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (a*Cot[c + d*x]^3)/(3*d)
 - (b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/(3*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
```

```
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx = -\int (-b-a\cos(c+dx))\csc^4(c+dx)\sec(c+dx)\,dx$$
$$= a\int \csc^4(c+dx)\,dx + b\int \csc^4(c+dx)\sec(c+dx)\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, \cot(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(\right.}{d}$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{b\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx, x, \text{c}\right.}{d}$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{b\csc(c+dx)}{d} - \frac{b\csc^3(c+dx)}{3d} - \frac{b\,\text{S}}{}$$
$$= \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{b\csc(c+dx)}{d}$$

Mathematica [C] time = 0.03, size = 69, normalized size = 1.00

$$-\frac{2a\cot(c+dx)}{3d} - \frac{a\cot(c+dx)\csc^2(c+dx)}{3d} - \frac{b\csc^3(c+dx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\sin^2(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*a*Cot[c + d*x])/(3*d) - (a*Cot[c + d*x]*Csc[c + d*x]^2)/(3*d) - (b*Csc[
c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[c + d*x]^2])/(3*d)
```

fricas [A] time = 0.70, size = 125, normalized size = 1.81

$$-\frac{4\,a\cos\left(dx+c\right)^3 + 6\,b\cos\left(dx+c\right)^2 - 3\left(b\cos\left(dx+c\right)^2 - b\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right) + 3\left(b\cos\left(dx\right.\right.}{6\left(d\cos\left(dx+c\right)^2 - d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(4*a*cos(d*x + c)^3 + 6*b*cos(d*x + c)^2 - 3*(b*cos(d*x + c)^2 - b)*lo
g(sin(d*x + c) + 1)*sin(d*x + c) + 3*(b*cos(d*x + c)^2 - b)*log(-sin(d*x +
c) + 1)*sin(d*x + c) - 6*a*cos(d*x + c) - 8*b)/((d*cos(d*x + c)^2 - d)*sin(
d*x + c))
```

giac [B] time = 0.25, size = 133, normalized size = 1.93

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 24\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 24\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 + 24*b*log(abs(ta
n(1/2*d*x + 1/2*c) + 1)) - 24*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 9*a*ta
n(1/2*d*x + 1/2*c) - 15*b*tan(1/2*d*x + 1/2*c) - (9*a*tan(1/2*d*x + 1/2*c)^
2 + 15*b*tan(1/2*d*x + 1/2*c)^2 + a + b)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.86, size = 81, normalized size = 1.17

$$-\frac{2a\cot(dx+c)}{3d}-\frac{a\cot(dx+c)\left(\csc^2(dx+c)\right)}{3d}-\frac{b}{3d\sin(dx+c)^3}-\frac{b}{d\sin(dx+c)}+\frac{b\ln(\sec(dx+c)+\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*sec(d*x+c)),x)
```

```
[Out] -2/3*a*cot(d*x+c)/d-1/3/d*a*cot(d*x+c)*csc(d*x+c)^2-1/3/d*b/sin(d*x+c)^3-1/
d*b/sin(d*x+c)+1/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.66, size = 76, normalized size = 1.10

$$-\frac{b\left(\frac{2\left(3\sin(dx+c)^2+1\right)}{\sin(dx+c)^3}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)+\frac{2\left(3\tan(dx+c)^2+1\right)a}{\tan(dx+c)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(b*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1)
+ 3*log(sin(d*x + c) - 1)) + 2*(3*tan(d*x + c)^2 + 1)*a/tan(d*x + c)^3)/d
```

mupad [B] time = 1.00, size = 101, normalized size = 1.46

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{a}{24}-\frac{b}{24}\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left((3a+5b)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+\frac{a}{3}+\frac{b}{3}\right)}{8d}+\frac{2b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{\tan\left(\frac{c}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/sin(c + d*x)^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(a/24 - b/24))/d - (cot(c/2 + (d*x)/2)^3*(a/3 + b/3 +
 tan(c/2 + (d*x)/2)^2*(3*a + 5*b)))/(8*d) + (2*b*atanh(tan(c/2 + (d*x)/2)))
/d + (tan(c/2 + (d*x)/2)*((3*a)/8 - (5*b)/8))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int(a+b\sec(c+dx))\csc^4(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*csc(c + d*x)**4, x)
```

3.173 $\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=101

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{b\csc^5(c+dx)}{5d}-\frac{b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(}{d}$$

```
[Out] b*arctanh(sin(d*x+c))/d-a*cot(d*x+c)/d-2/3*a*cot(d*x+c)^3/d-1/5*a*cot(d*x+c
)^5/d-b*csc(d*x+c)/d-1/3*b*csc(d*x+c)^3/d-1/5*b*csc(d*x+c)^5/d
```

Rubi [A] time = 0.11, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3872, 2838, 2621, 302, 207, 3767}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{b\csc^5(c+dx)}{5d}-\frac{b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (2*a*Cot[c + d*x]^3)/(3*
d) - (a*Cot[c + d*x]^5)/(5*d) - (b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/(3*
d) - (b*Csc[c + d*x]^5)/(5*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
```

```
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx = -\int(-b-a\cos(c+dx))\csc^6(c+dx)\sec(c+dx)\,dx$$
$$= a\int\csc^6(c+dx)\,dx + b\int\csc^6(c+dx)\sec(c+dx)\,dx$$
$$= -\frac{a\,\mathrm{Subst}\left(\int\left(1+2x^2+x^4\right)dx,x,\cot(c+dx)\right)}{d} - \frac{b\,\mathrm{Subst}\left(\int\frac{x^6}{-1+x^2}\,dx,x,c\right.}{d}$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{b\,\mathrm{Subst}\left(\int\left(1+x^2+\right.\right.}{}$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{b\csc(c+dx)}{d} - \frac{b\csc}{}$$
$$= \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d}$$

Mathematica [C] time = 0.03, size = 91, normalized size = 0.90

$$-\frac{8a\cot(c+dx)}{15d} - \frac{a\cot(c+dx)\csc^4(c+dx)}{5d} - \frac{4a\cot(c+dx)\csc^2(c+dx)}{15d} - \frac{b\csc^5(c+dx)\,{}_2F_1\left(-\frac{5}{2},1;-\frac{3}{2};\sin^2(c+\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-8*a*Cot[c + d*x])/(15*d) - (4*a*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) - (a*
Cot[c + d*x]*Csc[c + d*x]^4)/(5*d) - (b*Csc[c + d*x]^5*Hypergeometric2F1[-5
/2, 1, -3/2, Sin[c + d*x]^2])/(5*d)
```

fricas [A] time = 0.70, size = 174, normalized size = 1.72

$$-\frac{16\,a\cos(dx+c)^5 + 30\,b\cos(dx+c)^4 - 40\,a\cos(dx+c)^3 - 70\,b\cos(dx+c)^2 - 15\left(b\cos(dx+c)^4 - 2\,b\cos(dx\right.}{30\left(d\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/30*(16*a*cos(d*x + c)^5 + 30*b*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 70
*b*cos(d*x + c)^2 - 15*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)*log(sin(
d*x + c) + 1)*sin(d*x + c) + 15*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)
*log(-sin(d*x + c) + 1)*sin(d*x + c) + 30*a*cos(d*x + c) + 46*b)/((d*cos(d*
x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

giac [B] time = 1.72, size = 194, normalized size = 1.92

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 35\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 480\,b\log\left(\left|\tan\left(\frac{1}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/480*(3*a*tan(1/2*d*x + 1/2*c)^5 - 3*b*tan(1/2*d*x + 1/2*c)^5 + 25*a*tan(1
/2*d*x + 1/2*c)^3 - 35*b*tan(1/2*d*x + 1/2*c)^3 + 480*b*log(abs(tan(1/2*d*x
 + 1/2*c) + 1)) - 480*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 150*a*tan(1/2*
d*x + 1/2*c) - 330*b*tan(1/2*d*x + 1/2*c) - (150*a*tan(1/2*d*x + 1/2*c)^4 +
 330*b*tan(1/2*d*x + 1/2*c)^4 + 25*a*tan(1/2*d*x + 1/2*c)^2 + 35*b*tan(1/2*
d*x + 1/2*c)^2 + 3*a + 3*b)/tan(1/2*d*x + 1/2*c)^5)/d
```

maple [A] time = 0.87, size = 115, normalized size = 1.14

$$-\frac{8a\cot(dx+c)}{15d}-\frac{a\cot(dx+c)\left(\csc^4(dx+c)\right)}{5d}-\frac{4a\cot(dx+c)\left(\csc^2(dx+c)\right)}{15d}-\frac{b}{5d\sin(dx+c)^5}-\frac{b}{3d\sin(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+b*sec(d*x+c)),x)
```

```
[Out] -8/15*a*cot(d*x+c)/d-1/5/d*a*cot(d*x+c)*csc(d*x+c)^4-4/15/d*a*cot(d*x+c)*cs
c(d*x+c)^2-1/5/d*b/sin(d*x+c)^5-1/3/d*b/sin(d*x+c)^3-1/d*b/sin(d*x+c)+1/d*b
*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.32, size = 96, normalized size = 0.95

$$-\frac{b\left(\frac{2\left(15\sin(dx+c)^4+5\sin(dx+c)^2+3\right)}{\sin(dx+c)^5}-15\log(\sin(dx+c)+1)+15\log(\sin(dx+c)-1)\right)+\frac{2\left(15\tan(dx+c)^4+10\tan(dx}{\tan(dx+c)^5}}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*(b*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 15*
log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 2*(15*tan(d*x + c)^4 +
10*tan(d*x + c)^2 + 3)*a/tan(d*x + c)^5)/d
```

mupad [B] time = 1.09, size = 142, normalized size = 1.41

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{5\,a}{96}-\frac{7\,b}{96}\right)}{d}-\frac{\cot\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left((10\,a+22\,b)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(\frac{5\,a}{3}+\frac{7\,b}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+\frac{a}{5}+\frac{b}{5}\right)}{32\,d}+\frac{\mathrm{t}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/sin(c + d*x)^6,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*((5*a)/96 - (7*b)/96))/d - (cot(c/2 + (d*x)/2)^5*(a/5
 + b/5 + tan(c/2 + (d*x)/2)^2*((5*a)/3 + (7*b)/3) + tan(c/2 + (d*x)/2)^4*(1
0*a + 22*b)))/(32*d) + (tan(c/2 + (d*x)/2)^5*(a/160 - b/160))/d + (2*b*atan
h(tan(c/2 + (d*x)/2)))/d + (tan(c/2 + (d*x)/2)*((5*a)/16 - (11*b)/16))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.174 $\int (a+b\sec(c+dx))^2\sin^5(c+dx)\,dx$

Optimal. Leaf size=124

$$\frac{\left(2a^2-b^2\right)\cos^3(c+dx)}{3d}-\frac{\left(a^2-2b^2\right)\cos(c+dx)}{d}-\frac{a^2\cos^5(c+dx)}{5d}-\frac{ab\cos^4(c+dx)}{2d}+\frac{2ab\cos^2(c+dx)}{d}-\frac{2ab\log(\mathrm{co}}{d}$$

```
[Out] -(a^2-2*b^2)*cos(d*x+c)/d+2*a*b*cos(d*x+c)^2/d+1/3*(2*a^2-b^2)*cos(d*x+c)^3
/d-1/2*a*b*cos(d*x+c)^4/d-1/5*a^2*cos(d*x+c)^5/d-2*a*b*ln(cos(d*x+c))/d+b^2
*sec(d*x+c)/d
```

Rubi [A] time = 0.20, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2837, 12, 948}

$$\frac{\left(2a^2-b^2\right)\cos^3(c+dx)}{3d}-\frac{\left(a^2-2b^2\right)\cos(c+dx)}{d}-\frac{a^2\cos^5(c+dx)}{5d}-\frac{ab\cos^4(c+dx)}{2d}+\frac{2ab\cos^2(c+dx)}{d}-\frac{2ab\log(\mathrm{co}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^5,x]
```

```
[Out] -(((a^2 - 2*b^2)*Cos[c + d*x])/d) + (2*a*b*Cos[c + d*x]^2)/d + ((2*a^2 - b^
2)*Cos[c + d*x]^3)/(3*d) - (a*b*Cos[c + d*x]^4)/(2*d) - (a^2*Cos[c + d*x]^5
)/(5*d) - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\sin^5(c+dx)\,dx=\int(-b-a\cos(c+dx))^2\sin^3(c+dx)\tan^2(c+dx)\,dx$$
$$=\frac{\text{Subst}\left(\int\frac{a^2(-b+x)^2(a^2-x^2)^2}{x^2}dx,x,-a\cos(c+dx)\right)}{a^5d}$$
$$=\frac{\text{Subst}\left(\int\frac{(-b+x)^2(a^2-x^2)^2}{x^2}dx,x,-a\cos(c+dx)\right)}{a^3d}$$
$$=\frac{\text{Subst}\left(\int\left(a^4\left(1-\frac{2b^2}{a^2}\right)+\frac{a^4b^2}{x^2}-\frac{2a^4b}{x}+4a^2bx-\left(2a^2-b^2\right)x^2-2bx^3+\right.\right.}{a^3d}$$
$$=-\frac{\left(a^2-2b^2\right)\cos(c+dx)}{d}+\frac{2ab\cos^2(c+dx)}{d}+\frac{\left(2a^2-b^2\right)\cos^3(c+dx}{3d}$$

Mathematica [A] time = 0.36, size = 112, normalized size = 0.90

$$-\frac{30\left(5a^2-14b^2\right)\cos(c+dx)-25a^2\cos(3(c+dx))+3a^2\cos(5(c+dx))-180ab\cos(2(c+dx))+15ab\cos(4(}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^5,x]
```

```
[Out] -1/240*(30*(5*a^2 - 14*b^2)*Cos[c + d*x] - 180*a*b*Cos[2*(c + d*x)] - 25*a^
2*Cos[3*(c + d*x)] + 20*b^2*Cos[3*(c + d*x)] + 15*a*b*Cos[4*(c + d*x)] + 3*
a^2*Cos[5*(c + d*x)] + 480*a*b*Log[Cos[c + d*x]] - 240*b^2*Sec[c + d*x])/d
```

fricas [A] time = 0.73, size = 125, normalized size = 1.01

$$-\frac{48\,a^2\cos(dx+c)^6+120\,ab\cos(dx+c)^5-480\,ab\cos(dx+c)^3-80\left(2\,a^2-b^2\right)\cos(dx+c)^4+480\,ab\cos(}{240\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/240*(48*a^2*cos(d*x + c)^6 + 120*a*b*cos(d*x + c)^5 - 480*a*b*cos(d*x +
c)^3 - 80*(2*a^2 - b^2)*cos(d*x + c)^4 + 480*a*b*cos(d*x + c)*log(-cos(d*x
+ c)) + 195*a*b*cos(d*x + c) + 240*(a^2 - 2*b^2)*cos(d*x + c)^2 - 240*b^2)/
(d*cos(d*x + c))
```

giac [B] time = 0.43, size = 418, normalized size = 3.37

$$\frac{60\,ab\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,ab\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{60\left(ab+b^2+\frac{ab(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}+\frac{32\,a^2+137\,ab-100\,b^2-\frac{160\,a^2(\cos}{\cos(d}}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/30*(60*a*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a*b*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 60*(a*b + b^2 + a*b*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 1) + (32*a^2 + 137*a*b - 100*b^2 - 160*a^2*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) - 805*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 440*b^2*(cos(d
```

```
*x + c) - 1)/(cos(d*x + c) + 1) + 320*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2 + 1970*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 640*b^2*(co
s(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1970*a*b*(cos(d*x + c) - 1)^3/(cos
(d*x + c) + 1)^3 + 360*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805*
a*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 60*b^2*(cos(d*x + c) - 1)^4
/(cos(d*x + c) + 1)^4 - 137*a*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5)/
((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)/d
```

maple [A] time = 0.63, size = 184, normalized size = 1.48

$$-\frac{8a^2\cos(dx+c)}{15d}-\frac{\cos(dx+c)\,a^2\left(\sin^4(dx+c)\right)}{5d}-\frac{4\cos(dx+c)\,a^2\left(\sin^2(dx+c)\right)}{15d}-\frac{ab\left(\sin^4(dx+c)\right)}{2d}-\frac{ab\left(\sin^2(\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c)^5,x)
```

```
[Out] -8/15*a^2*cos(d*x+c)/d-1/5/d*cos(d*x+c)*a^2*sin(d*x+c)^4-4/15/d*cos(d*x+c)*
a^2*sin(d*x+c)^2-1/2/d*a*b*sin(d*x+c)^4-1/d*a*b*sin(d*x+c)^2-2*a*b*ln(cos(d
*x+c))/d+1/d*b^2*sin(d*x+c)^6/cos(d*x+c)+8/3*b^2*cos(d*x+c)/d+1/d*b^2*sin(d
*x+c)^4*cos(d*x+c)+4/3/d*b^2*cos(d*x+c)*sin(d*x+c)^2
```

maxima [A] time = 0.42, size = 105, normalized size = 0.85

$$-\frac{6\,a^2\cos(dx+c)^5+15\,ab\cos(dx+c)^4-60\,ab\cos(dx+c)^2-10\left(2\,a^2-b^2\right)\cos(dx+c)^3+60\,ab\log(\cos(dx+}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/30*(6*a^2*cos(d*x + c)^5 + 15*a*b*cos(d*x + c)^4 - 60*a*b*cos(d*x + c)^2
 - 10*(2*a^2 - b^2)*cos(d*x + c)^3 + 60*a*b*log(cos(d*x + c)) + 30*(a^2 - 2
*b^2)*cos(d*x + c) - 30*b^2/cos(d*x + c))/d
```

mupad [B] time = 0.95, size = 104, normalized size = 0.84

$$-\frac{\cos(c+d\,x)\left(a^2-2\,b^2\right)-\cos(c+d\,x)^3\left(\frac{2\,a^2}{3}-\frac{b^2}{3}\right)+\frac{a^2\cos(c+d\,x)^5}{5}-\frac{b^2}{\cos(c+d\,x)}-2\,a\,b\cos(c+d\,x)^2+\frac{a\,b\cos(c+d\,x)^4}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5*(a + b/cos(c + d*x))^2,x)
```

```
[Out] -(cos(c + d*x)*(a^2 - 2*b^2) - cos(c + d*x)^3*((2*a^2)/3 - b^2/3) + (a^2*co
s(c + d*x)^5)/5 - b^2/cos(c + d*x) - 2*a*b*cos(c + d*x)^2 + (a*b*cos(c + d*
x)^4)/2 + 2*a*b*log(cos(c + d*x)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.175 $\int (a+b\sec(c+dx))^2\sin^3(c+dx)\,dx$

Optimal. Leaf size=80

$$-\frac{\left(a^2-b^2\right)\cos(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{ab\cos^2(c+dx)}{d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

```
[Out] -(a^2-b^2)*cos(d*x+c)/d+a*b*cos(d*x+c)^2/d+1/3*a^2*cos(d*x+c)^3/d-2*a*b*ln(
cos(d*x+c))/d+b^2*sec(d*x+c)/d
```

Rubi [A] time = 0.14, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2837, 12, 894}

$$-\frac{\left(a^2-b^2\right)\cos(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{ab\cos^2(c+dx)}{d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^3,x]
```

```
[Out] -(((a^2 - b^2)*Cos[c + d*x])/d) + (a*b*Cos[c + d*x]^2)/d + (a^2*Cos[c + d*x
]^3)/(3*d) - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\sin^3(c+dx)\,dx = \int (-b-a\cos(c+dx))^2\sin(c+dx)\tan^2(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^2(-b+x)^2(a^2-x^2)}{x^2}dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-b+x)^2(a^2-x^2)}{x^2}dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{b^2}{a^2}\right)+\frac{a^2b^2}{x^2}-\frac{2a^2b}{x}+2bx-x^2\right)dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= -\frac{(a^2-b^2)\cos(c+dx)}{d}+\frac{ab\cos^2(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}-\frac{2ab\log(c\cdots}{\cdots}$$

Mathematica [A] time = 0.18, size = 72, normalized size = 0.90

$$\frac{(12b^2-9a^2)\cos(c+dx)+a^2\cos(3(c+dx))+6ab\cos(2(c+dx))-24ab\log(\cos(c+dx))+12b^2\sec(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^3,x]
```

```
[Out] ((-9*a^2 + 12*b^2)*Cos[c + d*x] + 6*a*b*Cos[2*(c + d*x)] + a^2*Cos[3*(c + d
*x)] - 24*a*b*Log[Cos[c + d*x]] + 12*b^2*Sec[c + d*x])/(12*d)
```

fricas [A] time = 0.59, size = 92, normalized size = 1.15

$$\frac{2a^2\cos(dx+c)^4+6ab\cos(dx+c)^3-12ab\cos(dx+c)\log(-\cos(dx+c))-3ab\cos(dx+c)-6(a^2-b^2)\cos\cdots}{6d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^2*cos(d*x + c)^4 + 6*a*b*cos(d*x + c)^3 - 12*a*b*cos(d*x + c)*log(
-cos(d*x + c)) - 3*a*b*cos(d*x + c) - 6*(a^2 - b^2)*cos(d*x + c)^2 + 6*b^2)
/(d*cos(d*x + c))
```

giac [A] time = 0.41, size = 100, normalized size = 1.25

$$-\frac{2ab\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{b^2}{d\cos(dx+c)}+\frac{a^2d^5\cos(dx+c)^3+3abd^5\cos(dx+c)^2-3a^2d^5\cos(dx+c)+3b^2d^5\cos\cdots}{3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -2*a*b*log(abs(cos(d*x + c))/abs(d))/d + b^2/(d*cos(d*x + c)) + 1/3*(a^2*d^
5*cos(d*x + c)^3 + 3*a*b*d^5*cos(d*x + c)^2 - 3*a^2*d^5*cos(d*x + c) + 3*b^
2*d^5*cos(d*x + c))/d^6
```

maple [A] time = 0.63, size = 125, normalized size = 1.56

$$-\frac{\cos(dx+c)a^2(\sin^2(dx+c))}{3d}-\frac{2a^2\cos(dx+c)}{3d}-\frac{ab(\sin^2(dx+c))}{d}-\frac{2ab\ln(\cos(dx+c))}{d}+\frac{b^2(\sin^4(dx+c))}{d\cos(dx+c)}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c)^3,x)
```

```
[Out] -1/3/d*cos(d*x+c)*a^2*sin(d*x+c)^2-2/3*a^2*cos(d*x+c)/d-1/d*a*b*sin(d*x+c)^
2-2*a*b*ln(cos(d*x+c))/d+1/d*b^2*sin(d*x+c)^4/cos(d*x+c)+1/d*b^2*cos(d*x+c)
*sin(d*x+c)^2+2*b^2*cos(d*x+c)/d
```

maxima [A] time = 0.35, size = 71, normalized size = 0.89

$$\frac{a^2\cos\left(dx+c\right)^3+3\,ab\cos\left(dx+c\right)^2-6\,ab\log\left(\cos\left(dx+c\right)\right)-3\left(a^2-b^2\right)\cos\left(dx+c\right)+\frac{3\,b^2}{\cos(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(a^2*cos(d*x + c)^3 + 3*a*b*cos(d*x + c)^2 - 6*a*b*log(cos(d*x + c)) -
3*(a^2 - b^2)*cos(d*x + c) + 3*b^2/cos(d*x + c))/d
```

mupad [B] time = 0.92, size = 69, normalized size = 0.86

$$\frac{\frac{a^2\cos(c+d\,x)^3}{3}-\cos\left(c+d\,x\right)\,\left(a^2-b^2\right)+\frac{b^2}{\cos(c+d\,x)}+a\,b\cos\left(c+d\,x\right)^2-2\,a\,b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((a^2*cos(c + d*x)^3)/3 - cos(c + d*x)*(a^2 - b^2) + b^2/cos(c + d*x) + a*b
*cos(c + d*x)^2 - 2*a*b*log(cos(c + d*x)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^2\sin^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sin(c + d*x)**3, x)
```

3.176 $\int (a+b\sec(c+dx))^2\sin(c+dx)\,dx$

Optimal. Leaf size=42

$$-\frac{a^2\cos(c+dx)}{d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

```
[Out] -a^2*cos(d*x+c)/d-2*a*b*ln(cos(d*x+c))/d+b^2*sec(d*x+c)/d
```

Rubi [A] time = 0.08, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 2833, 12, 43}

$$-\frac{a^2\cos(c+dx)}{d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x],x]
```

```
[Out] -((a^2*Cos[c + d*x])/d) - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Sec[c + d*x])/
d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\sin(c+dx)\,dx = \int (-b-a\cos(c+dx))^2\sec(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^2(-b+x)^2}{x^2}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(-b+x)^2}{x^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(1+\frac{b^2}{x^2}-\frac{2b}{x}\right)dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^2\cos(c+dx)}{d} - \frac{2ab\log(\cos(c+dx))}{d} + \frac{b^2\sec(c+dx)}{d}$$

Mathematica [A] time = 0.06, size = 37, normalized size = 0.88

$$\frac{b(b\sec(c+dx)-2a\log(\cos(c+dx)))-a^2\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x],x]
[Out] (-(a^2*Cos[c + d*x]) + b*(-2*a*Log[Cos[c + d*x]] + b*Sec[c + d*x]))/d
```

fricas [A] time = 0.63, size = 50, normalized size = 1.19

$$-\frac{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)\log(-\cos(dx+c))-b^2}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c),x, algorithm="fricas")
[Out] -(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)*log(-cos(d*x + c)) - b^2)/(d*cos(
d*x + c))
```

giac [A] time = 0.34, size = 50, normalized size = 1.19

$$-\frac{a^2\cos(dx+c)}{d} - \frac{2\,ab\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d} + \frac{b^2}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c),x, algorithm="giac")
[Out] -a^2*cos(d*x + c)/d - 2*a*b*log(abs(cos(d*x + c))/abs(d))/d + b^2/(d*cos(d*
x + c))
```

maple [A] time = 0.19, size = 45, normalized size = 1.07

$$\frac{b^2\sec(dx+c)}{d} + \frac{2ab\ln(\sec(dx+c))}{d} - \frac{a^2}{d\sec(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c),x)
[Out] b^2*sec(d*x+c)/d+2/d*a*b*ln(sec(d*x+c))-1/d*a^2/sec(d*x+c)
```

maxima [A] time = 0.63, size = 40, normalized size = 0.95

$$-\frac{a^2\cos\left(dx+c\right)+2\,ab\log\left(\cos\left(dx+c\right)\right)-\frac{b^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c),x, algorithm="maxima")
[Out] -(a^2*cos(d*x + c) + 2*a*b*log(cos(d*x + c)) - b^2/cos(d*x + c))/d
```

mupad [B] time = 0.05, size = 40, normalized size = 0.95

$$-\frac{a^2\,\cos\left(c+d\,x\right)-\frac{b^2}{\cos(c+d\,x)}+2\,a\,b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b/cos(c + d*x))^2,x)
[Out] -(a^2*cos(c + d*x) - b^2/cos(c + d*x) + 2*a*b*log(cos(c + d*x)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^2\sin\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c),x)
[Out] Integral((a + b*sec(c + d*x))**2*sin(c + d*x), x)
```

3.177 $\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=74

$$-\frac{(a-b)^2\log(\cos(c+dx)+1)}{2d}+\frac{(a+b)^2\log(1-\cos(c+dx))}{2d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

```
[Out] 1/2*(a+b)^2*ln(1-cos(d*x+c))/d-2*a*b*ln(cos(d*x+c))/d-1/2*(a-b)^2*ln(1+cos(
d*x+c))/d+b^2*sec(d*x+c)/d
```

Rubi [A] time = 0.18, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3872, 2837, 12, 1802}

$$-\frac{(a-b)^2\log(\cos(c+dx)+1)}{2d}+\frac{(a+b)^2\log(1-\cos(c+dx))}{2d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((a + b)^2*Log[1 - Cos[c + d*x]])/(2*d) - (2*a*b*Log[Cos[c + d*x]])/d - ((a
 - b)^2*Log[1 + Cos[c + d*x]])/(2*d) + (b^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx = \int (-b-a\cos(c+dx))^2\csc(c+dx)\sec^2(c+dx)\,dx$$
$$= \frac{a\,\mathrm{Subst}\left(\int \frac{a^2(-b+x)^2}{x^2(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^3\,\mathrm{Subst}\left(\int \frac{(-b+x)^2}{x^2(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^3\,\mathrm{Subst}\left(\int \left(\frac{(a-b)^2}{2a^3(a-x)}+\frac{b^2}{a^2x^2}-\frac{2b}{a^2x}+\frac{(a+b)^2}{2a^3(a+x)}\right)dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{(a+b)^2\log(1-\cos(c+dx))}{2d}-\frac{2ab\log(\cos(c+dx))}{d}-\frac{(a-b)^2\log(1+c}{2d}$$

Mathematica [A] time = 0.16, size = 91, normalized size = 1.23

$$\frac{a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-2ab\log(\cos(c+dx))-(a-b)^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+b^2}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-((a - b)^2*Log[Cos[(c + d*x)/2]]) - 2*a*b*Log[Cos[c + d*x]] + a^2*Log[Sin
[(c + d*x)/2]] + 2*a*b*Log[Sin[(c + d*x)/2]] + b^2*Log[Sin[(c + d*x)/2]] +
b^2*Sec[c + d*x])/d
```

fricas [A] time = 0.53, size = 97, normalized size = 1.31

$$-\frac{4ab\cos(dx+c)\log(-\cos(dx+c))+\left(a^2-2ab+b^2\right)\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-\left(a^2+2ab+b^2\right)c}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a*b*cos(d*x + c)*log(-cos(d*x + c)) + (a^2 - 2*a*b + b^2)*cos(d*x +
 c)*log(1/2*cos(d*x + c) + 1/2) - (a^2 + 2*a*b + b^2)*cos(d*x + c)*log(-1/2
*cos(d*x + c) + 1/2) - 2*b^2)/(d*cos(d*x + c))
```

giac [A] time = 0.51, size = 124, normalized size = 1.68

$$-\frac{4ab\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\left(a^2+2ab+b^2\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-\frac{4\left(ab+b^2+\frac{ab(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*a*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^2 + 2
*a*b + b^2)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 4*(a*b + b^
2 + a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) + 1))/d
```

maple [A] time = 0.38, size = 77, normalized size = 1.04

$$\frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{d}+\frac{2ab\ln(\tan(dx+c))}{d}+\frac{b^2}{d\cos(dx+c)}+\frac{b^2\ln(\csc(dx+c)-\cot(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*a^2*ln(csc(d*x+c)-cot(d*x+c))+2*a*b*ln(tan(d*x+c))/d+1/d*b^2/cos(d*x+c)
+1/d*b^2*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.74, size = 73, normalized size = 0.99

$$-\frac{4\,ab\log\left(\cos\left(dx+c\right)\right)+\left(a^2-2\,ab+b^2\right)\log\left(\cos\left(dx+c\right)+1\right)-\left(a^2+2\,ab+b^2\right)\log\left(\cos\left(dx+c\right)-1\right)-\frac{2}{\cos(}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(4*a*b*log(cos(d*x + c)) + (a^2 - 2*a*b + b^2)*log(cos(d*x + c) + 1) -
 (a^2 + 2*a*b + b^2)*log(cos(d*x + c) - 1) - 2*b^2/cos(d*x + c))/d
```

mupad [B] time = 0.97, size = 62, normalized size = 0.84

$$\frac{\frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,{\left(a+b\right)}^2}{2}-\frac{\ln\left(\cos\left(c+d\,x\right)+1\right)\,{\left(a-b\right)}^2}{2}+\frac{b^2}{\cos\left(c+d\,x\right)}-2\,a\,b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/sin(c + d*x),x)
```

```
[Out] ((log(cos(c + d*x) - 1)*(a + b)^2)/2 - (log(cos(c + d*x) + 1)*(a - b)^2)/2
+ b^2/cos(c + d*x) - 2*a*b*log(cos(c + d*x)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(c+dx\right)\right)^2\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*csc(c + d*x), x)
```

3.178 $\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=114

$$-\frac{\csc^2(c+dx)\left(\left(a^2+b^2\right)\cos(c+dx)+2ab\right)}{2d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{(a+b)(a+3b)\log(1-\cos(c+dx))}{4d}-\frac{(a-3b}{}$$

```
[Out] -1/2*(2*a*b+(a^2+b^2)*cos(d*x+c))*csc(d*x+c)^2/d+1/4*(a+b)*(a+3*b)*ln(1-cos
(d*x+c))/d-2*a*b*ln(cos(d*x+c))/d-1/4*(a-3*b)*(a-b)*ln(1+cos(d*x+c))/d+b^2*
sec(d*x+c)/d
```

Rubi [A] time = 0.29, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2837, 12, 1805, 1802}

$$-\frac{\csc^2(c+dx)\left(\left(a^2+b^2\right)\cos(c+dx)+2ab\right)}{2d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{(a+b)(a+3b)\log(1-\cos(c+dx))}{4d}-\frac{(a-3b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((2*a*b + (a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x]^2)/(2*d) + ((a + b)*(a +
3*b)*Log[1 - Cos[c + d*x]])/(4*d) - (2*a*b*Log[Cos[c + d*x]])/d - ((a - 3*b
)*(a - b)*Log[1 + Cos[c + d*x]])/(4*d) + (b^2*Sec[c + d*x])/d
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx = \int (-b-a\cos(c+dx))^2\csc^3(c+dx)\sec^2(c+dx)\,dx$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{a^2(-b+x)^2}{x^2\left(a^2-x^2\right)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^5\,\text{Subst}\left(\int \frac{(-b+x)^2}{x^2\left(a^2-x^2\right)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{a\left(2b+\frac{\left(a^2+b^2\right)\cos(c+dx)}{a}\right)\csc^2(c+dx)}{2d} - \frac{a^3\,\text{Subst}\left(\int \frac{-2b^2+4bx-\frac{\left(a^2+b^2\right)x^2}{a^2}}{x^2\left(a^2-x^2\right)}\right.}{2d}$$
$$= -\frac{a\left(2b+\frac{\left(a^2+b^2\right)\cos(c+dx)}{a}\right)\csc^2(c+dx)}{2d} - \frac{a^3\,\text{Subst}\left(\int\left(\frac{(a-3b)(-a+b)}{2a^3(a-x)} - \frac{2}{a^2}\right.\right.}{2d}$$
$$= -\frac{a\left(2b+\frac{\left(a^2+b^2\right)\cos(c+dx)}{a}\right)\csc^2(c+dx)}{2d} + \frac{(a+b)(a+3b)\log(1-\cos(c}{4d}$$

Mathematica [B] time = 0.63, size = 329, normalized size = 2.89

$$-\frac{\csc^4(c+dx)\left(2\left(a^2+3b^2\right)\cos(2(c+dx))+\cos(c+dx)\left(\left(a^2-4ab+3b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+a^2\left(-\log\left(\right.\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/2*(Csc[c + d*x]^4*(2*a^2 - 2*b^2 + 2*(a^2 + 3*b^2)*Cos[2*(c + d*x)] - a^
2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 4*a*b*Cos[3*(c + d*x)]*Log[Cos[(
c + d*x)/2]] - 3*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 4*a*b*Cos[3*(
c + d*x)]*Log[Cos[c + d*x]] + a^2*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] +
4*a*b*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 3*b^2*Cos[3*(c + d*x)]*Log[S
in[(c + d*x)/2]] + Cos[c + d*x]*(8*a*b + (a^2 - 4*a*b + 3*b^2)*Log[Cos[(c +
 d*x)/2]] + 4*a*b*Log[Cos[c + d*x]] - a^2*Log[Sin[(c + d*x)/2]] - 4*a*b*Log
[Sin[(c + d*x)/2]] - 3*b^2*Log[Sin[(c + d*x)/2]])))/(d*(Csc[(c + d*x)/2]^2
- Sec[(c + d*x)/2]^2))
```

fricas [A] time = 0.54, size = 205, normalized size = 1.80

$$\frac{4\,ab\cos(dx+c)+2\left(a^2+3\,b^2\right)\cos(dx+c)^2-4\,b^2-8\left(ab\cos(dx+c)^3-ab\cos(dx+c)\right)\log(-\cos(dx+c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*b*cos(d*x + c) + 2*(a^2 + 3*b^2)*cos(d*x + c)^2 - 4*b^2 - 8*(a*b*c
os(d*x + c)^3 - a*b*cos(d*x + c))*log(-cos(d*x + c)) - ((a^2 - 4*a*b + 3*b^
2)*cos(d*x + c)^3 - (a^2 - 4*a*b + 3*b^2)*cos(d*x + c))*log(1/2*cos(d*x + c
) + 1/2) + ((a^2 + 4*a*b + 3*b^2)*cos(d*x + c)^3 - (a^2 + 4*a*b + 3*b^2)*co
s(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^3 - d*cos(d*x + c
))
```

giac [B] time = 0.29, size = 314, normalized size = 2.75

$$-\frac{16\,ab\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-2\left(a^2+4\,ab+3\,b^2\right)\log\left(\frac{|-\cos(dx}{|\cos(dx+}\right.}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(16*a*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + a^2*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a*b*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*(a^2 + 4*a*b + 3*b^2)
*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - (a^2 + 2*a*b + b^2 + 6
*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 14*b^2*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4*a*b*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + (cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2))/d
```

maple [A] time = 0.74, size = 139, normalized size = 1.22

$$-\frac{a^2\cot(dx+c)\csc(dx+c)}{2d}+\frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{2d}-\frac{ab}{d\sin(dx+c)^2}+\frac{2ab\ln(\tan(dx+c))}{d}-\frac{}{2d\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/2*a^2*cot(d*x+c)*csc(d*x+c)/d+1/2/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-1/d*a*
b/sin(d*x+c)^2+2*a*b*ln(tan(d*x+c))/d-1/2/d*b^2/sin(d*x+c)^2/cos(d*x+c)+3/2
/d*b^2/cos(d*x+c)+3/2/d*b^2*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.67, size = 119, normalized size = 1.04

$$-\frac{8\,ab\log(\cos(dx+c))+\left(a^2-4\,ab+3\,b^2\right)\log(\cos(dx+c)+1)-\left(a^2+4\,ab+3\,b^2\right)\log(\cos(dx+c)-1)-\frac{2(2}{}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(8*a*b*log(cos(d*x + c)) + (a^2 - 4*a*b + 3*b^2)*log(cos(d*x + c) + 1)
 - (a^2 + 4*a*b + 3*b^2)*log(cos(d*x + c) - 1) - 2*(2*a*b*cos(d*x + c) + (a
^2 + 3*b^2)*cos(d*x + c)^2 - 2*b^2)/(cos(d*x + c)^3 - cos(d*x + c)))/d
```

mupad [B] time = 0.12, size = 120, normalized size = 1.05

$$\frac{\ln(\cos(c+d\,x)-1)\,(a+b)\,(a+3\,b)}{4\,d}-\frac{\ln(\cos(c+d\,x)+1)\,(a-b)\,(a-3\,b)}{4\,d}-\frac{2\,a\,b\,\ln(\cos(c+d\,x))}{d}-\frac{\cos(c+d}{d\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/sin(c + d*x)^3,x)
```

```
[Out] (log(cos(c + d*x) - 1)*(a + b)*(a + 3*b))/(4*d) - (log(cos(c + d*x) + 1)*(a
 - b)*(a - 3*b))/(4*d) - (2*a*b*log(cos(c + d*x)))/d - (cos(c + d*x)^2*(a^2
/2 + (3*b^2)/2) - b^2 + a*b*cos(c + d*x))/(d*(cos(c + d*x) - cos(c + d*x)^3
))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^2 \csc^3(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*csc(c + d*x)**3, x)
```

3.179 $\int (a+b\sec(c+dx))^2\sin^6(c+dx)\,dx$

Optimal. Leaf size=175

$$\frac{(13a^2-6b^2)\sin(c+dx)\cos^3(c+dx)}{24d}-\frac{(11a^2-18b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{5}{16}x\left(a^2-6b^2\right)-\frac{a^2\sin(c+dx)\cos^5}{6d}$$

```
[Out] 5/16*(a^2-6*b^2)*x+2*a*b*arctanh(sin(d*x+c))/d-2*a*b*sin(d*x+c)/d-1/16*(11*
a^2-18*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/24*(13*a^2-6*b^2)*cos(d*x+c)^3*sin(d*
x+c)/d-1/6*a^2*cos(d*x+c)^5*sin(d*x+c)/d-2/3*a*b*sin(d*x+c)^3/d-2/5*a*b*sin
(d*x+c)^5/d+b^2*tan(d*x+c)/d
```

Rubi [A] time = 0.46, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3872, 2911, 2592, 302, 206, 455, 1814, 1157, 388, 203}

$$\frac{(13a^2-6b^2)\sin(c+dx)\cos^3(c+dx)}{24d}-\frac{(11a^2-18b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{5}{16}x\left(a^2-6b^2\right)-\frac{a^2\sin(c+dx)\cos^5}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^6,x]
```

```
[Out] (5*(a^2 - 6*b^2)*x)/16 + (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Sin[c + d
*x])/d - ((11*a^2 - 18*b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + ((13*a^2 -
6*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (a^2*Cos[c + d*x]^5*Sin[c + d*
x])/(6*d) - (2*a*b*Sin[c + d*x]^3)/(3*d) - (2*a*b*Sin[c + d*x]^5)/(5*d) + (
b^2*Tan[c + d*x])/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
```

```
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\sin^6(c+dx)\,dx = \int (-b-a\cos(c+dx))^2\sin^4(c+dx)\tan^2(c+dx)\,dx$$
$$= (2ab)\int \sin^5(c+dx)\tan(c+dx)\,dx + \int \left(b^2+a^2\cos^2(c+dx)\right)\sin^4(c+$$
$$= \frac{\text{Subst}\left(\int \frac{x^6\left(a^2+b^2+b^2x^2\right)}{\left(1+x^2\right)^4}dx, x, \tan(c+dx)\right)}{d} + \frac{(2ab)\,\text{Subst}\left(\int \frac{x^6}{1-x^2}dx, x, \text{si}\right.}{d}$$
$$= -\frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{\text{Subst}\left(\int \frac{-a^2+6a^2x^2-6a^2x^4-6b^2x^6}{\left(1+x^2\right)^3}dx, x, \tan(c\right.}{6d}$$
$$= -\frac{2ab\sin(c+dx)}{d} + \frac{\left(13a^2-6b^2\right)\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{a^2\cos^5(c+}{}$$
$$= \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} - \frac{2ab\sin(c+dx)}{d} - \frac{\left(11a^2-18b^2\right)\cos(c+dx)\,\text{s}}{16d}$$
$$= \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} - \frac{2ab\sin(c+dx)}{d} - \frac{\left(11a^2-18b^2\right)\cos(c+dx)\,\text{s}}{16d}$$
$$= \frac{5}{16}\left(a^2-6b^2\right)x + \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} - \frac{2ab\sin(c+dx)}{d} - \frac{\left(11a^2-1\right.}{}$$

Mathematica [A] time = 1.68, size = 193, normalized size = 1.10

$$\tan(c+dx)\left(-5\left(29a^2-84b^2\right)\cos(2(c+dx)) + 35a^2\cos(4(c+dx)) - 5a^2\cos(6(c+dx)) - 185a^2 + 232ab\cos(3(c\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^6,x]
```

```
[Out] (60*(5*(a^2 - 6*b^2)*(c + d*x) - 32*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] + 32*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 2128*a*b*Sin[c +
 d*x] + (-185*a^2 + 1410*b^2 - 5*(29*a^2 - 84*b^2)*Cos[2*(c + d*x)] + 232*a
*b*Cos[3*(c + d*x)] + 35*a^2*Cos[4*(c + d*x)] - 30*b^2*Cos[4*(c + d*x)] - 2
4*a*b*Cos[5*(c + d*x)] - 5*a^2*Cos[6*(c + d*x)])*Tan[c + d*x])/(960*d)
```

fricas [A] time = 0.49, size = 176, normalized size = 1.01

$$75\left(a^2-6\,b^2\right)dx\cos\left(dx+c\right) + 240\,ab\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right) - 240\,ab\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(75*(a^2 - 6*b^2)*d*x*cos(d*x + c) + 240*a*b*cos(d*x + c)*log(sin(d*x
 + c) + 1) - 240*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) - (40*a^2*cos(d*x
+ c)^6 + 96*a*b*cos(d*x + c)^5 - 352*a*b*cos(d*x + c)^3 - 10*(13*a^2 - 6*b^
2)*cos(d*x + c)^4 + 736*a*b*cos(d*x + c) + 15*(11*a^2 - 18*b^2)*cos(d*x + c
)^2 - 240*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

giac [B] time = 0.74, size = 379, normalized size = 2.17

$$480\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 480\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + 75\left(a^2-6\,b^2\right)(dx+c) - \frac{480\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/240*(480*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 480*a*b*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) + 75*(a^2 - 6*b^2)*(d*x + c) - 480*b^2*tan(1/2*d*x + 1/
2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(75*a^2*tan(1/2*d*x + 1/2*c)^11 - 480
*a*b*tan(1/2*d*x + 1/2*c)^11 - 210*b^2*tan(1/2*d*x + 1/2*c)^11 + 425*a^2*ta
n(1/2*d*x + 1/2*c)^9 - 3040*a*b*tan(1/2*d*x + 1/2*c)^9 - 870*b^2*tan(1/2*d*
x + 1/2*c)^9 + 990*a^2*tan(1/2*d*x + 1/2*c)^7 - 8256*a*b*tan(1/2*d*x + 1/2*
c)^7 - 660*b^2*tan(1/2*d*x + 1/2*c)^7 - 990*a^2*tan(1/2*d*x + 1/2*c)^5 - 82
56*a*b*tan(1/2*d*x + 1/2*c)^5 + 660*b^2*tan(1/2*d*x + 1/2*c)^5 - 425*a^2*ta
n(1/2*d*x + 1/2*c)^3 - 3040*a*b*tan(1/2*d*x + 1/2*c)^3 + 870*b^2*tan(1/2*d*
x + 1/2*c)^3 - 75*a^2*tan(1/2*d*x + 1/2*c) - 480*a*b*tan(1/2*d*x + 1/2*c) +
 210*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

maple [A] time = 0.70, size = 246, normalized size = 1.41

$$-\frac{a^2\left(\sin^5(dx+c)\right)\cos(dx+c)}{6d}-\frac{5a^2\cos(dx+c)\left(\sin^3(dx+c)\right)}{24d}-\frac{5a^2\cos(dx+c)\sin(dx+c)}{16d}+\frac{5a^2x}{16}+\frac{5a^2c}{16d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c)^6,x)
```

```
[Out] -1/6/d*a^2*sin(d*x+c)^5*cos(d*x+c)-5/24/d*a^2*cos(d*x+c)*sin(d*x+c)^3-5/16*
a^2*cos(d*x+c)*sin(d*x+c)/d+5/16*a^2*x+5/16/d*a^2*c-2/5*a*b*sin(d*x+c)^5/d-
2/3*a*b*sin(d*x+c)^3/d-2*a*b*sin(d*x+c)/d+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c))
+1/d*b^2*sin(d*x+c)^7/cos(d*x+c)+1/d*b^2*sin(d*x+c)^5*cos(d*x+c)+5/4/d*b^2*
cos(d*x+c)*sin(d*x+c)^3+15/8/d*b^2*cos(d*x+c)*sin(d*x+c)-15/8*b^2*x-15/8/d*
c*b^2
```

maxima [A] time = 0.81, size = 173, normalized size = 0.99

$$\frac{5\left(4\sin(2dx+2c)^3+60dx+60c+9\sin(4dx+4c)-48\sin(2dx+2c)\right)a^2-64\left(6\sin(dx+c)^5+10\sin\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/960*(5*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*si
n(2*d*x + 2*c))*a^2 - 64*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin
(d*x + c) + 1) + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a*b - 120*(15*
d*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*
x + c)^2 + 1) - 8*tan(d*x + c))*b^2)/d
```

mupad [B] time = 3.10, size = 231, normalized size = 1.32

$$\frac{\frac{5\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)a^2}{8}+4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)a\,b-\frac{15\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)b^2}{4}}{d}-\frac{\frac{15\,a^2\,\sin(c+d\,x)}{128}-\frac{5\,b^2\,\sin(c+d\,x)}{4}+\frac{3\,a^2\,\sin(3\,c+3\,d\,x)}{32}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6*(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((5*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/8 - (15*b^2*atan(sin(c
/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 + 4*a*b*atanh(sin(c/2 + (d*x)/2)/cos(c
/2 + (d*x)/2)))/d - ((15*a^2*sin(c + d*x))/128 - (5*b^2*sin(c + d*x))/4 + (
```

```
3*a^2*sin(3*c + 3*d*x))/32 - (a^2*sin(5*c + 5*d*x))/48 + (a^2*sin(7*c + 7*d
*x))/384 - (15*b^2*sin(3*c + 3*d*x))/64 + (b^2*sin(5*c + 5*d*x))/64 + (59*a
*b*sin(2*c + 2*d*x))/48 - (2*a*b*sin(4*c + 4*d*x))/15 + (a*b*sin(6*c + 6*d*
x))/80)/(d*cos(c + d*x))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**6,x)
```

```
[Out] Timed out
```

3.180 $\int (a+b\sec(c+dx))^2\sin^4(c+dx)\,dx$

Optimal. Leaf size=178

$$-\frac{b\left(28a^2+b^2\right)\sin(c+dx)}{6ad}-\frac{\left(12a^2+b^2\right)\sin(c+dx)(a\cos(c+dx)+b)^2}{12abd}-\frac{\left(39a^2+2b^2\right)\sin(c+dx)\cos(c+dx)}{24d}$$

```
[Out] 3/8*(a^2-4*b^2)*x+2*a*b*arctanh(sin(d*x+c))/d-1/6*b*(28*a^2+b^2)*sin(d*x+c)
/a/d-1/24*(39*a^2+2*b^2)*cos(d*x+c)*sin(d*x+c)/d-1/12*(12*a^2+b^2)*(b+a*cos
(d*x+c))^2*sin(d*x+c)/a/b/d+1/4*(b+a*cos(d*x+c))^3*sin(d*x+c)/a/d+(b+a*cos(
d*x+c))^3*tan(d*x+c)/b/d
```

Rubi [A] time = 0.56, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2894, 3049, 3033, 3023, 2735, 3770}

$$-\frac{b\left(28a^2+b^2\right)\sin(c+dx)}{6ad}-\frac{\left(12a^2+b^2\right)\sin(c+dx)(a\cos(c+dx)+b)^2}{12abd}-\frac{\left(39a^2+2b^2\right)\sin(c+dx)\cos(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^4,x]
```

```
[Out] (3*(a^2 - 4*b^2)*x)/8 + (2*a*b*ArcTanh[Sin[c + d*x]])/d - (b*(28*a^2 + b^2)
*Sin[c + d*x])/(6*a*d) - ((39*a^2 + 2*b^2)*Cos[c + d*x]*Sin[c + d*x])/(24*d
) - ((12*a^2 + b^2)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(12*a*b*d) + ((b +
 a*Cos[c + d*x])^3*Sin[c + d*x])/(4*a*d) + ((b + a*Cos[c + d*x])^3*Tan[c +
d*x])/(b*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2894

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d
*Sin[e + f*x])^(n + 2))/(b*d^2*f*(m + n + 4)), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
```

```
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^2\sin^4(c+dx)\,dx &= \int (-b-a\cos(c+dx))^2\sin^2(c+dx)\tan^2(c+dx)\,dx\\
&= \frac{(b+a\cos(c+dx))^3\sin(c+dx)}{4ad}+\frac{(b+a\cos(c+dx))^3\tan(c+dx)}{bd}-\int(\\
&= -\frac{\left(12a^2+b^2\right)(b+a\cos(c+dx))^2\sin(c+dx)}{12abd}+\frac{(b+a\cos(c+dx))^3\sin(}{4ad}\\
&= -\frac{\left(39a^2+2b^2\right)\cos(c+dx)\sin(c+dx)}{24d}-\frac{\left(12a^2+b^2\right)(b+a\cos(c+dx))^2}{12abd}\\
&= -\frac{b\left(28a^2+b^2\right)\sin(c+dx)}{6ad}-\frac{\left(39a^2+2b^2\right)\cos(c+dx)\sin(c+dx)}{24d}-\frac{(12}{}\\
&= \frac{3}{8}\left(a^2-4b^2\right)x-\frac{b\left(28a^2+b^2\right)\sin(c+dx)}{6ad}-\frac{\left(39a^2+2b^2\right)\cos(c+dx)\sin}{24d}\\
&= \frac{3}{8}\left(a^2-4b^2\right)x+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{b\left(28a^2+b^2\right)\sin(c+dx)}{6ad}-\end{aligned}$$

Mathematica [A] time = 1.05, size = 157, normalized size = 0.88

$$\frac{\tan(c+dx)\left(-6\left(3a^2-4b^2\right)\cos(2(c+dx))+3\left(a^2\cos(4(c+dx))-7a^2+40b^2\right)+16ab\cos(3(c+dx))\right)+12\left(3\left(a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^4,x]
```

```
[Out] (12*(3*(a^2 - 4*b^2)*(c + d*x) - 16*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] + 16*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 208*a*b*Sin[c +
d*x] + (-6*(3*a^2 - 4*b^2)*Cos[2*(c + d*x)] + 16*a*b*Cos[3*(c + d*x)] + 3*(
-7*a^2 + 40*b^2 + a^2*Cos[4*(c + d*x)]))*Tan[c + d*x])/(96*d)
```

fricas [A] time = 0.62, size = 142, normalized size = 0.80

$$\frac{9\left(a^2-4\,b^2\right)dx\cos\left(dx+c\right)+24\,ab\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-24\,ab\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+\right.}{24\,d\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(9*(a^2 - 4*b^2)*d*x*cos(d*x + c) + 24*a*b*cos(d*x + c)*log(sin(d*x +
c) + 1) - 24*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) + (6*a^2*cos(d*x + c)^
4 + 16*a*b*cos(d*x + c)^3 - 64*a*b*cos(d*x + c) - 3*(5*a^2 - 4*b^2)*cos(d*x
 + c)^2 + 24*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

giac [A] time = 0.30, size = 285, normalized size = 1.60

$$48\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-48\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+9\left(a^2-4\,b^2\right)(dx+c)-\frac{48\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/24*(48*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 48*a*b*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) + 9*(a^2 - 4*b^2)*(d*x + c) - 48*b^2*tan(1/2*d*x + 1/2*c)/
(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(9*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*a*b*tan
(1/2*d*x + 1/2*c)^7 - 12*b^2*tan(1/2*d*x + 1/2*c)^7 + 33*a^2*tan(1/2*d*x +
1/2*c)^5 - 208*a*b*tan(1/2*d*x + 1/2*c)^5 - 12*b^2*tan(1/2*d*x + 1/2*c)^5 -
 33*a^2*tan(1/2*d*x + 1/2*c)^3 - 208*a*b*tan(1/2*d*x + 1/2*c)^3 + 12*b^2*ta
n(1/2*d*x + 1/2*c)^3 - 9*a^2*tan(1/2*d*x + 1/2*c) - 48*a*b*tan(1/2*d*x + 1/
2*c) + 12*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.64, size = 187, normalized size = 1.05

$$-\frac{a^2\cos\left(dx+c\right)\left(\sin^3\left(dx+c\right)\right)}{4d}-\frac{3a^2\cos\left(dx+c\right)\sin\left(dx+c\right)}{8d}+\frac{3a^2x}{8}+\frac{3a^2c}{8d}-\frac{2ab\left(\sin^3\left(dx+c\right)\right)}{3d}-\frac{2ab\sin\left(dx+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c)^4,x)
```

```
[Out] -1/4/d*a^2*cos(d*x+c)*sin(d*x+c)^3-3/8*a^2*cos(d*x+c)*sin(d*x+c)/d+3/8*a^2*
x+3/8/d*a^2*c-2/3*a*b*sin(d*x+c)^3/d-2*a*b*sin(d*x+c)/d+2/d*a*b*ln(sec(d*x+
c)+tan(d*x+c))+1/d*b^2*sin(d*x+c)^5/cos(d*x+c)+1/d*b^2*cos(d*x+c)*sin(d*x+c
)^3+3/2/d*b^2*cos(d*x+c)*sin(d*x+c)-3/2*b^2*x-3/2/d*c*b^2
```

maxima [A] time = 0.88, size = 125, normalized size = 0.70

$$\frac{3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)-8\,\sin\left(2\,dx+2\,c\right)\right)a^2-32\left(2\,\sin\left(dx+c\right)^3-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a^2 - 32*(2
*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin
(d*x + c))*a*b - 48*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*ta
n(d*x + c))*b^2)/d
```

mupad [B] time = 1.23, size = 207, normalized size = 1.16

$$\frac{3\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4\,d}-\frac{3\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^2\,{\cos\left(c+d\,x\right)}^3\,\sin\left(c+d\,x\right)}{4\,d}+\frac{b^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}-\frac{8\,a\,b\,\sin\left(c+d\,x\right)}{3\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (3*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(4*d) - (3*b^2*atan(sin
(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a^2*cos(c + d*x)^3*sin(c + d*x))/
(4*d) + (b^2*sin(c + d*x))/(d*cos(c + d*x)) - (8*a*b*sin(c + d*x))/(3*d) +
(4*a*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (5*a^2*cos(c + d*x
)*sin(c + d*x))/(8*d) + (b^2*cos(c + d*x)*sin(c + d*x))/(2*d) + (2*a*b*cos(
c + d*x)^2*sin(c + d*x))/(3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b \sec (c + dx))^2 \sin^4 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**4,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sin(c + d*x)**4, x)
```

3.181 $\int (a+b\sec(c+dx))^2\sin^2(c+dx)\,dx$

Optimal. Leaf size=77

$$-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^2x}{2}-\frac{2ab\sin(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}-b^2x$$

```
[Out] 1/2*a^2*x-b^2*x+2*a*b*arctanh(sin(d*x+c))/d-2*a*b*sin(d*x+c)/d-1/2*a^2*cos(
d*x+c)*sin(d*x+c)/d+b^2*tan(d*x+c)/d
```

Rubi [A] time = 0.13, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2722, 2635, 8, 2592, 321, 206, 3473}

$$-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^2x}{2}-\frac{2ab\sin(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}-b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^2,x]
```

```
[Out] (a^2*x)/2 - b^2*x + (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Sin[c + d*x])/
d - (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (b^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
```

```
&& IGtQ[m, 0]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^2\sin^2(c+dx)\,dx &= \int (-b-a\cos(c+dx))^2\tan^2(c+dx)\,dx\\ &= \int \left(a^2\sin^2(c+dx)+2ab\sin(c+dx)\tan(c+dx)+b^2\tan^2(c+dx)\right)\,dx\\ &= a^2\int \sin^2(c+dx)\,dx+(2ab)\int \sin(c+dx)\tan(c+dx)\,dx+b^2\int \tan^2(c\\ &= -\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{b^2\tan(c+dx)}{d}+\frac{1}{2}a^2\int 1\,dx-b^2\int 1\,dx\\ &= \frac{a^2x}{2}-b^2x-\frac{2ab\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{b^2\tan(c+d\\ &= \frac{a^2x}{2}-b^2x+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.60, size = 121, normalized size = 1.57

$$-\frac{a^2\sin(2(c+dx))-2a^2c-2a^2dx+8ab\sin(c+dx)+8ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-8ab\log\left(\sin\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^2,x]
```

```
[Out] -1/4*(-2*a^2*c + 4*b^2*c - 2*a^2*d*x + 4*b^2*d*x + 8*a*b*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]] - 8*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 8
*a*b*Sin[c + d*x] + a^2*Sin[2*(c + d*x)] - 4*b^2*Tan[c + d*x])/d
```

fricas [A] time = 0.50, size = 108, normalized size = 1.40

$$\frac{\left(a^2-2\,b^2\right)dx\cos\left(dx+c\right)+2\,ab\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-2\,ab\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)-\left(a\right.}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 - 2*b^2)*d*x*cos(d*x + c) + 2*a*b*cos(d*x + c)*log(sin(d*x + c) +
 1) - 2*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) - (a^2*cos(d*x + c)^2 + 4*a
*b*cos(d*x + c) - 2*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

giac [B] time = 0.28, size = 159, normalized size = 2.06

$$\frac{4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\left(a^2-2\,b^2\right)(dx+c)-\frac{4\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\left(\cdots\right.}{\cdots}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 4*a*b*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + (a^2 - 2*b^2)*(d*x + c) - 4*b^2*tan(1/2*d*x + 1/2*c)/(tan(1
/2*d*x + 1/2*c)^2 - 1) + 2*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x
+ 1/2*c)^3 - a^2*tan(1/2*d*x + 1/2*c) - 4*a*b*tan(1/2*d*x + 1/2*c))/(tan(1/
2*d*x + 1/2*c)^2 + 1)^2)/d
```

maple [A] time = 0.39, size = 99, normalized size = 1.29

$$-\frac{a^2\cos(dx+c)\sin(dx+c)}{2d}+\frac{a^2x}{2}+\frac{a^2c}{2d}-\frac{2ab\sin(dx+c)}{d}+\frac{2ab\ln(\sec(dx+c)+\tan(dx+c))}{d}-b^2x+\frac{b^2\tan(d\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c)^2,x)
```

```
[Out] -1/2*a^2*cos(d*x+c)*sin(d*x+c)/d+1/2*a^2*x+1/2/d*a^2*c-2*a*b*sin(d*x+c)/d+2
/d*a*b*ln(sec(d*x+c)+tan(d*x+c))-b^2*x+b^2*tan(d*x+c)/d-1/d*c*b^2
```

maxima [A] time = 0.91, size = 80, normalized size = 1.04

$$\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))a^2-4\,(dx+c-\tan(dx+c))b^2+4\,ab\left(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-\cdots\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a^2 - 4*(d*x + c - tan(d*x + c))*b^2
+ 4*a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

mupad [B] time = 1.17, size = 143, normalized size = 1.86

$$\frac{a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b^2\,\sin(c+d\,x)}{d\,\cos(c+d\,x)}-\frac{2\,a\,b\,\sin(c+d\,x)}{d}+\frac{4\,a\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{a^2\,\mathrm{c}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (2*b^2*atan(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^2*sin(c + d*x))/(d*cos(c + d*x)) - (2
*a*b*sin(c + d*x))/d + (4*a*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d - (a^2*cos(c + d*x)*sin(c + d*x))/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^2\sin^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sin(c + d*x)**2, x)
```

3.182 $\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=59

$$-\frac{\left(a^2+b^2\right)\cot(c+dx)}{d}-\frac{2ab\csc(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] 2*a*b*arctanh(sin(d*x+c))/d-(a^2+b^2)*cot(d*x+c)/d-2*a*b*csc(d*x+c)/d+b^2*t
an(d*x+c)/d
```

Rubi [A] time = 0.41, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3872, 2911, 2621, 321, 207, 14}

$$-\frac{\left(a^2+b^2\right)\cot(c+dx)}{d}-\frac{2ab\csc(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - ((a^2 + b^2)*Cot[c + d*x])/d - (2*a*b*Csc
[c + d*x])/d + (b^2*Tan[c + d*x])/d
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx = \int (-b-a\cos(c+dx))^2\csc^2(c+dx)\sec^2(c+dx)\,dx$$
$$= (2ab)\int \csc^2(c+dx)\sec(c+dx)\,dx + \int \left(b^2+a^2\cos^2(c+dx)\right)\csc^2(c$$
$$= \frac{\mathrm{Subst}\left(\int \frac{a^2+b^2+b^2x^2}{x^2}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\,\mathrm{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, c\right.}{d}$$
$$= -\frac{2ab\csc(c+dx)}{d} + \frac{\mathrm{Subst}\left(\int \left(b^2+\frac{a^2+b^2}{x^2}\right)dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab}{}$$
$$= \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} - \frac{\left(a^2+b^2\right)\cot(c+dx)}{d} - \frac{2ab\csc(c+dx)}{d} +$$

Mathematica [B] time = 0.49, size = 138, normalized size = 2.34

$$-\frac{\csc^3\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\left(a^2+2b^2\right)\cos(2(c+dx))+4ab\cos(c+dx)+a\left(a+2b\sin(2(c+dx))\right)\left(\log\right.\right.}{4d\left(\cot^2\left(\frac{1}{2}(c+dx)\right)-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/4*(Csc[(c + d*x)/2]^3*Sec[(c + d*x)/2]*(4*a*b*Cos[c + d*x] + (a^2 + 2*b^
2)*Cos[2*(c + d*x)] + a*(a + 2*b*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
- Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Sin[2*(c + d*x)])))/(d*(-1 + Co
t[(c + d*x)/2]^2))
```

fricas [A] time = 0.48, size = 104, normalized size = 1.76

$$\frac{ab\cos(dx+c)\log(\sin(dx+c)+1)\sin(dx+c) - ab\cos(dx+c)\log(-\sin(dx+c)+1)\sin(dx+c) - 2\,ab\,\mathrm{co}}{d\cos(dx+c)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a*b*cos(d*x + c)*log(sin(d*x + c) + 1)*sin(d*x + c) - a*b*cos(d*x + c)*log
(-sin(d*x + c) + 1)*sin(d*x + c) - 2*a*b*cos(d*x + c) - (a^2 + 2*b^2)*cos(d
*x + c)^2 + b^2)/(d*cos(d*x + c)*sin(d*x + c))
```

giac [B] time = 1.22, size = 167, normalized size = 2.83

$$\frac{4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 4*a*b*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + a^2*tan(1/2*d*x + 1/2*c) - 2*a*b*tan(1/2*d*x + 1/2*c) + b^2
*tan(1/2*d*x + 1/2*c) - (a^2*tan(1/2*d*x + 1/2*c)^2 + 2*a*b*tan(1/2*d*x + 1
/2*c)^2 + 5*b^2*tan(1/2*d*x + 1/2*c)^2 - a^2 - 2*a*b - b^2)/(tan(1/2*d*x +
1/2*c)^3 - tan(1/2*d*x + 1/2*c)))/d
```

maple [A] time = 0.65, size = 89, normalized size = 1.51

$$-\frac{a^2\cot(dx+c)}{d}-\frac{2ab}{d\sin(dx+c)}+\frac{2ab\ln(\sec(dx+c)+\tan(dx+c))}{d}+\frac{b^2}{d\sin(dx+c)\cos(dx+c)}-\frac{2b^2\cot(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -a^2*cot(d*x+c)/d-2/d*a*b/sin(d*x+c)+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*
b^2/sin(d*x+c)/cos(d*x+c)-2/d*b^2*cot(d*x+c)
```

maxima [A] time = 0.54, size = 73, normalized size = 1.24

$$-\frac{ab\left(\frac{2}{\sin(dx+c)}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+b^2\left(\frac{1}{\tan(dx+c)}-\tan(dx+c)\right)+\frac{a^2}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(a*b*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + b^
2*(1/tan(d*x + c) - tan(d*x + c)) + a^2/tan(d*x + c))/d
```

mupad [B] time = 1.07, size = 108, normalized size = 1.83

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{(a-b)}^2}{2\,d}-\frac{2\,a\,b+a^2+b^2-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(a^2+2\,a\,b+5\,b^2\right)}{d\,\left(2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\right)}+\frac{4\,a\,b\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/sin(c + d*x)^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(a - b)^2)/(2*d) - (2*a*b + a^2 + b^2 - tan(c/2 + (d*x)
/2)^2*(2*a*b + a^2 + 5*b^2))/(d*(2*tan(c/2 + (d*x)/2) - 2*tan(c/2 + (d*x)/2
)^3)) + (4*a*b*atanh(tan(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^2\csc^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*csc(c + d*x)**2, x)
```

3.183 $\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=100

$$-\frac{(a^2+b^2)\cot^3(c+dx)}{3d}-\frac{(a^2+2b^2)\cot(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] 2*a*b*arctanh(sin(d*x+c))/d-(a^2+2*b^2)*cot(d*x+c)/d-1/3*(a^2+b^2)*cot(d*x+
c)^3/d-2*a*b*csc(d*x+c)/d-2/3*a*b*csc(d*x+c)^3/d+b^2*tan(d*x+c)/d
```

Rubi [A] time = 0.32, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2911, 2621, 302, 207, 448}

$$-\frac{(a^2+b^2)\cot^3(c+dx)}{3d}-\frac{(a^2+2b^2)\cot(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - ((a^2 + 2*b^2)*Cot[c + d*x])/d - ((a^2 +
b^2)*Cot[c + d*x]^3)/(3*d) - (2*a*b*Csc[c + d*x])/d - (2*a*b*Csc[c + d*x]^3
)/(3*d) + (b^2*Tan[c + d*x])/d
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx = \int (-b-a\cos(c+dx))^2\csc^4(c+dx)\sec^2(c+dx)\,dx$$
$$= (2ab)\int \csc^4(c+dx)\sec(c+dx)\,dx + \int \left(b^2+a^2\cos^2(c+dx)\right)\csc^4(c+$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)(a^2+b^2+b^2x^2)}{x^4}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx\right.}{d}$$
$$= \frac{\text{Subst}\left(\int \left(b^2+\frac{a^2+b^2}{x^4}+\frac{a^2+2b^2}{x^2}\right)dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\,\text{Subst}\left(\int \left(1+\right.\right.}{}$$
$$= -\frac{(a^2+2b^2)\cot(c+dx)}{d} - \frac{(a^2+b^2)\cot^3(c+dx)}{3d} - \frac{2ab\csc(c+dx)}{d} - \frac{2a}{}$$
$$= \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} - \frac{(a^2+2b^2)\cot(c+dx)}{d} - \frac{(a^2+b^2)\cot^3(c+d}{3d}$$

Mathematica [B] time = 0.66, size = 259, normalized size = 2.59

$$\frac{\csc^5\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-2\left(a^2+4b^2\right)\cos(2(c+dx))+a^2\cos(4(c+dx))-3a^2-14ab\cos(c+dx)+6a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (Csc[(c + d*x)/2]^5*Sec[(c + d*x)/2]^3*(-3*a^2 - 14*a*b*Cos[c + d*x] - 2*(a
^2 + 4*b^2)*Cos[2*(c + d*x)] + 6*a*b*Cos[3*(c + d*x)] + a^2*Cos[4*(c + d*x)
] + 4*b^2*Cos[4*(c + d*x)] - 6*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
*Sin[2*(c + d*x)] + 6*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[2*(c
 + d*x)] + 3*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[4*(c + d*x)]
- 3*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[4*(c + d*x)]))/(96*d*(
-1 + Cot[(c + d*x)/2]^2))
```

fricas [A] time = 0.55, size = 178, normalized size = 1.78

$$-\frac{6\,ab\cos(dx+c)^3+2\left(a^2+4\,b^2\right)\cos(dx+c)^4-8\,ab\cos(dx+c)-3\left(a^2+4\,b^2\right)\cos(dx+c)^2-3\left(ab\cos(dx+\right.}{3\left(d\cos(d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a*b*cos(d*x + c)^3 + 2*(a^2 + 4*b^2)*cos(d*x + c)^4 - 8*a*b*cos(d*x
 + c) - 3*(a^2 + 4*b^2)*cos(d*x + c)^2 - 3*(a*b*cos(d*x + c)^3 - a*b*cos(d*
x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + 3*(a*b*cos(d*x + c)^3 - a*b*co
s(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*x + c) + 3*b^2)/((d*cos(d*x + c)^3
 - d*cos(d*x + c))*sin(d*x + c))
```

giac [B] time = 1.13, size = 226, normalized size = 2.26

$$a^2 \tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 2\,ab\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + b^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + 48\,ab\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right) - 48\,ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 - 2*a*b*tan(1/2*d*x + 1/2*c)^3 + b^2*tan(1
/2*d*x + 1/2*c)^3 + 48*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 48*a*b*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) + 9*a^2*tan(1/2*d*x + 1/2*c) - 30*a*b*tan(1/
2*d*x + 1/2*c) + 21*b^2*tan(1/2*d*x + 1/2*c) - 48*b^2*tan(1/2*d*x + 1/2*c)/
(tan(1/2*d*x + 1/2*c)^2 - 1) - (9*a^2*tan(1/2*d*x + 1/2*c)^2 + 30*a*b*tan(1
/2*d*x + 1/2*c)^2 + 21*b^2*tan(1/2*d*x + 1/2*c)^2 + a^2 + 2*a*b + b^2)/tan(
1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.92, size = 151, normalized size = 1.51

$$-\frac{2a^2\cot(dx+c)}{3d} - \frac{a^2\cot(dx+c)\left(\csc^2(dx+c)\right)}{3d} - \frac{2ab}{3d\sin(dx+c)^3} - \frac{2ab}{d\sin(dx+c)} + \frac{2ab\ln(\sec(dx+c)+\tan(}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/3*a^2*cot(d*x+c)/d-1/3/d*a^2*cot(d*x+c)*csc(d*x+c)^2-2/3/d*a*b/sin(d*x+c
)^3-2/d*a*b/sin(d*x+c)+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c))-1/3/d*b^2/sin(d*x+
c)^3/cos(d*x+c)+4/3/d*b^2/sin(d*x+c)/cos(d*x+c)-8/3/d*b^2*cot(d*x+c)
```

maxima [A] time = 0.75, size = 112, normalized size = 1.12

$$-\frac{ab\left(\frac{2\left(3\sin(dx+c)^2+1\right)}{\sin(dx+c)^3} - 3\log(\sin(dx+c)+1) + 3\log(\sin(dx+c)-1)\right) + b^2\left(\frac{6\tan(dx+c)^2+1}{\tan(dx+c)^3} - 3\tan(dx+c)\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(a*b*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)) + b^2*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 -
 3*tan(d*x + c)) + (3*tan(d*x + c)^2 + 1)*a^2/tan(d*x + c)^3)/d
```

mupad [B] time = 1.10, size = 182, normalized size = 1.82

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(a-b)^2}{24\,d} - \frac{\frac{2ab}{3} - \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\left(3a^2+10ab+23b^2\right) + \tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{8a^2}{3}+\frac{28ab}{3}+\frac{20b^2}{3}\right) + \frac{a^2}{3}}{d\left(8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3 - 8\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/sin(c + d*x)^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(a - b)^2)/(24*d) - ((2*a*b)/3 - tan(c/2 + (d*x)/2)^4
*(10*a*b + 3*a^2 + 23*b^2) + tan(c/2 + (d*x)/2)^2*((28*a*b)/3 + (8*a^2)/3 +
 (20*b^2)/3) + a^2/3 + b^2/3)/(d*(8*tan(c/2 + (d*x)/2)^3 - 8*tan(c/2 + (d*x
)/2)^5)) + (tan(c/2 + (d*x)/2)*(a^2/8 - (3*a*b)/4 + (5*b^2)/8 + (a - b)^2/4
))/d + (4*a*b*atanh(tan(c/2 + (d*x)/2)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^2 \csc^4(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*csc(c + d*x)**4, x)
```

3.184 $\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=143

$$-\frac{\left(a^2+b^2\right)\cot^5(c+dx)}{5d}-\frac{\left(2a^2+3b^2\right)\cot^3(c+dx)}{3d}-\frac{\left(a^2+3b^2\right)\cot(c+dx)}{d}-\frac{2ab\csc^5(c+dx)}{5d}-\frac{2ab\csc^3(c+dx)}{3d}$$

```
[Out] 2*a*b*arctanh(sin(d*x+c))/d-(a^2+3*b^2)*cot(d*x+c)/d-1/3*(2*a^2+3*b^2)*cot(
d*x+c)^3/d-1/5*(a^2+b^2)*cot(d*x+c)^5/d-2*a*b*csc(d*x+c)/d-2/3*a*b*csc(d*x+
c)^3/d-2/5*a*b*csc(d*x+c)^5/d+b^2*tan(d*x+c)/d
```

Rubi [A] time = 0.41, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2911, 2621, 302, 207, 448}

$$-\frac{\left(a^2+b^2\right)\cot^5(c+dx)}{5d}-\frac{\left(2a^2+3b^2\right)\cot^3(c+dx)}{3d}-\frac{\left(a^2+3b^2\right)\cot(c+dx)}{d}-\frac{2ab\csc^5(c+dx)}{5d}-\frac{2ab\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - ((a^2 + 3*b^2)*Cot[c + d*x])/d - ((2*a^2
+ 3*b^2)*Cot[c + d*x]^3)/(3*d) - ((a^2 + b^2)*Cot[c + d*x]^5)/(5*d) - (2*a*
b*Csc[c + d*x])/d - (2*a*b*Csc[c + d*x]^3)/(3*d) - (2*a*b*Csc[c + d*x]^5)/(
5*d) + (b^2*Tan[c + d*x])/d
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx = \int (-b-a\cos(c+dx))^2\csc^6(c+dx)\sec^2(c+dx)\,dx$$
$$= (2ab)\int \csc^6(c+dx)\sec(c+dx)\,dx + \int \left(b^2+a^2\cos^2(c+dx)\right)\csc^6(c+$$
$$= \frac{\text{Subst}\left(\int \frac{(1+x^2)^2(a^2+b^2+b^2x^2)}{x^6}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\,\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx\right.}{d}$$
$$= \frac{\text{Subst}\left(\int \left(b^2+\frac{a^2+b^2}{x^6}+\frac{2a^2+3b^2}{x^4}+\frac{a^2+3b^2}{x^2}\right)dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\,\text{Sub}}{}$$
$$= -\frac{(a^2+3b^2)\cot(c+dx)}{d} - \frac{(2a^2+3b^2)\cot^3(c+dx)}{3d} - \frac{(a^2+b^2)\cot^5(c+}{5d}$$
$$= \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} - \frac{(a^2+3b^2)\cot(c+dx)}{d} - \frac{(2a^2+3b^2)\cot^3(c+}{3d}$$

Mathematica [B] time = 0.74, size = 368, normalized size = 2.57

$$-\frac{\csc^7\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(20\left(a^2+6b^2\right)\cos(2(c+dx))-16a^2\cos(4(c+dx))+4a^2\cos(6(c+dx))+40a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/7680*(Csc[(c + d*x)/2]^7*Sec[(c + d*x)/2]^5*(40*a^2 + 196*a*b*Cos[c + d*
x] + 20*(a^2 + 6*b^2)*Cos[2*(c + d*x)] - 130*a*b*Cos[3*(c + d*x)] - 16*a^2*
Cos[4*(c + d*x)] - 96*b^2*Cos[4*(c + d*x)] + 30*a*b*Cos[5*(c + d*x)] + 4*a^
2*Cos[6*(c + d*x)] + 24*b^2*Cos[6*(c + d*x)] + 75*a*b*Log[Cos[(c + d*x)/2]
- Sin[(c + d*x)/2]]*Sin[2*(c + d*x)] - 75*a*b*Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]]*Sin[2*(c + d*x)] - 60*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]]*Sin[4*(c + d*x)] + 60*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[
4*(c + d*x)] + 15*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[6*(c + d
*x)] - 15*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[6*(c + d*x)]))/(
d*(-1 + Cot[(c + d*x)/2]^2))
```

fricas [A] time = 0.53, size = 241, normalized size = 1.69

$$-\frac{30\,ab\cos(dx+c)^5+8\left(a^2+6\,b^2\right)\cos(dx+c)^6-70\,ab\cos(dx+c)^3-20\left(a^2+6\,b^2\right)\cos(dx+c)^4+46\,ab\cos(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(30*a*b*cos(d*x + c)^5 + 8*(a^2 + 6*b^2)*cos(d*x + c)^6 - 70*a*b*cos(
d*x + c)^3 - 20*(a^2 + 6*b^2)*cos(d*x + c)^4 + 46*a*b*cos(d*x + c) + 15*(a^
2 + 6*b^2)*cos(d*x + c)^2 - 15*(a*b*cos(d*x + c)^5 - 2*a*b*cos(d*x + c)^3 +
```

```
 a*b*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + 15*(a*b*cos(d*x + c
)^5 - 2*a*b*cos(d*x + c)^3 + a*b*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d
*x + c) - 15*b^2)/((d*cos(d*x + c)^5 - 2*d*cos(d*x + c)^3 + d*cos(d*x + c))
*sin(d*x + c))
```

giac [B] time = 0.33, size = 326, normalized size = 2.28

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+25\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-70\,ab\tan\left(\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*a*b*tan(1/2*d*x + 1/2*c)^5 + 3*b^2*
tan(1/2*d*x + 1/2*c)^5 + 25*a^2*tan(1/2*d*x + 1/2*c)^3 - 70*a*b*tan(1/2*d*x
 + 1/2*c)^3 + 45*b^2*tan(1/2*d*x + 1/2*c)^3 + 960*a*b*log(abs(tan(1/2*d*x +
 1/2*c) + 1)) - 960*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 150*a^2*tan(1/
2*d*x + 1/2*c) - 660*a*b*tan(1/2*d*x + 1/2*c) + 570*b^2*tan(1/2*d*x + 1/2*c
) - 960*b^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (150*a^2*ta
n(1/2*d*x + 1/2*c)^4 + 660*a*b*tan(1/2*d*x + 1/2*c)^4 + 570*b^2*tan(1/2*d*x
 + 1/2*c)^4 + 25*a^2*tan(1/2*d*x + 1/2*c)^2 + 70*a*b*tan(1/2*d*x + 1/2*c)^2
 + 45*b^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2 + 6*a*b + 3*b^2)/tan(1/2*d*x + 1/2
*c)^5)/d
```

maple [A] time = 0.98, size = 212, normalized size = 1.48

$$-\frac{8a^2\cot(dx+c)}{15d}-\frac{a^2\cot(dx+c)\left(\csc^4(dx+c)\right)}{5d}-\frac{4a^2\cot(dx+c)\left(\csc^2(dx+c)\right)}{15d}-\frac{2ab}{5d\sin(dx+c)^5}-\frac{2ab}{3d\sin(d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -8/15*a^2*cot(d*x+c)/d-1/5/d*a^2*cot(d*x+c)*csc(d*x+c)^4-4/15/d*a^2*cot(d*x
+c)*csc(d*x+c)^2-2/5/d*a*b/sin(d*x+c)^5-2/3/d*a*b/sin(d*x+c)^3-2/d*a*b/sin(
d*x+c)+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c))-1/5/d*b^2/sin(d*x+c)^5/cos(d*x+c)-
2/5/d*b^2/sin(d*x+c)^3/cos(d*x+c)+8/5/d*b^2/sin(d*x+c)/cos(d*x+c)-16/5/d*b^
2*cot(d*x+c)
```

maxima [A] time = 0.71, size = 143, normalized size = 1.00

$$-\frac{ab\left(\frac{2\left(15\,\sin(dx+c)^4+5\,\sin(dx+c)^2+3\right)}{\sin(dx+c)^5}-15\,\log(\sin(dx+c)+1)+15\,\log(\sin(dx+c)-1)\right)+3\,b^2\left(\frac{15\,\tan(dx+c)^4+5\,\mathrm{ta}}{\tan(dx+c)}\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/15*(a*b*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 1
5*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 3*b^2*((15*tan(d*x +
c)^4 + 5*tan(d*x + c)^2 + 1)/tan(d*x + c)^5 - 5*tan(d*x + c)) + (15*tan(d*x
 + c)^4 + 10*tan(d*x + c)^2 + 3)*a^2/tan(d*x + c)^5)/d
```

mupad [B] time = 1.04, size = 248, normalized size = 1.73

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(a-b)^2}{160\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{a^2}{32}-\frac{5\,a\,b}{48}+\frac{7\,b^2}{96}+\frac{(a-b)^2}{48}\right)}{d}-\frac{\frac{2\,a\,b}{5}+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{22\,a^2}{15}+\frac{64\,a\,b}{15}+\frac{14\,b^2}{5}\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^2/sin(c + d*x)^6,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(a - b)^2)/(160*d) + (tan(c/2 + (d*x)/2)^3*(a^2/32 -
(5*a*b)/48 + (7*b^2)/96 + (a - b)^2/48))/d - ((2*a*b)/5 + tan(c/2 + (d*x)/2
)^2*((64*a*b)/15 + (22*a^2)/15 + (14*b^2)/5) - tan(c/2 + (d*x)/2)^6*(44*a*b
 + 10*a^2 + 102*b^2) + tan(c/2 + (d*x)/2)^4*((118*a*b)/3 + (25*a^2)/3 + 35*
b^2) + a^2/5 + b^2/5)/(d*(32*tan(c/2 + (d*x)/2)^5 - 32*tan(c/2 + (d*x)/2)^7
)) + (tan(c/2 + (d*x)/2)*((7*a^2)/32 - (19*a*b)/16 + (35*b^2)/32 + (3*(a -
b)^2)/32))/d + (4*a*b*atanh(tan(c/2 + (d*x)/2)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.185 $\int (a+b\sec(c+dx))^3\sin^5(c+dx)\,dx$

Optimal. Leaf size=170

$$-\frac{a^3\cos^5(c+dx)}{5d}+\frac{a\left(2a^2-3b^2\right)\cos^3(c+dx)}{3d}+\frac{b\left(6a^2-b^2\right)\cos^2(c+dx)}{2d}-\frac{a\left(a^2-6b^2\right)\cos(c+dx)}{d}-\frac{b\left(3a^2-2\right.}{}$$

```
[Out] -a*(a^2-6*b^2)*cos(d*x+c)/d+1/2*b*(6*a^2-b^2)*cos(d*x+c)^2/d+1/3*a*(2*a^2-3
*b^2)*cos(d*x+c)^3/d-3/4*a^2*b*cos(d*x+c)^4/d-1/5*a^3*cos(d*x+c)^5/d-b*(3*a
^2-2*b^2)*ln(cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)^2/d
```

Rubi [A] time = 0.26, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3872, 2837, 12, 948}

$$\frac{a\left(2a^2-3b^2\right)\cos^3(c+dx)}{3d}+\frac{b\left(6a^2-b^2\right)\cos^2(c+dx)}{2d}-\frac{a\left(a^2-6b^2\right)\cos(c+dx)}{d}-\frac{b\left(3a^2-2b^2\right)\log(\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^5,x]
```

```
[Out] -((a*(a^2 - 6*b^2)*Cos[c + d*x])/d) + (b*(6*a^2 - b^2)*Cos[c + d*x]^2)/(2*d
) + (a*(2*a^2 - 3*b^2)*Cos[c + d*x]^3)/(3*d) - (3*a^2*b*Cos[c + d*x]^4)/(4*
d) - (a^3*Cos[c + d*x]^5)/(5*d) - (b*(3*a^2 - 2*b^2)*Log[Cos[c + d*x]])/d +
 (3*a*b^2*Sec[c + d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))^3\sin^5(c+dx)\,dx = -\int (-b-a\cos(c+dx))^3\sin^2(c+dx)\tan^3(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^3(-b+x)^3(a^2-x^2)^2}{x^3}dx, x, -a\cos(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-b+x)^3(a^2-x^2)^2}{x^3}dx, x, -a\cos(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(a^4\left(1-\frac{6b^2}{a^2}\right)-\frac{a^4b^3}{x^3}+\frac{3a^4b^2}{x^2}+\frac{-3a^4b+2a^2b^3}{x}-b\left(-6a^2+b^2\right)x-\left(2\right.\right.\right.}{a^2d}$$
$$= -\frac{a\left(a^2-6b^2\right)\cos(c+dx)}{d}+\frac{b\left(6a^2-b^2\right)\cos^2(c+dx)}{2d}+\frac{a\left(2a^2-3b^2\right)\text{cc}}{3d}$$

Mathematica [A] time = 0.69, size = 154, normalized size = 0.91

$$\frac{50a^3\cos(3(c+dx))-6a^3\cos(5(c+dx))+60\left(9a^2b-2b^3\right)\cos(2(c+dx))-60a\left(5a^2-42b^2\right)\cos(c+dx)-45a^2b}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^5,x]
```

```
[Out] (-60*a*(5*a^2 - 42*b^2)*Cos[c + d*x] + 60*(9*a^2*b - 2*b^3)*Cos[2*(c + d*x)
] + 50*a^3*Cos[3*(c + d*x)] - 120*a*b^2*Cos[3*(c + d*x)] - 45*a^2*b*Cos[4*(
c + d*x)] - 6*a^3*Cos[5*(c + d*x)] - 1440*a^2*b*Log[Cos[c + d*x]] + 960*b^3
*Log[Cos[c + d*x]] + 1440*a*b^2*Sec[c + d*x] + 240*b^3*Sec[c + d*x]^2)/(480
*d)
```

fricas [A] time = 0.53, size = 175, normalized size = 1.03

$$-\frac{96\,a^3\cos(dx+c)^7+360\,a^2b\cos(dx+c)^6-160\left(2\,a^3-3\,ab^2\right)\cos(dx+c)^5-240\left(6\,a^2b-b^3\right)\cos(dx+c)^4-1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/480*(96*a^3*cos(d*x + c)^7 + 360*a^2*b*cos(d*x + c)^6 - 160*(2*a^3 - 3*a
*b^2)*cos(d*x + c)^5 - 240*(6*a^2*b - b^3)*cos(d*x + c)^4 - 1440*a*b^2*cos(
d*x + c) + 480*(a^3 - 6*a*b^2)*cos(d*x + c)^3 + 480*(3*a^2*b - 2*b^3)*cos(d
*x + c)^2*log(-cos(d*x + c)) - 240*b^3 + 15*(39*a^2*b - 8*b^3)*cos(d*x + c)
^2)/(d*cos(d*x + c)^2)
```

giac [B] time = 0.58, size = 695, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*(3*a^2*b - 2*b^3)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 1)) - 60*(3*a^2*b - 2*b^3)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
- 1)) + 30*(9*a^2*b + 12*a*b^2 - 6*b^3 + 18*a^2*b*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 12*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 16*b^3*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x +
```

```
 c) + 1)^2 - 6*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 1)^2 + (64*a^3 + 411*a^2*b - 600*a*b^2 - 274*b^
3 - 320*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2415*a^2*b*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 2640*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) + 1490*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 640*a^3*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 + 5910*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 - 3840*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3100*b^3*(
cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 5910*a^2*b*(cos(d*x + c) - 1)^3/
(cos(d*x + c) + 1)^3 + 2160*a*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3
 + 3100*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2415*a^2*b*(cos(d*x
 + c) - 1)^4/(cos(d*x + c) + 1)^4 - 360*a*b^2*(cos(d*x + c) - 1)^4/(cos(d*x
 + c) + 1)^4 - 1490*b^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 411*a^2
*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 274*b^3*(cos(d*x + c) - 1)^5
/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)/d
```

maple [A] time = 0.66, size = 266, normalized size = 1.56

$$-\frac{8a^3\cos(dx+c)}{15d}-\frac{a^3\cos(dx+c)\left(\sin^4(dx+c)\right)}{5d}-\frac{4a^3\cos(dx+c)\left(\sin^2(dx+c)\right)}{15d}-\frac{3a^2b\left(\sin^4(dx+c)\right)}{4d}-\frac{3a^2b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c)^5,x)
```

```
[Out] -8/15*a^3*cos(d*x+c)/d-1/5/d*a^3*cos(d*x+c)*sin(d*x+c)^4-4/15/d*a^3*cos(d*x
+c)*sin(d*x+c)^2-3/4/d*a^2*b*sin(d*x+c)^4-3/2/d*a^2*b*sin(d*x+c)^2-3*a^2*b*
ln(cos(d*x+c))/d+3/d*b^2*a*sin(d*x+c)^6/cos(d*x+c)+8*a*b^2*cos(d*x+c)/d+3/d
*b^2*a*sin(d*x+c)^4*cos(d*x+c)+4/d*b^2*a*cos(d*x+c)*sin(d*x+c)^2+1/2/d*b^3*
sin(d*x+c)^6/cos(d*x+c)^2+1/2/d*b^3*sin(d*x+c)^4+1/d*sin(d*x+c)^2*b^3+2*b^3
*ln(cos(d*x+c))/d
```

maxima [A] time = 0.34, size = 142, normalized size = 0.84

$$-\frac{12\,a^3\cos(dx+c)^5+45\,a^2b\cos(dx+c)^4-20\left(2\,a^3-3\,ab^2\right)\cos(dx+c)^3-30\left(6\,a^2b-b^3\right)\cos(dx+c)^2+6}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(12*a^3*cos(d*x + c)^5 + 45*a^2*b*cos(d*x + c)^4 - 20*(2*a^3 - 3*a*b^
2)*cos(d*x + c)^3 - 30*(6*a^2*b - b^3)*cos(d*x + c)^2 + 60*(a^3 - 6*a*b^2)*
cos(d*x + c) + 60*(3*a^2*b - 2*b^3)*log(cos(d*x + c)) - 30*(6*a*b^2*cos(d*x
 + c) + b^3)/cos(d*x + c)^2)/d
```

mupad [B] time = 0.96, size = 143, normalized size = 0.84

$$-\frac{\cos(c+d\,x)^3\left(a\,b^2-\frac{2\,a^3}{3}\right)-\cos(c+d\,x)^2\left(3\,a^2\,b-\frac{b^3}{2}\right)+\ln(\cos(c+d\,x))\left(3\,a^2\,b-2\,b^3\right)-\frac{\frac{b^3}{2}+3\,a\,\cos(c+d\,x)\,b^2}{\cos(c+d\,x)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5*(a + b/cos(c + d*x))^3,x)
```

```
[Out] -(cos(c + d*x)^3*(a*b^2 - (2*a^3)/3) - cos(c + d*x)^2*(3*a^2*b - b^3/2) + l
og(cos(c + d*x))*(3*a^2*b - 2*b^3) - (b^3/2 + 3*a*b^2*cos(c + d*x))/cos(c +
 d*x)^2 - cos(c + d*x)*(6*a*b^2 - a^3) + (a^3*cos(c + d*x)^5)/5 + (3*a^2*b*
cos(c + d*x)^4)/4)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**5,x)
```

```
[Out] Timed out
```

3.186 $\int (a+b\sec(c+dx))^3\sin^3(c+dx)\,dx$

Optimal. Leaf size=116

$$\frac{a^3\cos^3(c+dx)}{3d}-\frac{a\left(a^2-3b^2\right)\cos(c+dx)}{d}-\frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d}+\frac{3a^2b\cos^2(c+dx)}{2d}+\frac{3ab^2\sec(c+dx)}{d}$$

```
[Out] -a*(a^2-3*b^2)*cos(d*x+c)/d+3/2*a^2*b*cos(d*x+c)^2/d+1/3*a^3*cos(d*x+c)^3/d
-b*(3*a^2-b^2)*ln(cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)^2/d
```

Rubi [A] time = 0.13, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3872, 2721, 894}

$$-\frac{a\left(a^2-3b^2\right)\cos(c+dx)}{d}-\frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d}+\frac{3a^2b\cos^2(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{3ab^2\sec(c+d}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^3,x]
```

```
[Out] -((a*(a^2 - 3*b^2)*Cos[c + d*x])/d) + (3*a^2*b*Cos[c + d*x]^2)/(2*d) + (a^3
*Cos[c + d*x]^3)/(3*d) - (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/d + (3*a*b^2*S
ec[c + d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int (a+b\sec(c+dx))^3\sin^3(c+dx)\,dx &= -\int (-b-a\cos(c+dx))^3\tan^3(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{(-b+x)^3\left(a^2-x^2\right)}{x^3}\,dx,x,-a\cos(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{3b^2}{a^2}\right)-\frac{a^2b^3}{x^3}+\frac{3a^2b^2}{x^2}+\frac{-3a^2b+b^3}{x}+3bx-x^2\right)dx,x,-a\right.}{d}\\
&= -\frac{a\left(a^2-3b^2\right)\cos(c+dx)}{d}+\frac{3a^2b\cos^2(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{b}{}
\end{aligned}$$

Mathematica [A] time = 0.35, size = 102, normalized size = 0.88

$$\frac{a^3\cos(3(c+dx))-9a\left(a^2-4b^2\right)\cos(c+dx)+9a^2b\cos(2(c+dx))-36a^2b\log(\cos(c+dx))+36ab^2\sec(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^3,x]
```

```
[Out] (-9*a*(a^2 - 4*b^2)*Cos[c + d*x] + 9*a^2*b*Cos[2*(c + d*x)] + a^3*Cos[3*(c
+ d*x)] - 36*a^2*b*Log[Cos[c + d*x]] + 12*b^3*Log[Cos[c + d*x]] + 36*a*b^2*
Sec[c + d*x] + 6*b^3*Sec[c + d*x]^2)/(12*d)
```

fricas [A] time = 0.51, size = 123, normalized size = 1.06

$$\frac{4\,a^3\cos\left(dx+c\right)^5+18\,a^2b\cos\left(dx+c\right)^4-9\,a^2b\cos\left(dx+c\right)^2+36\,ab^2\cos\left(dx+c\right)-12\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)^3}{12\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a^3*cos(d*x + c)^5 + 18*a^2*b*cos(d*x + c)^4 - 9*a^2*b*cos(d*x + c)
^2 + 36*a*b^2*cos(d*x + c) - 12*(a^3 - 3*a*b^2)*cos(d*x + c)^3 - 12*(3*a^2*
b - b^3)*cos(d*x + c)^2*log(-cos(d*x + c)) + 6*b^3)/(d*cos(d*x + c)^2)
```

giac [A] time = 0.40, size = 128, normalized size = 1.10

$$-\frac{\left(3\,a^2b-b^3\right)\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{6\,ab^2\cos\left(dx+c\right)+b^3}{2\,d\cos\left(dx+c\right)^2}+\frac{2\,a^3d^8\cos\left(dx+c\right)^3+9\,a^2bd^8\cos\left(dx+c\right)^2-6\,a^3d^8\cos\left(\right.}{6\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -(3*a^2*b - b^3)*log(abs(cos(d*x + c))/abs(d))/d + 1/2*(6*a*b^2*cos(d*x + c
) + b^3)/(d*cos(d*x + c)^2) + 1/6*(2*a^3*d^8*cos(d*x + c)^3 + 9*a^2*b*d^8*c
os(d*x + c)^2 - 6*a^3*d^8*cos(d*x + c) + 18*a*b^2*d^8*cos(d*x + c))/d^9
```

maple [A] time = 0.65, size = 164, normalized size = 1.41

$$-\frac{a^3\cos\left(dx+c\right)\left(\sin^2\left(dx+c\right)\right)}{3d}-\frac{2a^3\cos\left(dx+c\right)}{3d}-\frac{3a^2b\left(\sin^2\left(dx+c\right)\right)}{2d}-\frac{3a^2b\ln\left(\cos\left(dx+c\right)\right)}{d}+\frac{3b^2a\left(\sin^4\left(dx+\right.\right.}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c)^3,x)
```

```
[Out] -1/3/d*a^3*cos(d*x+c)*sin(d*x+c)^2-2/3*a^3*cos(d*x+c)/d-3/2/d*a^2*b*sin(d*x
+c)^2-3*a^2*b*ln(cos(d*x+c))/d+3/d*b^2*a*sin(d*x+c)^4/cos(d*x+c)+3/d*b^2*a*
cos(d*x+c)*sin(d*x+c)^2+6*a*b^2*cos(d*x+c)/d+1/2*b^3*tan(d*x+c)^2/d+b^3*ln(
cos(d*x+c))/d
```

maxima [A] time = 0.61, size = 98, normalized size = 0.84

$$\frac{2\,a^3\cos\left(dx+c\right)^3+9\,a^2b\cos\left(dx+c\right)^2-6\left(a^3-3\,ab^2\right)\cos\left(dx+c\right)-6\left(3\,a^2b-b^3\right)\log\left(\cos\left(dx+c\right)\right)+\frac{3\left(6\,ab^2\cos\right.}{\cos(d}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^3*cos(d*x + c)^3 + 9*a^2*b*cos(d*x + c)^2 - 6*(a^3 - 3*a*b^2)*cos(
d*x + c) - 6*(3*a^2*b - b^3)*log(cos(d*x + c)) + 3*(6*a*b^2*cos(d*x + c) +
b^3)/cos(d*x + c)^2)/d
```

mupad [B] time = 0.07, size = 99, normalized size = 0.85

$$\frac{\frac{\frac{b^3}{2}+3\,a\,\cos(c+d\,x)\,b^2}{\cos(c+d\,x)^2}-\ln\left(\cos\left(c+d\,x\right)\right)\,\left(3\,a^2\,b-b^3\right)+\cos\left(c+d\,x\right)\,\left(3\,a\,b^2-a^3\right)+\frac{a^3\,\cos(c+d\,x)^3}{3}+\frac{3\,a^2\,b\,\cos(c+d\,x)^2}{2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + b/cos(c + d*x))^3,x)
```

```
[Out] ((b^3/2 + 3*a*b^2*cos(c + d*x))/cos(c + d*x)^2 - log(cos(c + d*x))*(3*a^2*b
 - b^3) + cos(c + d*x)*(3*a*b^2 - a^3) + (a^3*cos(c + d*x)^3)/3 + (3*a^2*b*
cos(c + d*x)^2)/2)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c + dx))^3 \sin^3(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*sin(c + d*x)**3, x)
```

3.187 $\int (a+b\sec(c+dx))^3\sin(c+dx)\,dx$

Optimal. Leaf size=64

$$-\frac{a^3\cos(c+dx)}{d}-\frac{3a^2b\log(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\sec^2(c+dx)}{2d}$$

```
[Out] -a^3*cos(d*x+c)/d-3*a^2*b*ln(cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec
(d*x+c)^2/d
```

Rubi [A] time = 0.10, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 2833, 12, 43}

$$-\frac{3a^2b\log(\cos(c+dx))}{d}-\frac{a^3\cos(c+dx)}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x],x]
```

```
[Out] -((a^3*Cos[c + d*x])/d) - (3*a^2*b*Log[Cos[c + d*x]])/d + (3*a*b^2*Sec[c +
d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))^3\sin(c+dx)\,dx = -\int (-b-a\cos(c+dx))^3\sec^2(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^3(-b+x)^3}{x^3}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{(-b+x)^3}{x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(1-\frac{b^3}{x^3}+\frac{3b^2}{x^2}-\frac{3b}{x}\right)dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^3\cos(c+dx)}{d}-\frac{3a^2b\log(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\sec^2}{2}$$

Mathematica [A] time = 0.12, size = 56, normalized size = 0.88

$$\frac{b\left(-6a^2\log(\cos(c+dx))+6ab\sec(c+dx)+b^2\sec^2(c+dx)\right)-2a^3\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x],x]
```

```
[Out] (-2*a^3*Cos[c + d*x] + b*(-6*a^2*Log[Cos[c + d*x]] + 6*a*b*Sec[c + d*x] + b
^2*Sec[c + d*x]^2))/(2*d)
```

fricas [A] time = 0.54, size = 67, normalized size = 1.05

$$-\frac{2\,a^3\cos\left(dx+c\right)^3+6\,a^2b\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)-6\,ab^2\cos\left(dx+c\right)-b^3}{2\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*cos(d*x + c)^3 + 6*a^2*b*cos(d*x + c)^2*log(-cos(d*x + c)) - 6*
a*b^2*cos(d*x + c) - b^3)/(d*cos(d*x + c)^2)
```

giac [A] time = 1.56, size = 66, normalized size = 1.03

$$-\frac{a^3\cos\left(dx+c\right)}{d}-\frac{3\,a^2b\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{6\,ab^2\cos\left(dx+c\right)+b^3}{2\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -a^3*cos(d*x + c)/d - 3*a^2*b*log(abs(cos(d*x + c))/abs(d))/d + 1/2*(6*a*b^
2*cos(d*x + c) + b^3)/(d*cos(d*x + c)^2)
```

maple [A] time = 0.19, size = 65, normalized size = 1.02

$$\frac{b^3\left(\sec^2\left(dx+c\right)\right)}{2d}+\frac{3a\,b^2\sec\left(dx+c\right)}{d}+\frac{3a^2b\ln\left(\sec\left(dx+c\right)\right)}{d}-\frac{a^3}{d\sec\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c),x)
```

```
[Out] 1/2*b^3*sec(d*x+c)^2/d+3*a*b^2*sec(d*x+c)/d+3/d*a^2*b*ln(sec(d*x+c))-1/d*a^
3/sec(d*x+c)
```

maxima [A] time = 1.18, size = 57, normalized size = 0.89

$$-\frac{2\,a^3\cos\left(dx+c\right)+6\,a^2b\log\left(\cos\left(dx+c\right)\right)-\frac{6\,ab^2}{\cos(dx+c)}-\frac{b^3}{\cos(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^3*cos(d*x + c) + 6*a^2*b*log(cos(d*x + c)) - 6*a*b^2/cos(d*x + c)
 - b^3/cos(d*x + c)^2)/d
```

mupad [B] time = 0.93, size = 57, normalized size = 0.89

$$-\frac{a^3\,\cos\left(c+d\,x\right)-\frac{\frac{b^3}{2}+3\,a\,\cos(c+d\,x)\,b^2}{\cos(c+d\,x)^2}+3\,a^2\,b\,\ln\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b/cos(c + d*x))^3,x)
```

```
[Out] -(a^3*cos(c + d*x) - (b^3/2 + 3*a*b^2*cos(c + d*x))/cos(c + d*x)^2 + 3*a^2*
b*log(cos(c + d*x)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^3\sin\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*sin(c + d*x), x)
```

3.188 $\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=102

$$-\frac{b\left(3a^2+b^2\right)\log(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}-\frac{(a-b)^3\log(\cos(c+dx)+1)}{2d}+\frac{(a+b)^3\log(1-\cos(c+dx)}{2d}$$

```
[Out] 1/2*(a+b)^3*ln(1-cos(d*x+c))/d-b*(3*a^2+b^2)*ln(cos(d*x+c))/d-1/2*(a-b)^3*l
n(1+cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)^2/d
```

Rubi [A] time = 0.22, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 2837, 12, 1802}

$$-\frac{b\left(3a^2+b^2\right)\log(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}-\frac{(a-b)^3\log(\cos(c+dx)+1)}{2d}+\frac{(a+b)^3\log(1-\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((a + b)^3*Log[1 - Cos[c + d*x]])/(2*d) - (b*(3*a^2 + b^2)*Log[Cos[c + d*x]
])/d - ((a - b)^3*Log[1 + Cos[c + d*x]])/(2*d) + (3*a*b^2*Sec[c + d*x])/d +
 (b^3*Sec[c + d*x]^2)/(2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx = -\int(-b-a\cos(c+dx))^3\csc(c+dx)\sec^3(c+dx)\,dx$$
$$= \frac{a\,\text{Subst}\left(\int\frac{a^3(-b+x)^3}{x^3(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^4\,\text{Subst}\left(\int\frac{(-b+x)^3}{x^3(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^4\,\text{Subst}\left(\int\left(\frac{(a-b)^3}{2a^4(a-x)}-\frac{b^3}{a^2x^3}+\frac{3b^2}{a^2x^2}+\frac{b(-3a^2-b^2)}{a^4x}+\frac{(a+b)^3}{2a^4(a+x)}\right)dx, x, -a\cos(c\right.}{d}$$
$$= \frac{(a+b)^3\log(1-\cos(c+dx))}{2d} - \frac{b(3a^2+b^2)\log(\cos(c+dx))}{d} - \frac{(a-b)^3\,\text{l}}{}$$

Mathematica [A] time = 0.30, size = 89, normalized size = 0.87

$$\frac{-2b(3a^2+b^2)\log(\cos(c+dx)) + 6ab^2\sec(c+dx) + 2(a+b)^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 2(a-b)^3\log\left(\cos\left(\frac{1}{2}(c+\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-2*(a - b)^3*Log[Cos[(c + d*x)/2]] - 2*b*(3*a^2 + b^2)*Log[Cos[c + d*x]] +
 2*(a + b)^3*Log[Sin[(c + d*x)/2]] + 6*a*b^2*Sec[c + d*x] + b^3*Sec[c + d*x
]^2)/(2*d)
```

fricas [A] time = 0.51, size = 139, normalized size = 1.36

$$\frac{6\,ab^2\cos(dx+c) - 2\left(3\,a^2b+b^3\right)\cos(dx+c)^2\log(-\cos(dx+c)) - \left(a^3-3\,a^2b+3\,ab^2-b^3\right)\cos(dx+c)^2\log\left(\right.}{2\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*cos(d*x + c) - 2*(3*a^2*b + b^3)*cos(d*x + c)^2*log(-cos(d*x +
 c)) - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(d*x + c)^2*log(1/2*cos(d*x + c)
+ 1/2) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c)^2*log(-1/2*cos(d*x +
c) + 1/2) + b^3)/(d*cos(d*x + c)^2)
```

giac [B] time = 0.50, size = 250, normalized size = 2.45

$$\frac{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 2\left(3\,a^2b+b^3\right)\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{9\,a^2b+12\,ab^2+3\,b^3+\frac{18\,a^2b(\cos(dx+c)-}{\cos(dx+c)+1}}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x
 + c) + 1)) - 2*(3*a^2*b + b^3)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) - 1)) + (9*a^2*b + 12*a*b^2 + 3*b^3 + 18*a^2*b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 12*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b^3*(cos
```

```
(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 + 3*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

maple [A] time = 0.39, size = 113, normalized size = 1.11

$$\frac{a^3 \ln(\csc(dx+c) - \cot(dx+c))}{d} + \frac{3a^2 b \ln(\tan(dx+c))}{d} + \frac{3b^2 a}{d\cos(dx+c)} + \frac{3b^2 a \ln(\csc(dx+c) - \cot(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*a^3*ln(csc(d*x+c)-cot(d*x+c))+3*a^2*b*ln(tan(d*x+c))/d+3/d*b^2*a/cos(d*
x+c)+3/d*b^2*a*ln(csc(d*x+c)-cot(d*x+c))+1/2/d*b^3/cos(d*x+c)^2+1/d*b^3*ln(
tan(d*x+c))
```

maxima [A] time = 0.50, size = 112, normalized size = 1.10

$$-\frac{\left(a^3 - 3\,a^2 b + 3\,ab^2 - b^3\right)\log(\cos(dx+c)+1) - \left(a^3 + 3\,a^2 b + 3\,ab^2 + b^3\right)\log(\cos(dx+c)-1) + 2\left(3\,a^2 b + \right.}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*log(cos(d*x + c) + 1) - (a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*log(cos(d*x + c) - 1) + 2*(3*a^2*b + b^3)*log(cos(d*x +
c)) - (6*a*b^2*cos(d*x + c) + b^3)/cos(d*x + c)^2)/d
```

mupad [B] time = 0.13, size = 85, normalized size = 0.83

$$\frac{\frac{\ln(\cos(c+d\,x)-1)\,(a+b)^3}{2} - \frac{\ln(\cos(c+d\,x)+1)\,(a-b)^3}{2} + \frac{\frac{b^3}{2}+3\,a\,\cos(c+d\,x)\,b^2}{\cos(c+d\,x)^2} - \ln(\cos(c+d\,x))\,\left(3\,a^2\,b+b^3\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/sin(c + d*x),x)
```

```
[Out] ((log(cos(c + d*x) - 1)*(a + b)^3)/2 - (log(cos(c + d*x) + 1)*(a - b)^3)/2
+ (b^3/2 + 3*a*b^2*cos(c + d*x))/cos(c + d*x)^2 - log(cos(c + d*x))*(3*a^2*
b + b^3))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c+dx))^3 \csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*csc(c + d*x), x)
```

3.189 $\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=162

$$-\frac{b\left(3a^2+2b^2\right)\log(\cos(c+dx))}{d}-\frac{a^2\csc^2(c+dx)\left(a\left(\frac{3b^2}{a^2}+1\right)\cos(c+dx)+b\left(\frac{b^2}{a^2}+3\right)\right)}{2d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{(a+b}{}$$

[Out] -1/2*a^2*(b*(3+b^2/a^2)+a*(1+3*b^2/a^2)*cos(d*x+c))*csc(d*x+c)^2/d+1/4*(a+b)^2*(a+4*b)*ln(1-cos(d*x+c))/d-b*(3*a^2+2*b^2)*ln(cos(d*x+c))/d-1/4*(a-4*b)*(a-b)^2*ln(1+cos(d*x+c))/d+3*a*b^2*sec(d*x+c)/d+1/2*b^3*sec(d*x+c)^2/d

Rubi [A] time = 0.35, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2837, 12, 1805, 1802}

$$-\frac{b\left(3a^2+2b^2\right)\log(\cos(c+dx))}{d}-\frac{a^2\csc^2(c+dx)\left(a\left(\frac{3b^2}{a^2}+1\right)\cos(c+dx)+b\left(\frac{b^2}{a^2}+3\right)\right)}{2d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{(a+b}{}$$

Antiderivative was successfully verified.

[In] Int[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]

[Out] -(a^2*(b*(3 + b^2/a^2) + a*(1 + (3*b^2)/a^2)*Cos[c + d*x])*Csc[c + d*x]^2)/(2*d) + ((a + b)^2*(a + 4*b)*Log[1 - Cos[c + d*x]])/(4*d) - (b*(3*a^2 + 2*b^2)*Log[Cos[c + d*x]])/d - ((a - 4*b)*(a - b)^2*Log[1 + Cos[c + d*x]])/(4*d) + (3*a*b^2*Sec[c + d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
```

```
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx = -\int(-b-a\cos(c+dx))^3\csc^3(c+dx)\sec^3(c+dx)\,dx$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{a^3(-b+x)^3}{x^3\left(a^2-x^2\right)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \frac{(-b+x)^3}{x^3\left(a^2-x^2\right)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^2\left(b\left(3+\frac{b^2}{a^2}\right)+a\left(1+\frac{3b^2}{a^2}\right)\cos(c+dx)\right)\csc^2(c+dx)}{2d} - \frac{a^4\,\text{Subst}\left(\int\right.}{}$$
$$= -\frac{a^2\left(b\left(3+\frac{b^2}{a^2}\right)+a\left(1+\frac{3b^2}{a^2}\right)\cos(c+dx)\right)\csc^2(c+dx)}{2d} - \frac{a^4\,\text{Subst}\left(\int\right.}{}$$
$$= -\frac{a^2\left(b\left(3+\frac{b^2}{a^2}\right)+a\left(1+\frac{3b^2}{a^2}\right)\cos(c+dx)\right)\csc^2(c+dx)}{2d} + \frac{(a+b)^2(a+}{}$$

Mathematica [B] time = 6.20, size = 669, normalized size = 4.13

$$\frac{\left(-3a^2b-2b^3\right)\cos^3(c+dx)\log(\cos(c+dx))(a+b\sec(c+dx))^3}{d(a\cos(c+dx)+b)^3} + \frac{\left(a^3-3a^2b+3ab^2-b^3\right)\cos^3(c+dx)\sec^2\left(\frac{1}{2}\right.}{8d(a\cos(c+dx)+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(d*(b + a*Cos[c + d*x])^3)
+ ((-a^3 - 3*a^2*b - 3*a*b^2 - b^3)*Cos[c + d*x]^3*Csc[(c + d*x)/2]^2*(a +
b*Sec[c + d*x])^3)/(8*d*(b + a*Cos[c + d*x])^3) + ((-a^3 + 6*a^2*b - 9*a*b^
2 + 4*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2]]*(a + b*Sec[c + d*x])^3)/(2*
d*(b + a*Cos[c + d*x])^3) + ((-3*a^2*b - 2*b^3)*Cos[c + d*x]^3*Log[Cos[c +
d*x]]*(a + b*Sec[c + d*x])^3)/(d*(b + a*Cos[c + d*x])^3) + ((a^3 + 6*a^2*b
+ 9*a*b^2 + 4*b^3)*Cos[c + d*x]^3*Log[Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x]
)^3)/(2*d*(b + a*Cos[c + d*x])^3) + ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*Cos[c
+ d*x]^3*Sec[(c + d*x)/2]^2*(a + b*Sec[c + d*x])^3)/(8*d*(b + a*Cos[c + d*x
])^3) + (b^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*Cos[c + d*x
])^3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c + d*x]^3*(a
+ b*Sec[c + d*x])^3*Sin[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3*(Cos[(c + d
*x)/2] - Sin[(c + d*x)/2])) + (b^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(
4*d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) - (3*a*
b^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[(c + d*x)/2])/(d*(b + a*Cos[c
 + d*x])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

fricas [A] time = 0.51, size = 290, normalized size = 1.79

$$-\frac{12\,ab^2\cos(dx+c) - 2\left(a^3+9\,ab^2\right)\cos(dx+c)^3 + 2\,b^3 - 2\left(3\,a^2b+2\,b^3\right)\cos(dx+c)^2 + 4\left(\left(3\,a^2b+2\,b^3\right)\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(12*a*b^2*cos(d*x + c) - 2*(a^3 + 9*a*b^2)*cos(d*x + c)^3 + 2*b^3 - 2*
(3*a^2*b + 2*b^3)*cos(d*x + c)^2 + 4*((3*a^2*b + 2*b^3)*cos(d*x + c)^4 - (3
*a^2*b + 2*b^3)*cos(d*x + c)^2)*log(-cos(d*x + c)) + ((a^3 - 6*a^2*b + 9*a*
b^2 - 4*b^3)*cos(d*x + c)^4 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(d*x + c
)^2)*log(1/2*cos(d*x + c) + 1/2) - ((a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cos(d
*x + c)^4 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cos(d*x + c)^2)*log(-1/2*cos(
d*x + c) + 1/2))/(d*cos(d*x + c)^4 - d*cos(d*x + c)^2)
```

giac [B] time = 1.26, size = 482, normalized size = 2.98

$$-\frac{\frac{a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{3\,a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{3\,ab^2(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{b^3(\cos(dx+c)-1)}{\cos(dx+c)+1} - 2\left(a^3 + 6\,a^2b + 9\,ab^2 + 4\,b^3\right)\log\left(\frac{|-\cos(dx+c)+}{|\cos(dx+c)+1}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a^2*b*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 3*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b^3*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)
*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + 8*(3*a^2*b + 2*b^3)*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^3 + 3*a^2*b + 3*a*b
^2 + b^3 - 2*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a^2*b*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - 18*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) - 8*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d
*x + c) - 1) - 4*(9*a^2*b + 12*a*b^2 + 6*b^3 + 18*a^2*b*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1) + 12*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*b^3
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^2*b*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 + 6*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

maple [A] time = 0.75, size = 201, normalized size = 1.24

$$-\frac{a^3\cot(dx+c)\csc(dx+c)}{2d} + \frac{a^3\ln(\csc(dx+c)-\cot(dx+c))}{2d} - \frac{3a^2b}{2d\sin(dx+c)^2} + \frac{3a^2b\ln(\tan(dx+c))}{d} - \frac{}{2d\sin(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*sec(d*x+c))^3,x)
```

```
[Out] -1/2*a^3*cot(d*x+c)*csc(d*x+c)/d+1/2/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-3/2/d*
a^2*b/sin(d*x+c)^2+3*a^2*b*ln(tan(d*x+c))/d-3/2/d*b^2*a/sin(d*x+c)^2/cos(d*
x+c)+9/2/d*b^2*a/cos(d*x+c)+9/2/d*b^2*a*ln(csc(d*x+c)-cot(d*x+c))+1/2/d*b^3
/sin(d*x+c)^2/cos(d*x+c)^2-1/d*b^3/sin(d*x+c)^2+2/d*b^3*ln(tan(d*x+c))
```

maxima [A] time = 0.71, size = 171, normalized size = 1.06

$$-\frac{\left(a^3 - 6\,a^2b + 9\,ab^2 - 4\,b^3\right)\log(\cos(dx+c)+1) - \left(a^3 + 6\,a^2b + 9\,ab^2 + 4\,b^3\right)\log(\cos(dx+c)-1) + 4\left(3\,a^2b + \right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*((a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*log(cos(d*x + c) + 1) - (a^3 + 6*a^
2*b + 9*a*b^2 + 4*b^3)*log(cos(d*x + c) - 1) + 4*(3*a^2*b + 2*b^3)*log(cos(
```

```
d*x + c)) + 2*(6*a*b^2*cos(d*x + c) - (a^3 + 9*a*b^2)*cos(d*x + c)^3 + b^3
- (3*a^2*b + 2*b^3)*cos(d*x + c)^2)/(cos(d*x + c)^4 - cos(d*x + c)^2))/d
```

mupad [B] time = 1.01, size = 159, normalized size = 0.98

$$\frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(a+b\right)^2\,\left(a+4\,b\right)}{4\,d}-\frac{\ln\left(\cos\left(c+d\,x\right)\right)\,\left(3\,a^2\,b+2\,b^3\right)}{d}-\frac{\cos\left(c+d\,x\right)^3\,\left(\frac{a^3}{2}+\frac{9\,a\,b^2}{2}\right)-\frac{b^3}{2}+\mathrm{co}}{d\,\left(\cos\left(c+a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/sin(c + d*x)^3,x)
```

```
[Out] (log(cos(c + d*x) - 1)*(a + b)^2*(a + 4*b))/(4*d) - (log(cos(c + d*x))*(3*a
^2*b + 2*b^3))/d - (cos(c + d*x)^3*((9*a*b^2)/2 + a^3/2) - b^3/2 + cos(c +
d*x)^2*((3*a^2*b)/2 + b^3) - 3*a*b^2*cos(c + d*x))/(d*(cos(c + d*x)^2 - cos
(c + d*x)^4)) - (log(cos(c + d*x) + 1)*(a - b)^2*(a - 4*b))/(4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^3\csc^3\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*csc(c + d*x)**3, x)
```

3.190 $\int (a+b\sec(c+dx))^3\sin^6(c+dx)\,dx$

Optimal. Leaf size=299

$$-\frac{a^3\sin^5(c+dx)\cos(c+dx)}{6d}-\frac{5a^3\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{5a^3\sin(c+dx)\cos(c+dx)}{16d}+\frac{5a^3x}{16}-\frac{3a^2b\sin^5(c+dx)}{5d}$$

```
[Out] 5/16*a^3*x-45/8*a*b^2*x+3*a^2*b*arctanh(sin(d*x+c))/d-5/2*b^3*arctanh(sin(d
*x+c))/d-3*a^2*b*sin(d*x+c)/d+5/2*b^3*sin(d*x+c)/d-5/16*a^3*cos(d*x+c)*sin(
d*x+c)/d-a^2*b*sin(d*x+c)^3/d+5/6*b^3*sin(d*x+c)^3/d-5/24*a^3*cos(d*x+c)*si
n(d*x+c)^3/d-3/5*a^2*b*sin(d*x+c)^5/d-1/6*a^3*cos(d*x+c)*sin(d*x+c)^5/d+45/
8*a*b^2*tan(d*x+c)/d-15/8*a*b^2*sin(d*x+c)^2*tan(d*x+c)/d-3/4*a*b^2*sin(d*x
+c)^4*tan(d*x+c)/d+1/2*b^3*sin(d*x+c)^3*tan(d*x+c)^2/d
```

Rubi [A] time = 0.34, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3872, 2912, 2635, 8, 2592, 302, 206, 2591, 288, 321, 203}

$$-\frac{3a^2b\sin^5(c+dx)}{5d}-\frac{a^2b\sin^3(c+dx)}{d}-\frac{3a^2b\sin(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\sin^5(c+dx)\cos(c+dx)}{6d}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^6,x]
```

```
[Out] (5*a^3*x)/16 - (45*a*b^2*x)/8 + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (5*b^3*
ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^2*b*Sin[c + d*x])/d + (5*b^3*Sin[c + d*
x])/(2*d) - (5*a^3*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (a^2*b*Sin[c + d*x]^
3)/d + (5*b^3*Sin[c + d*x]^3)/(6*d) - (5*a^3*Cos[c + d*x]*Sin[c + d*x]^3)/(
24*d) - (3*a^2*b*Sin[c + d*x]^5)/(5*d) - (a^3*Cos[c + d*x]*Sin[c + d*x]^5)/
(6*d) + (45*a*b^2*Tan[c + d*x])/(8*d) - (15*a*b^2*Sin[c + d*x]^2*Tan[c + d*
x])/(8*d) - (3*a*b^2*Sin[c + d*x]^4*Tan[c + d*x])/(4*d) + (b^3*Sin[c + d*x]
^3*Tan[c + d*x]^2)/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))^3\sin^6(c+dx)\,dx = -\int (-b-a\cos(c+dx))^3\sin^3(c+dx)\tan^3(c+dx)\,dx$$

$$= -\int \left(-a^3\sin^6(c+dx) - 3a^2b\sin^5(c+dx)\tan(c+dx) - 3ab^2\sin^4(c+d$$

$$= a^3\int \sin^6(c+dx)\,dx + \left(3a^2b\right)\int \sin^5(c+dx)\tan(c+dx)\,dx + \left(3ab^2\right)\int$$

$$= -\frac{a^3\cos(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{6}\left(5a^3\right)\int \sin^4(c+dx)\,dx + \frac{\left(3a^2b\right)\text{Subs}}{}$$

$$= -\frac{5a^3\cos(c+dx)\sin^3(c+dx)}{24d} - \frac{a^3\cos(c+dx)\sin^5(c+dx)}{6d} - \frac{3ab^2\sin^4(}{}$$

$$= -\frac{3a^2b\sin(c+dx)}{d} - \frac{5a^3\cos(c+dx)\sin(c+dx)}{16d} - \frac{a^2b\sin^3(c+dx)}{d} - \frac{5a}{}$$

$$= \frac{5a^3x}{16} + \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d} - \frac{3a^2b\sin(c+dx)}{d} + \frac{5b^3\sin(c+dx)}{2d} -$$

$$= \frac{5a^3x}{16} - \frac{45}{8}ab^2x + \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d} - \frac{5b^3\tanh^{-1}(\sin(c+dx))}{2d} -$$

Mathematica [B] time = 6.26, size = 818, normalized size = 2.74

$$\frac{\left(5b^3-6a^2b\right)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\sec(c+dx))^3}{2d(b+a\cos(c+dx))^3} + \frac{\left(6a^2b-5b^3\right)\cos^3(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^6,x]
```

```
[Out] (5*a*(a^2 - 18*b^2)*(c + d*x)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(16*d*
(b + a*Cos[c + d*x])^3) + ((-6*a^2*b + 5*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x])^3)/(2*d*(b + a*Cos[c + d*x]
)^3) + ((6*a^2*b - 5*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]]*(a + b*Sec[c + d*x])^3)/(2*d*(b + a*Cos[c + d*x])^3) + (b^3*Cos[c +
d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*S
in[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2] - Sin[(c + d*x
)/2])) - (b^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*Cos[c + d*
x])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c + d*x]^3*(a
 + b*Sec[c + d*x])^3*Sin[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])) + (3*b*(-11*a^2 + 6*b^2)*Cos[c + d*x]^3*(a + b
*Sec[c + d*x])^3*Sin[c + d*x])/(8*d*(b + a*Cos[c + d*x])^3) - (3*a*(5*a^2 -
 32*b^2)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[2*(c + d*x)])/(64*d*(b +
 a*Cos[c + d*x])^3) - (b*(-21*a^2 + 4*b^2)*Cos[c + d*x]^3*(a + b*Sec[c + d*
x])^3*Sin[3*(c + d*x)])/(48*d*(b + a*Cos[c + d*x])^3) + (3*a*(a^2 - 2*b^2)*
Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[4*(c + d*x)])/(64*d*(b + a*Cos[c
+ d*x])^3) - (3*a^2*b*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[5*(c + d*x)
])/(80*d*(b + a*Cos[c + d*x])^3) - (a^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])
^3*Sin[6*(c + d*x)])/(192*d*(b + a*Cos[c + d*x])^3)
```

fricas [A] time = 0.53, size = 241, normalized size = 0.81

$$\frac{75\left(a^3-18\,ab^2\right)dx\cos(dx+c)^2 + 60\left(6\,a^2b-5\,b^3\right)\cos(dx+c)^2\log(\sin(dx+c)+1) - 60\left(6\,a^2b-5\,b^3\right)\cos(dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(75*(a^3 - 18*a*b^2)*d*x*cos(d*x + c)^2 + 60*(6*a^2*b - 5*b^3)*cos(d*
x + c)^2*log(sin(d*x + c) + 1) - 60*(6*a^2*b - 5*b^3)*cos(d*x + c)^2*log(-s
in(d*x + c) + 1) - (40*a^3*cos(d*x + c)^7 + 144*a^2*b*cos(d*x + c)^6 - 10*(
13*a^3 - 18*a*b^2)*cos(d*x + c)^5 - 16*(33*a^2*b - 5*b^3)*cos(d*x + c)^4 -
720*a*b^2*cos(d*x + c) + 15*(11*a^3 - 54*a*b^2)*cos(d*x + c)^3 - 120*b^3 +
16*(69*a^2*b - 35*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [B] time = 0.46, size = 563, normalized size = 1.88

$$75\left(a^3-18\,ab^2\right)(dx+c)+120\left(6\,a^2b-5\,b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-120\left(6\,a^2b-5\,b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/240*(75*(a^3 - 18*a*b^2)*(d*x + c) + 120*(6*a^2*b - 5*b^3)*log(abs(tan(1/
2*d*x + 1/2*c) + 1)) - 120*(6*a^2*b - 5*b^3)*log(abs(tan(1/2*d*x + 1/2*c) -
 1)) - 240*(6*a*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6
*a*b^2*tan(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 - 1)^2 + 2*(75*a^3*tan(1/2*d*x + 1/2*c)^11 - 720*a^2*b*tan(1/2*d*x + 1
/2*c)^11 - 630*a*b^2*tan(1/2*d*x + 1/2*c)^11 + 480*b^3*tan(1/2*d*x + 1/2*c)
^11 + 425*a^3*tan(1/2*d*x + 1/2*c)^9 - 4560*a^2*b*tan(1/2*d*x + 1/2*c)^9 -
2610*a*b^2*tan(1/2*d*x + 1/2*c)^9 + 2720*b^3*tan(1/2*d*x + 1/2*c)^9 + 990*a
^3*tan(1/2*d*x + 1/2*c)^7 - 12384*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 1980*a*b^2
*tan(1/2*d*x + 1/2*c)^7 + 5760*b^3*tan(1/2*d*x + 1/2*c)^7 - 990*a^3*tan(1/2
*d*x + 1/2*c)^5 - 12384*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 1980*a*b^2*tan(1/2*d
*x + 1/2*c)^5 + 5760*b^3*tan(1/2*d*x + 1/2*c)^5 - 425*a^3*tan(1/2*d*x + 1/2
*c)^3 - 4560*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 2610*a*b^2*tan(1/2*d*x + 1/2*c)
^3 + 2720*b^3*tan(1/2*d*x + 1/2*c)^3 - 75*a^3*tan(1/2*d*x + 1/2*c) - 720*a^
2*b*tan(1/2*d*x + 1/2*c) + 630*a*b^2*tan(1/2*d*x + 1/2*c) + 480*b^3*tan(1/2
*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

maple [A] time = 0.70, size = 354, normalized size = 1.18

$$-\frac{a^3\cos(dx+c)\left(\sin^5(dx+c)\right)}{6d}-\frac{5a^3\cos(dx+c)\left(\sin^3(dx+c)\right)}{24d}-\frac{5a^3\cos(dx+c)\sin(dx+c)}{16d}+\frac{5a^3x}{16}+\frac{5a^3c}{16d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c)^6,x)
```

```
[Out] -1/6*a^3*cos(d*x+c)*sin(d*x+c)^5/d-5/24*a^3*cos(d*x+c)*sin(d*x+c)^3/d-5/16*
a^3*cos(d*x+c)*sin(d*x+c)/d+5/16*a^3*x+5/16/d*a^3*c-3/5*a^2*b*sin(d*x+c)^5/
d-a^2*b*sin(d*x+c)^3/d-3*a^2*b*sin(d*x+c)/d+3/d*a^2*b*ln(sec(d*x+c)+tan(d*x
+c))+3/d*b^2*a*sin(d*x+c)^7/cos(d*x+c)+3/d*b^2*a*sin(d*x+c)^5*cos(d*x+c)+15
/4/d*b^2*a*cos(d*x+c)*sin(d*x+c)^3+45/8*a*b^2*cos(d*x+c)*sin(d*x+c)/d-45/8*
a*b^2*x-45/8/d*a*b^2*c+1/2/d*b^3*sin(d*x+c)^7/cos(d*x+c)^2+1/2/d*b^3*sin(d*
x+c)^5+5/6*b^3*sin(d*x+c)^3/d+5/2*b^3*sin(d*x+c)/d-5/2/d*b^3*ln(sec(d*x+c)+
tan(d*x+c))
```

maxima [A] time = 0.91, size = 242, normalized size = 0.81

$$5\left(4\,\sin\left(2\,dx+2\,c\right)^3+60\,dx+60\,c+9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)a^3-96\left(6\,\sin\left(dx+c\right)^5+10\,\sin\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/960*(5*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*si
n(2*d*x + 2*c))*a^3 - 96*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin
(d*x + c) + 1) + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a^2*b - 360*(1
5*d*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(
d*x + c)^2 + 1) - 8*tan(d*x + c))*a*b^2 + 80*(4*sin(d*x + c)^3 - 6*sin(d*x
+ c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c)
- 1) + 24*sin(d*x + c))*b^3)/d
```

mupad [B] time = 1.69, size = 373, normalized size = 1.25

$$\frac{7\,b^3\,\sin\left(c+d\,x\right)}{3\,d}+\frac{5\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{8\,d}-\frac{5\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{13\,a^3\,{\cos\left(c+d\,x\right)}^3\,\sin\left(c+d\,x\right)}{24\,d}-\frac{a^3\cos\left(c+d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (7*b^3*sin(c + d*x))/(3*d) + (5*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)
/2)))/(8*d) - (5*b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (13*
a^3*cos(c + d*x)^3*sin(c + d*x))/(24*d) - (a^3*cos(c + d*x)^5*sin(c + d*x))
/(6*d) + (b^3*sin(c + d*x))/(2*d*cos(c + d*x)^2) - (b^3*cos(c + d*x)^2*sin(
c + d*x))/(3*d) - (11*a^3*cos(c + d*x)*sin(c + d*x))/(16*d) - (23*a^2*b*sin
(c + d*x))/(5*d) - (45*a*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(
4*d) + (6*a^2*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (27*a*b^2
*cos(c + d*x)*sin(c + d*x))/(8*d) + (3*a*b^2*sin(c + d*x))/(d*cos(c + d*x))
 + (11*a^2*b*cos(c + d*x)^2*sin(c + d*x))/(5*d) - (3*a*b^2*cos(c + d*x)^3*s
in(c + d*x))/(4*d) - (3*a^2*b*cos(c + d*x)^4*sin(c + d*x))/(5*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**6,x)
```

```
[Out] Timed out
```

3.191 $\int (a+b\sec(c+dx))^3\sin^4(c+dx)\,dx$

Optimal. Leaf size=236

$$-\frac{b\left(17a^2-b^2\right)\sin(c+dx)}{2d}+\frac{3b\left(2a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{\left(4a^2-b^2\right)\sin(c+dx)(a\cos(c+dx)+b)^3}{4b^2d}-\frac{(6a$$

```
[Out] 3/8*a*(a^2-12*b^2)*x+3/2*b*(2*a^2-b^2)*arctanh(sin(d*x+c))/d-1/2*b*(17*a^2-
b^2)*sin(d*x+c)/d-1/8*a*(21*a^2-2*b^2)*cos(d*x+c)*sin(d*x+c)/d-1/4*(6*a^2-b
^2)*(b+a*cos(d*x+c))^2*sin(d*x+c)/b/d-1/4*(4*a^2-b^2)*(b+a*cos(d*x+c))^3*si
n(d*x+c)/b^2/d+a*(b+a*cos(d*x+c))^4*tan(d*x+c)/b^2/d+1/2*(b+a*cos(d*x+c))^4
*sec(d*x+c)*tan(d*x+c)/b/d
```

Rubi [A] time = 0.75, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2893, 3049, 3033, 3023, 2735, 3770}

$$-\frac{b\left(17a^2-b^2\right)\sin(c+dx)}{2d}+\frac{3b\left(2a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{\left(4a^2-b^2\right)\sin(c+dx)(a\cos(c+dx)+b)^3}{4b^2d}-\frac{(6a$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^4,x]
```

```
[Out] (3*a*(a^2 - 12*b^2)*x)/8 + (3*b*(2*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d)
- (b*(17*a^2 - b^2)*Sin[c + d*x])/(2*d) - (a*(21*a^2 - 2*b^2)*Cos[c + d*x]*
Sin[c + d*x])/(8*d) - ((6*a^2 - b^2)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(
4*b*d) - ((4*a^2 - b^2)*(b + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*b^2*d) + (a
*(b + a*Cos[c + d*x])^4*Tan[c + d*x])/(b^2*d) + ((b + a*Cos[c + d*x])^4*Sec
[c + d*x]*Tan[c + d*x])/(2*b*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))^3\sin^4(c+dx)\,dx = -\int (-b-a\cos(c+dx))^3\sin(c+dx)\tan^3(c+dx)\,dx$$

$$= \frac{a(b+a\cos(c+dx))^4\tan(c+dx)}{b^2d} + \frac{(b+a\cos(c+dx))^4\sec(c+dx)\tan(}{2bd}$$

$$= -\frac{\left(4a^2-b^2\right)(b+a\cos(c+dx))^3\sin(c+dx)}{4b^2d} + \frac{a(b+a\cos(c+dx))^4\tan(}{b^2d}$$

$$= -\frac{\left(6a^2-b^2\right)(b+a\cos(c+dx))^2\sin(c+dx)}{4bd} - \frac{\left(4a^2-b^2\right)(b+a\cos(c+}{4b^2d}$$

$$= -\frac{a\left(21a^2-2b^2\right)\cos(c+dx)\sin(c+dx)}{8d} - \frac{\left(6a^2-b^2\right)(b+a\cos(c+dx))^2}{4bd}$$

$$= -\frac{b\left(17a^2-b^2\right)\sin(c+dx)}{2d} - \frac{a\left(21a^2-2b^2\right)\cos(c+dx)\sin(c+dx)}{8d} - \frac{(6}{}$$

$$= \frac{3}{8}a\left(a^2-12b^2\right)x - \frac{b\left(17a^2-b^2\right)\sin(c+dx)}{2d} - \frac{a\left(21a^2-2b^2\right)\cos(c+dx}{8d}$$

$$= \frac{3}{8}a\left(a^2-12b^2\right)x + \frac{3b\left(2a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{2d} - \frac{b\left(17a^2-b^2\right)\sin}{2d}$$

Mathematica [B] time = 6.19, size = 696, normalized size = 2.95

$$\frac{a^3\sin(4(c+dx))\cos^3(c+dx)(a+b\sec(c+dx))^3}{32d(a\cos(c+dx)+b)^3}+\frac{3\left(b^3-2a^2b\right)\cos^3(c+dx)(a+b\sec(c+dx))^3\log\left(\cos\left(\frac{1}{2}(\right.\right.}{2d(a\cos(c+dx)+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^4,x]
```

```
[Out] (3*a*(a^2 - 12*b^2)*(c + d*x)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(8*d*(
b + a*Cos[c + d*x])^3) + (3*(-2*a^2*b + b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x])^3)/(2*d*(b + a*Cos[c + d*x])
^3) - (3*(-2*a^2*b + b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]]*(a + b*Sec[c + d*x])^3)/(2*d*(b + a*Cos[c + d*x])^3) + (b^3*Cos[c +
d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*S
in[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2] - Sin[(c + d*x
)/2])) - (b^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*Cos[c + d*
x])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c + d*x]^3*(a
 + b*Sec[c + d*x])^3*Sin[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])) + (b*(-15*a^2 + 4*b^2)*Cos[c + d*x]^3*(a + b*S
ec[c + d*x])^3*Sin[c + d*x])/(4*d*(b + a*Cos[c + d*x])^3) - (a*(a^2 - 3*b^2
)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[2*(c + d*x)])/(4*d*(b + a*Cos[c
 + d*x])^3) + (a^2*b*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[3*(c + d*x)]
)/(4*d*(b + a*Cos[c + d*x])^3) + (a^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3
*Sin[4*(c + d*x)])/(32*d*(b + a*Cos[c + d*x])^3)
```

fricas [A] time = 0.57, size = 196, normalized size = 0.83

$$\frac{3\left(a^3-12\,ab^2\right)dx\cos\left(dx+c\right)^2+6\left(2\,a^2b-b^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-6\left(2\,a^2b-b^3\right)\cos\left(dx+c\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(3*(a^3 - 12*a*b^2)*d*x*cos(d*x + c)^2 + 6*(2*a^2*b - b^3)*cos(d*x + c)
^2*log(sin(d*x + c) + 1) - 6*(2*a^2*b - b^3)*cos(d*x + c)^2*log(-sin(d*x +
c) + 1) + (2*a^3*cos(d*x + c)^5 + 8*a^2*b*cos(d*x + c)^4 + 24*a*b^2*cos(d*x
 + c) - (5*a^3 - 12*a*b^2)*cos(d*x + c)^3 + 4*b^3 - 8*(4*a^2*b - b^3)*cos(d
*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [A] time = 0.40, size = 431, normalized size = 1.83

$$\frac{3\left(a^3-12\,ab^2\right)(dx+c)+12\left(2\,a^2b-b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-12\left(2\,a^2b-b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^3 - 12*a*b^2)*(d*x + c) + 12*(2*a^2*b - b^3)*log(abs(tan(1/2*d*x
+ 1/2*c) + 1)) - 12*(2*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 8*
(6*a*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(
1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2
 + 2*(3*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 12*a
*b^2*tan(1/2*d*x + 1/2*c)^7 + 8*b^3*tan(1/2*d*x + 1/2*c)^7 + 11*a^3*tan(1/2
```

```
*d*x + 1/2*c)^5 - 104*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 12*a*b^2*tan(1/2*d*x +
 1/2*c)^5 + 24*b^3*tan(1/2*d*x + 1/2*c)^5 - 11*a^3*tan(1/2*d*x + 1/2*c)^3 -
 104*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 24*b^
3*tan(1/2*d*x + 1/2*c)^3 - 3*a^3*tan(1/2*d*x + 1/2*c) - 24*a^2*b*tan(1/2*d*
x + 1/2*c) + 12*a*b^2*tan(1/2*d*x + 1/2*c) + 8*b^3*tan(1/2*d*x + 1/2*c))/(t
an(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

maple [A] time = 0.62, size = 276, normalized size = 1.17

$$-\frac{a^3\cos(dx+c)\left(\sin^3(dx+c)\right)}{4d}-\frac{3a^3\cos(dx+c)\sin(dx+c)}{8d}+\frac{3a^3x}{8}+\frac{3a^3c}{8d}-\frac{a^2b\left(\sin^3(dx+c)\right)}{d}-\frac{3a^2b\sin(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c)^4,x)
```

```
[Out] -1/4*a^3*cos(d*x+c)*sin(d*x+c)^3/d-3/8*a^3*cos(d*x+c)*sin(d*x+c)/d+3/8*a^3*
x+3/8/d*a^3*c-a^2*b*sin(d*x+c)^3/d-3*a^2*b*sin(d*x+c)/d+3/d*a^2*b*ln(sec(d*
x+c)+tan(d*x+c))+3/d*b^2*a*sin(d*x+c)^5/cos(d*x+c)+3/d*b^2*a*cos(d*x+c)*sin
(d*x+c)^3+9/2*a*b^2*cos(d*x+c)*sin(d*x+c)/d-9/2*a*b^2*x-9/2/d*a*b^2*c+1/2/d
*b^3*sin(d*x+c)^5/cos(d*x+c)^2+1/2*b^3*sin(d*x+c)^3/d+3/2*b^3*sin(d*x+c)/d-
3/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.73, size = 183, normalized size = 0.78

$$\frac{(12\,dx+12\,c+\sin(4\,dx+4\,c)-8\,\sin(2\,dx+2\,c))a^3-16\left(2\,\sin(dx+c)^3-3\,\log(\sin(dx+c)+1)+3\,\log(\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/32*((12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a^3 - 16*(2*s
in(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d
*x + c))*a^2*b - 48*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*ta
n(d*x + c))*a*b^2 - 8*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(
d*x + c) + 1) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)))/d
```

mupad [B] time = 1.47, size = 281, normalized size = 1.19

$$\frac{b^3\,\sin(c+d\,x)}{d}+\frac{3\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4\,d}-\frac{3\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{a^3\cos(c+d\,x)^3\,\sin(c+d\,x)}{4\,d}+\frac{b^3\,\sin(c+d\,x)}{2\,d\cos(c+d\,x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (b^3*sin(c + d*x))/d + (3*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
(4*d) - (3*b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (a^3*cos(c
 + d*x)^3*sin(c + d*x))/(4*d) + (b^3*sin(c + d*x))/(2*d*cos(c + d*x)^2) - (
5*a^3*cos(c + d*x)*sin(c + d*x))/(8*d) - (4*a^2*b*sin(c + d*x))/d - (9*a*b^
2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*a^2*b*atanh(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3*a*b^2*cos(c + d*x)*sin(c + d*x))/(2*d
) + (3*a*b^2*sin(c + d*x))/(d*cos(c + d*x)) + (a^2*b*cos(c + d*x)^2*sin(c +
 d*x))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.192 $\int (a+b\sec(c+dx))^3 \sin^2(c+dx)\,dx$

Optimal. Leaf size=138

$$-\frac{5a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{b\left(6a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{1}{2}ax\left(a^2-6b^2\right)-\frac{15a^2b\sin(c+dx)}{2d}+\frac{3a\tan(c+d$$

```
[Out] 1/2*a*(a^2-6*b^2)*x+1/2*b*(6*a^2-b^2)*arctanh(sin(d*x+c))/d-15/2*a^2*b*sin(
d*x+c)/d-5/2*a^3*cos(d*x+c)*sin(d*x+c)/d+3/2*a*(b+a*cos(d*x+c))^2*tan(d*x+c
)/d+1/2*(b+a*cos(d*x+c))^3*sec(d*x+c)*tan(d*x+c)/d
```

Rubi [A] time = 0.50, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2889, 3048, 3047, 3033, 3023, 2735, 3770}

$$\frac{b\left(6a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{1}{2}ax\left(a^2-6b^2\right)-\frac{15a^2b\sin(c+dx)}{2d}-\frac{5a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a\tan(c+dx$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^2,x]
```

```
[Out] (a*(a^2 - 6*b^2)*x)/2 + (b*(6*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d) - (15
*a^2*b*Sin[c + d*x])/(2*d) - (5*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (3*a
*(b + a*Cos[c + d*x])^2*Tan[c + d*x])/(2*d) + ((b + a*Cos[c + d*x])^3*Sec[c
 + d*x]*Tan[c + d*x])/(2*d)
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))^3\sin^2(c+dx)\,dx = -\int (-b-a\cos(c+dx))^3\sec(c+dx)\tan^2(c+dx)\,dx$$
$$= -\int (-b-a\cos(c+dx))^3\left(1-\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= \frac{(b+a\cos(c+dx))^3\sec(c+dx)\tan(c+dx)}{2d} - \frac{1}{2}\int (-b-a\cos(c+dx$$
$$= \frac{3a(b+a\cos(c+dx))^2\tan(c+dx)}{2d} + \frac{(b+a\cos(c+dx))^3\sec(c+dx)}{2d}$$
$$= -\frac{5a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a(b+a\cos(c+dx))^2\tan(c+dx)}{2d} +$$
$$= -\frac{15a^2b\sin(c+dx)}{2d} - \frac{5a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a(b+a\cos(c+}{2}$$
$$= \frac{1}{2}a\left(a^2-6b^2\right)x - \frac{15a^2b\sin(c+dx)}{2d} - \frac{5a^3\cos(c+dx)\sin(c+dx)}{2d} + $$
$$= \frac{1}{2}a\left(a^2-6b^2\right)x + \frac{b\left(6a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{2d} - \frac{15a^2b\sin(c+d}{2d}$$

Mathematica [B] time = 0.97, size = 327, normalized size = 2.37

$$\frac{\sec^2(c+dx)\left(-\frac{1}{2}a^3\sin(2(c+dx))-\frac{1}{4}a^3\sin(4(c+dx))+a^3c+a^3dx+\left(2b^3-3a^2b\right)\sin(c+dx)+\cos(2(c+dx))\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^2,x]
```

```
[Out] (Sec[c + d*x]^2*(a^3*c - 6*a*b^2*c + a^3*d*x - 6*a*b^2*d*x - 6*a^2*b*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]] + b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 6*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - b^3*Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]] + Cos[2*(c + d*x)]*(a*(a^2 - 6*b^2)*(c + d*x)
 + (-6*a^2*b + b^3)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - b*(-6*a^2 +
b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (-3*a^2*b + 2*b^3)*Sin[c +
 d*x] - (a^3*Sin[2*(c + d*x)])/2 + 6*a*b^2*Sin[2*(c + d*x)] - 3*a^2*b*Sin[3
*(c + d*x)] - (a^3*Sin[4*(c + d*x)])/4))/(4*d)
```

fricas [A] time = 0.51, size = 151, normalized size = 1.09

$$\frac{2\left(a^3-6\,ab^2\right)dx\cos\left(dx+c\right)^2+\left(6\,a^2b-b^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(6\,a^2b-b^3\right)\cos\left(dx+c\right)^2\log\left(-\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^3 - 6*a*b^2)*d*x*cos(d*x + c)^2 + (6*a^2*b - b^3)*cos(d*x + c)^2*
log(sin(d*x + c) + 1) - (6*a^2*b - b^3)*cos(d*x + c)^2*log(-sin(d*x + c) +
1) - 2*(a^3*cos(d*x + c)^3 + 6*a^2*b*cos(d*x + c)^2 - 6*a*b^2*cos(d*x + c)
- b^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

giac [B] time = 0.38, size = 346, normalized size = 2.51

$$\frac{\left(a^3-6\,ab^2\right)(dx+c)+\left(6\,a^2b-b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(6\,a^2b-b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(a^3\tan\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*((a^3 - 6*a*b^2)*(d*x + c) + (6*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - (6*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a^3*tan(
1/2*d*x + 1/2*c)^7 - 6*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 6*a*b^2*tan(1/2*d*x +
 1/2*c)^7 + b^3*tan(1/2*d*x + 1/2*c)^7 - 3*a^3*tan(1/2*d*x + 1/2*c)^5 + 6*a
^2*b*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 3*b^3*tan(1/
2*d*x + 1/2*c)^5 + 3*a^3*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b*tan(1/2*d*x + 1/2
*c)^3 + 6*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*b^3*tan(1/2*d*x + 1/2*c)^3 - a^3
*tan(1/2*d*x + 1/2*c) - 6*a^2*b*tan(1/2*d*x + 1/2*c) + 6*a*b^2*tan(1/2*d*x
+ 1/2*c) + b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/d
```

maple [A] time = 0.42, size = 167, normalized size = 1.21

$$-\frac{a^3\cos\left(dx+c\right)\sin\left(dx+c\right)}{2d}+\frac{a^3x}{2}+\frac{a^3c}{2d}-\frac{3a^2b\sin\left(dx+c\right)}{d}+\frac{3a^2b\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}-3a\,b^2x+\frac{3a\,b^2\,\mathrm{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c)^2,x)
```

```
[Out] -1/2*a^3*cos(d*x+c)*sin(d*x+c)/d+1/2*a^3*x+1/2/d*a^3*c-3*a^2*b*sin(d*x+c)/d
+3/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))-3*a*b^2*x+3*a*b^2*tan(d*x+c)/d-3/d*a*b
^2*c+1/2/d*b^3*sin(d*x+c)^3/cos(d*x+c)^2+1/2*b^3*sin(d*x+c)/d-1/2/d*b^3*ln(
sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 1.04, size = 129, normalized size = 0.93

$$\frac{(2\,dx + 2\,c - \sin\left(2\,dx + 2\,c\right))a^3 - 12\,(dx + c - \tan\left(dx + c\right))ab^2 - b^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} + \log\left(\sin\left(dx + c\right) + 1\right) - \log\left(\right.\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a^3 - 12*(d*x + c - tan(d*x + c))*a*b
^2 - b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log
(sin(d*x + c) - 1)) + 6*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
) - 2*sin(d*x + c)))/d
```

mupad [B] time = 1.27, size = 202, normalized size = 1.46

$$\frac{a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}-\frac{b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{b^3\,\sin\left(c+d\,x\right)}{2\,d\,{\cos\left(c+d\,x\right)}^2}-\frac{a^3\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}-\frac{3\,a^2\,b\,\sin\left(c+d\,\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (b^3*atanh(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2)))/d + (b^3*sin(c + d*x))/(2*d*cos(c + d*x)^2) -
 (a^3*cos(c + d*x)*sin(c + d*x))/(2*d) - (3*a^2*b*sin(c + d*x))/d - (6*a*b^
2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (6*a^2*b*atanh(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3*a*b^2*sin(c + d*x))/(d*cos(c + d*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sec\left(c + dx\right)\right)^3 \sin^2\left(c + dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*sin(c + d*x)**2, x)
```

3.193 $\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=133

$$-\frac{a^3\cot(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{3ab^2\tan(c+dx)}{d}-\frac{3ab^2\cot(c+dx)}{d}-\frac{3b^3\csc(c+d}{2d}$$

```
[Out] 3*a^2*b*arctanh(sin(d*x+c))/d+3/2*b^3*arctanh(sin(d*x+c))/d-a^3*cot(d*x+c)/
d-3*a*b^2*cot(d*x+c)/d-3*a^2*b*csc(d*x+c)/d-3/2*b^3*csc(d*x+c)/d+1/2*b^3*cs
c(d*x+c)*sec(d*x+c)^2/d+3*a*b^2*tan(d*x+c)/d
```

Rubi [A] time = 0.27, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3872, 2912, 3767, 8, 2621, 321, 207, 2620, 14, 288}

$$-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\tan(c+dx)}{d}-\frac{3ab^2\cot(c+dx)}{d}-\frac{3b^3\csc(c+d}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (3*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cot[c + d*x])/d - (3*a*b^2*Cot[c + d*x])/d - (3*a^2*b*Csc[c + d*x])/d -
 (3*b^3*Csc[c + d*x])/(2*d) + (b^3*Csc[c + d*x]*Sec[c + d*x]^2)/(2*d) + (3*
a*b^2*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx = -\int (-b-a\cos(c+dx))^3\csc^2(c+dx)\sec^3(c+dx)\,dx$$
$$= \int \left(a^3\csc^2(c+dx)+3a^2b\csc^2(c+dx)\sec(c+dx)+3ab^2\csc^2(c+dx\right.$$
$$= a^3\int\csc^2(c+dx)\,dx+\left(3a^2b\right)\int\csc^2(c+dx)\sec(c+dx)\,dx+\left(3ab^2\right.$$
$$= -\frac{a^3\,\mathrm{Subst}(\int 1\,dx,x,\cot(c+dx))}{d}-\frac{\left(3a^2b\right)\mathrm{Subst}\left(\int\frac{x^2}{-1+x^2}\,dx,x,\csc(c\right.}{d}$$
$$= -\frac{a^3\cot(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{b^3\csc(c+dx)\sec^2(c+dx)}{2d}-$$
$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}-\frac{3ab^2\cot(c+dx)}{d}-\frac{3a^2b}{}$$
$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{3b^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\cot(c+dx)}{d}$$

Mathematica [B] time = 0.66, size = 406, normalized size = 3.05

$$-\frac{\csc^5\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(2a^3\cos(3(c+dx))+6\left(2a^2b+b^3\right)\cos(2(c+dx))+6a\left(a^2+2b^2\right)\cos(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/16*(Csc[(c + d*x)/2]^5*Sec[(c + d*x)/2]*(12*a^2*b + 2*b^3 + 6*a*(a^2 + 2
*b^2)*Cos[c + d*x] + 6*(2*a^2*b + b^3)*Cos[2*(c + d*x)] + 2*a^3*Cos[3*(c +
d*x)] + 12*a*b^2*Cos[3*(c + d*x)] + 6*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2]]*Sin[c + d*x] + 3*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin
[c + d*x] - 6*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] -
 3*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 6*a^2*b*Log[
Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 3*b^3*Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 6*a^2*b*Log[Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 3*b^3*Log[Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]]*Sin[3*(c + d*x)]))/(d*(-1 + Cot[(c + d*x)/2]^2)^2)
```

fricas [A] time = 0.69, size = 151, normalized size = 1.14

$$\frac{3\left(2\,a^2b+b^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-3\left(2\,a^2b+b^3\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right.}{4\,d\cos\left(dx+c\right)^2\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*(2*a^2*b + b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1)*sin(d*x + c) -
3*(2*a^2*b + b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1)*sin(d*x + c) + 12*a
*b^2*cos(d*x + c) - 4*(a^3 + 6*a*b^2)*cos(d*x + c)^3 + 2*b^3 - 6*(2*a^2*b +
 b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^2*sin(d*x + c))
```

giac [A] time = 0.36, size = 225, normalized size = 1.69

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\left(2\,a^2b+b^3\right)\log\left(\left|\mathrm{ta}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(a^3*tan(1/2*d*x + 1/2*c) - 3*a^2*b*tan(1/2*d*x + 1/2*c) + 3*a*b^2*tan(
1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c) + 3*(2*a^2*b + b^3)*log(abs(tan
(1/2*d*x + 1/2*c) + 1)) - 3*(2*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) -
1)) - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)/tan(1/2*d*x + 1/2*c) - 2*(6*a*b^2*tan
(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/
2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

maple [A] time = 0.64, size = 158, normalized size = 1.19

$$-\frac{a^3\cot\left(dx+c\right)}{d}-\frac{3a^2b}{d\sin\left(dx+c\right)}+\frac{3a^2b\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{3b^2a}{d\sin\left(dx+c\right)\cos\left(dx+c\right)}-\frac{6a\,b^2\cot\left(dx+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*sec(d*x+c))^3,x)
```

```
[Out] -a^3*cot(d*x+c)/d-3/d*a^2*b/sin(d*x+c)+3/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+
3/d*b^2*a/sin(d*x+c)/cos(d*x+c)-6*a*b^2*cot(d*x+c)/d+1/2/d*b^3/sin(d*x+c)/c
os(d*x+c)^2-3/2/d*b^3/sin(d*x+c)+3/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.87, size = 139, normalized size = 1.05

$$-\frac{b^3\left(\frac{2\left(3\sin(dx+c)^2-2\right)}{\sin(dx+c)^3-\sin(dx+c)}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)+6a^2b\left(\frac{2}{\sin(dx+c)}-\log\left(\sin\left(dx+c\right.\right.\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*(2*(3*sin(d*x + c)^2 - 2)/(sin(d*x + c)^3 - sin(d*x + c)) - 3*log
(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 6*a^2*b*(2/sin(d*x + c) - l
og(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a*b^2*(1/tan(d*x + c) -
tan(d*x + c)) + 4*a^3/tan(d*x + c))/d
```

mupad [B] time = 1.47, size = 181, normalized size = 1.36

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(6\,a^2\,b+3\,b^3\right)}{d}-\frac{3\,a\,b^2-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(2\,a^3+6\,a^2\,b+18\,a\,b^2+4\,b^3\right)+3\,a^2\,b+a^3+b^3}{d\,\left(2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/sin(c + d*x)^2,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(6*a^2*b + 3*b^3))/d - (3*a*b^2 - tan(c/2 + (d*x
)/2)^2*(18*a*b^2 + 6*a^2*b + 2*a^3 + 4*b^3) + 3*a^2*b + a^3 + b^3 + tan(c/2
 + (d*x)/2)^4*(15*a*b^2 + 3*a^2*b + a^3 - b^3))/(d*(2*tan(c/2 + (d*x)/2) -
4*tan(c/2 + (d*x)/2)^3 + 2*tan(c/2 + (d*x)/2)^5)) + (tan(c/2 + (d*x)/2)*(a
- b)^3)/(2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sec\left(c+dx\right)\right)^3\csc^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*csc(c + d*x)**2, x)
```

3.194 $\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=205

$$-\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}-\frac{a^2b\csc^3(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{3ab^2\tan(c+dx)}{d}$$

```
[Out] 3*a^2*b*arctanh(sin(d*x+c))/d+5/2*b^3*arctanh(sin(d*x+c))/d-a^3*cot(d*x+c)/
d-6*a*b^2*cot(d*x+c)/d-1/3*a^3*cot(d*x+c)^3/d-a*b^2*cot(d*x+c)^3/d-3*a^2*b*
csc(d*x+c)/d-5/2*b^3*csc(d*x+c)/d-a^2*b*csc(d*x+c)^3/d-5/6*b^3*csc(d*x+c)^3
/d+1/2*b^3*csc(d*x+c)^3*sec(d*x+c)^2/d+3*a*b^2*tan(d*x+c)/d
```

Rubi [A] time = 0.29, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3872, 2912, 3767, 2621, 302, 207, 2620, 270, 288}

$$-\frac{a^2b\csc^3(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (5*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cot[c + d*x])/d - (6*a*b^2*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d)
 - (a*b^2*Cot[c + d*x]^3)/d - (3*a^2*b*Csc[c + d*x])/d - (5*b^3*Csc[c + d*x
])/(2*d) - (a^2*b*Csc[c + d*x]^3)/d - (5*b^3*Csc[c + d*x]^3)/(6*d) + (b^3*C
sc[c + d*x]^3*Sec[c + d*x]^2)/(2*d) + (3*a*b^2*Tan[c + d*x])/d
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx = -\int (-b-a\cos(c+dx))^3\csc^4(c+dx)\sec^3(c+dx)\,dx$$
$$= \int \left(a^3\csc^4(c+dx)+3a^2b\csc^4(c+dx)\sec(c+dx)+3ab^2\csc^4(c+dx\right.$$
$$= a^3\int \csc^4(c+dx)\,dx+\left(3a^2b\right)\int \csc^4(c+dx)\sec(c+dx)\,dx+\left(3ab^2\right)$$
$$= -\frac{a^3\operatorname{Subst}\left(\int\left(1+x^2\right)dx,x,\cot(c+dx)\right)}{d}-\frac{\left(3a^2b\right)\operatorname{Subst}\left(\int\frac{x^4}{-1+x^2}dx\right.}{d}$$
$$= -\frac{a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{b^3\csc^3(c+dx)\sec^2(c+dx)}{2d}-\frac{(3}{}$$
$$= -\frac{a^3\cot(c+dx)}{d}-\frac{6ab^2\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{ab^2\cot^3(c+}{d}$$
$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}-\frac{6ab^2\cot(c+dx)}{d}-\frac{a^3\cot}{}$$
$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{5b^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\cot(c+dx)}{d}$$

Mathematica [B] time = 0.97, size = 610, normalized size = 2.98

$$-\frac{\csc^7\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(4a^3\cos(3(c+dx))-4a^3\cos(5(c+dx))+8\left(6a^2b+5b^3\right)\cos(2(c+dx))+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/768*(Csc[(c + d*x)/2]^7*Sec[(c + d*x)/2]^3*(84*a^2*b + 22*b^3 + 32*a*(a^
2 + 3*b^2)*Cos[c + d*x] + 8*(6*a^2*b + 5*b^3)*Cos[2*(c + d*x)] + 4*a^3*Cos[
3*(c + d*x)] + 48*a*b^2*Cos[3*(c + d*x)] - 36*a^2*b*Cos[4*(c + d*x)] - 30*b
^3*Cos[4*(c + d*x)] - 4*a^3*Cos[5*(c + d*x)] - 48*a*b^2*Cos[5*(c + d*x)] +
36*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 30*b^3*Log
[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 36*a^2*b*Log[Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 30*b^3*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]*Sin[c + d*x] + 18*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]]*Sin[3*(c + d*x)] + 15*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Si
n[3*(c + d*x)] - 18*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c
 + d*x)] - 15*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)]
 - 18*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 15*
b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 18*a^2*b*Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 15*b^3*Log[Cos[(c
 + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)]))/(d*(-1 + Cot[(c + d*x)/2]
^2)^2)
```

fricas [A] time = 1.21, size = 260, normalized size = 1.27

$$-\frac{8\left(a^{3}+12\,ab^{2}\right)\cos\left(dx+c\right)^{5}+6\left(6\,a^{2}b+5\,b^{3}\right)\cos\left(dx+c\right)^{4}+36\,ab^{2}\cos\left(dx+c\right)-12\left(a^{3}+12\,ab^{2}\right)\cos\left(dx+c\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/12*(8*(a^3 + 12*a*b^2)*cos(d*x + c)^5 + 6*(6*a^2*b + 5*b^3)*cos(d*x + c)
^4 + 36*a*b^2*cos(d*x + c) - 12*(a^3 + 12*a*b^2)*cos(d*x + c)^3 + 6*b^3 - 8
*(6*a^2*b + 5*b^3)*cos(d*x + c)^2 - 3*((6*a^2*b + 5*b^3)*cos(d*x + c)^4 - (
6*a^2*b + 5*b^3)*cos(d*x + c)^2)*log(sin(d*x + c) + 1)*sin(d*x + c) + 3*((6
*a^2*b + 5*b^3)*cos(d*x + c)^4 - (6*a^2*b + 5*b^3)*cos(d*x + c)^2)*log(-sin
(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^4 - d*cos(d*x + c)^2)*sin(d*x
 + c))
```

giac [A] time = 0.52, size = 361, normalized size = 1.76

$$\frac{a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-3\,a^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+3\,ab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-b^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*tan(1/2*d*x + 1/2*c)^3 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 3*a*b^2
*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 + 9*a^3*tan(1/2*d*x +
1/2*c) - 45*a^2*b*tan(1/2*d*x + 1/2*c) + 63*a*b^2*tan(1/2*d*x + 1/2*c) - 27
*b^3*tan(1/2*d*x + 1/2*c) + 12*(6*a^2*b + 5*b^3)*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - 12*(6*a^2*b + 5*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 24*(6*
a*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2
*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 -
(9*a^3*tan(1/2*d*x + 1/2*c)^2 + 45*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 63*a*b^2*
tan(1/2*d*x + 1/2*c)^2 + 27*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3 + 3*a^2*b + 3*
a*b^2 + b^3)/tan(1/2*d*x + 1/2*c)^3)/d
```

maple [A] time = 0.92, size = 246, normalized size = 1.20

$$-\frac{2a^{3}\cot\left(dx+c\right)}{3d}-\frac{a^{3}\cot\left(dx+c\right)\left(\csc^{2}\left(dx+c\right)\right)}{3d}-\frac{a^{2}b}{d\sin\left(dx+c\right)^{3}}-\frac{3a^{2}b}{d\sin\left(dx+c\right)}+\frac{3a^{2}b\ln\left(\sec\left(dx+c\right)+\tan\left(dx\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*sec(d*x+c))^3,x)
```

```
[Out] -2/3*a^3*cot(d*x+c)/d-1/3/d*a^3*cot(d*x+c)*csc(d*x+c)^2-1/d*a^2*b/sin(d*x+c
)^3-3/d*a^2*b/sin(d*x+c)+3/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))-1/d*b^2*a/sin(
d*x+c)^3/cos(d*x+c)+4/d*b^2*a/sin(d*x+c)/cos(d*x+c)-8*a*b^2*cot(d*x+c)/d-1/
3/d*b^3/sin(d*x+c)^3/cos(d*x+c)^2+5/6/d*b^3/sin(d*x+c)/cos(d*x+c)^2-5/2/d*b
^3/sin(d*x+c)+5/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 1.56, size = 190, normalized size = 0.93

$$-\frac{b^3\left(\frac{2\left(15\sin(dx+c)^4-10\sin(dx+c)^2-2\right)}{\sin(dx+c)^5-\sin(dx+c)^3}-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)\right)+6a^2b\left(\frac{2\left(3\sin(dx+c)^2+1\right.}{\sin(dx+c)^3}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(b^3*(2*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 - 2)/(sin(d*x + c)^5 -
 sin(d*x + c)^3) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 6
*a^2*b*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) +
 3*log(sin(d*x + c) - 1)) + 12*a*b^2*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3
 - 3*tan(d*x + c)) + 4*(3*tan(d*x + c)^2 + 1)*a^3/tan(d*x + c)^3)/d
```

mupad [B] time = 1.17, size = 260, normalized size = 1.27

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(a-b)^3}{24d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{3b(a-b)^2}{4}-\frac{3(a-b)^3}{8}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\left(\frac{7a^3}{3}+13a^2b+19ab^2+\frac{25b^3}{3}\right)-\text{ta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/sin(c + d*x)^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*(a - b)^3)/(24*d) - (tan(c/2 + (d*x)/2)*((3*b*(a - b)
^2)/4 - (3*(a - b)^3)/8))/d - (atanh(tan(c/2 + (d*x)/2))*(a^2*b*6i + b^3*5i
)*1i)/d - (tan(c/2 + (d*x)/2)^2*(19*a*b^2 + 13*a^2*b + (7*a^3)/3 + (25*b^3)
/3) - tan(c/2 + (d*x)/2)^4*(89*a*b^2 + 29*a^2*b + (17*a^3)/3 + (77*b^3)/3)
+ a*b^2 + a^2*b + a^3/3 + b^3/3 + tan(c/2 + (d*x)/2)^6*(69*a*b^2 + 15*a^2*b
 + 3*a^3 + b^3))/(d*(8*tan(c/2 + (d*x)/2)^3 - 16*tan(c/2 + (d*x)/2)^5 + 8*t
an(c/2 + (d*x)/2)^7))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.195 $\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=279

$$-\frac{a^3\cot^5(c+dx)}{5d}-\frac{2a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}-\frac{3a^2b\csc^5(c+dx)}{5d}-\frac{a^2b\csc^3(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b}{}$$

```
[Out] 3*a^2*b*arctanh(sin(d*x+c))/d+7/2*b^3*arctanh(sin(d*x+c))/d-a^3*cot(d*x+c)/
d-9*a*b^2*cot(d*x+c)/d-2/3*a^3*cot(d*x+c)^3/d-3*a*b^2*cot(d*x+c)^3/d-1/5*a^
3*cot(d*x+c)^5/d-3/5*a*b^2*cot(d*x+c)^5/d-3*a^2*b*csc(d*x+c)/d-7/2*b^3*csc(
d*x+c)/d-a^2*b*csc(d*x+c)^3/d-7/6*b^3*csc(d*x+c)^3/d-3/5*a^2*b*csc(d*x+c)^5
/d-7/10*b^3*csc(d*x+c)^5/d+1/2*b^3*csc(d*x+c)^5*sec(d*x+c)^2/d+3*a*b^2*tan(
d*x+c)/d
```

Rubi [A] time = 0.32, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3872, 2912, 3767, 2621, 302, 207, 2620, 270, 288}

$$-\frac{3a^2b\csc^5(c+dx)}{5d}-\frac{a^2b\csc^3(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\cot^5(c+dx)}{5d}-\frac{2a^3\cot^3(c+}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (7*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cot[c + d*x])/d - (9*a*b^2*Cot[c + d*x])/d - (2*a^3*Cot[c + d*x]^3)/(3*
d) - (3*a*b^2*Cot[c + d*x]^3)/d - (a^3*Cot[c + d*x]^5)/(5*d) - (3*a*b^2*Cot
[c + d*x]^5)/(5*d) - (3*a^2*b*Csc[c + d*x])/d - (7*b^3*Csc[c + d*x])/(2*d)
- (a^2*b*Csc[c + d*x]^3)/d - (7*b^3*Csc[c + d*x]^3)/(6*d) - (3*a^2*b*Csc[c
+ d*x]^5)/(5*d) - (7*b^3*Csc[c + d*x]^5)/(10*d) + (b^3*Csc[c + d*x]^5*Sec[c
 + d*x]^2)/(2*d) + (3*a*b^2*Tan[c + d*x])/d
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx = -\int (-b-a\cos(c+dx))^3\csc^6(c+dx)\sec^3(c+dx)\,dx$$

$$= \int \left(a^3\csc^6(c+dx)+3a^2b\csc^6(c+dx)\sec(c+dx)+3ab^2\csc^6(c+dx\right.$$

$$= a^3\int \csc^6(c+dx)\,dx+\left(3a^2b\right)\int \csc^6(c+dx)\sec(c+dx)\,dx+\left(3ab^2\right)$$

$$= -\frac{a^3\,\text{Subst}\left(\int\left(1+2x^2+x^4\right)dx,x,\cot(c+dx)\right)}{d}-\frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{}{-1}\right.}{}$$

$$= -\frac{a^3\cot(c+dx)}{d}-\frac{2a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot^5(c+dx)}{5d}+\frac{b^3\csc^5(c+d}{2}$$

$$= -\frac{a^3\cot(c+dx)}{d}-\frac{9ab^2\cot(c+dx)}{d}-\frac{2a^3\cot^3(c+dx)}{3d}-\frac{3ab^2\cot^3(c}{d}$$

$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}-\frac{9ab^2\cot(c+dx)}{d}-\frac{2a^3c}{}$$

$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{7b^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\cot(c+dx)}{d}$$

Mathematica [B] time = 1.50, size = 812, normalized size = 2.91

$$-\frac{\csc^9\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(16\cos(3(c+dx))a^3-48\cos(5(c+dx))a^3+16\cos(7(c+dx))a^3+1176ba^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/61440*(Csc[(c + d*x)/2]^9*Sec[(c + d*x)/2]^5*(1176*a^2*b + 412*b^3 + 80*
a*(5*a^2 + 18*b^2)*Cos[c + d*x] + 66*(6*a^2*b + 7*b^3)*Cos[2*(c + d*x)] + 1
6*a^3*Cos[3*(c + d*x)] + 288*a*b^2*Cos[3*(c + d*x)] - 600*a^2*b*Cos[4*(c +
d*x)] - 700*b^3*Cos[4*(c + d*x)] - 48*a^3*Cos[5*(c + d*x)] - 864*a*b^2*Cos[
5*(c + d*x)] + 180*a^2*b*Cos[6*(c + d*x)] + 210*b^3*Cos[6*(c + d*x)] + 16*a
^3*Cos[7*(c + d*x)] + 288*a*b^2*Cos[7*(c + d*x)] + 450*a^2*b*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 525*b^3*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]]*Sin[c + d*x] - 450*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]]*Sin[c + d*x] - 525*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[
c + d*x] + 90*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x
)] + 105*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 90
*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 105*b^3*
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 270*a^2*b*Log[C
os[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 315*b^3*Log[Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 270*a^2*b*Log[Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 315*b^3*Log[Cos[(c + d*x)/2] + S
in[(c + d*x)/2]]*Sin[5*(c + d*x)] + 90*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]]*Sin[7*(c + d*x)] + 105*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]]*Sin[7*(c + d*x)] - 90*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Si
n[7*(c + d*x)] - 105*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[7*(c
+ d*x)]))/(d*(-1 + Cot[(c + d*x)/2]^2)^2)
```

fricas [A] time = 0.62, size = 354, normalized size = 1.27

$$-\frac{32\left(a^3+18\,ab^2\right)\cos\left(dx+c\right)^7+30\left(6\,a^2b+7\,b^3\right)\cos\left(dx+c\right)^6-80\left(a^3+18\,ab^2\right)\cos\left(dx+c\right)^5-70\left(6\,a^2b+7\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(32*(a^3 + 18*a*b^2)*cos(d*x + c)^7 + 30*(6*a^2*b + 7*b^3)*cos(d*x +
c)^6 - 80*(a^3 + 18*a*b^2)*cos(d*x + c)^5 - 70*(6*a^2*b + 7*b^3)*cos(d*x +
c)^4 - 180*a*b^2*cos(d*x + c) + 60*(a^3 + 18*a*b^2)*cos(d*x + c)^3 - 30*b^3
 + 46*(6*a^2*b + 7*b^3)*cos(d*x + c)^2 - 15*((6*a^2*b + 7*b^3)*cos(d*x + c)
^6 - 2*(6*a^2*b + 7*b^3)*cos(d*x + c)^4 + (6*a^2*b + 7*b^3)*cos(d*x + c)^2)
*log(sin(d*x + c) + 1)*sin(d*x + c) + 15*((6*a^2*b + 7*b^3)*cos(d*x + c)^6
- 2*(6*a^2*b + 7*b^3)*cos(d*x + c)^4 + (6*a^2*b + 7*b^3)*cos(d*x + c)^2)*lo
g(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^6 - 2*d*cos(d*x + c)^4
+ d*cos(d*x + c)^2)*sin(d*x + c))
```

giac [A] time = 0.38, size = 498, normalized size = 1.78

$$\frac{3\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-9\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+25\,a^3\tan\left(\frac{1}{2}\,dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 9*a*
b^2*tan(1/2*d*x + 1/2*c)^5 - 3*b^3*tan(1/2*d*x + 1/2*c)^5 + 25*a^3*tan(1/2*
d*x + 1/2*c)^3 - 105*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 135*a*b^2*tan(1/2*d*x +
 1/2*c)^3 - 55*b^3*tan(1/2*d*x + 1/2*c)^3 + 150*a^3*tan(1/2*d*x + 1/2*c) -
990*a^2*b*tan(1/2*d*x + 1/2*c) + 1710*a*b^2*tan(1/2*d*x + 1/2*c) - 870*b^3*
tan(1/2*d*x + 1/2*c) + 240*(6*a^2*b + 7*b^3)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) - 240*(6*a^2*b + 7*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 480*(6*a*
b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d
*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - (1
50*a^3*tan(1/2*d*x + 1/2*c)^4 + 990*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 1710*a*b
^2*tan(1/2*d*x + 1/2*c)^4 + 870*b^3*tan(1/2*d*x + 1/2*c)^4 + 25*a^3*tan(1/2
*d*x + 1/2*c)^2 + 105*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 135*a*b^2*tan(1/2*d*x
+ 1/2*c)^2 + 55*b^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3 + 9*a^2*b + 9*a*b^2 + 3*
b^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

maple [A] time = 0.97, size = 334, normalized size = 1.20

$$-\frac{8a^3\cot(dx+c)}{15d}-\frac{a^3\cot(dx+c)\left(\csc^4(dx+c)\right)}{5d}-\frac{4a^3\cot(dx+c)\left(\csc^2(dx+c)\right)}{15d}-\frac{3a^2b}{5d\sin(dx+c)^5}-\frac{a^2b}{d\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+b*sec(d*x+c))^3,x)
```

```
[Out] -8/15*a^3*cot(d*x+c)/d-1/5/d*a^3*cot(d*x+c)*csc(d*x+c)^4-4/15/d*a^3*cot(d*x
+c)*csc(d*x+c)^2-3/5/d*a^2*b/sin(d*x+c)^5-1/d*a^2*b/sin(d*x+c)^3-3/d*a^2*b/
sin(d*x+c)+3/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))-3/5/d*b^2*a/sin(d*x+c)^5/cos
(d*x+c)-6/5/d*b^2*a/sin(d*x+c)^3/cos(d*x+c)+24/5/d*b^2*a/sin(d*x+c)/cos(d*x
+c)-48/5*a*b^2*cot(d*x+c)/d-1/5/d*b^3/sin(d*x+c)^5/cos(d*x+c)^2-7/15/d*b^3/
sin(d*x+c)^3/cos(d*x+c)^2+7/6/d*b^3/sin(d*x+c)/cos(d*x+c)^2-7/2/d*b^3/sin(d
*x+c)+7/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

maxima [A] time = 0.80, size = 230, normalized size = 0.82

$$-\frac{b^3\left(\frac{2\left(105\sin(dx+c)^6-70\sin(dx+c)^4-14\sin(dx+c)^2-6\right)}{\sin(dx+c)^7-\sin(dx+c)^5}-105\log(\sin(dx+c)+1)+105\log(\sin(dx+c)-1)\right)+6a^2b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(b^3*(2*(105*sin(d*x + c)^6 - 70*sin(d*x + c)^4 - 14*sin(d*x + c)^2 -
 6)/(sin(d*x + c)^7 - sin(d*x + c)^5) - 105*log(sin(d*x + c) + 1) + 105*log
(sin(d*x + c) - 1)) + 6*a^2*b*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)
/sin(d*x + c)^5 - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 36
*a*b^2*((15*tan(d*x + c)^4 + 5*tan(d*x + c)^2 + 1)/tan(d*x + c)^5 - 5*tan(d
*x + c)) + 4*(15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 + 3)*a^3/tan(d*x + c)^5
)/d
```

mupad [B] time = 1.22, size = 363, normalized size = 1.30

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,{\left(a-b\right)}^3}{160\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{19\,a^3}{15}+\frac{29\,a^2\,b}{5}+\frac{39\,a\,b^2}{5}+\frac{49\,b^3}{15}\right)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8\,\left(10\,a^3+66\,a^2\,b+306\,a\right.}{d\,\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^3/sin(c + d*x)^6,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(a - b)^3)/(160*d) - (tan(c/2 + (d*x)/2)^2*((39*a*b^2
)/5 + (29*a^2*b)/5 + (19*a^3)/15 + (49*b^3)/15) + tan(c/2 + (d*x)/2)^8*(306
*a*b^2 + 66*a^2*b + 10*a^3 + 26*b^3) - tan(c/2 + (d*x)/2)^6*(411*a*b^2 + 12
5*a^2*b + (55*a^3)/3 + (433*b^3)/3) + tan(c/2 + (d*x)/2)^4*((483*a*b^2)/5 +
 (263*a^2*b)/5 + (103*a^3)/15 + (763*b^3)/15) + (3*a*b^2)/5 + (3*a^2*b)/5 +
 a^3/5 + b^3/5)/(d*(32*tan(c/2 + (d*x)/2)^5 - 64*tan(c/2 + (d*x)/2)^7 + 32*
tan(c/2 + (d*x)/2)^9)) - (atanh(tan(c/2 + (d*x)/2))*(a^2*b*6i + b^3*7i)*1i)
/d + (tan(c/2 + (d*x)/2)*((21*a*b^2)/16 - (3*a^2*b)/8 - a^3/16 - (7*b^3)/8
+ (3*(a - b)^2*(a - 4*b))/16 + (3*(a - b)^3)/16))/d + (tan(c/2 + (d*x)/2)^3
*(((a - b)^2*(a - 4*b))/48 + (a - b)^3/32))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.196 $\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=223

$$-\frac{b\cos^6(c+dx)}{6a^2d}+\frac{b\left(a^2-b^2\right)^3\log(a\cos(c+dx)+b)}{a^8d}-\frac{\left(a^2-b^2\right)^3\cos(c+dx)}{a^7d}+\frac{b\left(3a^2-b^2\right)\cos^4(c+dx)}{4a^4d}-\frac{(3a^2$$

```
[Out] -(a^2-b^2)^3*cos(d*x+c)/a^7/d-1/2*b*(3*a^4-3*a^2*b^2+b^4)*cos(d*x+c)^2/a^6/
d+1/3*(3*a^4-3*a^2*b^2+b^4)*cos(d*x+c)^3/a^5/d+1/4*b*(3*a^2-b^2)*cos(d*x+c)
^4/a^4/d-1/5*(3*a^2-b^2)*cos(d*x+c)^5/a^3/d-1/6*b*cos(d*x+c)^6/a^2/d+1/7*co
s(d*x+c)^7/a/d+b*(a^2-b^2)^3*ln(b+a*cos(d*x+c))/a^8/d
```

Rubi [A] time = 0.25, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2837, 12, 772}

$$-\frac{\left(3a^2-b^2\right)\cos^5(c+dx)}{5a^3d}+\frac{b\left(3a^2-b^2\right)\cos^4(c+dx)}{4a^4d}+\frac{\left(-3a^2b^2+3a^4+b^4\right)\cos^3(c+dx)}{3a^5d}-\frac{b\left(-3a^2b^2+3a^4+b^4\right.}{2a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)^3*Cos[c + d*x])/(a^7*d)) - (b*(3*a^4 - 3*a^2*b^2 + b^4)*Cos[
c + d*x]^2)/(2*a^6*d) + ((3*a^4 - 3*a^2*b^2 + b^4)*Cos[c + d*x]^3)/(3*a^5*d
) + (b*(3*a^2 - b^2)*Cos[c + d*x]^4)/(4*a^4*d) - ((3*a^2 - b^2)*Cos[c + d*x
]^5)/(5*a^3*d) - (b*Cos[c + d*x]^6)/(6*a^2*d) + Cos[c + d*x]^7/(7*a*d) + (b
*(a^2 - b^2)^3*Log[b + a*Cos[c + d*x]])/(a^8*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)\sin^7(c+dx)}{-b-a\cos(c+dx)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x(a^2-x^2)^3}{a(-b+x)}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{x(a^2-x^2)^3}{-b+x}\,dx, x, -a\cos(c+dx)\right)}{a^8 d}$$
$$= \frac{\text{Subst}\left(\int \left((a^2-b^2)^3 + \frac{b(-a^2+b^2)^3}{b-x} - b\left(3a^4-3a^2b^2+b^4\right)x - \left(3a^4-3a^2b^2+b^4\right)x^2 - b\left(\right.\right.\right.}{a^8 d}$$
$$= -\frac{(a^2-b^2)^3\cos(c+dx)}{a^7 d} - \frac{b\left(3a^4-3a^2b^2+b^4\right)\cos^2(c+dx)}{2a^6 d} + \frac{\left(3a^4-3a^2b^2+b^4\right)\cos^3}{3a^5 d}$$

Mathematica [A] time = 1.56, size = 282, normalized size = 1.26

$$\frac{735a^7\cos(3(c+dx)) - 147a^7\cos(5(c+dx)) + 15a^7\cos(7(c+dx)) + 420a^6b\cos(4(c+dx)) - 35a^6b\cos(6(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-105*a*(35*a^6 - 152*a^4*b^2 + 176*a^2*b^4 - 64*b^6)*Cos[c + d*x] - 105*(2
9*a^6*b - 40*a^4*b^3 + 16*a^2*b^5)*Cos[2*(c + d*x)] + 735*a^7*Cos[3*(c + d*
x)] - 1260*a^5*b^2*Cos[3*(c + d*x)] + 560*a^3*b^4*Cos[3*(c + d*x)] + 420*a^
6*b*Cos[4*(c + d*x)] - 210*a^4*b^3*Cos[4*(c + d*x)] - 147*a^7*Cos[5*(c + d*
x)] + 84*a^5*b^2*Cos[5*(c + d*x)] - 35*a^6*b*Cos[6*(c + d*x)] + 15*a^7*Cos[
7*(c + d*x)] + 6720*a^6*b*Log[b + a*Cos[c + d*x]] - 20160*a^4*b^3*Log[b + a
*Cos[c + d*x]] + 20160*a^2*b^5*Log[b + a*Cos[c + d*x]] - 6720*b^7*Log[b + a
*Cos[c + d*x]])/(6720*a^8*d)
```

fricas [A] time = 0.61, size = 222, normalized size = 1.00

$$\frac{60\,a^7\cos(dx+c)^7 - 70\,a^6b\cos(dx+c)^6 - 84\left(3\,a^7 - a^5b^2\right)\cos(dx+c)^5 + 105\left(3\,a^6b - a^4b^3\right)\cos(dx+c)^4 + 140}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/420*(60*a^7*cos(d*x + c)^7 - 70*a^6*b*cos(d*x + c)^6 - 84*(3*a^7 - a^5*b^
2)*cos(d*x + c)^5 + 105*(3*a^6*b - a^4*b^3)*cos(d*x + c)^4 + 140*(3*a^7 - 3
*a^5*b^2 + a^3*b^4)*cos(d*x + c)^3 - 210*(3*a^6*b - 3*a^4*b^3 + a^2*b^5)*co
s(d*x + c)^2 - 420*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c) + 420
*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*log(a*cos(d*x + c) + b))/(a^8*d)
```

giac [B] time = 1.62, size = 1559, normalized size = 6.99

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/420*(420*(a^7*b - a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 - 3*a^2*b^6
 - a*b^7 + b^8)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b
```

```
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^9 - a^8*b) - 420*(a^6*b - 3*a^4
*b^3 + 3*a^2*b^5 - b^7)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)
)/a^8 + (384*a^7 - 1089*a^6*b - 1848*a^5*b^2 + 3267*a^4*b^3 + 2240*a^3*b^4
- 3267*a^2*b^5 - 840*a*b^6 + 1089*b^7 - 2688*a^7*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) + 8463*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12096*a^5*
b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 24549*a^4*b^3*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) - 14000*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 23709*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 5040*a*b^6*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) - 7623*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + 8064*a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 28749*a^6*b*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 32088*a^5*b^2*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 + 78687*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2 + 35280*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 72807*a^2*b
^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 12600*a*b^6*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2 + 22869*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2 - 13440*a^7*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*a^6*b*(c
os(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 40320*a^5*b^2*(cos(d*x + c) - 1)^
3/(cos(d*x + c) + 1)^3 - 136185*a^4*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c)
+ 1)^3 - 45920*a^3*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 122745*a
^2*b^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 16800*a*b^6*(cos(d*x + c
) - 1)^3/(cos(d*x + c) + 1)^3 - 38115*b^7*(cos(d*x + c) - 1)^3/(cos(d*x + c
) + 1)^3 - 56035*a^6*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 24360*a^
5*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 136185*a^4*b^3*(cos(d*x +
 c) - 1)^4/(cos(d*x + c) + 1)^4 + 32480*a^3*b^4*(cos(d*x + c) - 1)^4/(cos(d
*x + c) + 1)^4 - 122745*a^2*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 -
 12600*a*b^6*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 38115*b^7*(cos(d*x
 + c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a^6*b*(cos(d*x + c) - 1)^5/(cos(d
*x + c) + 1)^5 + 6720*a^5*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 7
8687*a^4*b^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 11760*a^3*b^4*(cos
(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 72807*a^2*b^5*(cos(d*x + c) - 1)^5/
(cos(d*x + c) + 1)^5 + 5040*a*b^6*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5
 - 22869*b^7*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 8463*a^6*b*(cos(d*
x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 840*a^5*b^2*(cos(d*x + c) - 1)^6/(cos(
d*x + c) + 1)^6 + 24549*a^4*b^3*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 +
 1680*a^3*b^4*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 23709*a^2*b^5*(co
s(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 840*a*b^6*(cos(d*x + c) - 1)^6/(co
s(d*x + c) + 1)^6 + 7623*b^7*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 10
89*a^6*b*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 3267*a^4*b^3*(cos(d*x
+ c) - 1)^7/(cos(d*x + c) + 1)^7 + 3267*a^2*b^5*(cos(d*x + c) - 1)^7/(cos(d
*x + c) + 1)^7 - 1089*b^7*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/(a^8*(
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7))/d
```

maple [A] time = 0.54, size = 363, normalized size = 1.63

$$\frac{\cos^{7}\left(dx+c\right)}{7da}-\frac{b\left(\cos^{6}\left(dx+c\right)\right)}{6a^{2}d}-\frac{3\left(\cos^{5}\left(dx+c\right)\right)}{5da}+\frac{\left(\cos^{5}\left(dx+c\right)\right)b^{2}}{5d\,a^{3}}+\frac{3b\left(\cos^{4}\left(dx+c\right)\right)}{4a^{2}d}-\frac{\left(\cos^{4}\left(dx+c\right)\right)b^{3}}{4d\,a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/7*cos(d*x+c)^7/d/a-1/6*b*cos(d*x+c)^6/a^2/d-3/5*cos(d*x+c)^5/d/a+1/5/d/a^
3*cos(d*x+c)^5*b^2+3/4*b*cos(d*x+c)^4/a^2/d-1/4/d/a^4*cos(d*x+c)^4*b^3+cos(
d*x+c)^3/d/a-1/d/a^3*cos(d*x+c)^3*b^2+1/3/d/a^5*cos(d*x+c)^3*b^4-3/2*b*cos(
d*x+c)^2/a^2/d+3/2/d/a^4*cos(d*x+c)^2*b^3-1/2/d/a^6*cos(d*x+c)^2*b^5-cos(d*
x+c)/d/a+3/d/a^3*cos(d*x+c)*b^2-3/d/a^5*cos(d*x+c)*b^4+1/d/a^7*cos(d*x+c)*b
^6+b*ln(b+a*cos(d*x+c))/a^2/d-3/d*b^3/a^4*ln(b+a*cos(d*x+c))+3/d*b^5/a^6*ln
(b+a*cos(d*x+c))-1/d*b^7/a^8*ln(b+a*cos(d*x+c))
```

maxima [A] time = 0.58, size = 224, normalized size = 1.00

$$\frac{\frac{60\,a^6\cos(dx+c)^7-70\,a^5b\cos(dx+c)^6-84\left(3\,a^6-a^4b^2\right)\cos(dx+c)^5+105\left(3\,a^5b-a^3b^3\right)\cos(dx+c)^4+140\left(3\,a^6-3\,a^4b^2+a^2b^4\right)\cos(dx+c)^3-210\left(3\,a^5b-3\,a^3b^3\right.}{a^7}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/420*((60*a^6*cos(d*x + c)^7 - 70*a^5*b*cos(d*x + c)^6 - 84*(3*a^6 - a^4*b
^2)*cos(d*x + c)^5 + 105*(3*a^5*b - a^3*b^3)*cos(d*x + c)^4 + 140*(3*a^6 -
3*a^4*b^2 + a^2*b^4)*cos(d*x + c)^3 - 210*(3*a^5*b - 3*a^3*b^3 + a*b^5)*cos
(d*x + c)^2 - 420*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos(d*x + c))/a^7 + 4
20*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*log(a*cos(d*x + c) + b)/a^8)/d
```

mupad [B] time = 0.16, size = 249, normalized size = 1.12

$$\frac{\cos\left(c+d\,x\right)^3\left(\frac{1}{a}-\frac{b^2\left(\frac{1}{a}-\frac{b^2}{3\,a^3}\right)}{a^2}\right)-\cos\left(c+d\,x\right)^5\left(\frac{3}{5\,a}-\frac{b^2}{5\,a^3}\right)-\cos\left(c+d\,x\right)\left(\frac{1}{a}-\frac{b^2\left(\frac{3}{a}-\frac{b^2\left(\frac{3}{a}-\frac{b^2}{a^3}\right)}{a^2}\right)}{a^2}\right)+\frac{\cos(c+d\,x)^7}{7\,a}+\frac{\ln(b+}{}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a + b/cos(c + d*x)),x)
```

```
[Out] (cos(c + d*x)^3*(1/a - (b^2*(1/a - b^2/(3*a^3)))/a^2) - cos(c + d*x)^5*(3/(
5*a) - b^2/(5*a^3)) - cos(c + d*x)*(1/a - (b^2*(3/a - (b^2*(3/a - b^2/a^3))
/a^2))/a^2) + cos(c + d*x)^7/(7*a) + (log(b + a*cos(c + d*x))*(a^6*b - b^7
+ 3*a^2*b^5 - 3*a^4*b^3))/a^8 - (b*cos(c + d*x)^6)/(6*a^2) - (b*cos(c + d*x
)^2*(3/a - (b^2*(3/a - b^2/a^3))/a^2))/(2*a) + (b*cos(c + d*x)^4*(3/a - b^2
/a^3))/(4*a))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.197 $\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=152

$$\frac{b\cos^4(c+dx)}{4a^2d}+\frac{b\left(a^2-b^2\right)^2\log(a\cos(c+dx)+b)}{a^6d}-\frac{\left(a^2-b^2\right)^2\cos(c+dx)}{a^5d}-\frac{b\left(2a^2-b^2\right)\cos^2(c+dx)}{2a^4d}+\frac{\left(2a^2-\right.}{}$$

[Out] -(a^2-b^2)^2*cos(d*x+c)/a^5/d-1/2*b*(2*a^2-b^2)*cos(d*x+c)^2/a^4/d+1/3*(2*a^2-b^2)*cos(d*x+c)^3/a^3/d+1/4*b*cos(d*x+c)^4/a^2/d-1/5*cos(d*x+c)^5/a/d+b*(a^2-b^2)^2*ln(b+a*cos(d*x+c))/a^6/d

Rubi [A] time = 0.19, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2837, 12, 772}

$$\frac{\left(2a^2-b^2\right)\cos^3(c+dx)}{3a^3d}-\frac{b\left(2a^2-b^2\right)\cos^2(c+dx)}{2a^4d}-\frac{\left(a^2-b^2\right)^2\cos(c+dx)}{a^5d}+\frac{b\left(a^2-b^2\right)^2\log(a\cos(c+dx)+b)}{a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)^2*Cos[c + d*x])/(a^5*d)) - (b*(2*a^2 - b^2)*Cos[c + d*x]^2)/(2*a^4*d) + ((2*a^2 - b^2)*Cos[c + d*x]^3)/(3*a^3*d) + (b*Cos[c + d*x]^4)/(4*a^2*d) - Cos[c + d*x]^5/(5*a*d) + (b*(a^2 - b^2)^2*Log[b + a*Cos[c + d*x]])/(a^6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)\sin^5(c+dx)}{-b-a\cos(c+dx)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x\left(a^2-x^2\right)^2}{a(-b+x)}dx, x, -a\cos(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{x\left(a^2-x^2\right)^2}{-b+x}dx, x, -a\cos(c+dx)\right)}{a^6 d}$$
$$= \frac{\text{Subst}\left(\int \left(\left(a^2-b^2\right)^2 - \frac{b\left(-a^2+b^2\right)^2}{b-x} + b\left(-2a^2+b^2\right)x - \left(2a^2-b^2\right)x^2 + bx^3 + x^4\right)dx, x, -\right.}{a^6 d}$$
$$= -\frac{\left(a^2-b^2\right)^2\cos(c+dx)}{a^5 d} - \frac{b\left(2a^2-b^2\right)\cos^2(c+dx)}{2a^4 d} + \frac{\left(2a^2-b^2\right)\cos^3(c+dx)}{3a^3 d} + \frac{b\cos^4}{4}$$

Mathematica [A] time = 0.38, size = 172, normalized size = 1.13

$$50a^5\cos(3(c+dx)) - 6a^5\cos(5(c+dx)) + 15a^4b\cos(4(c+dx)) + 480a^4b\log(a\cos(c+dx)+b) - 40a^3b^2\cos(3($$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-60*a*(5*a^4 - 14*a^2*b^2 + 8*b^4)*Cos[c + d*x] - 60*(3*a^4*b - 2*a^2*b^3)
*Cos[2*(c + d*x)] + 50*a^5*Cos[3*(c + d*x)] - 40*a^3*b^2*Cos[3*(c + d*x)] +
 15*a^4*b*Cos[4*(c + d*x)] - 6*a^5*Cos[5*(c + d*x)] + 480*a^4*b*Log[b + a*C
os[c + d*x]] - 960*a^2*b^3*Log[b + a*Cos[c + d*x]] + 480*b^5*Log[b + a*Cos[
c + d*x]])/(480*a^6*d)
```

fricas [A] time = 0.61, size = 140, normalized size = 0.92

$$-\frac{12\,a^5\cos(dx+c)^5 - 15\,a^4b\cos(dx+c)^4 - 20\left(2\,a^5 - a^3b^2\right)\cos(dx+c)^3 + 30\left(2\,a^4b - a^2b^3\right)\cos(dx+c)^2 + 60}{60\,a^6 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(12*a^5*cos(d*x + c)^5 - 15*a^4*b*cos(d*x + c)^4 - 20*(2*a^5 - a^3*b^
2)*cos(d*x + c)^3 + 30*(2*a^4*b - a^2*b^3)*cos(d*x + c)^2 + 60*(a^5 - 2*a^3
*b^2 + a*b^4)*cos(d*x + c) - 60*(a^4*b - 2*a^2*b^3 + b^5)*log(a*cos(d*x + c
) + b))/(a^6*d)
```

giac [B] time = 0.44, size = 867, normalized size = 5.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(60*(a^5*b - a^4*b^2 - 2*a^3*b^3 + 2*a^2*b^4 + a*b^5 - b^6)*log(abs(a
+ b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1)))/(a^7 - a^6*b) - 60*(a^4*b - 2*a^2*b^3 + b^5)*log(abs(-(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^6 + (64*a^5 - 137*a^4*b - 200*a^3*b
^2 + 274*a^2*b^3 + 120*a*b^4 - 137*b^5 - 320*a^5*(cos(d*x + c) - 1)/(cos(d*
```

```
x + c) + 1) + 805*a^4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 880*a^3*b^2
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1490*a^2*b^3*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 480*a*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 685*b^
5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 640*a^5*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2 - 1970*a^4*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1
280*a^3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3100*a^2*b^3*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 720*a*b^4*(cos(d*x + c) - 1)^2/(cos(d
*x + c) + 1)^2 - 1370*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970*
a^4*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 720*a^3*b^2*(cos(d*x + c)
 - 1)^3/(cos(d*x + c) + 1)^3 - 3100*a^2*b^3*(cos(d*x + c) - 1)^3/(cos(d*x +
 c) + 1)^3 - 480*a*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1370*b^5
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 805*a^4*b*(cos(d*x + c) - 1)^4
/(cos(d*x + c) + 1)^4 - 120*a^3*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)
^4 + 1490*a^2*b^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 120*a*b^4*(co
s(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 685*b^5*(cos(d*x + c) - 1)^4/(cos(
d*x + c) + 1)^4 + 137*a^4*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 274
*a^2*b^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 137*b^5*(cos(d*x + c)
- 1)^5/(cos(d*x + c) + 1)^5)/(a^6*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
1)^5))/d
```

maple [A] time = 0.40, size = 216, normalized size = 1.42

$$-\frac{\cos^5(dx+c)}{5da}+\frac{b\left(\cos^4(dx+c)\right)}{4a^2d}+\frac{2\left(\cos^3(dx+c)\right)}{3da}-\frac{\left(\cos^3(dx+c)\right)b^2}{3d\,a^3}-\frac{b\left(\cos^2(dx+c)\right)}{a^2d}+\frac{\left(\cos^2(dx+c)\right)b^3}{2d\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/5*cos(d*x+c)^5/d/a+1/4*b*cos(d*x+c)^4/a^2/d+2/3*cos(d*x+c)^3/d/a-1/3/d/a
^3*cos(d*x+c)^3*b^2-b*cos(d*x+c)^2/a^2/d+1/2/d/a^4*cos(d*x+c)^2*b^3-cos(d*x
+c)/d/a+2/d/a^3*cos(d*x+c)*b^2-1/d/a^5*cos(d*x+c)*b^4+b*ln(b+a*cos(d*x+c))/
a^2/d-2/d*b^3/a^4*ln(b+a*cos(d*x+c))+1/d*b^5/a^6*ln(b+a*cos(d*x+c))
```

maxima [A] time = 0.72, size = 141, normalized size = 0.93

$$-\frac{\frac{12\,a^4\cos(dx+c)^5-15\,a^3b\cos(dx+c)^4-20\left(2\,a^4-a^2b^2\right)\cos(dx+c)^3+30\left(2\,a^3b-ab^3\right)\cos(dx+c)^2+60\left(a^4-2\,a^2b^2+b^4\right)\cos(dx+c)}{a^5}-\frac{60\left(a^4b-2\,a^2b^3+b\right.}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*((12*a^4*cos(d*x + c)^5 - 15*a^3*b*cos(d*x + c)^4 - 20*(2*a^4 - a^2*b
^2)*cos(d*x + c)^3 + 30*(2*a^3*b - a*b^3)*cos(d*x + c)^2 + 60*(a^4 - 2*a^2*
b^2 + b^4)*cos(d*x + c))/a^5 - 60*(a^4*b - 2*a^2*b^3 + b^5)*log(a*cos(d*x +
 c) + b)/a^6)/d
```

mupad [B] time = 0.09, size = 151, normalized size = 0.99

$$-\frac{\cos(c+d\,x)\left(\frac{1}{a}-\frac{b^2\left(\frac{2}{a}-\frac{b^2}{a^3}\right)}{a^2}\right)-\cos(c+d\,x)^3\,\left(\frac{2}{3\,a}-\frac{b^2}{3\,a^3}\right)+\frac{\cos(c+d\,x)^5}{5\,a}-\frac{b\cos(c+d\,x)^4}{4\,a^2}-\frac{\ln(b+a\,\cos(c+d\,x))\left(a^4\,b-2\,a^2\,b^3+\right.}{a^6}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + b/cos(c + d*x)),x)
```

```
[Out] -(cos(c + d*x)*(1/a - (b^2*(2/a - b^2/a^3))/a^2) - cos(c + d*x)^3*(2/(3*a)
- b^2/(3*a^3)) + cos(c + d*x)^5/(5*a) - (b*cos(c + d*x)^4)/(4*a^2) - (log(b
```

```
 + a*cos(c + d*x))*(a^4*b + b^5 - 2*a^2*b^3))/a^6 + (b*cos(c + d*x)^2*(2/a
- b^2/a^3))/(2*a))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.198 $\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=89

$$-\frac{b\cos^2(c+dx)}{2a^2d}+\frac{b\left(a^2-b^2\right)\log(a\cos(c+dx)+b)}{a^4d}-\frac{\left(a^2-b^2\right)\cos(c+dx)}{a^3d}+\frac{\cos^3(c+dx)}{3ad}$$

```
[Out] -(a^2-b^2)*cos(d*x+c)/a^3/d-1/2*b*cos(d*x+c)^2/a^2/d+1/3*cos(d*x+c)^3/a/d+b
*(a^2-b^2)*ln(b+a*cos(d*x+c))/a^4/d
```

Rubi [A] time = 0.16, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = $\{3872, 2837, 12, 772\}$

$$-\frac{\left(a^2-b^2\right)\cos(c+dx)}{a^3d}+\frac{b\left(a^2-b^2\right)\log(a\cos(c+dx)+b)}{a^4d}-\frac{b\cos^2(c+dx)}{2a^2d}+\frac{\cos^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)*Cos[c + d*x])/(a^3*d)) - (b*Cos[c + d*x]^2)/(2*a^2*d) + Cos[
c + d*x]^3/(3*a*d) + (b*(a^2 - b^2)*Log[b + a*Cos[c + d*x]])/(a^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)\sin^3(c+dx)}{-b-a\cos(c+dx)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x\left(a^2-x^2\right)}{a(-b+x)}\,dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{x\left(a^2-x^2\right)}{-b+x}\,dx, x, -a\cos(c+dx)\right)}{a^4d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{b^2}{a^2}\right)+\frac{-a^2b+b^3}{b-x}-bx-x^2\right)dx, x, -a\cos(c+dx)\right)}{a^4d}$$
$$= -\frac{\left(a^2-b^2\right)\cos(c+dx)}{a^3d}-\frac{b\cos^2(c+dx)}{2a^2d}+\frac{\cos^3(c+dx)}{3ad}+\frac{b\left(a^2-b^2\right)\log(b+a\cos(c+ \ldots}{a^4d}$$

Mathematica [A] time = 0.21, size = 89, normalized size = 1.00

$$\frac{\left(12ab^2-9a^3\right)\cos(c+dx)+a^3\cos(3(c+dx))-3a^2b\cos(2(c+dx))+12a^2b\log(a\cos(c+dx)+b)-12b^3\log(a \ldots}{12a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((-9*a^3 + 12*a*b^2)*Cos[c + d*x] - 3*a^2*b*Cos[2*(c + d*x)] + a^3*Cos[3*(c
 + d*x)] + 12*a^2*b*Log[b + a*Cos[c + d*x]] - 12*b^3*Log[b + a*Cos[c + d*x]
])/(12*a^4*d)
```

fricas [A] time = 0.56, size = 78, normalized size = 0.88

$$\frac{2\,a^3\cos\left(dx+c\right)^3-3\,a^2b\cos\left(dx+c\right)^2-6\left(a^3-ab^2\right)\cos\left(dx+c\right)+6\left(a^2b-b^3\right)\log\left(a\cos\left(dx+c\right)+b\right)}{6\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*cos(d*x + c)^3 - 3*a^2*b*cos(d*x + c)^2 - 6*(a^3 - a*b^2)*cos(d*
x + c) + 6*(a^2*b - b^3)*log(a*cos(d*x + c) + b))/(a^4*d)
```

giac [A] time = 0.23, size = 102, normalized size = 1.15

$$\frac{\left(a^2b-b^3\right)\log\left(|-a\cos\left(dx+c\right)-b|\right)}{a^4d}+\frac{2\,a^2d^2\cos\left(dx+c\right)^3-3\,abd^2\cos\left(dx+c\right)^2-6\,a^2d^2\cos\left(dx+c\right)+6\,b^2d^2\cos \ldots}{6\,a^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (a^2*b - b^3)*log(abs(-a*cos(d*x + c) - b))/(a^4*d) + 1/6*(2*a^2*d^2*cos(d*
x + c)^3 - 3*a*b*d^2*cos(d*x + c)^2 - 6*a^2*d^2*cos(d*x + c) + 6*b^2*d^2*co
s(d*x + c))/(a^3*d^3)
```

maple [A] time = 0.40, size = 106, normalized size = 1.19

$$\frac{\cos^3\left(dx+c\right)}{3da}-\frac{b\left(\cos^2\left(dx+c\right)\right)}{2a^2d}-\frac{\cos\left(dx+c\right)}{da}+\frac{\cos\left(dx+c\right)b^2}{d\,a^3}+\frac{b\ln\left(b+a\cos\left(dx+c\right)\right)}{a^2d}-\frac{b^3\ln\left(b+a\cos\left(dx+c\right) \ldots}{d\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/3*cos(d*x+c)^3/d/a-1/2*b*cos(d*x+c)^2/a^2/d-cos(d*x+c)/d/a+1/d/a^3*cos(d*
x+c)*b^2+b*ln(b+a*cos(d*x+c))/a^2/d-1/d*b^3/a^4*ln(b+a*cos(d*x+c))
```

maxima [A] time = 0.58, size = 80, normalized size = 0.90

$$\frac{\frac{2\,a^2\cos(dx+c)^3-3\,ab\cos(dx+c)^2-6\left(a^2-b^2\right)\cos(dx+c)}{a^3}+\frac{6\left(a^2b-b^3\right)\log(a\cos(dx+c)+b)}{a^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*((2*a^2*cos(d*x + c)^3 - 3*a*b*cos(d*x + c)^2 - 6*(a^2 - b^2)*cos(d*x +
 c))/a^3 + 6*(a^2*b - b^3)*log(a*cos(d*x + c) + b)/a^4)/d
```

mupad [B] time = 1.02, size = 79, normalized size = 0.89

$$-\frac{\cos\left(c+d\,x\right)\left(\frac{1}{a}-\frac{b^2}{a^3}\right)-\frac{\cos(c+d\,x)^3}{3\,a}+\frac{b\cos(c+d\,x)^2}{2\,a^2}-\frac{\ln(b+a\,\cos(c+d\,x))\left(a^2\,b-b^3\right)}{a^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + b/cos(c + d*x)),x)
```

```
[Out] -(cos(c + d*x)*(1/a - b^2/a^3) - cos(c + d*x)^3/(3*a) + (b*cos(c + d*x)^2)/
(2*a^2) - (log(b + a*cos(c + d*x))*(a^2*b - b^3))/a^4)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.199 $\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{b\log(a\cos(c+dx)+b)}{a^2d}-\frac{\cos(c+dx)}{ad}$$

```
[Out] -cos(d*x+c)/a/d+b*ln(b+a*cos(d*x+c))/a^2/d
```

Rubi [A] time = 0.08, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 2833, 12, 43}

$$\frac{b\log(a\cos(c+dx)+b)}{a^2d}-\frac{\cos(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Cos[c + d*x]/(a*d)) + (b*Log[b + a*Cos[c + d*x]])/(a^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)\sin(c+dx)}{-b-a\cos(c+dx)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x}{a(-b+x)}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{-b+x}\,dx, x, -a\cos(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(1-\frac{b}{b-x}\right)dx, x, -a\cos(c+dx)\right)}{a^2d}$$
$$= -\frac{\cos(c+dx)}{ad} + \frac{b\log(b+a\cos(c+dx))}{a^2d}$$

Mathematica [A] time = 0.02, size = 30, normalized size = 0.88

$$\frac{b\log(a\cos(c+dx)+b) - a\cos(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Sec[c + d*x]),x]
[Out] (-(a*Cos[c + d*x]) + b*Log[b + a*Cos[c + d*x]])/(a^2*d)
```

fricas [A] time = 0.51, size = 31, normalized size = 0.91

$$-\frac{a\cos(dx+c) - b\log(a\cos(dx+c)+b)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] -(a*cos(d*x + c) - b*log(a*cos(d*x + c) + b))/(a^2*d)
```

giac [A] time = 0.24, size = 38, normalized size = 1.12

$$-\frac{\cos(dx+c)}{ad} + \frac{b\log(|-a\cos(dx+c)-b|)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] -cos(d*x + c)/(a*d) + b*log(abs(-a*cos(d*x + c) - b))/(a^2*d)
```

maple [A] time = 0.06, size = 53, normalized size = 1.56

$$\frac{b\ln(a+b\sec(dx+c))}{d\,a^2} - \frac{1}{da\sec(dx+c)} - \frac{b\ln(\sec(dx+c))}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+b*sec(d*x+c)),x)
[Out] 1/d*b/a^2*ln(a+b*sec(d*x+c))-1/d/a/sec(d*x+c)-1/d*b/a^2*ln(sec(d*x+c))
```

maxima [A] time = 0.53, size = 33, normalized size = 0.97

$$-\frac{\frac{\cos(dx+c)}{a} - \frac{b\log(a\cos(dx+c)+b)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(cos(d*x + c)/a - b*log(a*cos(d*x + c) + b)/a^2)/d
```

mupad [B] time = 0.06, size = 30, normalized size = 0.88

$$\frac{b\ \ln\left(b+a\ \cos\left(c+d\,x\right)\right)-a\ \cos\left(c+d\,x\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b/cos(c + d*x)),x)
```

```
[Out] (b*log(b + a*cos(c + d*x)) - a*cos(c + d*x))/(a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*sec(c + d*x)), x)
```

3.200 $\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{b\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)}$$

```
[Out] 1/2*ln(1-cos(d*x+c))/(a+b)/d-1/2*ln(1+cos(d*x+c))/(a-b)/d+b*ln(b+a*cos(d*x+
c))/(a^2-b^2)/d
```

Rubi [A] time = 0.11, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3872, 2721, 801}

$$\frac{b\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) - Log[1 + Cos[c + d*x]]/(2*(a - b)*d) +
 (b*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)*d)
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)}{-b-a\cos(c+dx)}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{x}{(-b+x)\left(a^2-x^2\right)}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{2(a-b)(a-x)}-\frac{b}{(a-b)(a+b)(b-x)}+\frac{1}{2(a+b)(a+x)}\right)dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{\log(1-\cos(c+dx))}{2(a+b)d}-\frac{\log(1+\cos(c+dx))}{2(a-b)d}+\frac{b\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)d}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 63, normalized size = 0.85

$$\frac{(a-b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\left((a+b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+b\log(a\cos(c+dx)+b)}{d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-((a + b)*Log[Cos[(c + d*x)/2]]) + b*Log[b + a*Cos[c + d*x]] + (a - b)*Log
[Sin[(c + d*x)/2]])/((a - b)*(a + b)*d)
```

fricas [A] time = 0.52, size = 64, normalized size = 0.86

$$\frac{2\,b\log\left(a\cos\left(dx+c\right)+b\right)-(a+b)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)+(a-b)\log\left(-\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(a^2-b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*log(a*cos(d*x + c) + b) - (a + b)*log(1/2*cos(d*x + c) + 1/2) + (a
 - b)*log(-1/2*cos(d*x + c) + 1/2))/((a^2 - b^2)*d)
```

giac [A] time = 0.30, size = 100, normalized size = 1.35

$$\frac{\frac{2\,b\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^2-b^2}+\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^2 - b^2) + log(abs(-cos(d*x + c) + 1)
/abs(cos(d*x + c) + 1))/(a + b))/d
```

maple [A] time = 0.42, size = 75, normalized size = 1.01

$$\frac{b\ln\left(b+a\cos\left(dx+c\right)\right)}{d\left(a-b\right)\left(a+b\right)}+\frac{\ln\left(-1+\cos\left(dx+c\right)\right)}{d\left(2a+2b\right)}-\frac{\ln\left(1+\cos\left(dx+c\right)\right)}{d\left(2a-2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*b/(a-b)/(a+b)*ln(b+a*cos(d*x+c))+1/d/(2*a+2*b)*ln(-1+cos(d*x+c))-1/d/(2
*a-2*b)*ln(1+cos(d*x+c))
```

maxima [A] time = 0.75, size = 64, normalized size = 0.86

$$\frac{\frac{2\,b\log(a\cos(dx+c)+b)}{a^2-b^2}-\frac{\log(\cos(dx+c)+1)}{a-b}+\frac{\log(\cos(dx+c)-1)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*log(a*cos(d*x + c) + b)/(a^2 - b^2) - log(cos(d*x + c) + 1)/(a - b
) + log(cos(d*x + c) - 1)/(a + b))/d
```

mupad [B] time = 1.15, size = 68, normalized size = 0.92

$$\frac{\ln\left(\cos\left(c+d\,x\right)-1\right)}{2\,d\,\left(a+b\right)}-\frac{\ln\left(\cos\left(c+d\,x\right)+1\right)}{2\,d\,\left(a-b\right)}+\frac{b\,\ln\left(b+a\,\cos\left(c+d\,x\right)\right)}{d\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + b/cos(c + d*x))),x)
```

```
[Out] log(cos(c + d*x) - 1)/(2*d*(a + b)) - log(cos(c + d*x) + 1)/(2*d*(a - b)) +
 (b*log(b + a*cos(c + d*x)))/(d*(a^2 - b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)/(a + b*sec(c + d*x)), x)
```

3.201 $\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=116

$$\frac{a^2b\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^2}+\frac{\csc^2(c+dx)(b-a\cos(c+dx))}{2d\left(a^2-b^2\right)}+\frac{a\log(1-\cos(c+dx))}{4d(a+b)^2}-\frac{a\log(\cos(c+dx)+1)}{4d(a-b)^2}$$

```
[Out] 1/2*(b-a*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)/d+1/4*a*ln(1-cos(d*x+c))/(a+b)^
2/d-1/4*a*ln(1+cos(d*x+c))/(a-b)^2/d+a^2*b*ln(b+a*cos(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A] time = 0.21, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = $\{3872, 2837, 12, 823, 801\}$

$$\frac{a^2b\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^2}+\frac{\csc^2(c+dx)(b-a\cos(c+dx))}{2d\left(a^2-b^2\right)}+\frac{a\log(1-\cos(c+dx))}{4d(a+b)^2}-\frac{a\log(\cos(c+dx)+1)}{4d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((b - a*Cos[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)*d) + (a*Log[1 - Cos[c
+ d*x]])/(4*(a + b)^2*d) - (a*Log[1 + Cos[c + d*x]])/(4*(a - b)^2*d) + (a^2
*b*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
```

```
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cot(c+dx)\csc^2(c+dx)}{-b-a\cos(c+dx)}\,dx$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{x}{a(-b+x)\left(a^2-x^2\right)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{x}{(-b+x)\left(a^2-x^2\right)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)d} + \frac{\text{Subst}\left(\int \frac{a^2b+a^2x}{(-b+x)\left(a^2-x^2\right)}\,dx, x, -a\cos(c+dx)\right)}{2\left(a^2-b^2\right)d}$$
$$= \frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)d} + \frac{\text{Subst}\left(\int \left(\frac{a(a+b)}{2(a-b)(a-x)} - \frac{2a^2b}{(a-b)(a+b)(b-x)} + \frac{a(a-b)}{2(a+b)(a+x)}\right)\right.}{2\left(a^2-b^2\right)d}$$
$$= \frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)d} + \frac{a\log(1-\cos(c+dx))}{4(a+b)^2d} - \frac{a\log(1+\cos(c+dx))}{4(a-b)^2d} +$$

Mathematica [A] time = 0.63, size = 123, normalized size = 1.06

$$\frac{-(a-b)^2(a+b)\csc^2\left(\frac{1}{2}(c+dx)\right) + (a-b)(a+b)^2\sec^2\left(\frac{1}{2}(c+dx)\right) - 4a\left((a-b)^2\left(-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right) + (\right.}{8d(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-((a - b)^2*(a + b)*Csc[(c + d*x)/2]^2) - 4*a*((a + b)^2*Log[Cos[(c + d*x)
/2]] - 2*a*b*Log[b + a*Cos[c + d*x]] - (a - b)^2*Log[Sin[(c + d*x)/2]]) + (
a - b)*(a + b)^2*Sec[(c + d*x)/2]^2)/(8*(a - b)^2*(a + b)^2*d)
```

fricas [A] time = 0.55, size = 216, normalized size = 1.86

$$-\frac{2\,a^2b - 2\,b^3 - 2\left(a^3-ab^2\right)\cos\left(dx+c\right) - 4\left(a^2b\cos\left(dx+c\right)^2 - a^2b\right)\log\left(a\cos\left(dx+c\right)+b\right) - \left(a^3+2\,a^2b+a\right.}{4\left(\left(a^4-2\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*b - 2*b^3 - 2*(a^3 - a*b^2)*cos(d*x + c) - 4*(a^2*b*cos(d*x + c
)^2 - a^2*b)*log(a*cos(d*x + c) + b) - (a^3 + 2*a^2*b + a*b^2 - (a^3 + 2*a^
2*b + a*b^2)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + (a^3 - 2*a^2*b +
 a*b^2 - (a^3 - 2*a^2*b + a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/
2))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2 - (a^4 - 2*a^2*b^2 + b^4)*d)
```

giac [A] time = 0.40, size = 202, normalized size = 1.74

$$\frac{\frac{8\,a^2b\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^4-2\,a^2b^2+b^4} + \frac{2\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2+2\,ab+b^2} + \frac{\left(a+b-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\left(a^2+2\,ab+b^2\right)(\cos(dx+c)-1)} - \frac{\cos(dx+c)-1}{(a-b)(\cos(dx+c)+1)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^4 - 2*a^2*b^2 + b^4) + 2*a*log(ab
s(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^2 + 2*a*b + b^2) + (a + b -
2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/((a^2 + 2*a*b
 + b^2)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a - b)*(cos(d*x + c) + 1
)))/d
```

maple [A] time = 0.55, size = 121, normalized size = 1.04

$$\frac{a^2 b \ln(b + a\cos(dx+c))}{d(a+b)^2(a-b)^2} + \frac{1}{d(4a+4b)(-1+\cos(dx+c))} + \frac{a\ln(-1+\cos(dx+c))}{4d(a+b)^2} + \frac{1}{d(4a-4b)(1+\cos(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*a^2*b/(a+b)^2/(a-b)^2*ln(b+a*cos(d*x+c))+1/d/(4*a+4*b)/(-1+cos(d*x+c))+
1/4/d*a/(a+b)^2*ln(-1+cos(d*x+c))+1/d/(4*a-4*b)/(1+cos(d*x+c))-1/4*a*ln(1+c
os(d*x+c))/(a-b)^2/d
```

maxima [A] time = 0.73, size = 132, normalized size = 1.14

$$\frac{\frac{4a^2 b\log(a\cos(dx+c)+b)}{a^4-2a^2b^2+b^4} - \frac{a\log(\cos(dx+c)+1)}{a^2-2ab+b^2} + \frac{a\log(\cos(dx+c)-1)}{a^2+2ab+b^2} + \frac{2(a\cos(dx+c)-b)}{(a^2-b^2)\cos(dx+c)^2-a^2+b^2}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a^2*b*log(a*cos(d*x + c) + b)/(a^4 - 2*a^2*b^2 + b^4) - a*log(cos(d*
x + c) + 1)/(a^2 - 2*a*b + b^2) + a*log(cos(d*x + c) - 1)/(a^2 + 2*a*b + b^
2) + 2*(a*cos(d*x + c) - b)/((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2))/d
```

mupad [B] time = 0.30, size = 133, normalized size = 1.15

$$\frac{a\ln(\cos(c+dx)-1)}{4d(a+b)^2} - \frac{\ln(b+a\cos(c+dx))\left(\frac{a}{4(a+b)^2} - \frac{a}{4(a-b)^2}\right)}{d} - \frac{\frac{b}{2(a^2-b^2)} - \frac{a\cos(c+dx)}{2(a^2-b^2)}}{d\left(\cos(c+dx)^2-1\right)} - \frac{a\ln(\cos(c+dx)+1}{4d(a-b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + b/cos(c + d*x))),x)
```

```
[Out] (a*log(cos(c + d*x) - 1))/(4*d*(a + b)^2) - (log(b + a*cos(c + d*x))*(a/(4*
(a + b)^2) - a/(4*(a - b)^2)))/d - (b/(2*(a^2 - b^2)) - (a*cos(c + d*x))/(2
*(a^2 - b^2)))/(d*(cos(c + d*x)^2 - 1)) - (a*log(cos(c + d*x) + 1))/(4*d*(a
 - b)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

3.202 $\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=179

$$\frac{\csc^4(c+dx)(b-a\cos(c+dx))}{4d\left(a^2-b^2\right)}+\frac{\csc^2(c+dx)\left(4a^2b-a\left(3a^2+b^2\right)\cos(c+dx)\right)}{8d\left(a^2-b^2\right)^2}+\frac{a^4b\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^3}+$$

```
[Out] 1/8*(4*a^2*b-a*(3*a^2+b^2)*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)^2/d+1/4*(b-a*
cos(d*x+c))*csc(d*x+c)^4/(a^2-b^2)/d+1/16*a*(3*a+b)*ln(1-cos(d*x+c))/(a+b)^
3/d-1/16*a*(3*a-b)*ln(1+cos(d*x+c))/(a-b)^3/d+a^4*b*ln(b+a*cos(d*x+c))/(a^2
-b^2)^3/d
```

Rubi [A] time = 0.30, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2837, 12, 823, 801}

$$\frac{a^4b\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^3}+\frac{\csc^4(c+dx)(b-a\cos(c+dx))}{4d\left(a^2-b^2\right)}+\frac{\csc^2(c+dx)\left(4a^2b-a\left(3a^2+b^2\right)\cos(c+dx)\right)}{8d\left(a^2-b^2\right)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((4*a^2*b - a*(3*a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x]^2)/(8*(a^2 - b^2)^2*
d) + ((b - a*Cos[c + d*x])*Csc[c + d*x]^4)/(4*(a^2 - b^2)*d) + (a*(3*a + b)
*Log[1 - Cos[c + d*x]])/(16*(a + b)^3*d) - (a*(3*a - b)*Log[1 + Cos[c + d*x
]])/(16*(a - b)^3*d) + (a^4*b*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cot(c+dx)\csc^4(c+dx)}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{a^5\,\text{Subst}\left(\int \frac{x}{a(-b+x)\left(a^2-x^2\right)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^4\,\text{Subst}\left(\int \frac{x}{(-b+x)\left(a^2-x^2\right)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{(b-a\cos(c+dx))\csc^4(c+dx)}{4\left(a^2-b^2\right)d} + \frac{a^2\,\text{Subst}\left(\int \frac{a^2b+3a^2x}{(-b+x)\left(a^2-x^2\right)^2}\,dx, x, -a\cos(c+dx)\right)}{4\left(a^2-b^2\right)d}$$

$$= \frac{\left(4a^2b-a\left(3a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{8\left(a^2-b^2\right)^2 d} + \frac{(b-a\cos(c+dx))\csc^4(c+dx)}{4\left(a^2-b^2\right)d} + \text{Su}$$

$$= \frac{\left(4a^2b-a\left(3a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{8\left(a^2-b^2\right)^2 d} + \frac{(b-a\cos(c+dx))\csc^4(c+dx)}{4\left(a^2-b^2\right)d} + \text{Su}$$

$$= \frac{\left(4a^2b-a\left(3a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{8\left(a^2-b^2\right)^2 d} + \frac{(b-a\cos(c+dx))\csc^4(c+dx)}{4\left(a^2-b^2\right)d} + a(3$$

Mathematica [A] time = 5.22, size = 207, normalized size = 1.16

$$8a\left(8a^3b\log(a\cos(c+dx)+b)+(a-b)^3(3a+b)\log\left(\sin\left(\tfrac{1}{2}(c+dx)\right)\right)-(3a-b)(a+b)^3\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*(a - b)^3*(3*a^2 + 4*a*b + b^2)*Csc[(c + d*x)/2]^2 - (a - b)^3*(a + b)^
2*Csc[(c + d*x)/2]^4 + 8*a*(-((3*a - b)*(a + b)^3*Log[Cos[(c + d*x)/2]]) +
8*a^3*b*Log[b + a*Cos[c + d*x]] + (a - b)^3*(3*a + b)*Log[Sin[(c + d*x)/2]]
) + 2*(a + b)^3*(3*a^2 - 4*a*b + b^2)*Sec[(c + d*x)/2]^2 + (a - b)^2*(a + b
)^3*Sec[(c + d*x)/2]^4)/(64*(a - b)^3*(a + b)^3*d)
```

fricas [B] time = 0.60, size = 469, normalized size = 2.62

$$12\,a^4b-16\,a^2b^3+4\,b^5+2\left(3\,a^5-2\,a^3b^2-ab^4\right)\cos\left(dx+c\right)^3-8\left(a^4b-a^2b^3\right)\cos\left(dx+c\right)^2-2\left(5\,a^5-6\,a^3b^2+a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(12*a^4*b - 16*a^2*b^3 + 4*b^5 + 2*(3*a^5 - 2*a^3*b^2 - a*b^4)*cos(d*x
 + c)^3 - 8*(a^4*b - a^2*b^3)*cos(d*x + c)^2 - 2*(5*a^5 - 6*a^3*b^2 + a*b^4
)*cos(d*x + c) + 16*(a^4*b*cos(d*x + c)^4 - 2*a^4*b*cos(d*x + c)^2 + a^4*b)
*log(a*cos(d*x + c) + b) - (3*a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4 + (3*a^5 +
8*a^4*b + 6*a^3*b^2 - a*b^4)*cos(d*x + c)^4 - 2*(3*a^5 + 8*a^4*b + 6*a^3*b^
2 - a*b^4)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + (3*a^5 - 8*a^4*b +
 6*a^3*b^2 - a*b^4 + (3*a^5 - 8*a^4*b + 6*a^3*b^2 - a*b^4)*cos(d*x + c)^4 -
 2*(3*a^5 - 8*a^4*b + 6*a^3*b^2 - a*b^4)*cos(d*x + c)^2)*log(-1/2*cos(d*x +
 c) + 1/2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4 - 2*(a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*
b^4 - b^6)*d)
```

giac [B] time = 0.75, size = 419, normalized size = 2.34

$$\frac{\frac{64\,a^4 b\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{4\left(3\,a^2+ab\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{\frac{8\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{b(\cos(dx+c}{(\cos(dx+c}}{a^2-2\,ab+b^2}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/64*(64*a^4*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
) + 4*(3*a^2 + a*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^3
+ 3*a^2*b + 3*a*b^2 + b^3) - (8*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4
*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 + b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(a^2 - 2*a*b + b
^2) - (a^2 + 2*a*b + b^2 - 8*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12
*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b^2*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 18*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 6*a*b*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/((a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*(cos(d*x + c) - 1)^2))/d
```

maple [A] time = 0.55, size = 259, normalized size = 1.45

$$\frac{a^4b\ln\left(b+a\cos\left(dx+c\right)\right)}{d\left(a+b\right)^3\left(a-b\right)^3}-\frac{1}{2d\left(8a+8b\right)\left(-1+\cos\left(dx+c\right)\right)^2}+\frac{3a}{16d\left(a+b\right)^2\left(-1+\cos\left(dx+c\right)\right)}+\frac{}{16d\left(a+b\right)^2\left(-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*a^4*b/(a+b)^3/(a-b)^3*ln(b+a*cos(d*x+c))-1/2/d/(8*a+8*b)/(-1+cos(d*x+c)
)^2+3/16/d/(a+b)^2/(-1+cos(d*x+c))*a+1/16/d/(a+b)^2/(-1+cos(d*x+c))*b+3/16/
d/(a+b)^3*a^2*ln(-1+cos(d*x+c))+1/16/d/(a+b)^3*a*ln(-1+cos(d*x+c))*b+1/2/d/
(8*a-8*b)/(1+cos(d*x+c))^2+3/16/d/(a-b)^2/(1+cos(d*x+c))*a-1/16/d/(a-b)^2/(
1+cos(d*x+c))*b-3/16/d*a^2/(a-b)^3*ln(1+cos(d*x+c))+1/16/d*a/(a-b)^3*ln(1+c
os(d*x+c))*b
```

maxima [A] time = 0.82, size = 268, normalized size = 1.50

$$\frac{\frac{16\,a^4b\log(a\cos(dx+c)+b)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{\left(3\,a^2-ab\right)\log(\cos(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(3\,a^2+ab\right)\log(\cos(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\left(4\,a^2b\cos(dx+c)^2-\left(3\,a^3+ab^2\right)\cos(dx+c)^3-6\,a^2b+\right.}{\left(a^4-2\,a^2b^2+b^4\right)\cos(dx+c)^4+a^4-2\,a^2b^2+b^4-2\,\left(a^4\right.}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^4*b*log(a*cos(d*x + c) + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
- (3*a^2 - a*b)*log(cos(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (3*
```

```
a^2 + a*b)*log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(4*a^2
*b*cos(d*x + c)^2 - (3*a^3 + a*b^2)*cos(d*x + c)^3 - 6*a^2*b + 2*b^3 + (5*a
^3 - a*b^2)*cos(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2
*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2))/d
```

mupad [B] time = 1.61, size = 297, normalized size = 1.66

$$\frac{\frac{3a^2b-b^3}{4\left(a^2-b^2\right)^2}+\frac{\cos(c+dx)^3\left(3a^3+ab^2\right)}{8\left(a^4-2a^2b^2+b^4\right)}-\frac{a^2b\cos(c+dx)^2}{2\left(a^2-b^2\right)^2}-\frac{a\cos(c+dx)\left(5a^2-b^2\right)}{8\left(a^4-2a^2b^2+b^4\right)}}{d\left(\cos(c+dx)^4-2\cos(c+dx)^2+1\right)}+\frac{\ln(\cos(c+dx)-1)\left(\frac{3}{16(a+b)}-\frac{5b}{16(a+b)^2}+\frac{b^2}{8(a+b)^3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^5*(a + b/cos(c + d*x))),x)
```

```
[Out] ((3*a^2*b - b^3)/(4*(a^2 - b^2)^2) + (cos(c + d*x)^3*(a*b^2 + 3*a^3))/(8*(a
^4 + b^4 - 2*a^2*b^2)) - (a^2*b*cos(c + d*x)^2)/(2*(a^2 - b^2)^2) - (a*cos(
c + d*x)*(5*a^2 - b^2))/(8*(a^4 + b^4 - 2*a^2*b^2)))/(d*(cos(c + d*x)^4 - 2
*cos(c + d*x)^2 + 1)) + (log(cos(c + d*x) - 1)*(3/(16*(a + b)) - (5*b)/(16*
(a + b)^2) + b^2/(8*(a + b)^3)))/d - (log(cos(c + d*x) + 1)*(b^2/(8*(a - b)
^3) + (5*b)/(16*(a - b)^2) + 3/(16*(a - b))))/d + (a^4*b*log(b + a*cos(c +
d*x)))/(d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**5/(a + b*sec(c + d*x)), x)
```

3.203 $\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=230

$$-\frac{2b(a-b)^{5/2}(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^7d}+\frac{\sin^5(c+dx)(6b-5a\cos(c+dx))}{30a^2d}+\frac{\sin^3(c+dx)\left(8b\left(a^2-b^2\right)\right.}{}$$

```
[Out] 1/16*(5*a^6-30*a^4*b^2+40*a^2*b^4-16*b^6)*x/a^7-2*(a-b)^(5/2)*b*(a+b)^(5/2)
*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^7/d+1/16*(16*b*(a^2-
b^2)^2-a*(5*a^4-14*a^2*b^2+8*b^4)*cos(d*x+c))*sin(d*x+c)/a^6/d+1/24*(8*b*(a
^2-b^2)-a*(5*a^2-6*b^2)*cos(d*x+c))*sin(d*x+c)^3/a^4/d+1/30*(6*b-5*a*cos(d*
x+c))*sin(d*x+c)^5/a^2/d
```

Rubi [A] time = 0.61, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3872, 2865, 2735, 2659, 208}

$$\frac{\sin^3(c+dx)\left(8b\left(a^2-b^2\right)-a\left(5a^2-6b^2\right)\cos(c+dx)\right)}{24a^4d}+\frac{\sin(c+dx)\left(16b\left(a^2-b^2\right)^2-a\left(-14a^2b^2+5a^4+8b^4\right)\right.}{16a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*x)/(16*a^7) - (2*(a - b)^(5/2)*
b*(a + b)^(5/2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^7*d
) + ((16*b*(a^2 - b^2)^2 - a*(5*a^4 - 14*a^2*b^2 + 8*b^4)*Cos[c + d*x])*Sin
[c + d*x])/(16*a^6*d) + ((8*b*(a^2 - b^2) - a*(5*a^2 - 6*b^2)*Cos[c + d*x])
*Sin[c + d*x]^3)/(24*a^4*d) + ((6*b - 5*a*Cos[c + d*x])*Sin[c + d*x]^5)/(30
*a^2*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
```

```
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)\sin^6(c+dx)}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{(6b-5a\cos(c+dx))\sin^5(c+dx)}{30a^2d} - \frac{\int \frac{\left(-ab+\left(5a^2-6b^2\right)\cos(c+dx)\right)\sin^4(c+dx)}{-b-a\cos(c+dx)}\,dx}{6a^2}$$

$$= \frac{\left(8b\left(a^2-b^2\right)-a\left(5a^2-6b^2\right)\cos(c+dx)\right)\sin^3(c+dx)}{24a^4d} + \frac{(6b-5a\cos(c+dx))\sin^5(c+$$

$$= \frac{\left(16b\left(a^2-b^2\right)^2-a\left(5a^4-14a^2b^2+8b^4\right)\cos(c+dx)\right)\sin(c+dx)}{16a^6d} + \frac{\left(8b\left(a^2-b^2\right)-a\left(\right.\right.}{}$$

$$= \frac{\left(5a^6-30a^4b^2+40a^2b^4-16b^6\right)x}{16a^7} + \frac{\left(16b\left(a^2-b^2\right)^2-a\left(5a^4-14a^2b^2+8b^4\right)\cos(c+d\right.}{16a^6d}$$

$$= \frac{\left(5a^6-30a^4b^2+40a^2b^4-16b^6\right)x}{16a^7} + \frac{\left(16b\left(a^2-b^2\right)^2-a\left(5a^4-14a^2b^2+8b^4\right)\cos(c+d\right.}{16a^6d}$$

$$= \frac{\left(5a^6-30a^4b^2+40a^2b^4-16b^6\right)x}{16a^7} - \frac{2(a-b)^{5/2}b(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^7d} +$$

Mathematica [A] time = 2.40, size = 268, normalized size = 1.17

$$\frac{45a^6\sin(4(c+dx))-5a^6\sin(6(c+dx))+300a^6c+300a^6dx-140a^5b\sin(3(c+dx))+12a^5b\sin(5(c+dx))-30}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

```
[Out] (300*a^6*c - 1800*a^4*b^2*c + 2400*a^2*b^4*c - 960*b^6*c + 300*a^6*d*x - 18
00*a^4*b^2*d*x + 2400*a^2*b^4*d*x - 960*b^6*d*x + 1920*b*(a^2 - b^2)^(5/2)*
ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + 120*a*b*(11*a^4 - 18
*a^2*b^2 + 8*b^4)*Sin[c + d*x] - 15*(15*a^6 - 32*a^4*b^2 + 16*a^2*b^4)*Sin[
2*(c + d*x)] - 140*a^5*b*Sin[3*(c + d*x)] + 80*a^3*b^3*Sin[3*(c + d*x)] + 4
5*a^6*Sin[4*(c + d*x)] - 30*a^4*b^2*Sin[4*(c + d*x)] + 12*a^5*b*Sin[5*(c +
d*x)] - 5*a^6*Sin[6*(c + d*x)])/(960*a^7*d)
```

fricas [A] time = 0.60, size = 553, normalized size = 2.40

$$\left[\frac{15\left(5a^6-30a^4b^2+40a^2b^4-16b^6\right)dx+120\left(a^4b-2a^2b^3+b^5\right)\sqrt{a^2-b^2}\log\left(\frac{2ab\cos(dx+c)-\left(a^2-2b^2\right)\cos(dx+c)^2-2\sqrt{a}}{a^2\cos(dx+c)^2+2a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/240*(15*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*d*x + 120*(a^4*b - 2*
a^2*b^3 + b^5)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(
d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 -
b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (40*a^6*cos(d*x + c
)^5 - 48*a^5*b*cos(d*x + c)^4 - 368*a^5*b + 560*a^3*b^3 - 240*a*b^5 - 10*(1
3*a^6 - 6*a^4*b^2)*cos(d*x + c)^3 + 16*(11*a^5*b - 5*a^3*b^3)*cos(d*x + c)^
2 + 15*(11*a^6 - 18*a^4*b^2 + 8*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/(a^7*d
), 1/240*(15*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*d*x - 240*(a^4*b -
2*a^2*b^3 + b^5)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c)
+ a)/((a^2 - b^2)*sin(d*x + c))) - (40*a^6*cos(d*x + c)^5 - 48*a^5*b*cos(d*
x + c)^4 - 368*a^5*b + 560*a^3*b^3 - 240*a*b^5 - 10*(13*a^6 - 6*a^4*b^2)*co
s(d*x + c)^3 + 16*(11*a^5*b - 5*a^3*b^3)*cos(d*x + c)^2 + 15*(11*a^6 - 18*a
^4*b^2 + 8*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/(a^7*d)]
```

giac [B] time = 0.29, size = 781, normalized size = 3.40

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/240*(15*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*(d*x + c)/a^7 - 480*(a
^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-
2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt
(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^7) + 2*(75*a^5*tan(1/2*d*x + 1/2*c)^11 +
 240*a^4*b*tan(1/2*d*x + 1/2*c)^11 - 210*a^3*b^2*tan(1/2*d*x + 1/2*c)^11 -
480*a^2*b^3*tan(1/2*d*x + 1/2*c)^11 + 120*a*b^4*tan(1/2*d*x + 1/2*c)^11 + 2
40*b^5*tan(1/2*d*x + 1/2*c)^11 + 425*a^5*tan(1/2*d*x + 1/2*c)^9 + 1520*a^4*
b*tan(1/2*d*x + 1/2*c)^9 - 870*a^3*b^2*tan(1/2*d*x + 1/2*c)^9 - 2720*a^2*b^
3*tan(1/2*d*x + 1/2*c)^9 + 360*a*b^4*tan(1/2*d*x + 1/2*c)^9 + 1200*b^5*tan(
1/2*d*x + 1/2*c)^9 + 990*a^5*tan(1/2*d*x + 1/2*c)^7 + 4128*a^4*b*tan(1/2*d*
x + 1/2*c)^7 - 660*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 - 5760*a^2*b^3*tan(1/2*d*
x + 1/2*c)^7 + 240*a*b^4*tan(1/2*d*x + 1/2*c)^7 + 2400*b^5*tan(1/2*d*x + 1/
2*c)^7 - 990*a^5*tan(1/2*d*x + 1/2*c)^5 + 4128*a^4*b*tan(1/2*d*x + 1/2*c)^5
 + 660*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 5760*a^2*b^3*tan(1/2*d*x + 1/2*c)^5
 - 240*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 2400*b^5*tan(1/2*d*x + 1/2*c)^5 - 425
*a^5*tan(1/2*d*x + 1/2*c)^3 + 1520*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 870*a^3*b
^2*tan(1/2*d*x + 1/2*c)^3 - 2720*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 360*a*b^4
*tan(1/2*d*x + 1/2*c)^3 + 1200*b^5*tan(1/2*d*x + 1/2*c)^3 - 75*a^5*tan(1/2*
d*x + 1/2*c) + 240*a^4*b*tan(1/2*d*x + 1/2*c) + 210*a^3*b^2*tan(1/2*d*x + 1
/2*c) - 480*a^2*b^3*tan(1/2*d*x + 1/2*c) - 120*a*b^4*tan(1/2*d*x + 1/2*c) +
 240*b^5*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^6*a^6))/d
```

maple [B] time = 0.52, size = 1566, normalized size = 6.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*b^4-48/d/a^4/(1+tan(
1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*b^3-6/d*b^5/a^5/((a-b)*(a+b))^(1/2
)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+2/d/a^6/(1+tan(1/2*
d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*b^5-2/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^
6*tan(1/2*d*x+1/2*c)^5*b^4-29/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*
x+1/2*c)^9*b^2-2/d/a^7*arctan(tan(1/2*d*x+1/2*c))*b^6-15/4/d/a^3*arctan(tan
```

```
(1/2*d*x+1/2*c))*b^2+5/d/a^5*arctan(tan(1/2*d*x+1/2*c))*b^4+5/8/d/a/(1+tan(
1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11+85/24/d/a/(1+tan(1/2*d*x+1/2*c)^2
)^6*tan(1/2*d*x+1/2*c)^9+33/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/
2*c)^7-33/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5+11/2/d/a^3/
(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*b^2+172/5/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*b+20/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6*
tan(1/2*d*x+1/2*c)^7*b^5-85/24/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1
/2*c)^3-5/8/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)+7/4/d/a^3/(1+
tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*b^2-68/3/d/a^4/(1+tan(1/2*d*x+1/
2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*b^3+172/5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*t
an(1/2*d*x+1/2*c)^7*b-7/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*
c)^11*b^2-4/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*b^3+3/d/
a^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*b^4+2/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*b+2/d*b^7/a^7/((a-b)*(a+b))^(1/2)*arc
tanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-2/d*b/a/((a-b)*(a+b))^(1
/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-4/d/a^4/(1+tan(1/
2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*b^3+5/8/a/d*arctan(tan(1/2*d*x+1/2*c))
+38/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*b-11/2/d/a^3/(1
+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*b^2-48/d/a^4/(1+tan(1/2*d*x+1
/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*b^3+10/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan
(1/2*d*x+1/2*c)^3*b^5+2/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)
*b^5+38/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b+29/4/d/a^
3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b^2+2/d/a^5/(1+tan(1/2*d*
x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*b^4+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*t
an(1/2*d*x+1/2*c)*b-68/3/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c
)^3*b^3-3/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b^4+1/d/a^5
/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*b^4+20/d/a^6/(1+tan(1/2*d
*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*b^5+6/d*b^3/a^3/((a-b)*(a+b))^(1/2)*arc
tanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+10/d/a^6/(1+tan(1/2*d*x+
1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*b^5
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 3.88, size = 3341, normalized size = 14.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + b/cos(c + d*x)),x)
```

```
[Out] (atan((((((((42*a^21*b - 10*a^22 + 32*a^14*b^8 - 48*a^15*b^7 - 80*a^16*b^6 +
 140*a^17*b^5 + 52*a^18*b^4 - 134*a^19*b^3 + 6*a^20*b^2)/a^18 - (tan(c/2 +
(d*x)/2)*(512*a^16*b + 512*a^14*b^3 - 1024*a^15*b^2)*(a^6*5i - b^6*16i + a^
2*b^4*40i - a^4*b^2*30i))/(128*a^19))*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4
*b^2*30i))/(16*a^7) + (tan(c/2 + (d*x)/2)*(1024*a*b^14 - 75*a^14*b + 25*a^1
5 - 512*b^15 + 2048*a^2*b^13 - 5120*a^3*b^12 - 2560*a^4*b^11 + 10240*a^5*b^
10 - 10240*a^7*b^8 + 2540*a^8*b^7 + 5180*a^9*b^6 - 2064*a^10*b^5 - 1136*a^1
1*b^4 + 619*a^12*b^3 + 31*a^13*b^2))/(8*a^12))*(a^6*5i - b^6*16i + a^2*b^4*
40i - a^4*b^2*30i)*1i)/(16*a^7) - (((((42*a^21*b - 10*a^22 + 32*a^14*b^8 -
48*a^15*b^7 - 80*a^16*b^6 + 140*a^17*b^5 + 52*a^18*b^4 - 134*a^19*b^3 + 6*a
```

^20*b^2)/a^18 + (tan(c/2 + (d*x)/2)*(512*a^16*b + 512*a^14*b^3 - 1024*a^15*b^2)*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(128*a^19))*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(16*a^7) - (tan(c/2 + (d*x)/2)*(1024*a*b^14 - 75*a^14*b + 25*a^15 - 512*b^15 + 2048*a^2*b^13 - 5120*a^3*b^12 - 2560*a^4*b^11 + 10240*a^5*b^10 - 10240*a^7*b^8 + 2540*a^8*b^7 + 5180*a^9*b^6 - 2064*a^10*b^5 - 1136*a^11*b^4 + 619*a^12*b^3 + 31*a^13*b^2))/(8*a^12))*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i)*1i)/(16*a^7))/(((25*a^19*b)/4 - 96*a*b^19 + 64*b^20 - 480*a^2*b^18 + 760*a^3*b^17 + 1544*a^4*b^16 - 2628*a^5*b^15 - 2748*a^6*b^14 + 5179*a^7*b^13 + 2890*a^8*b^12 - 6359*a^9*b^11 - 1736*a^10*b^10 + (19951*a^11*b^9)/4 + (937*a^12*b^8)/2 - (4915*a^13*b^7)/2 + (85*a^14*b^6)/2 + 715*a^15*b^5 - (105*a^16*b^4)/2 - (215*a^17*b^3)/2 + (15*a^18*b^2)/2)/a^18 + (((((42*a^21*b - 10*a^22 + 32*a^14*b^8 - 48*a^15*b^7 - 80*a^16*b^6 + 140*a^17*b^5 + 52*a^18*b^4 - 134*a^19*b^3 + 6*a^20*b^2)/a^18 - (tan(c/2 + (d*x)/2)*(512*a^16*b + 512*a^14*b^3 - 1024*a^15*b^2)*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(128*a^19))*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(16*a^7) + (tan(c/2 + (d*x)/2)*(1024*a*b^14 - 75*a^14*b + 25*a^15 - 512*b^15 + 2048*a^2*b^13 - 5120*a^3*b^12 - 2560*a^4*b^11 + 10240*a^5*b^10 - 10240*a^7*b^8 + 2540*a^8*b^7 + 5180*a^9*b^6 - 2064*a^10*b^5 - 1136*a^11*b^4 + 619*a^12*b^3 + 31*a^13*b^2))/(8*a^12))*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(16*a^7) + (((((42*a^21*b - 10*a^22 + 32*a^14*b^8 - 48*a^15*b^7 - 80*a^16*b^6 + 140*a^17*b^5 + 52*a^18*b^4 - 134*a^19*b^3 + 6*a^20*b^2)/a^18 + (tan(c/2 + (d*x)/2)*(512*a^16*b + 512*a^14*b^3 - 1024*a^15*b^2)*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(128*a^19))*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(16*a^7) - (tan(c/2 + (d*x)/2)*(1024*a*b^14 - 75*a^14*b + 25*a^15 - 512*b^15 + 2048*a^2*b^13 - 5120*a^3*b^12 - 2560*a^4*b^11 + 10240*a^5*b^10 - 10240*a^7*b^8 + 2540*a^8*b^7 + 5180*a^9*b^6 - 2064*a^10*b^5 - 1136*a^11*b^4 + 619*a^12*b^3 + 31*a^13*b^2))/(8*a^12))*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i))/(16*a^7)))*(a^6*5i - b^6*16i + a^2*b^4*40i - a^4*b^2*30i)*1i)/(8*a^7*d) - ((tan(c/2 + (d*x)/2)*(8*a*b^4 - 16*a^4*b + 5*a^5 - 16*b^5 + 32*a^2*b^3 - 14*a^3*b^2))/(8*a^6) - (tan(c/2 + (d*x)/2)^11*(8*a*b^4 + 16*a^4*b + 5*a^5 + 16*b^5 - 32*a^2*b^3 - 14*a^3*b^2))/(8*a^6) + (tan(c/2 + (d*x)/2)^3*(72*a*b^4 - 304*a^4*b + 85*a^5 - 240*b^5 + 544*a^2*b^3 - 174*a^3*b^2))/(24*a^6) - (tan(c/2 + (d*x)/2)^9*(72*a*b^4 + 304*a^4*b + 85*a^5 + 240*b^5 - 544*a^2*b^3 - 174*a^3*b^2))/(24*a^6) + (tan(c/2 + (d*x)/2)^5*(40*a*b^4 - 688*a^4*b + 165*a^5 - 400*b^5 + 960*a^2*b^3 - 110*a^3*b^2))/(20*a^6) - (tan(c/2 + (d*x)/2)^7*(40*a*b^4 + 688*a^4*b + 165*a^5 + 400*b^5 - 960*a^2*b^3 - 110*a^3*b^2))/(20*a^6))/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) + (b*atan(((b*((tan(c/2 + (d*x)/2)*(1024*a*b^14 - 75*a^14*b + 25*a^15 - 512*b^15 + 2048*a^2*b^13 - 5120*a^3*b^12 - 2560*a^4*b^11 + 10240*a^5*b^10 - 10240*a^7*b^8 + 2540*a^8*b^7 + 5180*a^9*b^6 - 2064*a^10*b^5 - 1136*a^11*b^4 + 619*a^12*b^3 + 31*a^13*b^2))/(8*a^12) + (b*((a + b)^5*(a - b)^5)^(1/2)*((42*a^21*b - 10*a^22 + 32*a^14*b^8 - 48*a^15*b^7 - 80*a^16*b^6 + 140*a^17*b^5 + 52*a^18*b^4 - 134*a^19*b^3 + 6*a^20*b^2)/a^18 - (b*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(512*a^16*b + 512*a^14*b^3 - 1024*a^15*b^2))/(8*a^19)))/a^7)*((a + b)^5*(a - b)^5)^(1/2)*1i)/a^7 + (b*((tan(c/2 + (d*x)/2)*(1024*a*b^14 - 75*a^14*b + 25*a^15 - 512*b^15 + 2048*a^2*b^13 - 5120*a^3*b^12 - 2560*a^4*b^11 + 10240*a^5*b^10 - 10240*a^7*b^8 + 2540*a^8*b^7 + 5180*a^9*b^6 - 2064*a^10*b^5 - 1136*a^11*b^4 + 619*a^12*b^3 + 31*a^13*b^2))/(8*a^12) - (b*((a + b)^5*(a - b)^5)^(1/2)*((42*a^21*b - 10*a^22 + 32*a^14*b^8 - 48*a^15*b^7 - 80*a^16*b^6 + 140*a^17*b^5 + 52*a^18*b^4 - 134*a^19*b^3 + 6*a^20*b^2)/a^18 + (b*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(512*a^16*b + 512*a^14*b^3 - 1024*a^15*b^2))/(8*a^19)))/a^7)*((a + b)^5*(a - b)^5)^(1/2)*1i)/a^7)/(((25*a^19*b)/4 - 96*a*b^19 + 64*b^20 - 480*a^2*b^18 + 760*a^3*b^17 + 1544*a^4*b^16 - 2628*a^5*b^15 - 2748*a^6*b^14 + 5179*a^7*b^13 + 2890*a^8*b^12 - 6359*a^9*b^11 - 1736*a^10*b^10 + (19951*a^11*b^9)/4 + (937*a^12*b^8)/2 - (4915*a^13*b^7)/2 + (85*a^14*b^6)/2 + 715*a^15*b^5 - (105*a^16*b^4)/2 - (215*a^17*b^3)/2 + (15*a^18*b^2)/2)/a^18 + (b*((tan(c/2 + (d*

```
x)/2)*(1024*a*b^14 - 75*a^14*b + 25*a^15 - 512*b^15 + 2048*a^2*b^13 - 5120*
a^3*b^12 - 2560*a^4*b^11 + 10240*a^5*b^10 - 10240*a^7*b^8 + 2540*a^8*b^7 +
5180*a^9*b^6 - 2064*a^10*b^5 - 1136*a^11*b^4 + 619*a^12*b^3 + 31*a^13*b^2))
/(8*a^12) + (b*((a + b)^5*(a - b)^5)^(1/2)*((42*a^21*b - 10*a^22 + 32*a^14*
b^8 - 48*a^15*b^7 - 80*a^16*b^6 + 140*a^17*b^5 + 52*a^18*b^4 - 134*a^19*b^3
 + 6*a^20*b^2)/a^18 - (b*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(51
2*a^16*b + 512*a^14*b^3 - 1024*a^15*b^2))/(8*a^19)))/a^7)*((a + b)^5*(a - b
)^5)^(1/2))/a^7 - (b*((tan(c/2 + (d*x)/2)*(1024*a*b^14 - 75*a^14*b + 25*a^1
5 - 512*b^15 + 2048*a^2*b^13 - 5120*a^3*b^12 - 2560*a^4*b^11 + 10240*a^5*b^
10 - 10240*a^7*b^8 + 2540*a^8*b^7 + 5180*a^9*b^6 - 2064*a^10*b^5 - 1136*a^1
1*b^4 + 619*a^12*b^3 + 31*a^13*b^2))/(8*a^12) - (b*((a + b)^5*(a - b)^5)^(1
/2)*((42*a^21*b - 10*a^22 + 32*a^14*b^8 - 48*a^15*b^7 - 80*a^16*b^6 + 140*a
^17*b^5 + 52*a^18*b^4 - 134*a^19*b^3 + 6*a^20*b^2)/a^18 + (b*tan(c/2 + (d*x
)/2)*((a + b)^5*(a - b)^5)^(1/2)*(512*a^16*b + 512*a^14*b^3 - 1024*a^15*b^2
))/(8*a^19)))/a^7)*((a + b)^5*(a - b)^5)^(1/2))/a^7))*((a + b)^5*(a - b)^5)
^(1/2)*2i)/(a^7*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**6/(a + b*sec(c + d*x)), x)
```

3.204 $\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=161

$$-\frac{2b(a-b)^{3/2}(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d}+\frac{\sin^3(c+dx)(4b-3a\cos(c+dx))}{12a^2d}+\frac{\sin(c+dx)\left(8b\left(a^2-b^2\right)\right.}{}$$

```
[Out] 1/8*(3*a^4-12*a^2*b^2+8*b^4)*x/a^5-2*(a-b)^(3/2)*b*(a+b)^(3/2)*arctanh((a-b
)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/d+1/8*(8*b*(a^2-b^2)-a*(3*a^2-4
*b^2)*cos(d*x+c))*sin(d*x+c)/a^4/d+1/12*(4*b-3*a*cos(d*x+c))*sin(d*x+c)^3/a
^2/d
```

Rubi [A] time = 0.38, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2865, 2735, 2659, 208}

$$\frac{\sin(c+dx)\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cos(c+dx)\right)}{8a^4d}+\frac{x\left(-12a^2b^2+3a^4+8b^4\right)}{8a^5}+\frac{\sin^3(c+dx)(4b-3a\cos(c+}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((3*a^4 - 12*a^2*b^2 + 8*b^4)*x)/(8*a^5) - (2*(a - b)^(3/2)*b*(a + b)^(3/2)
*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*d) + ((8*b*(a^2
- b^2) - a*(3*a^2 - 4*b^2)*Cos[c + d*x])*Sin[c + d*x])/(8*a^4*d) + ((4*b -
3*a*Cos[c + d*x])*Sin[c + d*x]^3)/(12*a^2*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
```

```
0] && IntegerQ[2*m]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)\sin^4(c+dx)}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{(4b-3a\cos(c+dx))\sin^3(c+dx)}{12a^2d} - \frac{\int \frac{\left(-ab+\left(3a^2-4b^2\right)\cos(c+dx)\right)\sin^2(c+dx)}{-b-a\cos(c+dx)}\,dx}{4a^2}$$

$$= \frac{\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cos(c+dx)\right)\sin(c+dx)}{8a^4d} + \frac{(4b-3a\cos(c+dx))\sin^3(c+$$

$$= \frac{\left(3a^4-12a^2b^2+8b^4\right)x}{8a^5} + \frac{\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cos(c+dx)\right)\sin(c+dx)}{8a^4d} + \frac{(4b\cdot$$

$$= \frac{\left(3a^4-12a^2b^2+8b^4\right)x}{8a^5} + \frac{\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cos(c+dx)\right)\sin(c+dx)}{8a^4d} + \frac{(4b\cdot$$

$$= \frac{\left(3a^4-12a^2b^2+8b^4\right)x}{8a^5} - \frac{2(a-b)^{3/2}b(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d} + \frac{\left(8b\left(a^2-b\right.$$

Mathematica [A] time = 0.82, size = 172, normalized size = 1.07

$$\frac{3a^4\sin(4(c+dx))+36a^4c+36a^4dx-8a^3b\sin(3(c+dx))+24ab\left(5a^2-4b^2\right)\sin(c+dx)+192b\left(a^2-b^2\right)^{3/2}\tan}{96a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] (36*a^4*c - 144*a^2*b^2*c + 96*b^4*c + 36*a^4*d*x - 144*a^2*b^2*d*x + 96*b^
4*d*x + 192*b*(a^2 - b^2)^(3/2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]] + 24*a*b*(5*a^2 - 4*b^2)*Sin[c + d*x] - 24*(a^4 - a^2*b^2)*Sin[2*
(c + d*x)] - 8*a^3*b*Sin[3*(c + d*x)] + 3*a^4*Sin[4*(c + d*x)])/(96*a^5*d)
```

fricas [A] time = 0.66, size = 393, normalized size = 2.44

$$\left[\frac{3\left(3a^4-12a^2b^2+8b^4\right)dx-12\left(a^2b-b^3\right)\sqrt{a^2-b^2}\log\left(\frac{2ab\cos(dx+c)-\left(a^2-2b^2\right)\cos(dx+c)^2+2\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right.}{24}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*d*x - 12*(a^2*b - b^3)*sqrt(a^2 - b^2
)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2
```

```
)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*
a*b*cos(d*x + c) + b^2)) + (6*a^4*cos(d*x + c)^3 - 8*a^3*b*cos(d*x + c)^2 +
 32*a^3*b - 24*a*b^3 - 3*(5*a^4 - 4*a^2*b^2)*cos(d*x + c))*sin(d*x + c))/(a
^5*d), 1/24*(3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*d*x - 24*(a^2*b - b^3)*sqrt(-a^
2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x
 + c))) + (6*a^4*cos(d*x + c)^3 - 8*a^3*b*cos(d*x + c)^2 + 32*a^3*b - 24*a*
b^3 - 3*(5*a^4 - 4*a^2*b^2)*cos(d*x + c))*sin(d*x + c))/(a^5*d)]
```

giac [B] time = 0.30, size = 407, normalized size = 2.53

$$\frac{\frac{3\left(3a^4-12a^2b^2+8b^4\right)(dx+c)}{a^5} - \frac{48\left(a^4b-2a^2b^3+b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^5} + \frac{2\left(9a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*(d*x + c)/a^5 - 48*(a^4*b - 2*a^2*b^3
+ b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b
^2)*a^5) + 2*(9*a^3*tan(1/2*d*x + 1/2*c)^7 + 24*a^2*b*tan(1/2*d*x + 1/2*c)^
7 - 12*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 24*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*a^
3*tan(1/2*d*x + 1/2*c)^5 + 104*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 12*a*b^2*tan(
1/2*d*x + 1/2*c)^5 - 72*b^3*tan(1/2*d*x + 1/2*c)^5 - 33*a^3*tan(1/2*d*x + 1
/2*c)^3 + 104*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^
3 - 72*b^3*tan(1/2*d*x + 1/2*c)^3 - 9*a^3*tan(1/2*d*x + 1/2*c) + 24*a^2*b*t
an(1/2*d*x + 1/2*c) + 12*a*b^2*tan(1/2*d*x + 1/2*c) - 24*b^3*tan(1/2*d*x +
1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^4))/d
```

maple [B] time = 0.45, size = 769, normalized size = 4.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/d*b/a/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))
^(1/2))+4/d*b^3/a^3/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((
a-b)*(a+b))^(1/2))-2/d*b^5/a^5/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*
c)*(a-b)/((a-b)*(a+b))^(1/2))+3/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*
x+1/2*c)^7+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*b-1/d/a^
3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*b^2-2/d/a^4/(1+tan(1/2*d*
x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*b^3+26/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^
4*tan(1/2*d*x+1/2*c)^5*b-1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2
*c)^5*b^2-6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*b^3+11/4/
d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-11/4/d/a/(1+tan(1/2*d*x
+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3+1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/
2*d*x+1/2*c)^3*b^2+26/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)
^3*b-6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*b^3+2/d/a^2/(1
+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*b-2/d/a^4/(1+tan(1/2*d*x+1/2*c)
^2)^4*tan(1/2*d*x+1/2*c)*b^3-3/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x
+1/2*c)+1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*b^2-3/d/a^3*a
rctan(tan(1/2*d*x+1/2*c))*b^2+2/d/a^5*arctan(tan(1/2*d*x+1/2*c))*b^4+3/4/a/
d*arctan(tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 2.29, size = 317, normalized size = 1.97

$$\frac{\frac{5\,b\,\sin(c+d\,x)}{4}-\frac{b\,\sin(3\,c+3\,d\,x)}{12}}{a^2\,d}-\frac{3\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-\frac{b^2\,\sin(2\,c+2\,d\,x)}{4}}{a^3\,d}+\frac{\frac{3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4}-\frac{\sin(2\,c+2\,d\,x)}{4}+\frac{\sin(4\,c+4\,d\,x)}{32}}{a\,d}-\frac{b^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + b/cos(c + d*x)),x)
```

```
[Out] ((5*b*sin(c + d*x))/4 - (b*sin(3*c + 3*d*x))/12)/(a^2*d) - (3*b^2*atan(sin(
c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - (b^2*sin(2*c + 2*d*x))/4)/(a^3*d) + ((
3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/4 - sin(2*c + 2*d*x)/4 + sin
(4*c + 4*d*x)/32)/(a*d) - (b^3*sin(c + d*x))/(a^4*d) + (2*b^4*atan(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^5*d) - (2*b*atanh((sin(c/2 + (d*x)/2)*(a
^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/(a^3*cos(c/2 + (d*x)/2) - b^3*cos(
c/2 + (d*x)/2) - a*b^2*cos(c/2 + (d*x)/2) + a^2*b*cos(c/2 + (d*x)/2)))*(a^6
 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)^(1/2))/(a^5*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

3.205 $\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=100

$$-\frac{2b\sqrt{a-b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d}+\frac{\sin(c+dx)(2b-a\cos(c+dx))}{2a^2d}+\frac{x\left(a^2-2b^2\right)}{2a^3}$$

```
[Out] 1/2*(a^2-2*b^2)*x/a^3+1/2*(2*b-a*cos(d*x+c))*sin(d*x+c)/a^2/d-2*b*arctanh((
a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))*(a-b)^(1/2)*(a+b)^(1/2)/a^3/d
```

Rubi [A] time = 0.21, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2865, 2735, 2659, 208}

$$\frac{x\left(a^2-2b^2\right)}{2a^3}+\frac{\sin(c+dx)(2b-a\cos(c+dx))}{2a^2d}-\frac{2b\sqrt{a-b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((a^2 - 2*b^2)*x)/(2*a^3) - (2*Sqrt[a - b]*b*Sqrt[a + b]*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*d) + ((2*b - a*Cos[c + d*x])*Sin[c
+ d*x])/(2*a^2*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)\sin^2(c+dx)}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{(2b-a\cos(c+dx))\sin(c+dx)}{2a^2d} - \frac{\int \frac{-ab+\left(a^2-2b^2\right)\cos(c+dx)}{-b-a\cos(c+dx)}\,dx}{2a^2}$$

$$= \frac{\left(a^2-2b^2\right)x}{2a^3} + \frac{(2b-a\cos(c+dx))\sin(c+dx)}{2a^2d} + \frac{\left(b\left(a^2-b^2\right)\right)\int \frac{1}{-b-a\cos(c+dx)}\,dx}{a^3}$$

$$= \frac{\left(a^2-2b^2\right)x}{2a^3} + \frac{(2b-a\cos(c+dx))\sin(c+dx)}{2a^2d} + \frac{\left(2b\left(a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,d\right.}{a^3d}$$

$$= \frac{\left(a^2-2b^2\right)x}{2a^3} - \frac{2\sqrt{a-b}\,b\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d} + \frac{(2b-a\cos(c+dx))\sin(c}{2a^2d}$$

Mathematica [A] time = 0.32, size = 96, normalized size = 0.96

$$\frac{2\left(a^2-2b^2\right)(c+dx)+8b\sqrt{a^2-b^2}\,\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+a^2(-\sin(2(c+dx)))+4ab\sin(c+dx)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(a^2 - 2*b^2)*(c + d*x) + 8*b*Sqrt[a^2 - b^2]*ArcTanh[((-a + b)*Tan[(c +
 d*x)/2])/Sqrt[a^2 - b^2]] + 4*a*b*Sin[c + d*x] - a^2*Sin[2*(c + d*x)])/(4*
a^3*d)
```

fricas [A] time = 0.49, size = 258, normalized size = 2.58

$$\left[\frac{\left(a^2-2\,b^2\right)dx+\sqrt{a^2-b^2}\,b\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)-\left(a^2\cos(dx+\right.}{2\,a^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2 - 2*b^2)*d*x + sqrt(a^2 - b^2)*b*log((2*a*b*cos(d*x + c) - (a^2
- 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x +
c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (a^2*c
os(d*x + c) - 2*a*b)*sin(d*x + c))/(a^3*d), 1/2*((a^2 - 2*b^2)*d*x - 2*sqrt
(-a^2 + b^2)*b*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*s
in(d*x + c))) - (a^2*cos(d*x + c) - 2*a*b)*sin(d*x + c))/(a^3*d)]
```

giac [B] time = 0.27, size = 185, normalized size = 1.85

$$\frac{\frac{\left(a^2-2\,b^2\right)(dx+c)}{a^3}-\frac{4\left(a^2b-b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\text{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^3}+\frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-a\tan\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((a^2 - 2*b^2)*(d*x + c)/a^3 - 4*(a^2*b - b^3)*(pi*floor(1/2*(d*x + c)/
pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
 + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^3) + 2*(a*tan(1/2*d*x + 1
/2*c)^3 + 2*b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) + 2*b*tan(1/2
*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

maple [B] time = 0.36, size = 269, normalized size = 2.69

$$-\frac{2b\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{da\sqrt{(a-b)\,(a+b)}}+\frac{2b^{3}\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,a^{3}\sqrt{(a-b)\,(a+b)}}+\frac{\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}+\frac{2\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{d\,a^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*sec(d*x+c)),x)
```

```
[Out] -2/d*b/a/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))
^(1/2))+2/d*b^3/a^3/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((
a-b)*(a+b))^(1/2))+1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+2/
d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*b+2/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*b-1/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1
/2*d*x+1/2*c)+1/a/d*arctan(tan(1/2*d*x+1/2*c))-2/d/a^3*arctan(tan(1/2*d*x+1
/2*c))*b^2
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.48, size = 147, normalized size = 1.47

$$\frac{\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-\frac{\sin(2\,c+2\,d\,x)}{4}}{a\,d}-\frac{2\,b^{2}\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a^{3}\,d}+\frac{b\,\sin\left(c+d\,x\right)}{a^{2}\,d}-\frac{2\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)(a+b)}\right)\sqrt{a^{2}-b^{2}}}{a^{3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b/cos(c + d*x)),x)
```

```
[Out] (atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - sin(2*c + 2*d*x)/4)/(a*d) -
(2*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^3*d) + (b*sin(c + d*
x))/(a^2*d) - (2*b*atanh((sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))/(cos(c/2 +
(d*x)/2)*(a + b)))*(a^2 - b^2)^(1/2))/(a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^{2}\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

3.206 $\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=84

$$\frac{\csc(c+dx)(b-a\cos(c+dx))}{d\left(a^2-b^2\right)}-\frac{2ab\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] -2*a*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b
)^(3/2)/d+(b-a*cos(d*x+c))*csc(d*x+c)/(a^2-b^2)/d
```

Rubi [A] time = 0.15, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2866, 12, 2659, 208}

$$\frac{\csc(c+dx)(b-a\cos(c+dx))}{d\left(a^2-b^2\right)}-\frac{2ab\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*a*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)
*(a + b)^(3/2)*d) + ((b - a*Cos[c + d*x])*Csc[c + d*x])/((a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
```

```
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc(c+dx)}{-b-a\cos(c+dx)}\,dx\\ &= \frac{(b-a\cos(c+dx))\csc(c+dx)}{\left(a^2-b^2\right)d}+\frac{\int \frac{ab}{-b-a\cos(c+dx)}\,dx}{a^2-b^2}\\ &= \frac{(b-a\cos(c+dx))\csc(c+dx)}{\left(a^2-b^2\right)d}+\frac{(ab)\int \frac{1}{-b-a\cos(c+dx)}\,dx}{a^2-b^2}\\ &= \frac{(b-a\cos(c+dx))\csc(c+dx)}{\left(a^2-b^2\right)d}+\frac{(2ab)\,\text{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2-b^2\right)d}\\ &= -\frac{2ab\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}+\frac{(b-a\cos(c+dx))\csc(c+dx)}{\left(a^2-b^2\right)d}\end{aligned}$$

Mathematica [A] time = 0.20, size = 118, normalized size = 1.40

$$\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{a^2-b^2}\,(b-a\cos(c+dx))+2ab\sin(c+dx)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\right)}{2d(a-b)(a+b)\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] (Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Sqrt[a^2 - b^2]*(b - a*Cos[c + d*x]) +
2*a*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Sin[c + d*x]))/(
2*(a - b)*(a + b)*Sqrt[a^2 - b^2]*d)
```

fricas [A] time = 0.56, size = 300, normalized size = 3.57

$$\left[-\frac{\sqrt{a^2-b^2}\,ab\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)\sin\left(dx+c\right)-2\,a^2b+2\,b^3+}{2\left(a^4-2\,a^2b^2+b^4\right)d\sin\left(dx+c\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a^2 - b^2)*a*b*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
+ c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)
/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*a^2*b +
2*b^3 + 2*(a^3 - a*b^2)*cos(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*sin(d*x +
c)), -(sqrt(-a^2 + b^2)*a*b*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/(
(a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - a^2*b + b^3 + (a^3 - a*b^2)*cos(d
*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*sin(d*x + c))]
```

giac [A] time = 0.25, size = 129, normalized size = 1.54

$$-\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)ab}{\left(a^2-b^2\right)\sqrt{-a^2+b^2}}-\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a-b}+\frac{1}{(a+b)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a*b/((a^2 - b
^2)*sqrt(-a^2 + b^2)) - tan(1/2*d*x + 1/2*c)/(a - b) + 1/((a + b)*tan(1/2*d
*x + 1/2*c)))/d
```

maple [A] time = 0.47, size = 96, normalized size = 1.14

$$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a-2b}-\frac{2ab\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)(a+b)\sqrt{(a-b)(a+b)}}-\frac{1}{2(a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/2/(a-b)*tan(1/2*d*x+1/2*c)-2/(a-b)/(a+b)*a*b/((a-b)*(a+b))^(1/2)*arc
tanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/2/(a+b)/tan(1/2*d*x+1/
2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.30, size = 109, normalized size = 1.30

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(2\,a-2\,b\right)}-\frac{a-b}{d\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a+b\right)\,\left(2\,a-2\,b\right)}-\frac{2\,a\,b\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(a^2-b^2\right)}{{\left(a+b\right)}^{3/2}\,\sqrt{a-b}}\right)}{d\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b/cos(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(d*(2*a - 2*b)) - (a - b)/(d*tan(c/2 + (d*x)/2)*(a + b)*
(2*a - 2*b)) - (2*a*b*atanh((tan(c/2 + (d*x)/2)*(a^2 - b^2))/((a + b)^(3/2)
*(a - b)^(1/2))))/(d*(a + b)^(3/2)*(a - b)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

3.207 $\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=140

$$-\frac{2a^3b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}+\frac{\csc^3(c+dx)(b-a\cos(c+dx))}{3d\left(a^2-b^2\right)}+\frac{\csc(c+dx)\left(3a^2b-a\left(2a^2+b^2\right)\cos(c+dx)\right)}{3d\left(a^2-b^2\right)^2}$$

```
[Out] -2*a^3*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(a
+b)^(5/2)/d+1/3*(3*a^2*b-a*(2*a^2+b^2)*cos(d*x+c))*csc(d*x+c)/(a^2-b^2)^2/d
+1/3*(b-a*cos(d*x+c))*csc(d*x+c)^3/(a^2-b^2)/d
```

Rubi [A] time = 0.31, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2866, 12, 2659, 208}

$$\frac{\csc^3(c+dx)(b-a\cos(c+dx))}{3d\left(a^2-b^2\right)}+\frac{\csc(c+dx)\left(3a^2b-a\left(2a^2+b^2\right)\cos(c+dx)\right)}{3d\left(a^2-b^2\right)^2}-\frac{2a^3b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*a^3*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/
2)*(a + b)^(5/2)*d) + ((3*a^2*b - a*(2*a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x
])/(3*(a^2 - b^2)^2*d) + ((b - a*Cos[c + d*x])*Csc[c + d*x]^3)/(3*(a^2 - b^
2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cot(c+dx)\csc^3(c+dx)}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{(b-a\cos(c+dx))\csc^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{\int \frac{\left(ab-2a^2\cos(c+dx)\right)\csc^2(c+dx)}{-b-a\cos(c+dx)}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{\left(3a^2b-a\left(2a^2+b^2\right)\cos(c+dx)\right)\csc(c+dx)}{3\left(a^2-b^2\right)^2 d} + \frac{(b-a\cos(c+dx))\csc^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{\int \frac{}{-b}}{3}$$

$$= \frac{\left(3a^2b-a\left(2a^2+b^2\right)\cos(c+dx)\right)\csc(c+dx)}{3\left(a^2-b^2\right)^2 d} + \frac{(b-a\cos(c+dx))\csc^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{\left(a^3b\right.}{}$$

$$= \frac{\left(3a^2b-a\left(2a^2+b^2\right)\cos(c+dx)\right)\csc(c+dx)}{3\left(a^2-b^2\right)^2 d} + \frac{(b-a\cos(c+dx))\csc^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{\left(2a\right.}{}$$

$$= -\frac{2a^3b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{\left(3a^2b-a\left(2a^2+b^2\right)\cos(c+dx)\right)\csc(c+dx)}{3\left(a^2-b^2\right)^2 d} + \frac{(b}{}$$

Mathematica [A] time = 0.92, size = 162, normalized size = 1.16

$$\frac{24a^3b\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right) + \sqrt{a^2-b^2}\,\csc^3(c+dx)\left(\left(3ab^2-6a^3\right)\cos(c+dx)+2a^3\cos(3(c+dx))-6a^2b\cos\right.}{12d(a-b)^2(a+b)^2\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] (24*a^3*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + Sqrt[a^2 -
 b^2]*(10*a^2*b - 4*b^3 + (-6*a^3 + 3*a*b^2)*Cos[c + d*x] - 6*a^2*b*Cos[2*(
c + d*x)] + 2*a^3*Cos[3*(c + d*x)] + a*b^2*Cos[3*(c + d*x)])*Csc[c + d*x]^3
)/(12*(a - b)^2*(a + b)^2*Sqrt[a^2 - b^2]*d)
```

fricas [A] time = 0.85, size = 558, normalized size = 3.99

$$\left[-\frac{8\,a^4b-10\,a^2b^3+2\,b^5+2\left(2\,a^5-a^3b^2-ab^4\right)\cos\left(dx+c\right)^3-3\left(a^3b\cos\left(dx+c\right)^2-a^3b\right)\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos(dx}{}\right.}{6\left(\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)d\cos\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(8*a^4*b - 10*a^2*b^3 + 2*b^5 + 2*(2*a^5 - a^3*b^2 - a*b^4)*cos(d*x +
 c)^3 - 3*(a^3*b*cos(d*x + c)^2 - a^3*b)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x
```

```
 + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) +
a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b
^2))*sin(d*x + c) - 6*(a^4*b - a^2*b^3)*cos(d*x + c)^2 - 6*(a^5 - a^3*b^2)*
cos(d*x + c))/(((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^2 - (a^6
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d)*sin(d*x + c)), -1/3*(4*a^4*b - 5*a^2*b^3
 + b^5 + (2*a^5 - a^3*b^2 - a*b^4)*cos(d*x + c)^3 + 3*(a^3*b*cos(d*x + c)^2
 - a^3*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((
a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - 3*(a^4*b - a^2*b^3)*cos(d*x + c)^2
 - 3*(a^5 - a^3*b^2)*cos(d*x + c))/(((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*
cos(d*x + c)^2 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d)*sin(d*x + c))]
```

giac [B] time = 0.27, size = 269, normalized size = 1.92

$$\frac{\frac{48\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^3b}{\left(a^4-2a^2b^2+b^4\right)\sqrt{-a^2+b^2}}+\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9a^2\tan}{a^3-3a^2b+3ab^2-}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(48*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^3*b/((a^4 -
2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + (a^2*tan(1/2*d*x + 1/2*c)^3 - 2*a*b*ta
n(1/2*d*x + 1/2*c)^3 + b^2*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*tan(1/2*d*x + 1/2
*c) - 12*a*b*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(a^3 - 3*a^
2*b + 3*a*b^2 - b^3) - (9*a*tan(1/2*d*x + 1/2*c)^2 + 3*b*tan(1/2*d*x + 1/2*
c)^2 + a + b)/((a^2 + 2*a*b + b^2)*tan(1/2*d*x + 1/2*c)^3))/d
```

maple [A] time = 0.54, size = 165, normalized size = 1.18

$$\frac{\frac{\frac{a\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{3}+3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8(a-b)^2}-\frac{2a^3b\,\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)^2(a+b)^2\sqrt{(a-b)(a+b)}}-\frac{1}{24(a+b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{3a+b}{8(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/8/(a-b)^2*(1/3*a*tan(1/2*d*x+1/2*c)^3-1/3*tan(1/2*d*x+1/2*c)^3*b+3*a
*tan(1/2*d*x+1/2*c)-tan(1/2*d*x+1/2*c)*b)-2/(a-b)^2/(a+b)^2*a^3*b/((a-b)*(a
+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/24/(a+b)
/tan(1/2*d*x+1/2*c)^3-1/8*(3*a+b)/(a+b)^2/tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.38, size = 219, normalized size = 1.56

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{2}{8\,a-8\,b}+\frac{8\,a+8\,b}{(8\,a-8\,b)^2}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3\,d\,(8\,a-8\,b)}-\frac{\frac{a^2-2\,a\,b+b^2}{3\,(a+b)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(3\,a^3-5\,a^2\,b+a\,b^2+b^3\right)}{(a+b)^2}}{d\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(8\,a^2-16\,a\,b+8\,b^2\right)}-\frac{2\,a^3\,b\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\ldots}{(a+b)\ldots}\right.}{d\,(a+b)^{5/2}\,(a\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + b/cos(c + d*x))),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(2/(8*a - 8*b) + (8*a + 8*b)/(8*a - 8*b)^2))/d + tan(c/
2 + (d*x)/2)^3/(3*d*(8*a - 8*b)) - ((a^2 - 2*a*b + b^2)/(3*(a + b)) + (tan(
c/2 + (d*x)/2)^2*(a*b^2 - 5*a^2*b + 3*a^3 + b^3))/(a + b)^2)/(d*tan(c/2 + (
d*x)/2)^3*(8*a^2 - 16*a*b + 8*b^2)) - (2*a^3*b*atanh((tan(c/2 + (d*x)/2)*(a
^4 + b^4 - 2*a^2*b^2))/((a + b)^(5/2)*(a - b)^(3/2))))/(d*(a + b)^(5/2)*(a
- b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

3.208 $\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=201

$$-\frac{2a^5b\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\csc^5(c+dx)(b-a\cos(c+dx))}{5d\left(a^2-b^2\right)}+\frac{\csc^3(c+dx)\left(5a^2b-a\left(4a^2+b^2\right)\cos(c+d\right.}{15d\left(a^2-b^2\right)^2}$$

```
[Out] -2*a^5*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/(a
+b)^(7/2)/d+1/15*(15*a^4*b-a*(8*a^4+9*a^2*b^2-2*b^4)*cos(d*x+c))*csc(d*x+c)
/(a^2-b^2)^3/d+1/15*(5*a^2*b-a*(4*a^2+b^2)*cos(d*x+c))*csc(d*x+c)^3/(a^2-b^
2)^2/d+1/5*(b-a*cos(d*x+c))*csc(d*x+c)^5/(a^2-b^2)/d
```

Rubi [A] time = 0.52, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2866, 12, 2659, 208}

$$\frac{\csc^5(c+dx)(b-a\cos(c+dx))}{5d\left(a^2-b^2\right)}+\frac{\csc^3(c+dx)\left(5a^2b-a\left(4a^2+b^2\right)\cos(c+dx)\right)}{15d\left(a^2-b^2\right)^2}+\frac{\csc(c+dx)\left(15a^4b-a\left(9a^2b^2\right.\right.}{15d\left(a^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*a^5*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/
2)*(a + b)^(7/2)*d) + ((15*a^4*b - a*(8*a^4 + 9*a^2*b^2 - 2*b^4)*Cos[c + d*
x])*Csc[c + d*x])/(15*(a^2 - b^2)^3*d) + ((5*a^2*b - a*(4*a^2 + b^2)*Cos[c
+ d*x])*Csc[c + d*x]^3)/(15*(a^2 - b^2)^2*d) + ((b - a*Cos[c + d*x])*Csc[c
+ d*x]^5)/(5*(a^2 - b^2)*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
```

```
[p, -1] && IntegerQ[2*m]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cot(c+dx)\csc^5(c+dx)}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{(b-a\cos(c+dx))\csc^5(c+dx)}{5\left(a^2-b^2\right)d} + \frac{\int \frac{\left(ab-4a^2\cos(c+dx)\right)\csc^4(c+dx)}{-b-a\cos(c+dx)}\,dx}{5\left(a^2-b^2\right)}$$

$$= \frac{\left(5a^2b-a\left(4a^2+b^2\right)\cos(c+dx)\right)\csc^3(c+dx)}{15\left(a^2-b^2\right)^2 d} + \frac{(b-a\cos(c+dx))\csc^5(c+dx)}{5\left(a^2-b^2\right)d} + \int$$

$$= \frac{\left(15a^4b-a\left(8a^4+9a^2b^2-2b^4\right)\cos(c+dx)\right)\csc(c+dx)}{15\left(a^2-b^2\right)^3 d} + \frac{\left(5a^2b-a\left(4a^2+b^2\right)\cos(c+\right.}{15\left(a^2-b^2\right)}$$

$$= \frac{\left(15a^4b-a\left(8a^4+9a^2b^2-2b^4\right)\cos(c+dx)\right)\csc(c+dx)}{15\left(a^2-b^2\right)^3 d} + \frac{\left(5a^2b-a\left(4a^2+b^2\right)\cos(c+\right.}{15\left(a^2-b^2\right)}$$

$$= \frac{\left(15a^4b-a\left(8a^4+9a^2b^2-2b^4\right)\cos(c+dx)\right)\csc(c+dx)}{15\left(a^2-b^2\right)^3 d} + \frac{\left(5a^2b-a\left(4a^2+b^2\right)\cos(c+\right.}{15\left(a^2-b^2\right)}$$

$$= -\frac{2a^5b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} + \frac{\left(15a^4b-a\left(8a^4+9a^2b^2-2b^4\right)\cos(c+dx)\right)\csc(c+\right.}{15\left(a^2-b^2\right)^3 d}$$

Mathematica [A] time = 1.29, size = 277, normalized size = 1.38

$$\frac{\sec(c+dx)(a\cos(c+dx)+b)\left(\frac{2\left(64a^2-43ab+9b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^3} - \frac{2\left(64a^2+43ab+9b^2\right)\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3} + \frac{960a^5b\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right.}{\left(a^2-b^2\right)^{7/2}}\right.}{480d(a+b\,\mathrm{se}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((960*a^5*b*ArcTanh[((-a + b)*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2) - (2*(64*a^2 + 43*a*b + 9*b^2)*
Cot[(c + d*x)/2])/(a + b)^3 + (8*(19*a - 9*b)*Csc[c + d*x]^3*Sin[(c + d*x)/
2]^4)/(a - b)^2 + (96*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6)/(a - b) - ((19*a +
 9*b)*Csc[(c + d*x)/2]^4*Sin[c + d*x])/(2*(a + b)^2) - (3*Csc[(c + d*x)/2]^
6*Sin[c + d*x])/(2*(a + b)) + (2*(64*a^2 - 43*a*b + 9*b^2)*Tan[(c + d*x)/2]
)/(a - b)^3))/(480*d*(a + b*Sec[c + d*x]))
```

fricas [B] time = 0.89, size = 861, normalized size = 4.28

$$\left[\frac{46\,a^6b-68\,a^4b^3+28\,a^2b^5-6\,b^7-2\left(8\,a^7+a^5b^2-11\,a^3b^4+2\,ab^6\right)\cos\left(dx+c\right)^5+30\left(a^6b-a^4b^3\right)\cos\left(dx+\right.}{30}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/30*(46*a^6*b - 68*a^4*b^3 + 28*a^2*b^5 - 6*b^7 - 2*(8*a^7 + a^5*b^2 - 11
*a^3*b^4 + 2*a*b^6)*cos(d*x + c)^5 + 30*(a^6*b - a^4*b^3)*cos(d*x + c)^4 +
10*(4*a^7 - a^5*b^2 - 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 15*(a^5*b*cos(d*x
 + c)^4 - 2*a^5*b*cos(d*x + c)^2 + a^5*b)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*
x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) +
 a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) +
b^2))*sin(d*x + c) - 10*(7*a^6*b - 8*a^4*b^3 + a^2*b^5)*cos(d*x + c)^2 - 30
*(a^7 - a^5*b^2)*cos(d*x + c))/(((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 +
 b^8)*d*cos(d*x + c)^4 - 2*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*
d*cos(d*x + c)^2 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d)*sin(d
*x + c)), 1/15*(23*a^6*b - 34*a^4*b^3 + 14*a^2*b^5 - 3*b^7 - (8*a^7 + a^5*b
^2 - 11*a^3*b^4 + 2*a*b^6)*cos(d*x + c)^5 + 15*(a^6*b - a^4*b^3)*cos(d*x +
c)^4 + 5*(4*a^7 - a^5*b^2 - 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 15*(a^5*b*c
os(d*x + c)^4 - 2*a^5*b*cos(d*x + c)^2 + a^5*b)*sqrt(-a^2 + b^2)*arctan(-sq
rt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c
) - 5*(7*a^6*b - 8*a^4*b^3 + a^2*b^5)*cos(d*x + c)^2 - 15*(a^7 - a^5*b^2)*c
os(d*x + c))/(((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d*cos(d*x +
c)^4 - 2*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d*cos(d*x + c)^2 +
 (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d)*sin(d*x + c))]
```

giac [B] time = 0.35, size = 541, normalized size = 2.69

$$-\frac{\frac{960\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^5b}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\sqrt{-a^2+b^2}}-\frac{3\,a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-12\,a^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18\,a^2b^2\tan\left(\frac{1}{2}\,dx+\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/480*(960*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*
tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^5*b/((a
^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - (3*a^4*tan(1/2*d*x +
1/2*c)^5 - 12*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 18*a^2*b^2*tan(1/2*d*x + 1/2*c
)^5 - 12*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*b^4*tan(1/2*d*x + 1/2*c)^5 + 25*a
^4*tan(1/2*d*x + 1/2*c)^3 - 90*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*b^2*t
an(1/2*d*x + 1/2*c)^3 - 70*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 15*b^4*tan(1/2*d*
x + 1/2*c)^3 + 150*a^4*tan(1/2*d*x + 1/2*c) - 420*a^3*b*tan(1/2*d*x + 1/2*c
) + 420*a^2*b^2*tan(1/2*d*x + 1/2*c) - 180*a*b^3*tan(1/2*d*x + 1/2*c) + 30*
b^4*tan(1/2*d*x + 1/2*c))/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^
4 - b^5) + (150*a^2*tan(1/2*d*x + 1/2*c)^4 + 120*a*b*tan(1/2*d*x + 1/2*c)^4
 + 30*b^2*tan(1/2*d*x + 1/2*c)^4 + 25*a^2*tan(1/2*d*x + 1/2*c)^2 + 40*a*b*t
an(1/2*d*x + 1/2*c)^2 + 15*b^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2 + 6*a*b + 3*b
^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(1/2*d*x + 1/2*c)^5))/d
```

maple [A] time = 0.60, size = 282, normalized size = 1.40

$$\frac{\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{5}-\frac{2\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab}{5}+\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2}{5}+\frac{5\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}{3}-\frac{8\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)ab}{3}+\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b^2+10a^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-8ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2b^2\tan}{32(a-b)^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/32/(a-b)^3*(1/5*tan(1/2*d*x+1/2*c)^5*a^2-2/5*tan(1/2*d*x+1/2*c)^5*a*
b+1/5*tan(1/2*d*x+1/2*c)^5*b^2+5/3*tan(1/2*d*x+1/2*c)^3*a^2-8/3*tan(1/2*d*x
+1/2*c)^3*a*b+tan(1/2*d*x+1/2*c)^3*b^2+10*a^2*tan(1/2*d*x+1/2*c)-8*a*b*tan(
1/2*d*x+1/2*c)+2*b^2*tan(1/2*d*x+1/2*c))-2/(a-b)^3/(a+b)^3*a^5*b/((a-b)*(a+
b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/160/(a+b)
/tan(1/2*d*x+1/2*c)^5-1/96*(5*a+3*b)/(a+b)^2/tan(1/2*d*x+1/2*c)^3-1/32/(a+b
)^3*(10*a^2+8*a*b+2*b^2)/tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.69, size = 387, normalized size = 1.93

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{5\,d\,\left(32\,a-32\,b\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{4}{3\,\left(32\,a-32\,b\right)}+\frac{32\,a+32\,b}{3\,{\left(32\,a-32\,b\right)}^2}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{5}{32\,a-32\,b}+\frac{\left(\frac{4}{32\,a-32\,b}+\frac{32\,a+32\,b}{{\left(32\,a-32\,b\right)}^2}\right)\,\left(32\,a+32\,b\right)}{32\,a-32\,b}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^6*(a + b/cos(c + d*x))),x)
```

```
[Out] tan(c/2 + (d*x)/2)^5/(5*d*(32*a - 32*b)) + (tan(c/2 + (d*x)/2)^3*(4/(3*(32*
a - 32*b)) + (32*a + 32*b)/(3*(32*a - 32*b)^2)))/d + (tan(c/2 + (d*x)/2)*(5
/(32*a - 32*b) + ((4/(32*a - 32*b) + (32*a + 32*b)/(32*a - 32*b)^2)*(32*a +
 32*b))/(32*a - 32*b)))/d - ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(5*(a + b)) -
(2*tan(c/2 + (d*x)/2)^4*(a*b^4 + 11*a^4*b - 5*a^5 + b^5 - 4*a^2*b^3 - 4*a^3
*b^2))/(a + b)^3 + (tan(c/2 + (d*x)/2)^2*(4*a*b^3 - 12*a^3*b + 5*a^4 - 3*b^
4 + 6*a^2*b^2))/(3*(a + b)^2))/(d*tan(c/2 + (d*x)/2)^5*(96*a*b^2 - 96*a^2*b
 + 32*a^3 - 32*b^3)) - (2*a^5*b*atanh((tan(c/2 + (d*x)/2)*(a^6 - b^6 + 3*a^
2*b^4 - 3*a^4*b^2))/((a + b)^(7/2)*(a - b)^(5/2))))/(d*(a + b)^(7/2)*(a - b
)^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^6\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+b*sec(d*x+c)),x)
[Out] Integral(csc(c + d*x)**6/(a + b*sec(c + d*x)), x)
```

3.209 $\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=267

$$-\frac{b\cos^6(c+dx)}{3a^3d}+\frac{\cos^7(c+dx)}{7a^2d}+\frac{b^2\left(a^2-b^2\right)^3}{a^9d(a\cos(c+dx)+b)}+\frac{2b\left(a^2-4b^2\right)\left(a^2-b^2\right)^2\log(a\cos(c+dx)+b)}{a^9d}-\frac{\left(a^2-7b\right.}{}$$

```
[Out] -(a^2-7*b^2)*(a^2-b^2)^2*cos(d*x+c)/a^8/d-3*b*(a^2-b^2)^2*cos(d*x+c)^2/a^7/
d+1/3*(3*a^4-9*a^2*b^2+5*b^4)*cos(d*x+c)^3/a^6/d+1/2*b*(3*a^2-2*b^2)*cos(d*
x+c)^4/a^5/d-3/5*(a^2-b^2)*cos(d*x+c)^5/a^4/d-1/3*b*cos(d*x+c)^6/a^3/d+1/7*
cos(d*x+c)^7/a^2/d+b^2*(a^2-b^2)^3/a^9/d/(b+a*cos(d*x+c))+2*b*(a^2-4*b^2)*(
a^2-b^2)^2*ln(b+a*cos(d*x+c))/a^9/d
```

Rubi [A] time = 0.37, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3872, 2837, 12, 948}

$$-\frac{3\left(a^2-b^2\right)\cos^5(c+dx)}{5a^4d}+\frac{b\left(3a^2-2b^2\right)\cos^4(c+dx)}{2a^5d}+\frac{\left(-9a^2b^2+3a^4+5b^4\right)\cos^3(c+dx)}{3a^6d}-\frac{3b\left(a^2-b^2\right)^2\cos^2(c\ldots}{a^7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((a^2 - 7*b^2)*(a^2 - b^2)^2*Cos[c + d*x])/(a^8*d)) - (3*b*(a^2 - b^2)^2*
Cos[c + d*x]^2)/(a^7*d) + ((3*a^4 - 9*a^2*b^2 + 5*b^4)*Cos[c + d*x]^3)/(3*a
^6*d) + (b*(3*a^2 - 2*b^2)*Cos[c + d*x]^4)/(2*a^5*d) - (3*(a^2 - b^2)*Cos[c
 + d*x]^5)/(5*a^4*d) - (b*Cos[c + d*x]^6)/(3*a^3*d) + Cos[c + d*x]^7/(7*a^2
*d) + (b^2*(a^2 - b^2)^3)/(a^9*d*(b + a*Cos[c + d*x])) + (2*b*(a^2 - 4*b^2)
*(a^2 - b^2)^2*Log[b + a*Cos[c + d*x]])/(a^9*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^7(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\left(a^2-x^2\right)^3}{a^2(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\left(a^2-x^2\right)^3}{(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^9 d}$$

$$= \frac{\text{Subst}\left(\int \left(\left(a^2-7b^2\right)\left(a^2-b^2\right)^2 - \frac{b^2\left(-a^2+b^2\right)^3}{(b-x)^2} + \frac{2b\left(-a^2+b^2\right)^2\left(-a^2+4b^2\right)}{b-x} - 6b\left(-a^2+b^2\right.\right.\right.}{}$$

$$= -\frac{\left(a^2-7b^2\right)\left(a^2-b^2\right)^2\cos(c+dx)}{a^8 d} - \frac{3b\left(a^2-b^2\right)^2\cos^2(c+dx)}{a^7 d} + \frac{\left(3a^4-9a^2b^2+\right.}{3}$$

Mathematica [A] time = 3.61, size = 417, normalized size = 1.56

$$\frac{588a^8\cos(4(c+dx)) - 132a^8\cos(6(c+dx)) + 15a^8\cos(8(c+dx)) - 3675a^8 - 3780a^7b\cos(3(c+dx)) + 476a}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-3675*a^8 + 61320*a^6*b^2 - 132720*a^4*b^4 + 87360*a^2*b^6 - 13440*b^8 - 1
40*(21*a^8 - 228*a^6*b^2 + 400*a^4*b^4 - 192*a^2*b^6)*Cos[2*(c + d*x)] - 37
80*a^7*b*Cos[3*(c + d*x)] + 8400*a^5*b^3*Cos[3*(c + d*x)] - 4480*a^3*b^5*Co
s[3*(c + d*x)] + 588*a^8*Cos[4*(c + d*x)] - 1848*a^6*b^2*Cos[4*(c + d*x)] +
 1120*a^4*b^4*Cos[4*(c + d*x)] + 476*a^7*b*Cos[5*(c + d*x)] - 336*a^5*b^3*C
os[5*(c + d*x)] - 132*a^8*Cos[6*(c + d*x)] + 112*a^6*b^2*Cos[6*(c + d*x)] -
 40*a^7*b*Cos[7*(c + d*x)] + 15*a^8*Cos[8*(c + d*x)] + 26880*a^6*b^2*Log[b
+ a*Cos[c + d*x]] - 161280*a^4*b^4*Log[b + a*Cos[c + d*x]] + 241920*a^2*b^6
*Log[b + a*Cos[c + d*x]] - 107520*b^8*Log[b + a*Cos[c + d*x]] + 1680*a*b*Co
s[c + d*x]*(-8*a^6 + 67*a^4*b^2 - 116*a^2*b^4 + 56*b^6 + 16*(a^2 - 4*b^2)*(
a^2 - b^2)^2*Log[b + a*Cos[c + d*x]]))/(13440*a^9*d*(b + a*Cos[c + d*x]))
```

fricas [A] time = 0.82, size = 344, normalized size = 1.29

$$\frac{120\,a^8\cos(dx+c)^8 - 160\,a^7b\cos(dx+c)^7 + 1715\,a^6b^2 - 4725\,a^4b^4 + 3780\,a^2b^6 - 840\,b^8 - 56\left(9\,a^8 - 4\,a^6b^2\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/840*(120*a^8*cos(d*x + c)^8 - 160*a^7*b*cos(d*x + c)^7 + 1715*a^6*b^2 - 4
725*a^4*b^4 + 3780*a^2*b^6 - 840*b^8 - 56*(9*a^8 - 4*a^6*b^2)*cos(d*x + c)^
6 + 84*(9*a^7*b - 4*a^5*b^3)*cos(d*x + c)^5 + 140*(6*a^8 - 9*a^6*b^2 + 4*a^
4*b^4)*cos(d*x + c)^4 - 280*(6*a^7*b - 9*a^5*b^3 + 4*a^3*b^5)*cos(d*x + c)^
3 - 840*(a^8 - 6*a^6*b^2 + 9*a^4*b^4 - 4*a^2*b^6)*cos(d*x + c)^2 + 35*(a^7*
b + 153*a^5*b^3 - 324*a^3*b^5 + 168*a*b^7)*cos(d*x + c) + 1680*(a^6*b^2 - 6
*a^4*b^4 + 9*a^2*b^6 - 4*b^8 + (a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*co
s(d*x + c))*log(a*cos(d*x + c) + b))/(a^10*d*cos(d*x + c) + a^9*b*d)
```

giac [B] time = 0.77, size = 1861, normalized size = 6.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/210*(420*(a^7*b - a^6*b^2 - 6*a^5*b^3 + 6*a^4*b^4 + 9*a^3*b^5 - 9*a^2*b^6
 - 4*a*b^7 + 4*b^8)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^10 - a^9*b) - 420*(a^6*b -
6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 1))/a^9 - 420*(a^7*b - 7*a^5*b^3 - 4*a^4*b^4 + 11*a^3*b^5 + 8*a^2*b^6
- 5*a*b^7 - 4*b^8 + a^7*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a^6*b^2*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 6*a^5*b^3*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) + 6*a^4*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^3*b^5*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 9*a^2*b^6*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - 4*a*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*b^8*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1))/((a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*a^9) + (192*a^7 - 1089*a^
6*b - 2772*a^5*b^2 + 6534*a^4*b^3 + 5600*a^3*b^4 - 9801*a^2*b^5 - 2940*a*b^
6 + 4356*b^7 - 1344*a^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8463*a^6*b*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 18144*a^5*b^2*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 49098*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 35
000*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 71127*a^2*b^5*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 17640*a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - 30492*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4032*a^7*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2 - 28749*a^6*b*(cos(d*x + c) - 1)^2/(cos(d
*x + c) + 1)^2 - 48132*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
157374*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 88200*a^3*b^4*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 218421*a^2*b^5*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 - 44100*a*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2 + 91476*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6720*a^7*(cos(
d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*a^6*b*(cos(d*x + c) - 1)^3/(co
s(d*x + c) + 1)^3 + 60480*a^5*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3
 - 272370*a^4*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 114800*a^3*b^
4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 368235*a^2*b^5*(cos(d*x + c)
- 1)^3/(cos(d*x + c) + 1)^3 + 58800*a*b^6*(cos(d*x + c) - 1)^3/(cos(d*x + c
) + 1)^3 - 152460*b^7*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 56035*a^6
*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 36540*a^5*b^2*(cos(d*x + c)
- 1)^4/(cos(d*x + c) + 1)^4 + 272370*a^4*b^3*(cos(d*x + c) - 1)^4/(cos(d*x
+ c) + 1)^4 + 81200*a^3*b^4*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 368
235*a^2*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 44100*a*b^6*(cos(d*
x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 152460*b^7*(cos(d*x + c) - 1)^4/(cos(d
*x + c) + 1)^4 + 28749*a^6*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 10
080*a^5*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 157374*a^4*b^3*(cos
(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 29400*a^3*b^4*(cos(d*x + c) - 1)^5/
(cos(d*x + c) + 1)^5 + 218421*a^2*b^5*(cos(d*x + c) - 1)^5/(cos(d*x + c) +
1)^5 + 17640*a*b^6*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 91476*b^7*(c
os(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 8463*a^6*b*(cos(d*x + c) - 1)^6/(
cos(d*x + c) + 1)^6 - 1260*a^5*b^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^
6 + 49098*a^4*b^3*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 4200*a^3*b^4*
(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 71127*a^2*b^5*(cos(d*x + c) - 1
)^6/(cos(d*x + c) + 1)^6 - 2940*a*b^6*(cos(d*x + c) - 1)^6/(cos(d*x + c) +
1)^6 + 30492*b^7*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 1089*a^6*b*(co
s(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 6534*a^4*b^3*(cos(d*x + c) - 1)^7/
(cos(d*x + c) + 1)^7 + 9801*a^2*b^5*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)
^7 - 4356*b^7*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/(a^9*((cos(d*x + c
) - 1)/(cos(d*x + c) + 1) - 1)^7))/d
```

maple [A] time = 0.59, size = 456, normalized size = 1.71

$$\frac{\cos^7(dx+c)}{7a^2d}-\frac{b\left(\cos^6(dx+c)\right)}{3a^3d}-\frac{3\left(\cos^5(dx+c)\right)}{5a^2d}+\frac{3\left(\cos^5(dx+c)\right)b^2}{5d\,a^4}+\frac{3b\left(\cos^4(dx+c)\right)}{2a^3d}-\frac{\left(\cos^4(dx+c)\right)b}{d\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/7*cos(d*x+c)^7/a^2/d-1/3*b*cos(d*x+c)^6/a^3/d-3/5*cos(d*x+c)^5/a^2/d+3/5/
d/a^4*cos(d*x+c)^5*b^2+3/2*b*cos(d*x+c)^4/a^3/d-1/d/a^5*cos(d*x+c)^4*b^3+co
s(d*x+c)^3/a^2/d-3/d/a^4*cos(d*x+c)^3*b^2+5/3/d/a^6*cos(d*x+c)^3*b^4-3*b*co
s(d*x+c)^2/a^3/d+6/d/a^5*cos(d*x+c)^2*b^3-3/d/a^7*cos(d*x+c)^2*b^5-cos(d*x+
c)/a^2/d+9/d/a^4*cos(d*x+c)*b^2-15/d/a^6*cos(d*x+c)*b^4+7/d/a^8*cos(d*x+c)*
b^6+2*b*ln(b+a*cos(d*x+c))/a^3/d-12/d/a^5*b^3*ln(b+a*cos(d*x+c))+18/d/a^7*b
^5*ln(b+a*cos(d*x+c))-8/d/a^9*b^7*ln(b+a*cos(d*x+c))+b^2/a^3/d/(b+a*cos(d*x
+c))-3/d*b^4/a^5/(b+a*cos(d*x+c))+3/d*b^6/a^7/(b+a*cos(d*x+c))-1/d*b^8/a^9/
(b+a*cos(d*x+c))
```

maxima [A] time = 0.79, size = 271, normalized size = 1.01

$$\frac{\frac{210\left(a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8\right)}{a^{10}\cos(dx+c)+a^9b}+\frac{30\,a^6\cos(dx+c)^7-70\,a^5b\cos(dx+c)^6-126\left(a^6-a^4b^2\right)\cos(dx+c)^5+105\left(3\,a^5b-2\,a^3b^3\right)\cos(dx+c)^4+70\left(3\,a^6-9\,a^4b^2\right.}{a^8}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/210*(210*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)/(a^10*cos(d*x + c) + a^9
*b) + (30*a^6*cos(d*x + c)^7 - 70*a^5*b*cos(d*x + c)^6 - 126*(a^6 - a^4*b^2
)*cos(d*x + c)^5 + 105*(3*a^5*b - 2*a^3*b^3)*cos(d*x + c)^4 + 70*(3*a^6 - 9
*a^4*b^2 + 5*a^2*b^4)*cos(d*x + c)^3 - 630*(a^5*b - 2*a^3*b^3 + a*b^5)*cos(
d*x + c)^2 - 210*(a^6 - 9*a^4*b^2 + 15*a^2*b^4 - 7*b^6)*cos(d*x + c))/a^8 +
 420*(a^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*log(a*cos(d*x + c) + b)/a^9)/d
```

mupad [B] time = 0.19, size = 588, normalized size = 2.20

$$\frac{\cos(c+d\,x)^4\left(\frac{b^3}{2\,a^5}+\frac{b\left(\frac{3}{a^2}-\frac{3\,b^2}{a^4}\right)}{2\,a}\right)}{d}-\frac{\cos(c+d\,x)^2\left(\frac{b^2\left(\frac{2\,b^3}{a^5}+\frac{2\,b\left(\frac{3}{a^2}-\frac{3\,b^2}{a^4}\right)}{a}\right)}{2\,a^2}+\frac{b\left(\frac{3}{a^2}+\frac{b^2\left(\frac{3}{a^2}-\frac{3\,b^2}{a^4}\right)}{a^2}-\frac{2\,b\left(\frac{2\,b^3}{a^5}+\frac{2\,b\left(\frac{3}{a^2}-\frac{3\,b^2}{a^4}\right)}{a}\right)}{a}\right)}{a}\right)}{d}-\mathrm{co}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a + b/cos(c + d*x))^2,x)
```

```
[Out] (cos(c + d*x)^4*(b^3/(2*a^5) + (b*(3/a^2 - (3*b^2)/a^4))/(2*a)))/d - (cos(c
 + d*x)^2*((b^2*((2*b^3)/a^5 + (2*b*(3/a^2 - (3*b^2)/a^4))/a))/(2*a^2) + (b
*(3/a^2 + (b^2*(3/a^2 - (3*b^2)/a^4))/a^2 - (2*b*((2*b^3)/a^5 + (2*b*(3/a^2
 - (3*b^2)/a^4))/a))/a))/a))/d - (cos(c + d*x)^5*(3/(5*a^2) - (3*b^2)/(5*a^
4)))/d + cos(c + d*x)^7/(7*a^2*d) - (cos(c + d*x)*(1/a^2 + (b^2*(3/a^2 + (b
^2*(3/a^2 - (3*b^2)/a^4))/a^2 - (2*b*((2*b^3)/a^5 + (2*b*(3/a^2 - (3*b^2)/a
^4))/a))/a))/a^2 - (2*b*((b^2*((2*b^3)/a^5 + (2*b*(3/a^2 - (3*b^2)/a^4))/a)
)/a^2 + (2*b*(3/a^2 + (b^2*(3/a^2 - (3*b^2)/a^4))/a^2 - (2*b*((2*b^3)/a^5 +
 (2*b*(3/a^2 - (3*b^2)/a^4))/a))/a))/a))/a))/d + (cos(c + d*x)^3*(1/a^2 + (
b^2*(3/a^2 - (3*b^2)/a^4))/(3*a^2) - (2*b*((2*b^3)/a^5 + (2*b*(3/a^2 - (3*b
^2)/a^4))/a))/(3*a)))/d - (b*cos(c + d*x)^6)/(3*a^3*d) - (b^8 - 3*a^2*b^6 +
 3*a^4*b^4 - a^6*b^2)/(a*d*(a^9*cos(c + d*x) + a^8*b)) + (log(b + a*cos(c +
 d*x))*(2*a^6*b - 8*b^7 + 18*a^2*b^5 - 12*a^4*b^3))/(a^9*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.210 $\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=194

$$\frac{b\cos^4(c+dx)}{2a^3d}-\frac{\cos^5(c+dx)}{5a^2d}+\frac{b^2\left(a^2-b^2\right)^2}{a^7d(a\cos(c+dx)+b)}-\frac{2b\left(a^2-b^2\right)\cos^2(c+dx)}{a^5d}+\frac{\left(2a^2-3b^2\right)\cos^3(c+dx)}{3a^4d}+-2$$

```
[Out] -(a^4-6*a^2*b^2+5*b^4)*cos(d*x+c)/a^6/d-2*b*(a^2-b^2)*cos(d*x+c)^2/a^5/d+1/
3*(2*a^2-3*b^2)*cos(d*x+c)^3/a^4/d+1/2*b*cos(d*x+c)^4/a^3/d-1/5*cos(d*x+c)^
5/a^2/d+b^2*(a^2-b^2)^2/a^7/d/(b+a*cos(d*x+c))+2*b*(a^4-4*a^2*b^2+3*b^4)*ln
(b+a*cos(d*x+c))/a^7/d
```

Rubi [A] time = 0.30, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2837, 12, 948}

$$\frac{\left(2a^2-3b^2\right)\cos^3(c+dx)}{3a^4d}-\frac{2b\left(a^2-b^2\right)\cos^2(c+dx)}{a^5d}-\frac{\left(-6a^2b^2+a^4+5b^4\right)\cos(c+dx)}{a^6d}+\frac{b^2\left(a^2-b^2\right)^2}{a^7d(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((a^4 - 6*a^2*b^2 + 5*b^4)*Cos[c + d*x])/(a^6*d)) - (2*b*(a^2 - b^2)*Cos[
c + d*x]^2)/(a^5*d) + ((2*a^2 - 3*b^2)*Cos[c + d*x]^3)/(3*a^4*d) + (b*Cos[c
 + d*x]^4)/(2*a^3*d) - Cos[c + d*x]^5/(5*a^2*d) + (b^2*(a^2 - b^2)^2)/(a^7*
d*(b + a*Cos[c + d*x])) + (2*b*(a^4 - 4*a^2*b^2 + 3*b^4)*Log[b + a*Cos[c +
d*x]])/(a^7*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^5(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{x^2(a^2-x^2)^2}{a^2(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2(a^2-x^2)^2}{(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}$$

$$= \frac{\text{Subst}\left(\int \left(a^4\left(1+\frac{-6a^2b^2+5b^4}{a^4}\right)+\frac{b^2(a^2-b^2)^2}{(b-x)^2}-\frac{2b(a^4-4a^2b^2+3b^4)}{b-x}+4b\left(-a^2+b^2\right)x-\left(2a^2\right.\right.\right.}{a^7 d}$$

$$= -\frac{\left(a^4-6a^2b^2+5b^4\right)\cos(c+dx)}{a^6 d}-\frac{2b\left(a^2-b^2\right)\cos^2(c+dx)}{a^5 d}+\frac{\left(2a^2-3b^2\right)\cos^3(c+}{3a^4 d}$$

Mathematica [A] time = 1.64, size = 280, normalized size = 1.44

$$\frac{22a^6\cos(4(c+dx))-3a^6\cos(6(c+dx))-150a^6-115a^5b\cos(3(c+dx))+9a^5b\cos(5(c+dx))-30a^4b^2\cos(4(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-150*a^6 + 1740*a^4*b^2 - 2160*a^2*b^4 + 480*b^6 - 5*(25*a^6 - 168*a^4*b^2
 + 144*a^2*b^4)*Cos[2*(c + d*x)] - 115*a^5*b*Cos[3*(c + d*x)] + 120*a^3*b^3
*Cos[3*(c + d*x)] + 22*a^6*Cos[4*(c + d*x)] - 30*a^4*b^2*Cos[4*(c + d*x)] +
 9*a^5*b*Cos[5*(c + d*x)] - 3*a^6*Cos[6*(c + d*x)] + 960*a^4*b^2*Log[b + a*
Cos[c + d*x]] - 3840*a^2*b^4*Log[b + a*Cos[c + d*x]] + 2880*b^6*Log[b + a*C
os[c + d*x]] + 120*a*b*Cos[c + d*x]*(-4*a^4 + 23*a^2*b^2 - 20*b^4 + 8*(a^4
- 4*a^2*b^2 + 3*b^4)*Log[b + a*Cos[c + d*x]]))/(480*a^7*d*(b + a*Cos[c + d*
x]))
```

fricas [A] time = 0.58, size = 240, normalized size = 1.24

$$-\frac{48\,a^6\cos\left(dx+c\right)^6-72\,a^5b\cos\left(dx+c\right)^5-435\,a^4b^2+720\,a^2b^4-240\,b^6-40\left(4\,a^6-3\,a^4b^2\right)\cos\left(dx+c\right)^4+80}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/240*(48*a^6*cos(d*x + c)^6 - 72*a^5*b*cos(d*x + c)^5 - 435*a^4*b^2 + 720
*a^2*b^4 - 240*b^6 - 40*(4*a^6 - 3*a^4*b^2)*cos(d*x + c)^4 + 80*(4*a^5*b -
3*a^3*b^3)*cos(d*x + c)^3 + 240*(a^6 - 4*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^
2 + 15*(3*a^5*b - 80*a^3*b^3 + 80*a*b^5)*cos(d*x + c) - 480*(a^4*b^2 - 4*a^
2*b^4 + 3*b^6 + (a^5*b - 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c))*log(a*cos(d*x +
 c) + b))/(a^8*d*cos(d*x + c) + a^7*b*d)
```

giac [B] time = 0.37, size = 1102, normalized size = 5.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(60*(a^5*b - a^4*b^2 - 4*a^3*b^3 + 4*a^2*b^4 + 3*a*b^5 - 3*b^6)*log(ab
s(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1)))/(a^8 - a^7*b) - 60*(a^4*b - 4*a^2*b^3 + 3*b^5)*log(abs(-
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^7 - 60*(a^5*b - 5*a^3*b^3 - 3
*a^2*b^4 + 4*a*b^5 + 3*b^6 + a^5*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
a^4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*a^3*b^3*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 4*a^2*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*
a*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*b^6*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1))/((a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1))*a^7) + (32*a^5 - 137*a^4*b - 300*a^3*b^2
+ 548*a^2*b^3 + 300*a*b^4 - 411*b^5 - 160*a^5*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) + 805*a^4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1320*a^3*b^2*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2980*a^2*b^3*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - 1200*a*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2055*b^
5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 320*a^5*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2 - 1970*a^4*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1
920*a^3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 6200*a^2*b^3*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1800*a*b^4*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 - 4110*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970
*a^4*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1080*a^3*b^2*(cos(d*x +
c) - 1)^3/(cos(d*x + c) + 1)^3 - 6200*a^2*b^3*(cos(d*x + c) - 1)^3/(cos(d*x
 + c) + 1)^3 - 1200*a*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 4110*
b^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 805*a^4*b*(cos(d*x + c) - 1
)^4/(cos(d*x + c) + 1)^4 - 180*a^3*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) +
 1)^4 + 2980*a^2*b^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 300*a*b^4*
(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 2055*b^5*(cos(d*x + c) - 1)^4/(
cos(d*x + c) + 1)^4 + 137*a^4*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 -
 548*a^2*b^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 411*b^5*(cos(d*x +
 c) - 1)^5/(cos(d*x + c) + 1)^5)/(a^7*((cos(d*x + c) - 1)/(cos(d*x + c) + 1
) - 1)^5))/d
```

maple [A] time = 0.52, size = 285, normalized size = 1.47

$$-\frac{\cos^5(dx+c)}{5a^2d}+\frac{b\left(\cos^4(dx+c)\right)}{2a^3d}+\frac{2\left(\cos^3(dx+c)\right)}{3a^2d}-\frac{\left(\cos^3(dx+c)\right)b^2}{d\,a^4}-\frac{2b\left(\cos^2(dx+c)\right)}{a^3d}+\frac{2\left(\cos^2(dx+c)\right)}{d\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/5*cos(d*x+c)^5/a^2/d+1/2*b*cos(d*x+c)^4/a^3/d+2/3*cos(d*x+c)^3/a^2/d-1/d
/a^4*cos(d*x+c)^3*b^2-2*b*cos(d*x+c)^2/a^3/d+2/d/a^5*cos(d*x+c)^2*b^3-cos(d
*x+c)/a^2/d+6/d/a^4*cos(d*x+c)*b^2-5/d/a^6*cos(d*x+c)*b^4+2*b*ln(b+a*cos(d*
x+c))/a^3/d-8/d/a^5*b^3*ln(b+a*cos(d*x+c))+6/d/a^7*b^5*ln(b+a*cos(d*x+c))+b
^2/a^3/d/(b+a*cos(d*x+c))-2/d*b^4/a^5/(b+a*cos(d*x+c))+1/d*b^6/a^7/(b+a*cos
(d*x+c))
```

maxima [A] time = 0.42, size = 184, normalized size = 0.95

$$\frac{\frac{30\left(a^4b^2-2a^2b^4+b^6\right)}{a^8\cos(dx+c)+a^7b}-\frac{6a^4\cos(dx+c)^5-15a^3b\cos(dx+c)^4-10\left(2a^4-3a^2b^2\right)\cos(dx+c)^3+60\left(a^3b-ab^3\right)\cos(dx+c)^2+30\left(a^4-6a^2b^2+5b^4\right)\cos(dx+c)}{a^6}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(30*(a^4*b^2 - 2*a^2*b^4 + b^6)/(a^8*cos(d*x + c) + a^7*b) - (6*a^4*co
s(d*x + c)^5 - 15*a^3*b*cos(d*x + c)^4 - 10*(2*a^4 - 3*a^2*b^2)*cos(d*x + c
)^3 + 60*(a^3*b - a*b^3)*cos(d*x + c)^2 + 30*(a^4 - 6*a^2*b^2 + 5*b^4)*cos(
d*x + c))/a^6 + 60*(a^4*b - 4*a^2*b^3 + 3*b^5)*log(a*cos(d*x + c) + b)/a^7)
/d
```

mupad [B] time = 0.12, size = 253, normalized size = 1.30

$$\frac{\cos\left(c+d\,x\right)^3\,\left(\frac{2}{3\,a^2}-\frac{b^2}{a^4}\right)}{d}-\frac{\cos\left(c+d\,x\right)^2\,\left(\frac{b^3}{a^5}+\frac{b\,\left(\frac{2}{a^2}-\frac{3\,b^2}{a^4}\right)}{a}\right)}{d}-\frac{\cos\left(c+d\,x\right)\,\left(\frac{1}{a^2}+\frac{b^2\,\left(\frac{2}{a^2}-\frac{3\,b^2}{a^4}\right)}{a^2}-\frac{2\,b\,\left(\frac{2\,b^3}{a^5}+\frac{2\,b\,\left(\frac{2}{a^2}-\frac{3\,b^2}{a^4}\right)}{a}\right)}{a}\right)}{d}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + b/cos(c + d*x))^2,x)
```

```
[Out] (cos(c + d*x)^3*(2/(3*a^2) - b^2/a^4))/d - (cos(c + d*x)^2*(b^3/a^5 + (b*(2
/a^2 - (3*b^2)/a^4))/a))/d - (cos(c + d*x)*(1/a^2 + (b^2*(2/a^2 - (3*b^2)/a
^4))/a^2 - (2*b*((2*b^3)/a^5 + (2*b*(2/a^2 - (3*b^2)/a^4))/a))/a))/d - cos(
c + d*x)^5/(5*a^2*d) + (b*cos(c + d*x)^4)/(2*a^3*d) + (log(b + a*cos(c + d*
x))*(2*a^4*b + 6*b^5 - 8*a^2*b^3))/(a^7*d) + (b^6 - 2*a^2*b^4 + a^4*b^2)/(a
*d*(a^7*cos(c + d*x) + a^6*b))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.211 $\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=119

$$-\frac{b\cos^2(c+dx)}{a^3d}+\frac{\cos^3(c+dx)}{3a^2d}+\frac{b^2\left(a^2-b^2\right)}{a^5d(a\cos(c+dx)+b)}+\frac{2b\left(a^2-2b^2\right)\log(a\cos(c+dx)+b)}{a^5d}-\frac{\left(a^2-3b^2\right)\cos(}{a^4d}$$

```
[Out] -(a^2-3*b^2)*cos(d*x+c)/a^4/d-b*cos(d*x+c)^2/a^3/d+1/3*cos(d*x+c)^3/a^2/d+b
^2*(a^2-b^2)/a^5/d/(b+a*cos(d*x+c))+2*b*(a^2-2*b^2)*ln(b+a*cos(d*x+c))/a^5/
d
```

Rubi [A] time = 0.23, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2837, 12, 894}

$$-\frac{\left(a^2-3b^2\right)\cos(c+dx)}{a^4d}+\frac{b^2\left(a^2-b^2\right)}{a^5d(a\cos(c+dx)+b)}+\frac{2b\left(a^2-2b^2\right)\log(a\cos(c+dx)+b)}{a^5d}-\frac{b\cos^2(c+dx)}{a^3d}+\frac{\cos^3(}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((a^2 - 3*b^2)*Cos[c + d*x])/(a^4*d)) - (b*Cos[c + d*x]^2)/(a^3*d) + Cos[
c + d*x]^3/(3*a^2*d) + (b^2*(a^2 - b^2))/(a^5*d*(b + a*Cos[c + d*x])) + (2*
b*(a^2 - 2*b^2)*Log[b + a*Cos[c + d*x]])/(a^5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^2(a^2-x^2)}{a^2(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2(a^2-x^2)}{(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{3b^2}{a^2}\right) - \frac{b^2(-a^2+b^2)}{(b-x)^2} + \frac{2b(-a^2+2b^2)}{b-x} - 2bx - x^2\right) dx, x, -a\cos(c+dx)\right)}{a^5 d}$$
$$= -\frac{(a^2-3b^2)\cos(c+dx)}{a^4 d} - \frac{b\cos^2(c+dx)}{a^3 d} + \frac{\cos^3(c+dx)}{3a^2 d} + \frac{b^2(a^2-b^2)}{a^5 d(b+a\cos(c+dx))} +$$

Mathematica [A] time = 0.56, size = 167, normalized size = 1.40

$$\frac{a^4\cos(4(c+dx)) - 9a^4 - 4a^3 b\cos(3(c+dx)) + 48a^2b^2\log(a\cos(c+dx)+b) + 24ab\cos(c+dx)\left(2\left(a^2-2b^2\right)\text{lo}\right.}{24a^5 d(a\cos(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-9*a^4 + 60*a^2*b^2 - 24*b^4 - 8*(a^4 - 3*a^2*b^2)*Cos[2*(c + d*x)] - 4*a^
3*b*Cos[3*(c + d*x)] + a^4*Cos[4*(c + d*x)] + 48*a^2*b^2*Log[b + a*Cos[c +
d*x]] - 96*b^4*Log[b + a*Cos[c + d*x]] + 24*a*b*Cos[c + d*x]*(-a^2 + 3*b^2
+ 2*(a^2 - 2*b^2)*Log[b + a*Cos[c + d*x]]))/(24*a^5*d*(b + a*Cos[c + d*x]))
```

fricas [A] time = 0.55, size = 150, normalized size = 1.26

$$\frac{2a^4\cos(dx+c)^4 - 4a^3b\cos(dx+c)^3 + 9a^2b^2 - 6b^4 - 6\left(a^4-2a^2b^2\right)\cos(dx+c)^2 - 3\left(a^3b-6ab^3\right)\cos(dx+c}{6\left(a^6 d\cos(dx+c) + a^5bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^4*cos(d*x + c)^4 - 4*a^3*b*cos(d*x + c)^3 + 9*a^2*b^2 - 6*b^4 - 6*
(a^4 - 2*a^2*b^2)*cos(d*x + c)^2 - 3*(a^3*b - 6*a*b^3)*cos(d*x + c) + 12*(a
^2*b^2 - 2*b^4 + (a^3*b - 2*a*b^3)*cos(d*x + c))*log(a*cos(d*x + c) + b))/(
a^6*d*cos(d*x + c) + a^5*b*d)
```

giac [A] time = 0.26, size = 139, normalized size = 1.17

$$\frac{2\left(a^2b-2b^3\right)\log\left(|-a\cos(dx+c)-b|\right)}{a^5 d} + \frac{a^2b^2-b^4}{(a\cos(dx+c)+b)a^5 d} + \frac{a^4d^5\cos(dx+c)^3 - 3a^3bd^5\cos(dx+c)^2 - 3a}{3a^6d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(a^2*b - 2*b^3)*log(abs(-a*cos(d*x + c) - b))/(a^5*d) + (a^2*b^2 - b^4)/(
(a*cos(d*x + c) + b)*a^5*d) + 1/3*(a^4*d^5*cos(d*x + c)^3 - 3*a^3*b*d^5*cos
(d*x + c)^2 - 3*a^4*d^5*cos(d*x + c) + 9*a^2*b^2*d^5*cos(d*x + c))/(a^6*d^6
)
```

maple [A] time = 0.44, size = 153, normalized size = 1.29

$$\frac{\cos^3(dx+c)}{3a^2d}-\frac{b\left(\cos^2(dx+c)\right)}{a^3d}-\frac{\cos(dx+c)}{a^2d}+\frac{3\cos(dx+c)\,b^2}{d\,a^4}+\frac{2b\ln(b+a\cos(dx+c))}{a^3d}-\frac{4b^3\ln(b+a\cos(}{d\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/3*cos(d*x+c)^3/a^2/d-b*cos(d*x+c)^2/a^3/d-cos(d*x+c)/a^2/d+3/d/a^4*cos(d*
x+c)*b^2+2*b*ln(b+a*cos(d*x+c))/a^3/d-4/d/a^5*b^3*ln(b+a*cos(d*x+c))+b^2/a^
3/d/(b+a*cos(d*x+c))-1/d*b^4/a^5/(b+a*cos(d*x+c))
```

maxima [A] time = 0.51, size = 112, normalized size = 0.94

$$\frac{\frac{3\left(a^2b^2-b^4\right)}{a^6\cos(dx+c)+a^5b}+\frac{a^2\cos(dx+c)^3-3\,ab\cos(dx+c)^2-3\left(a^2-3\,b^2\right)\cos(dx+c)}{a^4}+\frac{6\left(a^2b-2\,b^3\right)\log(a\cos(dx+c)+b)}{a^5}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a^2*b^2 - b^4)/(a^6*cos(d*x + c) + a^5*b) + (a^2*cos(d*x + c)^3 - 3
*a*b*cos(d*x + c)^2 - 3*(a^2 - 3*b^2)*cos(d*x + c))/a^4 + 6*(a^2*b - 2*b^3)
*log(a*cos(d*x + c) + b)/a^5)/d
```

mupad [B] time = 0.09, size = 113, normalized size = 0.95

$$-\frac{\cos(c+d\,x)\left(\frac{1}{a^2}-\frac{3\,b^2}{a^4}\right)-\frac{\cos(c+d\,x)^3}{3\,a^2}+\frac{b\cos(c+d\,x)^2}{a^3}-\frac{\ln(b+a\,\cos(c+d\,x))\left(2\,a^2\,b-4\,b^3\right)}{a^5}+\frac{b^4-a^2\,b^2}{a\left(\cos(c+d\,x)\,a^5+b\,a^4\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + b/cos(c + d*x))^2,x)
```

```
[Out] -(cos(c + d*x)*(1/a^2 - (3*b^2)/a^4) - cos(c + d*x)^3/(3*a^2) + (b*cos(c +
d*x)^2)/a^3 - (log(b + a*cos(c + d*x))*(2*a^2*b - 4*b^3))/a^5 + (b^4 - a^2*
b^2)/(a*(a^5*cos(c + d*x) + a^4*b)))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.212 $\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=57

$$\frac{b^2}{a^3d(a\cos(c+dx)+b)}+\frac{2b\log(a\cos(c+dx)+b)}{a^3d}-\frac{\cos(c+dx)}{a^2d}$$

```
[Out] -cos(d*x+c)/a^2/d+b^2/a^3/d/(b+a*cos(d*x+c))+2*b*ln(b+a*cos(d*x+c))/a^3/d
```

Rubi [A] time = 0.11, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3872, 2833, 12, 43}

$$\frac{b^2}{a^3d(a\cos(c+dx)+b)}+\frac{2b\log(a\cos(c+dx)+b)}{a^3d}-\frac{\cos(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Cos[c + d*x]/(a^2*d)) + b^2/(a^3*d*(b + a*Cos[c + d*x])) + (2*b*Log[b + a
*Cos[c + d*x]])/(a^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{a^2(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{b^2}{(b-x)^2}-\frac{2b}{b-x}\right)dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= -\frac{\cos(c+dx)}{a^2d}+\frac{b^2}{a^3d(b+a\cos(c+dx))}+\frac{2b\log(b+a\cos(c+dx))}{a^3d}$$

Mathematica [A] time = 0.18, size = 76, normalized size = 1.33

$$\frac{-a^2\cos^2(c+dx)+b^2(2\log(a\cos(c+dx)+b)+1)+ab\cos(c+dx)(2\log(a\cos(c+dx)+b)-1)}{a^3d(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-(a^2*Cos[c + d*x]^2) + a*b*Cos[c + d*x]*(-1 + 2*Log[b + a*Cos[c + d*x]])
+ b^2*(1 + 2*Log[b + a*Cos[c + d*x]]))/(a^3*d*(b + a*Cos[c + d*x]))
```

fricas [A] time = 0.61, size = 75, normalized size = 1.32

$$-\frac{a^2\cos(dx+c)^2+ab\cos(dx+c)-b^2-2\left(ab\cos(dx+c)+b^2\right)\log(a\cos(dx+c)+b)}{a^4d\cos(dx+c)+a^3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*cos(d*x + c)^2 + a*b*cos(d*x + c) - b^2 - 2*(a*b*cos(d*x + c) + b^2)*
log(a*cos(d*x + c) + b))/(a^4*d*cos(d*x + c) + a^3*b*d)
```

giac [A] time = 0.25, size = 61, normalized size = 1.07

$$-\frac{\cos(dx+c)}{a^2d}+\frac{2b\log(|-a\cos(dx+c)-b|)}{a^3d}+\frac{b^2}{(a\cos(dx+c)+b)a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -cos(d*x + c)/(a^2*d) + 2*b*log(abs(-a*cos(d*x + c) - b))/(a^3*d) + b^2/((a
*cos(d*x + c) + b)*a^3*d)
```

maple [A] time = 0.07, size = 75, normalized size = 1.32

$$-\frac{b}{d\,a^2\,(a+b\sec(dx+c))}+\frac{2b\ln(a+b\sec(dx+c))}{d\,a^3}-\frac{1}{d\,a^2\sec(dx+c)}-\frac{2b\ln(\sec(dx+c))}{d\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/d*b/a^2/(a+b*sec(d*x+c))+2/d/a^3*b*ln(a+b*sec(d*x+c))-1/d/a^2/sec(d*x+c)
-2/d/a^3*b*ln(sec(d*x+c))
```

maxima [A] time = 0.71, size = 55, normalized size = 0.96

$$\frac{\frac{b^2}{a^4 \cos(dx+c)+a^3 b} - \frac{\cos(dx+c)}{a^2} + \frac{2\,b\log(a\cos(dx+c)+b)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (b^2/(a^4*cos(d*x + c) + a^3*b) - cos(d*x + c)/a^2 + 2*b*log(a*cos(d*x + c)
 + b)/a^3)/d
```

mupad [B] time = 1.02, size = 60, normalized size = 1.05

$$\frac{b^2}{d\left(\cos(c+d\,x)\,a^4 + b\,a^3\right)} - \frac{\cos(c+d\,x)}{a^2\,d} + \frac{2\,b\,\ln(b+a\,\cos(c+d\,x))}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b/cos(c + d*x))^2,x)
```

```
[Out] b^2/(d*(a^4*cos(c + d*x) + a^3*b)) - cos(c + d*x)/(a^2*d) + (2*b*log(b + a*
cos(c + d*x)))/(a^3*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

3.213 $\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$\frac{b^2}{ad\left(a^2-b^2\right)(a\cos(c+dx)+b)}+\frac{2ab\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^2}+\frac{\log(1-\cos(c+dx))}{2d(a+b)^2}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)^2}$$

```
[Out] b^2/a/(a^2-b^2)/d/(b+a*cos(d*x+c))+1/2*ln(1-cos(d*x+c))/(a+b)^2/d-1/2*ln(1+
cos(d*x+c))/(a-b)^2/d+2*a*b*ln(b+a*cos(d*x+c))/(a^2-b^2)^2/d
```

Rubi [A] time = 0.23, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 2837, 12, 1629}

$$\frac{b^2}{ad\left(a^2-b^2\right)(a\cos(c+dx)+b)}+\frac{2ab\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^2}+\frac{\log(1-\cos(c+dx))}{2d(a+b)^2}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] b^2/(a*(a^2 - b^2)*d*(b + a*Cos[c + d*x])) + Log[1 - Cos[c + d*x]]/(2*(a +
b)^2*d) - Log[1 + Cos[c + d*x]]/(2*(a - b)^2*d) + (2*a*b*Log[b + a*Cos[c +
d*x]])/((a^2 - b^2)^2*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)\cot(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$
$$= \frac{a\,\text{Subst}\left(\int \frac{x^2}{a^2(-b+x)^2(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(-b+x)^2(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a}{2(a-b)^2(a-x)} + \frac{b^2}{(a-b)(a+b)(b-x)^2} - \frac{2a^2b}{(a-b)^2(a+b)^2(b-x)} + \frac{a}{2(a+b)^2(a+x)}\right)dx, x, -a\cos(c$$
$$= \frac{b^2}{a\left(a^2-b^2\right)d(b+a\cos(c+dx))} + \frac{\log(1-\cos(c+dx))}{2(a+b)^2d} - \frac{\log(1+\cos(c+dx))}{2(a-b)^2d} + \frac{2a$$

Mathematica [A] time = 0.35, size = 165, normalized size = 1.51

$$\frac{b\left(2a^2b\log(a\cos(c+dx)+b)+(a-b)\left(a(a-b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+b(a+b)\right)-a(a+b)^2\log\left(\cos\left(\frac{1}{2}(c+dx\right.\right.\right.}{ad(a-b)^2(a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-(a^2*Cos[c + d*x]*((a + b)^2*Log[Cos[(c + d*x)/2]] - 2*a*b*Log[b + a*Cos[
c + d*x]] - (a - b)^2*Log[Sin[(c + d*x)/2]])) + b*(-(a*(a + b)^2*Log[Cos[(c
 + d*x)/2]]) + 2*a^2*b*Log[b + a*Cos[c + d*x]] + (a - b)*(b*(a + b) + a*(a
- b)*Log[Sin[(c + d*x)/2]])))/(a*(a - b)^2*(a + b)^2*d*(b + a*Cos[c + d*x])
)
```

fricas [A] time = 0.64, size = 210, normalized size = 1.93

$$\frac{2\,a^2b^2-2\,b^4+4\left(a^3b\cos\left(dx+c\right)+a^2b^2\right)\log\left(a\cos\left(dx+c\right)+b\right)-\left(a^3b+2\,a^2b^2+ab^3+\left(a^4+2\,a^3b+a^2b^2\right)\cos\right.}{2\left(\left(a^6-2\,a^4b^2+a^2b^4\right)d\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*b^2 - 2*b^4 + 4*(a^3*b*cos(d*x + c) + a^2*b^2)*log(a*cos(d*x + c
) + b) - (a^3*b + 2*a^2*b^2 + a*b^3 + (a^4 + 2*a^3*b + a^2*b^2)*cos(d*x + c
))*log(1/2*cos(d*x + c) + 1/2) + (a^3*b - 2*a^2*b^2 + a*b^3 + (a^4 - 2*a^3*
b + a^2*b^2)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^6 - 2*a^4*b^2
+ a^2*b^4)*d*cos(d*x + c) + (a^5*b - 2*a^3*b^3 + a*b^5)*d)
```

giac [B] time = 0.27, size = 213, normalized size = 1.95

$$\frac{\frac{4\,ab\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^4-2\,a^2b^2+b^4}+\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2+2\,ab+b^2}-\frac{4\left(ab+b^2+\frac{ab(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\left(a^3+a^2b-ab^2-b^3\right)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^4 - 2*a^2*b^2 + b^4) + log(abs(-cos
(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^2 + 2*a*b + b^2) - 4*(a*b + b^2 +
a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a^3 + a^2*b - a*b^2 - b^3)*(a
+ b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1))))/d
```

maple [A] time = 0.44, size = 106, normalized size = 0.97

$$\frac{b^2}{d\,(a+b)\,(a-b)\,a\,(b+a\cos(dx+c))}+\frac{2ab\ln(b+a\cos(dx+c))}{d\,(a-b)^2\,(a+b)^2}+\frac{\ln(-1+\cos(dx+c))}{2d\,(a+b)^2}-\frac{\ln(1+\cos(dx+c)}{2\,(a-b)^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*b^2/(a+b)/(a-b)/a/(b+a*cos(d*x+c))+2/d*a*b/(a-b)^2/(a+b)^2*ln(b+a*cos(d
*x+c))+1/2/d/(a+b)^2*ln(-1+cos(d*x+c))-1/2*ln(1+cos(d*x+c))/(a-b)^2/d
```

maxima [A] time = 0.85, size = 123, normalized size = 1.13

$$\frac{\frac{4\,ab\log(a\cos(dx+c)+b)}{a^4-2\,a^2b^2+b^4}+\frac{2\,b^2}{a^3b-ab^3+\left(a^4-a^2b^2\right)\cos(dx+c)}-\frac{\log(\cos(dx+c)+1)}{a^2-2\,ab+b^2}+\frac{\log(\cos(dx+c)-1)}{a^2+2\,ab+b^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b*log(a*cos(d*x + c) + b)/(a^4 - 2*a^2*b^2 + b^4) + 2*b^2/(a^3*b -
 a*b^3 + (a^4 - a^2*b^2)*cos(d*x + c)) - log(cos(d*x + c) + 1)/(a^2 - 2*a*b
 + b^2) + log(cos(d*x + c) - 1)/(a^2 + 2*a*b + b^2))/d
```

mupad [B] time = 0.23, size = 103, normalized size = 0.94

$$\frac{\ln(\cos(c+d\,x)-1)}{2\,d\,(a+b)^2}-\frac{\ln(\cos(c+d\,x)+1)}{2\,d\,(a-b)^2}+\frac{b^2}{a\,d\,\left(a^2-b^2\right)\,(b+a\,\cos(c+d\,x))}+\frac{2\,a\,b\,\ln(b+a\,\cos(c+d\,x))}{d\,\left(a^2-b^2\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + b/cos(c + d*x))^2),x)
```

```
[Out] log(cos(c + d*x) - 1)/(2*d*(a + b)^2) - log(cos(c + d*x) + 1)/(2*d*(a - b)^
2) + b^2/(a*d*(a^2 - b^2)*(b + a*cos(c + d*x))) + (2*a*b*log(b + a*cos(c +
d*x)))/(d*(a^2 - b^2)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

3.214 $\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=168

$$\frac{ab^2}{d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{2ab\left(a^2+b^2\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^3}+\frac{\csc^2(c+dx)\left(2ab-\left(a^2+b^2\right)\cos(c+dx)\right)}{2d\left(a^2-b^2\right)^2}$$

```
[Out] a*b^2/(a^2-b^2)^2/d/(b+a*cos(d*x+c))+1/2*(2*a*b-(a^2+b^2)*cos(d*x+c))*csc(d
*x+c)^2/(a^2-b^2)^2/d+1/4*(a-b)*ln(1-cos(d*x+c))/(a+b)^3/d-1/4*(a+b)*ln(1+c
os(d*x+c))/(a-b)^3/d+2*a*b*(a^2+b^2)*ln(b+a*cos(d*x+c))/(a^2-b^2)^3/d
```

Rubi [A] time = 0.43, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2837, 12, 1647, 1629}

$$\frac{ab^2}{d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{2ab\left(a^2+b^2\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^3}+\frac{\csc^2(c+dx)\left(2ab-\left(a^2+b^2\right)\cos(c+dx)\right)}{2d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a*b^2)/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x])) + ((2*a*b - (a^2 + b^2)*Cos[
c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^2*d) + ((a - b)*Log[1 - Cos[c + d*
x]])/(4*(a + b)^3*d) - ((a + b)*Log[1 + Cos[c + d*x]])/(4*(a - b)^3*d) + (2
*a*b*(a^2 + b^2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)\csc(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \frac{a^3\,\text{Subst}\left(\int \frac{x^2}{a^2(-b+x)^2\left(a^2-x^2\right)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{x^2}{(-b+x)^2\left(a^2-x^2\right)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{\left(2ab-\left(a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^2 d} + \frac{\text{Subst}\left(\int \frac{-\frac{a^2b^2\left(a^2+b^2\right)}{\left(a^2-b^2\right)^2}+\frac{2a^2bx}{a^2-b^2}+\frac{a^2\left(a^2+b^2\right)x^2}{\left(a^2-b^2\right)^2}}{(-b+x)^2\left(a^2-x^2\right)}\right.}{2ad}$$

$$= \frac{\left(2ab-\left(a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^2 d} + \frac{\text{Subst}\left(\int \left(\frac{a(a+b)}{2(a-b)^3(a-x)} + \frac{2a^2b^2}{(a-b)^2(a+b)^2(b\right.\right.}{}$$

$$= \frac{ab^2}{\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))} + \frac{\left(2ab-\left(a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^2 d} + \frac{(a}{}$$

Mathematica [A] time = 1.31, size = 224, normalized size = 1.33

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)\left(\frac{16ab\left(a^2+b^2\right)(a\cos(c+dx)+b)\log(a\cos(c+dx)+b)}{\left(a^2-b^2\right)^3} + \frac{8ab^2}{(a-b)^2(a+b)^2} + \frac{4(a+b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)(a\cos}{(b-a)^3}\right.}{8d(a+b\sec(c+a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*((8*a*b^2)/((a - b)^2*(a + b)^2) - ((b + a*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/(a + b)^2 + (4*(a + b)*(b + a*Cos[c + d*x])*Log[Cos
[(c + d*x)/2]])/(-a + b)^3 + (16*a*b*(a^2 + b^2)*(b + a*Cos[c + d*x])*Log[b
 + a*Cos[c + d*x]])/(a^2 - b^2)^3 + (4*(a - b)*(b + a*Cos[c + d*x])*Log[Sin
[(c + d*x)/2]])/(a + b)^3 + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a -
b)^2)*Sec[c + d*x]^2)/(8*d*(a + b*Sec[c + d*x])^2)
```

fricas [B] time = 0.76, size = 630, normalized size = 3.75

$$-\frac{8a^3b^2 - 8ab^4 - 2\left(a^5+2a^3b^2-3ab^4\right)\cos(dx+c)^2 + 2\left(a^4b-2a^2b^3+b^5\right)\cos(dx+c) + 8\left(a^3b^2+ab^4-\left(a\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a^3*b^2 - 8*a*b^4 - 2*(a^5 + 2*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 +
2*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c) + 8*(a^3*b^2 + a*b^4 - (a^4*b + a^
```

```
2*b^3)*cos(d*x + c)^3 - (a^3*b^2 + a*b^4)*cos(d*x + c)^2 + (a^4*b + a^2*b^3
)*cos(d*x + c))*log(a*cos(d*x + c) + b) - (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 +
4*a*b^4 + b^5 - (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(d*x + c
)^3 - (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*cos(d*x + c)^2 + (a^5
 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(d*x + c))*log(1/2*cos(d*x +
 c) + 1/2) + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5 - (a^5 - 4*a^4*
b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*cos(d*x + c)^3 - (a^4*b - 4*a^3*b^2 + 6*
a^2*b^3 - 4*a*b^4 + b^5)*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^
2*b^3 + a*b^4)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^7 - 3*a^5*b^
2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^3 + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 -
b^7)*d*cos(d*x + c)^2 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c
) - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d)
```

giac [B] time = 0.32, size = 456, normalized size = 2.71

$$\frac{\frac{2\,(a-b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{16\left(a^3b+ab^3\right)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{a^3-a^2b-ab^2+b^3-\frac{8\,a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8\,ab^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a^3(\cos}{(\cos}}{\left(a^4-2\,a^2b^2+b^4\right)\left(\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos}{\cos}\right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(2*(a - b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^3 + 3*a
^2*b + 3*a*b^2 + b^3) + 16*(a^3*b + a*b^3)*log(abs(-a - b - a*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^6 -
 3*a^4*b^2 + 3*a^2*b^4 - b^6) + (a^3 - a^2*b - a*b^2 + b^3 - 8*a^2*b*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 8*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3*a^2*b*(cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2 - 3*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((a^4 - 2*a^2*b^2 +
 b^4)*(a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2)) - (cos(d*x + c) - 1)/((a^2 - 2*a*b + b^2)*
(cos(d*x + c) + 1)))/d
```

maple [A] time = 0.65, size = 224, normalized size = 1.33

$$\frac{b^2a}{d\,(a+b)^2\,(a-b)^2\,(b+a\cos(dx+c))}+\frac{2a^3b\ln(b+a\cos(dx+c))}{d\,(a+b)^3\,(a-b)^3}+\frac{2a\,b^3\ln(b+a\cos(dx+c))}{d\,(a+b)^3\,(a-b)^3}+\frac{1}{4d\,(a+b)^2\,(-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*b^2/(a+b)^2*a/(a-b)^2/(b+a*cos(d*x+c))+2/d*a^3*b/(a+b)^3/(a-b)^3*ln(b+a
*cos(d*x+c))+2/d*a*b^3/(a+b)^3/(a-b)^3*ln(b+a*cos(d*x+c))+1/4/d/(a+b)^2/(-1
+cos(d*x+c))+1/4/d/(a+b)^3*ln(-1+cos(d*x+c))*a-1/4/d/(a+b)^3*ln(-1+cos(d*x+
c))*b+1/4/d/(a-b)^2/(1+cos(d*x+c))-1/4/d/(a-b)^3*ln(1+cos(d*x+c))*a-1/4/d/(
a-b)^3*ln(1+cos(d*x+c))*b
```

maxima [A] time = 0.80, size = 274, normalized size = 1.63

$$\frac{\frac{8\left(a^3b+ab^3\right)\log(a\cos(dx+c)+b)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{(a+b)\log(\cos(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{(a-b)\log(\cos(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\left(4\,ab^2-\left(a^3+3\,ab^2\right)\cos(dx+c)^2+\right.}{a^4b-2\,a^2b^3+b^5-\left(a^5-2\,a^3b^2+ab^4\right)\cos(dx+c)^3-\left(a^4b-2\,a^2b^3\right.}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(8*(a^3*b + a*b^3)*log(a*cos(d*x + c) + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6) - (a + b)*log(cos(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (
a - b)*log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(4*a*b^2 -
 (a^3 + 3*a*b^2)*cos(d*x + c)^2 + (a^2*b - b^3)*cos(d*x + c))/(a^4*b - 2*a^
2*b^3 + b^5 - (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c)^3 - (a^4*b - 2*a^2*b^3
 + b^5)*cos(d*x + c)^2 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c)))/d
```

mupad [B] time = 1.47, size = 228, normalized size = 1.36

$$\frac{\frac{2\,a\,b^2}{\left(a^2-b^2\right)^2}+\frac{b\,\cos(c+d\,x)}{2\left(a^2-b^2\right)}-\frac{\cos(c+d\,x)^2\left(a^3+3\,a\,b^2\right)}{2\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\left(-a\cos\left(c+d\,x\right)^3-b\cos\left(c+d\,x\right)^2+a\,\cos\left(c+d\,x\right)+b\right)}-\frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\left(\frac{b}{2\,(a+b)^3}-\frac{1}{4\,(a+b)^2}\right)}{d}+\frac{\ln\left(b+\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + b/cos(c + d*x))^2),x)
```

```
[Out] ((2*a*b^2)/(a^2 - b^2)^2 + (b*cos(c + d*x))/(2*(a^2 - b^2)) - (cos(c + d*x)
^2*(3*a*b^2 + a^3))/(2*(a^4 + b^4 - 2*a^2*b^2)))/(d*(b + a*cos(c + d*x) - a
*cos(c + d*x)^3 - b*cos(c + d*x)^2)) - (log(cos(c + d*x) - 1)*(b/(2*(a + b)
^3) - 1/(4*(a + b)^2)))/d + (log(b + a*cos(c + d*x))*(2*a*b^3 + 2*a^3*b))/(
d*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (log(cos(c + d*x) + 1)*(a + b))/(4
*d*(a - b)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^3\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

3.215 $\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=259

$$\frac{(3a^2-4ab-b^2)\log(1-\cos(c+dx))}{16d(a+b)^4}-\frac{(3a^2+4ab-b^2)\log(\cos(c+dx)+1)}{16d(a-b)^4}+\frac{\csc^4(c+dx)\left(2ab-(a^2+b^2)\cos(\right.}{4d(a^2-b^2)^2}$$

```
[Out] a^3*b^2/(a^2-b^2)^3/d/(b+a*cos(d*x+c))+1/8*(8*a*b*(a^2+b^2)-(3*a^4+12*a^2*b
^2+b^4)*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)^3/d+1/4*(2*a*b-(a^2+b^2)*cos(d*x
+c))*csc(d*x+c)^4/(a^2-b^2)^2/d+1/16*(3*a^2-4*a*b-b^2)*ln(1-cos(d*x+c))/(a+
b)^4/d-1/16*(3*a^2+4*a*b-b^2)*ln(1+cos(d*x+c))/(a-b)^4/d+2*a^3*b*(a^2+2*b^2
)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A] time = 0.74, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2837, 12, 1647, 1629}

$$\frac{a^3b^2}{d(a^2-b^2)^3(a\cos(c+dx)+b)}+\frac{(3a^2-4ab-b^2)\log(1-\cos(c+dx))}{16d(a+b)^4}-\frac{(3a^2+4ab-b^2)\log(\cos(c+dx)+1)}{16d(a-b)^4}+$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a^3*b^2)/((a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) + ((8*a*b*(a^2 + b^2) - (3
*a^4 + 12*a^2*b^2 + b^4)*Cos[c + d*x])*Csc[c + d*x]^2)/(8*(a^2 - b^2)^3*d)
+ ((2*a*b - (a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x]^4)/(4*(a^2 - b^2)^2*d) +
 ((3*a^2 - 4*a*b - b^2)*Log[1 - Cos[c + d*x]])/(16*(a + b)^4*d) - ((3*a^2 +
 4*a*b - b^2)*Log[1 + Cos[c + d*x]])/(16*(a - b)^4*d) + (2*a^3*b*(a^2 + 2*b
^2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
```

```
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)\csc^3(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \frac{a^5\,\text{Subst}\left(\int \frac{x^2}{a^2(-b+x)^2\left(a^2-x^2\right)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^3\,\text{Subst}\left(\int \frac{x^2}{(-b+x)^2\left(a^2-x^2\right)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{\left(2ab-\left(a^2+b^2\right)\cos(c+dx)\right)\csc^4(c+dx)}{4\left(a^2-b^2\right)^2 d} + \frac{a\,\text{Subst}\left(\int \frac{-\frac{a^2b^2\left(a^2+b^2\right)}{\left(a^2-b^2\right)^2}+\frac{2a^2b\left(a^2-3b^2\right)x}{\left(a^2-b^2\right)^2}+\frac{3a}{}\cdots}{(-b+x)^2\left(a^2-x^2\right)^2}\cdots\right.}{4d}$$

$$= \frac{\left(8ab\left(a^2+b^2\right)-\left(3a^4+12a^2b^2+b^4\right)\cos(c+dx)\right)\csc^2(c+dx)}{8\left(a^2-b^2\right)^3 d} + \frac{\left(2ab-\left(a^2+b^2\right.\right.\cdots}{4\cdots}$$

$$= \frac{\left(8ab\left(a^2+b^2\right)-\left(3a^4+12a^2b^2+b^4\right)\cos(c+dx)\right)\csc^2(c+dx)}{8\left(a^2-b^2\right)^3 d} + \frac{\left(2ab-\left(a^2+b^2\right.\right.\cdots}{4\cdots}$$

$$= \frac{a^3b^2}{\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))} + \frac{\left(8ab\left(a^2+b^2\right)-\left(3a^4+12a^2b^2+b^4\right)\cos(c+d\cdots\right.}{8\left(a^2-b^2\right)^3 d}$$

Mathematica [A] time = 1.47, size = 320, normalized size = 1.24

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)\left(\frac{64a^3b^2}{(a-b)^3(a+b)^3}+\frac{8\left(-3a^2-4ab+b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b)}{(a-b)^4}+\frac{8\left(3a^2-4ab-b^2\right)\log\left(\sin\left(\frac{1}{2}(c\cdots\right.\right.}{(a+b)^4}\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*((64*a^3*b^2)/((a - b)^3*(a + b)^3) + (2*(-3*a + b)*(
b + a*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/(a + b)^3 - ((b + a*Cos[c + d*x])*C
sc[(c + d*x)/2]^4)/(a + b)^2 + (8*(-3*a^2 - 4*a*b + b^2)*(b + a*Cos[c + d*x
])*Log[Cos[(c + d*x)/2]])/(a - b)^4 + (128*a^3*b*(a^2 + 2*b^2)*(b + a*Cos[c
 + d*x])*Log[b + a*Cos[c + d*x]])/(a^2 - b^2)^4 + (8*(3*a^2 - 4*a*b - b^2)*
(b + a*Cos[c + d*x])*Log[Sin[(c + d*x)/2]])/(a + b)^4 + (2*(3*a + b)*(b + a
```

```
*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a - b)^3 + ((b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^4)/(a - b)^2)*Sec[c + d*x]^2)/(64*d*(a + b*Sec[c + d*x])^2)
```

fricas [B] time = 0.87, size = 1205, normalized size = 4.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(40*a^5*b^2 - 32*a^3*b^4 - 8*a*b^6 + 2*(3*a^7 + 17*a^5*b^2 - 19*a^3*b^
4 - a*b^6)*cos(d*x + c)^4 - 2*(5*a^6*b - 9*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d
*x + c)^3 - 2*(5*a^7 + 31*a^5*b^2 - 29*a^3*b^4 - 7*a*b^6)*cos(d*x + c)^2 +
2*(7*a^6*b - 15*a^4*b^3 + 9*a^2*b^5 - b^7)*cos(d*x + c) + 32*(a^5*b^2 + 2*a
^3*b^4 + (a^6*b + 2*a^4*b^3)*cos(d*x + c)^5 + (a^5*b^2 + 2*a^3*b^4)*cos(d*x
 + c)^4 - 2*(a^6*b + 2*a^4*b^3)*cos(d*x + c)^3 - 2*(a^5*b^2 + 2*a^3*b^4)*co
s(d*x + c)^2 + (a^6*b + 2*a^4*b^3)*cos(d*x + c))*log(a*cos(d*x + c) + b) -
(3*a^6*b + 16*a^5*b^2 + 33*a^4*b^3 + 32*a^3*b^4 + 13*a^2*b^5 - b^7 + (3*a^7
 + 16*a^6*b + 33*a^5*b^2 + 32*a^4*b^3 + 13*a^3*b^4 - a*b^6)*cos(d*x + c)^5
+ (3*a^6*b + 16*a^5*b^2 + 33*a^4*b^3 + 32*a^3*b^4 + 13*a^2*b^5 - b^7)*cos(d
*x + c)^4 - 2*(3*a^7 + 16*a^6*b + 33*a^5*b^2 + 32*a^4*b^3 + 13*a^3*b^4 - a*
b^6)*cos(d*x + c)^3 - 2*(3*a^6*b + 16*a^5*b^2 + 33*a^4*b^3 + 32*a^3*b^4 + 1
3*a^2*b^5 - b^7)*cos(d*x + c)^2 + (3*a^7 + 16*a^6*b + 33*a^5*b^2 + 32*a^4*b
^3 + 13*a^3*b^4 - a*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (3*a^6
*b - 16*a^5*b^2 + 33*a^4*b^3 - 32*a^3*b^4 + 13*a^2*b^5 - b^7 + (3*a^7 - 16*
a^6*b + 33*a^5*b^2 - 32*a^4*b^3 + 13*a^3*b^4 - a*b^6)*cos(d*x + c)^5 + (3*a
^6*b - 16*a^5*b^2 + 33*a^4*b^3 - 32*a^3*b^4 + 13*a^2*b^5 - b^7)*cos(d*x + c
)^4 - 2*(3*a^7 - 16*a^6*b + 33*a^5*b^2 - 32*a^4*b^3 + 13*a^3*b^4 - a*b^6)*c
os(d*x + c)^3 - 2*(3*a^6*b - 16*a^5*b^2 + 33*a^4*b^3 - 32*a^3*b^4 + 13*a^2*
b^5 - b^7)*cos(d*x + c)^2 + (3*a^7 - 16*a^6*b + 33*a^5*b^2 - 32*a^4*b^3 + 1
3*a^3*b^4 - a*b^6)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^9 - 4*a^
7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^5 + (a^8*b - 4*a^6*b^
3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^4 - 2*(a^9 - 4*a^7*b^2 + 6*
a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 - 2*(a^8*b - 4*a^6*b^3 + 6*a^
4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^2 + (a^9 - 4*a^7*b^2 + 6*a^5*b^4 -
4*a^3*b^6 + a*b^8)*d*cos(d*x + c) + (a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*
b^7 + b^9)*d)
```

giac [B] time = 0.45, size = 710, normalized size = 2.74

$$\frac{4\left(3a^2-4ab-b^2\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}+\frac{128\left(a^5b+2a^3b^3\right)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}-\frac{\frac{8a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{8ab(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a^2(\cos(dx+c)}{(\cos(dx+c)+}}{a^4-4a^3b+6a^2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/64*(4*(3*a^2 - 4*a*b - b^2)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) +
 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 128*(a^5*b + 2*a^3*b^3)*
log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) -
 (8*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 8*a*b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 2*a*b*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b^2*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (a^2 + 2*a*b +
b^2 - 8*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 8*a*b*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 18*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 2
4*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*b^2*(cos(d*x + c) - 1)^
```

```
2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/((a^4 + 4*a^3*b + 6*a^2*b^2 +
4*a*b^3 + b^4)*(cos(d*x + c) - 1)^2) - 128*(a^6*b + a^4*b^3 + 2*a^3*b^4 + a
^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a^5*b^2*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 2*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a^3*b
^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4
*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1))))/d
```

maple [A] time = 0.66, size = 368, normalized size = 1.42

$$\frac{b^2a^3}{d(a+b)^3(a-b)^3(b+a\cos(dx+c))}+\frac{2a^5b\ln(b+a\cos(dx+c))}{d(a+b)^4(a-b)^4}+\frac{4a^3b^3\ln(b+a\cos(dx+c))}{d(a+b)^4(a-b)^4}-\frac{}{16d(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*b^2*a^3/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))+2/d*a^5*b/(a+b)^4/(a-b)^4*ln(b
+a*cos(d*x+c))+4/d*a^3*b^3/(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))-1/16/d/(a+b)^
2/(-1+cos(d*x+c))^2+3/16/d/(a+b)^3/(-1+cos(d*x+c))*a-1/16/d/(a+b)^3/(-1+cos
(d*x+c))*b+3/16/d/(a+b)^4*ln(-1+cos(d*x+c))*a^2-1/4/d/(a+b)^4*ln(-1+cos(d*x
+c))*a*b-1/16/d/(a+b)^4*ln(-1+cos(d*x+c))*b^2+1/16/d/(a-b)^2/(1+cos(d*x+c))
^2+3/16/d/(a-b)^3/(1+cos(d*x+c))*a+1/16/d/(a-b)^3/(1+cos(d*x+c))*b-3/16/d/(
a-b)^4*ln(1+cos(d*x+c))*a^2-1/4/d/(a-b)^4*ln(1+cos(d*x+c))*a*b+1/16/d/(a-b)
^4*ln(1+cos(d*x+c))*b^2
```

maxima [B] time = 0.73, size = 511, normalized size = 1.97

$$\frac{\frac{32\left(a^5b+2a^3b^3\right)\log(a\cos(dx+c)+b)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}-\frac{\left(3a^2+4ab-b^2\right)\log(\cos(dx+c)+1)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}+\frac{\left(3a^2-4ab-b^2\right)\log(\cos(dx+c)-1)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}+\frac{}{a^6b-3a^4b^3+3a^2b^5-b^7+\left(a^7-3\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/16*(32*(a^5*b + 2*a^3*b^3)*log(a*cos(d*x + c) + b)/(a^8 - 4*a^6*b^2 + 6*a
^4*b^4 - 4*a^2*b^6 + b^8) - (3*a^2 + 4*a*b - b^2)*log(cos(d*x + c) + 1)/(a^
4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + (3*a^2 - 4*a*b - b^2)*log(cos(d*
x + c) - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(20*a^3*b^2 + 4
*a*b^4 + (3*a^5 + 20*a^3*b^2 + a*b^4)*cos(d*x + c)^4 - (5*a^4*b - 4*a^2*b^3
 - b^5)*cos(d*x + c)^3 - (5*a^5 + 36*a^3*b^2 + 7*a*b^4)*cos(d*x + c)^2 + (7
*a^4*b - 8*a^2*b^3 + b^5)*cos(d*x + c))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^
7 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c)^5 + (a^6*b - 3*a^4*b
^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^4 - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b
^6)*cos(d*x + c)^3 - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^2
 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c)))/d
```

mupad [B] time = 1.91, size = 447, normalized size = 1.73

$$\frac{\ln(\cos(c+dx)-1)\left(\frac{3}{16(a+b)^2}-\frac{5b}{8(a+b)^3}+\frac{3b^2}{8(a+b)^4}\right)}{d}-\frac{\ln(\cos(c+dx)+1)\left(\frac{3b^2}{8(a-b)^4}+\frac{5b}{8(a-b)^3}+\frac{3}{16(a-b)^2}\right)}{d}+\frac{\frac{\cos(c+d}{8\left(a^6\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^5*(a + b/cos(c + d*x))^2),x)
```

```
[Out] (log(cos(c + d*x) - 1)*(3/(16*(a + b)^2) - (5*b)/(8*(a + b)^3) + (3*b^2)/(8
*(a + b)^4)))/d - (log(cos(c + d*x) + 1)*((3*b^2)/(8*(a - b)^4) + (5*b)/(8*
(a - b)^3) + 3/(16*(a - b)^2)))/d + ((cos(c + d*x)^4*(a*b^4 + 3*a^5 + 20*a^
```

```
3*b^2))/(8*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (cos(c + d*x)*(7*a^2*b -
b^3))/(8*(a^4 + b^4 - 2*a^2*b^2)) - (cos(c + d*x)^3*(5*a^2*b + b^3))/(8*(a^
4 + b^4 - 2*a^2*b^2)) - (cos(c + d*x)^2*(7*a*b^4 + 5*a^5 + 36*a^3*b^2))/(8*
(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (b*(a*b^3 + 5*a^3*b))/(2*(a^2 - b^2)
*(a^4 + b^4 - 2*a^2*b^2)))/(d*(b + a*cos(c + d*x) - 2*a*cos(c + d*x)^3 + a*
cos(c + d*x)^5 - 2*b*cos(c + d*x)^2 + b*cos(c + d*x)^4)) + (log(b + a*cos(c
 + d*x))*(2*a^5*b + 4*a^3*b^3))/(d*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a
^6*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**5/(a + b*sec(c + d*x))**2, x)
```

3.216 $\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=473

$$\frac{7b\sin(c+dx)\cos^5(c+dx)}{30a^2d(a\cos(c+dx)+b)}-\frac{2b(a-b)^{3/2}(a+b)^{3/2}\left(2a^2-7b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^8d}-\frac{\left(16a^4-61a^2b^2+42\right.}{2}$$

```
[Out] 1/16*(5*a^6-90*a^4*b^2+200*a^2*b^4-112*b^6)*x/a^8-2*(a-b)^(3/2)*b*(a+b)^(3/
2)*(2*a^2-7*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^8/d+
1/15*b*(61*a^4-170*a^2*b^2+105*b^4)*sin(d*x+c)/a^7/d-1/16*(27*a^4-86*a^2*b^
2+56*b^4)*cos(d*x+c)*sin(d*x+c)/a^6/d+1/15*(15*a^4-52*a^2*b^2+35*b^4)*cos(d
*x+c)^2*sin(d*x+c)/a^5/b/d-1/24*(16*a^4-61*a^2*b^2+42*b^4)*cos(d*x+c)^3*sin
(d*x+c)/a^4/b^2/d-1/3*cos(d*x+c)^3*sin(d*x+c)/b/d/(b+a*cos(d*x+c))+1/6*a*co
s(d*x+c)^4*sin(d*x+c)/b^2/d/(b+a*cos(d*x+c))+1/10*(5*a^4-20*a^2*b^2+14*b^4)
*cos(d*x+c)^4*sin(d*x+c)/a^3/b^2/d/(b+a*cos(d*x+c))+7/30*b*cos(d*x+c)^5*sin
(d*x+c)/a^2/d/(b+a*cos(d*x+c))-1/6*cos(d*x+c)^6*sin(d*x+c)/a/d/(b+a*cos(d*x
+c))
```

Rubi [A] time = 1.71, antiderivative size = 473, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2896, 3047, 3049, 3023, 2735, 2659, 208}

$$\frac{b\left(-170a^2b^2+61a^4+105b^4\right)\sin(c+dx)}{15a^7d}+\frac{\left(-20a^2b^2+5a^4+14b^4\right)\sin(c+dx)\cos^4(c+dx)}{10a^3b^2d(a\cos(c+dx)+b)}-\frac{\left(-61a^2b^2+16a^4\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*x)/(16*a^8) - (2*(a - b)^(3/2
)*b*(a + b)^(3/2)*(2*a^2 - 7*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sq
rt[a + b]])/(a^8*d) + (b*(61*a^4 - 170*a^2*b^2 + 105*b^4)*Sin[c + d*x])/(15
*a^7*d) - ((27*a^4 - 86*a^2*b^2 + 56*b^4)*Cos[c + d*x]*Sin[c + d*x])/(16*a^
6*d) + ((15*a^4 - 52*a^2*b^2 + 35*b^4)*Cos[c + d*x]^2*Sin[c + d*x])/(15*a^5
*b*d) - ((16*a^4 - 61*a^2*b^2 + 42*b^4)*Cos[c + d*x]^3*Sin[c + d*x])/(24*a^
4*b^2*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(3*b*d*(b + a*Cos[c + d*x])) + (a*
Cos[c + d*x]^4*Sin[c + d*x])/(6*b^2*d*(b + a*Cos[c + d*x])) + ((5*a^4 - 20*
a^2*b^2 + 14*b^4)*Cos[c + d*x]^4*Sin[c + d*x])/(10*a^3*b^2*d*(b + a*Cos[c +
 d*x])) + (7*b*Cos[c + d*x]^5*Sin[c + d*x])/(30*a^2*d*(b + a*Cos[c + d*x]))
 - (Cos[c + d*x]^6*Sin[c + d*x])/(6*a*d*(b + a*Cos[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^6(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{3bd(b+a\cos(c+dx))} + \frac{a\cos^4(c+dx)\sin(c+dx)}{6b^2d(b+a\cos(c+dx))} + \frac{7b\cos^5(c+dx)\sin(c+}{30a^2d(b+a\cos(c+d}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{3bd(b+a\cos(c+dx))} + \frac{a\cos^4(c+dx)\sin(c+dx)}{6b^2d(b+a\cos(c+dx))} + \frac{\left(5a^4-20a^2b^2+14b^4\right)}{10a^3b^2d(b+}$$

$$= -\frac{\left(16a^4-61a^2b^2+42b^4\right)\cos^3(c+dx)\sin(c+dx)}{24a^4b^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{3bd(b+a\cos(c+dx))} +$$

$$= \frac{\left(15a^4-52a^2b^2+35b^4\right)\cos^2(c+dx)\sin(c+dx)}{15a^5bd} - \frac{\left(16a^4-61a^2b^2+42b^4\right)\cos^3(}{24a^4b^2d}$$

$$= -\frac{\left(27a^4-86a^2b^2+56b^4\right)\cos(c+dx)\sin(c+dx)}{16a^6d} + \frac{\left(15a^4-52a^2b^2+35b^4\right)\cos^2(}{15a^5bd}$$

$$= \frac{b\left(61a^4-170a^2b^2+105b^4\right)\sin(c+dx)}{15a^7d} - \frac{\left(27a^4-86a^2b^2+56b^4\right)\cos(c+dx)\sin}{16a^6d}$$

$$= \frac{\left(5a^6-90a^4b^2+200a^2b^4-112b^6\right)x}{16a^8} + \frac{b\left(61a^4-170a^2b^2+105b^4\right)\sin(c+dx)}{15a^7d} -$$

$$= \frac{\left(5a^6-90a^4b^2+200a^2b^4-112b^6\right)x}{16a^8} + \frac{b\left(61a^4-170a^2b^2+105b^4\right)\sin(c+dx)}{15a^7d} -$$

$$= \frac{\left(5a^6-90a^4b^2+200a^2b^4-112b^6\right)x}{16a^8} - \frac{2(a-b)^{3/2}b(a+b)^{3/2}\left(2a^2-7b^2\right)\tanh^{-1}\Big(}{a^8d}$$

Mathematica [A] time = 7.19, size = 402, normalized size = 0.85

$$3840b\left(2a^2-7b^2\right)\left(a^2-b^2\right)^{3/2}\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right) + \frac{-180a^7\sin(3(c+dx))+40a^7\sin(5(c+dx))-5a^7\sin(7(c+dx))+1910a^6b\,s}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (3840*b*(2*a^2 - 7*b^2)*(a^2 - b^2)^(3/2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2
])/Sqrt[a^2 - b^2]] + (600*a^6*b*c - 10800*a^4*b^3*c + 24000*a^2*b^5*c - 13
440*b^7*c + 600*a^6*b*d*x - 10800*a^4*b^3*d*x + 24000*a^2*b^5*d*x - 13440*b
^7*d*x + 120*a*(5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*(c + d*x)*Cos[c
 + d*x] - 15*a*(15*a^6 - 576*a^4*b^2 + 1488*a^2*b^4 - 896*b^6)*Sin[c + d*x]
 + 1910*a^6*b*Sin[2*(c + d*x)] - 5440*a^4*b^3*Sin[2*(c + d*x)] + 3360*a^2*b
^5*Sin[2*(c + d*x)] - 180*a^7*Sin[3*(c + d*x)] + 790*a^5*b^2*Sin[3*(c + d*x
```

```
)] - 560*a^3*b^4*Sin[3*(c + d*x)] - 166*a^6*b*Sin[4*(c + d*x)] + 140*a^4*b^
3*Sin[4*(c + d*x)] + 40*a^7*Sin[5*(c + d*x)] - 42*a^5*b^2*Sin[5*(c + d*x)]
+ 14*a^6*b*Sin[6*(c + d*x)] - 5*a^7*Sin[7*(c + d*x)])/(b + a*Cos[c + d*x]))
/(1920*a^8*d)
```

fricas [A] time = 0.65, size = 793, normalized size = 1.68

$$\left[\frac{15\left(5\,a^{7}-90\,a^{5}b^{2}+200\,a^{3}b^{4}-112\,ab^{6}\right)dx\cos\left(dx+c\right)+15\left(5\,a^{6}b-90\,a^{4}b^{3}+200\,a^{2}b^{5}-112\,b^{7}\right)dx+120\left(2\,a^{4}b\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/240*(15*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*d*x*cos(d*x + c)
+ 15*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*d*x + 120*(2*a^4*b^2 -
9*a^2*b^4 + 7*b^6 + (2*a^5*b - 9*a^3*b^3 + 7*a*b^5)*cos(d*x + c))*sqrt(a^2
- b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2
- b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2
 + 2*a*b*cos(d*x + c) + b^2)) - (40*a^7*cos(d*x + c)^6 - 56*a^6*b*cos(d*x +
 c)^5 - 976*a^5*b^2 + 2720*a^3*b^4 - 1680*a*b^6 - 2*(65*a^7 - 42*a^5*b^2)*c
os(d*x + c)^4 + 2*(111*a^6*b - 70*a^4*b^3)*cos(d*x + c)^3 + (165*a^7 - 458*
a^5*b^2 + 280*a^3*b^4)*cos(d*x + c)^2 - (571*a^6*b - 1430*a^4*b^3 + 840*a^2
*b^5)*cos(d*x + c))*sin(d*x + c))/(a^9*d*cos(d*x + c) + a^8*b*d), 1/240*(15
*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*d*x*cos(d*x + c) + 15*(5*a^
6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*d*x - 240*(2*a^4*b^2 - 9*a^2*b^4
+ 7*b^6 + (2*a^5*b - 9*a^3*b^3 + 7*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*ar
ctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (
40*a^7*cos(d*x + c)^6 - 56*a^6*b*cos(d*x + c)^5 - 976*a^5*b^2 + 2720*a^3*b^
4 - 1680*a*b^6 - 2*(65*a^7 - 42*a^5*b^2)*cos(d*x + c)^4 + 2*(111*a^6*b - 70
*a^4*b^3)*cos(d*x + c)^3 + (165*a^7 - 458*a^5*b^2 + 280*a^3*b^4)*cos(d*x +
c)^2 - (571*a^6*b - 1430*a^4*b^3 + 840*a^2*b^5)*cos(d*x + c))*sin(d*x + c))
/(a^9*d*cos(d*x + c) + a^8*b*d)]
```

giac [A] time = 0.37, size = 870, normalized size = 1.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(15*(5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*(d*x + c)/a^8 - 480*
(2*a^6*b - 11*a^4*b^3 + 16*a^2*b^5 - 7*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/
2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*
c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^8) - 480*(a^4*b^2*tan(1/2*d*x +
1/2*c) - 2*a^2*b^4*tan(1/2*d*x + 1/2*c) + b^6*tan(1/2*d*x + 1/2*c))/((a*tan
(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)*a^7) + 2*(75*a^5*ta
n(1/2*d*x + 1/2*c)^11 + 480*a^4*b*tan(1/2*d*x + 1/2*c)^11 - 630*a^3*b^2*tan
(1/2*d*x + 1/2*c)^11 - 1920*a^2*b^3*tan(1/2*d*x + 1/2*c)^11 + 600*a*b^4*tan
(1/2*d*x + 1/2*c)^11 + 1440*b^5*tan(1/2*d*x + 1/2*c)^11 + 425*a^5*tan(1/2*d
*x + 1/2*c)^9 + 3040*a^4*b*tan(1/2*d*x + 1/2*c)^9 - 2610*a^3*b^2*tan(1/2*d*
x + 1/2*c)^9 - 10880*a^2*b^3*tan(1/2*d*x + 1/2*c)^9 + 1800*a*b^4*tan(1/2*d*
x + 1/2*c)^9 + 7200*b^5*tan(1/2*d*x + 1/2*c)^9 + 990*a^5*tan(1/2*d*x + 1/2*
c)^7 + 8256*a^4*b*tan(1/2*d*x + 1/2*c)^7 - 1980*a^3*b^2*tan(1/2*d*x + 1/2*c
)^7 - 23040*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 1200*a*b^4*tan(1/2*d*x + 1/2*c
)^7 + 14400*b^5*tan(1/2*d*x + 1/2*c)^7 - 990*a^5*tan(1/2*d*x + 1/2*c)^5 + 8
256*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 1980*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 23
040*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 1200*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 14
```

```
400*b^5*tan(1/2*d*x + 1/2*c)^5 - 425*a^5*tan(1/2*d*x + 1/2*c)^3 + 3040*a^4*
b*tan(1/2*d*x + 1/2*c)^3 + 2610*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 10880*a^2*
b^3*tan(1/2*d*x + 1/2*c)^3 - 1800*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 7200*b^5*t
an(1/2*d*x + 1/2*c)^3 - 75*a^5*tan(1/2*d*x + 1/2*c) + 480*a^4*b*tan(1/2*d*x
 + 1/2*c) + 630*a^3*b^2*tan(1/2*d*x + 1/2*c) - 1920*a^2*b^3*tan(1/2*d*x + 1
/2*c) - 600*a*b^4*tan(1/2*d*x + 1/2*c) + 1440*b^5*tan(1/2*d*x + 1/2*c))/((t
an(1/2*d*x + 1/2*c)^2 + 1)^6*a^7))/d
```

maple [B] time = 0.64, size = 1735, normalized size = 3.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 76/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*b-87/4/d/a^4/(1+
tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*b^2+15/d/a^6/(1+tan(1/2*d*x+1/
2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*b^4-15/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(
1/2*d*x+1/2*c)^3*b^4+22/d*b^3/a^4/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1
/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-32/d*b^5/a^6/((a-b)*(a+b))^(1/2)*arctanh(t
an(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+14/d*b^7/a^8/((a-b)*(a+b))^(1/
2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+60/d/a^7/(1+tan(1/
2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*b^5-272/3/d/a^5/(1+tan(1/2*d*x+1/2*c
)^2)^6*tan(1/2*d*x+1/2*c)^9*b^3+4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*
d*x+1/2*c)^11*b-21/4/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11
*b^2+76/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b+87/4/d/a^
4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b^2-272/3/d/a^5/(1+tan(1/
2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b^3-192/d/a^5/(1+tan(1/2*d*x+1/2*c)^
2)^6*tan(1/2*d*x+1/2*c)^7*b^3+10/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d
*x+1/2*c)^7*b^4+120/d/a^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*b
^5+33/2/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*b^2-10/d/a^6/
(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*b^4-16/d/a^5/(1+tan(1/2*d*x
+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*b^3-33/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*t
an(1/2*d*x+1/2*c)^5+25/d/a^6*arctan(tan(1/2*d*x+1/2*c))*b^4-14/d/a^8*arctan
(tan(1/2*d*x+1/2*c))*b^6-45/4/d/a^4*arctan(tan(1/2*d*x+1/2*c))*b^2-5/8/d/a^
2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)+33/4/d/a^2/(1+tan(1/2*d*x+1
/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7+5/8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/
2*d*x+1/2*c)^11+85/24/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9
-85/24/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3+12/d/a^7/(1+ta
n(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*b^5+5/8/d/a^2*arctan(tan(1/2*d*x+1
/2*c))-2/d*b^2/a^3*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1
/2*c)^2*b-a-b)+4/d*b^4/a^5*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1
/2*d*x+1/2*c)^2*b-a-b)+21/4/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/
2*c)*b^2-5/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*b^4+344/5/d/
a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*b-192/d/a^5/(1+tan(1/2*
d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*b^3+344/5/d/a^3/(1+tan(1/2*d*x+1/2*c)^
2)^6*tan(1/2*d*x+1/2*c)^7*b-33/2/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d
*x+1/2*c)^7*b^2-2/d*b^6/a^7*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b-a-b)-4/d*b/a^2/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1
/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+120/d/a^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1
/2*d*x+1/2*c)^5*b^5+60/d/a^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^
3*b^5+4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*b-16/d/a^5/(1+t
an(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*b^3+5/d/a^6/(1+tan(1/2*d*x+1/2
*c)^2)^6*tan(1/2*d*x+1/2*c)^11*b^4+12/d/a^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(
1/2*d*x+1/2*c)^11*b^5
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 4.87, size = 3692, normalized size = 7.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + b/cos(c + d*x))^2,x)
```

```
[Out] (atan((((((((74*a^23*b - 10*a^24 + 224*a^16*b^8 - 336*a^17*b^7 - 400*a^18*b^
6 + 740*a^19*b^5 + 124*a^20*b^4 - 478*a^21*b^3 + 62*a^22*b^2)/a^21 - (tan(c
/2 + (d*x)/2)*(512*a^18*b + 512*a^16*b^3 - 1024*a^17*b^2)*(a^6*5i - b^6*112
i + a^2*b^4*200i - a^4*b^2*90i))/(128*a^22))*(a^6*5i - b^6*112i + a^2*b^4*2
00i - a^4*b^2*90i))/(16*a^8) + (tan(c/2 + (d*x)/2)*(50176*a*b^14 - 75*a^14*
b + 25*a^15 - 25088*b^15 + 64512*a^2*b^13 - 179200*a^3*b^12 - 30720*a^4*b^1
1 + 240640*a^5*b^10 - 46080*a^6*b^9 - 148480*a^7*b^8 + 53900*a^8*b^7 + 4054
0*a^9*b^6 - 18136*a^10*b^5 - 3864*a^11*b^4 + 1651*a^12*b^3 + 199*a^13*b^2))
/(8*a^14))*(a^6*5i - b^6*112i + a^2*b^4*200i - a^4*b^2*90i)*1i)/(16*a^8) -
(((((74*a^23*b - 10*a^24 + 224*a^16*b^8 - 336*a^17*b^7 - 400*a^18*b^6 + 740
*a^19*b^5 + 124*a^20*b^4 - 478*a^21*b^3 + 62*a^22*b^2)/a^21 + (tan(c/2 + (d
*x)/2)*(512*a^18*b + 512*a^16*b^3 - 1024*a^17*b^2)*(a^6*5i - b^6*112i + a^2
*b^4*200i - a^4*b^2*90i))/(128*a^22))*(a^6*5i - b^6*112i + a^2*b^4*200i - a
^4*b^2*90i))/(16*a^8) - (tan(c/2 + (d*x)/2)*(50176*a*b^14 - 75*a^14*b + 25*
a^15 - 25088*b^15 + 64512*a^2*b^13 - 179200*a^3*b^12 - 30720*a^4*b^11 + 240
640*a^5*b^10 - 46080*a^6*b^9 - 148480*a^7*b^8 + 53900*a^8*b^7 + 40540*a^9*b
^6 - 18136*a^10*b^5 - 3864*a^11*b^4 + 1651*a^12*b^3 + 199*a^13*b^2))/(8*a^1
4))*(a^6*5i - b^6*112i + a^2*b^4*200i - a^4*b^2*90i)*1i)/(16*a^8))/((((((74
*a^23*b - 10*a^24 + 224*a^16*b^8 - 336*a^17*b^7 - 400*a^18*b^6 + 740*a^19*b
^5 + 124*a^20*b^4 - 478*a^21*b^3 + 62*a^22*b^2)/a^21 - (tan(c/2 + (d*x)/2)*
(512*a^18*b + 512*a^16*b^3 - 1024*a^17*b^2)*(a^6*5i - b^6*112i + a^2*b^4*20
0i - a^4*b^2*90i))/(128*a^22))*(a^6*5i - b^6*112i + a^2*b^4*200i - a^4*b^2*
90i))/(16*a^8) + (tan(c/2 + (d*x)/2)*(50176*a*b^14 - 75*a^14*b + 25*a^15 -
25088*b^15 + 64512*a^2*b^13 - 179200*a^3*b^12 - 30720*a^4*b^11 + 240640*a^5
*b^10 - 46080*a^6*b^9 - 148480*a^7*b^8 + 53900*a^8*b^7 + 40540*a^9*b^6 - 18
136*a^10*b^5 - 3864*a^11*b^4 + 1651*a^12*b^3 + 199*a^13*b^2))/(8*a^14))*(a^
6*5i - b^6*112i + a^2*b^4*200i - a^4*b^2*90i))/(16*a^8) - (32928*a*b^19 - (
25*a^19*b)/2 - 21952*b^20 + 117600*a^2*b^18 - 190120*a^3*b^17 - 257432*a^4*
b^16 + 463764*a^5*b^15 + 290284*a^6*b^14 - 620037*a^7*b^13 - 169030*a^8*b^1
2 + 492572*a^9*b^11 + 35558*a^10*b^10 - (941393*a^11*b^9)/4 + (22469*a^12*b
^8)/2 + (260375*a^13*b^7)/4 - 7490*a^14*b^6 - (37705*a^15*b^5)/4 + (2565*a^
16*b^4)/2 + (2345*a^17*b^3)/4 - 55*a^18*b^2)/a^21 + (((((74*a^23*b - 10*a^2
4 + 224*a^16*b^8 - 336*a^17*b^7 - 400*a^18*b^6 + 740*a^19*b^5 + 124*a^20*b^
4 - 478*a^21*b^3 + 62*a^22*b^2)/a^21 + (tan(c/2 + (d*x)/2)*(512*a^18*b + 51
2*a^16*b^3 - 1024*a^17*b^2)*(a^6*5i - b^6*112i + a^2*b^4*200i - a^4*b^2*90i
))/(128*a^22))*(a^6*5i - b^6*112i + a^2*b^4*200i - a^4*b^2*90i))/(16*a^8) -
 (tan(c/2 + (d*x)/2)*(50176*a*b^14 - 75*a^14*b + 25*a^15 - 25088*b^15 + 645
12*a^2*b^13 - 179200*a^3*b^12 - 30720*a^4*b^11 + 240640*a^5*b^10 - 46080*a^
6*b^9 - 148480*a^7*b^8 + 53900*a^8*b^7 + 40540*a^9*b^6 - 18136*a^10*b^5 - 3
864*a^11*b^4 + 1651*a^12*b^3 + 199*a^13*b^2))/(8*a^14))*(a^6*5i - b^6*112i
+ a^2*b^4*200i - a^4*b^2*90i))/(16*a^8)))*(a^6*5i - b^6*112i + a^2*b^4*200i
 - a^4*b^2*90i)*1i)/(8*a^8*d) - ((tan(c/2 + (d*x)/2)^11*(336*a*b^5 + 206*a^
5*b + 35*a^6 - 1008*b^6 + 1688*a^2*b^4 - 572*a^3*b^3 - 694*a^4*b^2))/(12*a^
7) - (tan(c/2 + (d*x)/2)^3*(336*a*b^5 + 206*a^5*b - 35*a^6 + 1008*b^6 - 168
8*a^2*b^4 - 572*a^3*b^3 + 694*a^4*b^2))/(12*a^7) - (tan(c/2 + (d*x)/2)^5*(4
200*a*b^5 + 3801*a^5*b - 565*a^6 + 25200*b^6 - 40520*a^2*b^4 - 7570*a^3*b^3
```

```
 + 14266*a^4*b^2))/(120*a^7) + (tan(c/2 + (d*x)/2)^9*(4200*a*b^5 + 3801*a^5
*b + 565*a^6 - 25200*b^6 + 40520*a^2*b^4 - 7570*a^3*b^3 - 14266*a^4*b^2))/(
120*a^7) - (tan(c/2 + (d*x)/2)^7*(165*a^6 + 2800*b^6 - 4440*a^2*b^4 + 1446*
a^4*b^2))/(10*a^7) + (tan(c/2 + (d*x)/2)^13*(a - b)*(56*a*b^4 + 32*a^4*b +
5*a^5 + 112*b^5 - 144*a^2*b^3 - 58*a^3*b^2))/(8*a^7) + (tan(c/2 + (d*x)/2)*
(a + b)*(56*a*b^4 - 32*a^4*b + 5*a^5 - 112*b^5 + 144*a^2*b^3 - 58*a^3*b^2))
/(8*a^7))/(d*(a + b - tan(c/2 + (d*x)/2)^14*(a - b) + tan(c/2 + (d*x)/2)^2*
(5*a + 7*b) - tan(c/2 + (d*x)/2)^12*(5*a - 7*b) + tan(c/2 + (d*x)/2)^4*(9*a
 + 21*b) - tan(c/2 + (d*x)/2)^10*(9*a - 21*b) + tan(c/2 + (d*x)/2)^6*(5*a +
 35*b) - tan(c/2 + (d*x)/2)^8*(5*a - 35*b))) - (b*atan(((b*((tan(c/2 + (d*x
)/2)*(50176*a*b^14 - 75*a^14*b + 25*a^15 - 25088*b^15 + 64512*a^2*b^13 - 17
9200*a^3*b^12 - 30720*a^4*b^11 + 240640*a^5*b^10 - 46080*a^6*b^9 - 148480*a
^7*b^8 + 53900*a^8*b^7 + 40540*a^9*b^6 - 18136*a^10*b^5 - 3864*a^11*b^4 + 1
651*a^12*b^3 + 199*a^13*b^2))/(8*a^14) + (b*((74*a^23*b - 10*a^24 + 224*a^1
6*b^8 - 336*a^17*b^7 - 400*a^18*b^6 + 740*a^19*b^5 + 124*a^20*b^4 - 478*a^2
1*b^3 + 62*a^22*b^2)/a^21 - (b*tan(c/2 + (d*x)/2)*(2*a^2 - 7*b^2)*((a + b)^
3*(a - b)^3)^(1/2)*(512*a^18*b + 512*a^16*b^3 - 1024*a^17*b^2))/(8*a^22))*(
2*a^2 - 7*b^2)*((a + b)^3*(a - b)^3)^(1/2))/a^8)*(2*a^2 - 7*b^2)*((a + b)^3
*(a - b)^3)^(1/2)*1i)/a^8 + (b*((tan(c/2 + (d*x)/2)*(50176*a*b^14 - 75*a^14
*b + 25*a^15 - 25088*b^15 + 64512*a^2*b^13 - 179200*a^3*b^12 - 30720*a^4*b^
11 + 240640*a^5*b^10 - 46080*a^6*b^9 - 148480*a^7*b^8 + 53900*a^8*b^7 + 405
40*a^9*b^6 - 18136*a^10*b^5 - 3864*a^11*b^4 + 1651*a^12*b^3 + 199*a^13*b^2)
)/(8*a^14) - (b*((74*a^23*b - 10*a^24 + 224*a^16*b^8 - 336*a^17*b^7 - 400*a
^18*b^6 + 740*a^19*b^5 + 124*a^20*b^4 - 478*a^21*b^3 + 62*a^22*b^2)/a^21 +
(b*tan(c/2 + (d*x)/2)*(2*a^2 - 7*b^2)*((a + b)^3*(a - b)^3)^(1/2)*(512*a^18
*b + 512*a^16*b^3 - 1024*a^17*b^2))/(8*a^22))*(2*a^2 - 7*b^2)*((a + b)^3*(a
 - b)^3)^(1/2))/a^8)*(2*a^2 - 7*b^2)*((a + b)^3*(a - b)^3)^(1/2)*1i)/a^8)/(
(32928*a*b^19 - (25*a^19*b)/2 - 21952*b^20 + 117600*a^2*b^18 - 190120*a^3*b
^17 - 257432*a^4*b^16 + 463764*a^5*b^15 + 290284*a^6*b^14 - 620037*a^7*b^13
 - 169030*a^8*b^12 + 492572*a^9*b^11 + 35558*a^10*b^10 - (941393*a^11*b^9)/
4 + (22469*a^12*b^8)/2 + (260375*a^13*b^7)/4 - 7490*a^14*b^6 - (37705*a^15*
b^5)/4 + (2565*a^16*b^4)/2 + (2345*a^17*b^3)/4 - 55*a^18*b^2)/a^21 - (b*((t
an(c/2 + (d*x)/2)*(50176*a*b^14 - 75*a^14*b + 25*a^15 - 25088*b^15 + 64512*
a^2*b^13 - 179200*a^3*b^12 - 30720*a^4*b^11 + 240640*a^5*b^10 - 46080*a^6*b
^9 - 148480*a^7*b^8 + 53900*a^8*b^7 + 40540*a^9*b^6 - 18136*a^10*b^5 - 3864
*a^11*b^4 + 1651*a^12*b^3 + 199*a^13*b^2))/(8*a^14) + (b*((74*a^23*b - 10*a
^24 + 224*a^16*b^8 - 336*a^17*b^7 - 400*a^18*b^6 + 740*a^19*b^5 + 124*a^20*
b^4 - 478*a^21*b^3 + 62*a^22*b^2)/a^21 - (b*tan(c/2 + (d*x)/2)*(2*a^2 - 7*b
^2)*((a + b)^3*(a - b)^3)^(1/2)*(512*a^18*b + 512*a^16*b^3 - 1024*a^17*b^2)
)/(8*a^22))*(2*a^2 - 7*b^2)*((a + b)^3*(a - b)^3)^(1/2))/a^8)*(2*a^2 - 7*b^
2)*((a + b)^3*(a - b)^3)^(1/2))/a^8 + (b*((tan(c/2 + (d*x)/2)*(50176*a*b^14
 - 75*a^14*b + 25*a^15 - 25088*b^15 + 64512*a^2*b^13 - 179200*a^3*b^12 - 30
720*a^4*b^11 + 240640*a^5*b^10 - 46080*a^6*b^9 - 148480*a^7*b^8 + 53900*a^8
*b^7 + 40540*a^9*b^6 - 18136*a^10*b^5 - 3864*a^11*b^4 + 1651*a^12*b^3 + 199
*a^13*b^2))/(8*a^14) - (b*((74*a^23*b - 10*a^24 + 224*a^16*b^8 - 336*a^17*b
^7 - 400*a^18*b^6 + 740*a^19*b^5 + 124*a^20*b^4 - 478*a^21*b^3 + 62*a^22*b^
2)/a^21 + (b*tan(c/2 + (d*x)/2)*(2*a^2 - 7*b^2)*((a + b)^3*(a - b)^3)^(1/2)
*(512*a^18*b + 512*a^16*b^3 - 1024*a^17*b^2))/(8*a^22))*(2*a^2 - 7*b^2)*((a
 + b)^3*(a - b)^3)^(1/2))/a^8)*(2*a^2 - 7*b^2)*((a + b)^3*(a - b)^3)^(1/2))
/a^8))*(2*a^2 - 7*b^2)*((a + b)^3*(a - b)^3)^(1/2)*2i)/(a^8*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**6/(a + b*sec(c + d*x))**2, x)
```

3.217 $\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=261

$$-\frac{(a^2-b^2)\sin(c+dx)\cos^3(c+dx)}{a^2bd(a\cos(c+dx)+b)}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}-\frac{2b\sqrt{a-b}\,\sqrt{a+b}\,(2a^2-5b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c\right.}{\sqrt{a+b}}\right.}{a^6d}$$

```
[Out] 1/8*(3*a^4-36*a^2*b^2+40*b^4)*x/a^6+1/3*b*(11*a^2-15*b^2)*sin(d*x+c)/a^5/d-
1/8*(13*a^2-20*b^2)*cos(d*x+c)*sin(d*x+c)/a^4/d+1/3*(3*a^2-5*b^2)*cos(d*x+c
)^2*sin(d*x+c)/a^3/b/d+1/4*cos(d*x+c)^3*sin(d*x+c)/a^2/d-(a^2-b^2)*cos(d*x+
c)^3*sin(d*x+c)/a^2/b/d/(b+a*cos(d*x+c))-2*b*(2*a^2-5*b^2)*arctanh((a-b)^(1
/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))*(a-b)^(1/2)*(a+b)^(1/2)/a^6/d
```

Rubi [A] time = 0.83, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2892, 3049, 3023, 2735, 2659, 208}

$$\frac{b(11a^2-15b^2)\sin(c+dx)}{3a^5d}-\frac{(a^2-b^2)\sin(c+dx)\cos^3(c+dx)}{a^2bd(a\cos(c+dx)+b)}+\frac{(3a^2-5b^2)\sin(c+dx)\cos^2(c+dx)}{3a^3bd}-\frac{(13a^2-20\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((3*a^4 - 36*a^2*b^2 + 40*b^4)*x)/(8*a^6) - (2*Sqrt[a - b]*b*Sqrt[a + b]*(2
*a^2 - 5*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^6*d)
+ (b*(11*a^2 - 15*b^2)*Sin[c + d*x])/(3*a^5*d) - ((13*a^2 - 20*b^2)*Cos[c +
 d*x]*Sin[c + d*x])/(8*a^4*d) + ((3*a^2 - 5*b^2)*Cos[c + d*x]^2*Sin[c + d*x
])/(3*a^3*b*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a^2*d) - ((a^2 - b^2)*Cos
[c + d*x]^3*Sin[c + d*x])/(a^2*b*d*(b + a*Cos[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2892

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a*b^2*(m + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m +
n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3)*
```

```
(m + n + 4))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e +
 f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/(b^2*d*f*(m + n + 4)), x]) /; Free
Q[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ
[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^4(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} - \frac{(a^2-b^2)\cos^3(c+dx)\sin(c+dx)}{a^2bd(b+a\cos(c+dx))} - \int \frac{\cos^2(c+dx)(-8a^2+15b^2}{}$$

$$= \frac{(3a^2-5b^2)\cos^2(c+dx)\sin(c+dx)}{3a^3bd} + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} - \frac{(a^2-b^2)\cos^3(c+}{a^2bd(b+a\,c}$$

$$= -\frac{(13a^2-20b^2)\cos(c+dx)\sin(c+dx)}{8a^4d} + \frac{(3a^2-5b^2)\cos^2(c+dx)\sin(c+dx)}{3a^3bd} + \mathrm{co}$$

$$= \frac{b(11a^2-15b^2)\sin(c+dx)}{3a^5d} - \frac{(13a^2-20b^2)\cos(c+dx)\sin(c+dx)}{8a^4d} + \frac{(3a^2-5b^2)\,\mathrm{c}}{}$$

$$= \frac{(3a^4-36a^2b^2+40b^4)x}{8a^6} + \frac{b(11a^2-15b^2)\sin(c+dx)}{3a^5d} - \frac{(13a^2-20b^2)\cos(c+dx)\,\mathrm{s}}{8a^4d}$$

$$= \frac{(3a^4-36a^2b^2+40b^4)x}{8a^6} + \frac{b(11a^2-15b^2)\sin(c+dx)}{3a^5d} - \frac{(13a^2-20b^2)\cos(c+dx)\,\mathrm{s}}{8a^4d}$$

$$= \frac{(3a^4-36a^2b^2+40b^4)x}{8a^6} - \frac{2\sqrt{a-b}\,b\sqrt{a+b}\,(2a^2-5b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^6d} +$$

Mathematica [A] time = 3.36, size = 282, normalized size = 1.08

$$\frac{\frac{384b(2a^4-7a^2b^2+5b^4)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{-21a^5\sin(3(c+dx))+3a^5\sin(5(c+dx))+176a^4b\sin(2(c+dx))-10a^4b\sin(4(c+dx))+72a^4bc+72a^4bd}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((384*b*(2*a^4 - 7*a^2*b^2 + 5*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqr
t[a^2 - b^2]])/Sqrt[a^2 - b^2] + (72*a^4*b*c - 864*a^2*b^3*c + 960*b^5*c +
72*a^4*b*d*x - 864*a^2*b^3*d*x + 960*b^5*d*x + 24*a*(3*a^4 - 36*a^2*b^2 + 4
0*b^4)*(c + d*x)*Cos[c + d*x] - 24*a*(a^4 - 31*a^2*b^2 + 40*b^4)*Sin[c + d*
x] + 176*a^4*b*Sin[2*(c + d*x)] - 240*a^2*b^3*Sin[2*(c + d*x)] - 21*a^5*Sin
[3*(c + d*x)] + 40*a^3*b^2*Sin[3*(c + d*x)] - 10*a^4*b*Sin[4*(c + d*x)] + 3
*a^5*Sin[5*(c + d*x)])/(b + a*Cos[c + d*x]))/(192*a^6*d)
```

fricas [A] time = 0.57, size = 581, normalized size = 2.23

$$\left[\frac{3\left(3a^5-36a^3b^2+40ab^4\right)dx\cos(dx+c)+3\left(3a^4b-36a^2b^3+40b^5\right)dx-12\left(2a^2b^2-5b^4+\left(2a^3b-5ab^3\right)\mathrm{co}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/24*(3*(3*a^5 - 36*a^3*b^2 + 40*a*b^4)*d*x*cos(d*x + c) + 3*(3*a^4*b - 36
*a^2*b^3 + 40*b^5)*d*x - 12*(2*a^2*b^2 - 5*b^4 + (2*a^3*b - 5*a*b^3)*cos(d*
```

```
x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c
)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a
^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (6*a^5*cos(d*x + c)^4 - 10
*a^4*b*cos(d*x + c)^3 + 88*a^3*b^2 - 120*a*b^4 - 5*(3*a^5 - 4*a^3*b^2)*cos(
d*x + c)^2 + (49*a^4*b - 60*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(a^7*d*cos
(d*x + c) + a^6*b*d), 1/24*(3*(3*a^5 - 36*a^3*b^2 + 40*a*b^4)*d*x*cos(d*x +
 c) + 3*(3*a^4*b - 36*a^2*b^3 + 40*b^5)*d*x - 24*(2*a^2*b^2 - 5*b^4 + (2*a^
3*b - 5*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*c
os(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (6*a^5*cos(d*x + c)^4 - 10*a
^4*b*cos(d*x + c)^3 + 88*a^3*b^2 - 120*a*b^4 - 5*(3*a^5 - 4*a^3*b^2)*cos(d*
x + c)^2 + (49*a^4*b - 60*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(a^7*d*cos(d
*x + c) + a^6*b*d)]
```

giac [A] time = 0.89, size = 482, normalized size = 1.85

$$\frac{\frac{3\left(3a^4-36a^2b^2+40b^4\right)(dx+c)}{a^6}-\frac{48\left(2a^4b-7a^2b^3+5b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^6}-\frac{48\left(a^2b^2\tan\left(\frac{1}{2}\right.\right.}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^4 - 36*a^2*b^2 + 40*b^4)*(d*x + c)/a^6 - 48*(2*a^4*b - 7*a^2*b
^3 + 5*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*
tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^
2 + b^2)*a^6) - 48*(a^2*b^2*tan(1/2*d*x + 1/2*c) - b^4*tan(1/2*d*x + 1/2*c)
)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)*a^5) + 2*(
9*a^3*tan(1/2*d*x + 1/2*c)^7 + 48*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 36*a*b^2*t
an(1/2*d*x + 1/2*c)^7 - 96*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*a^3*tan(1/2*d*x
+ 1/2*c)^5 + 208*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 36*a*b^2*tan(1/2*d*x + 1/2*
c)^5 - 288*b^3*tan(1/2*d*x + 1/2*c)^5 - 33*a^3*tan(1/2*d*x + 1/2*c)^3 + 208
*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 288*b^3*t
an(1/2*d*x + 1/2*c)^3 - 9*a^3*tan(1/2*d*x + 1/2*c) + 48*a^2*b*tan(1/2*d*x +
 1/2*c) + 36*a*b^2*tan(1/2*d*x + 1/2*c) - 96*b^3*tan(1/2*d*x + 1/2*c))/((ta
n(1/2*d*x + 1/2*c)^2 + 1)^4*a^5))/d
```

maple [B] time = 0.46, size = 883, normalized size = 3.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/d*b^2/a^3*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^
2*b-a-b)+2/d*b^4/a^5*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x
+1/2*c)^2*b-a-b)-4/d*b/a^2/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(
a-b)/((a-b)*(a+b))^(1/2))+14/d*b^3/a^4/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*
d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-10/d*b^5/a^6/((a-b)*(a+b))^(1/2)*arct
anh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+3/4/d/a^2/(1+tan(1/2*d*x+
1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2
*d*x+1/2*c)^7*b-3/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*b^2
-8/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*b^3+52/3/d/a^3/(1+
tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*b-3/d/a^4/(1+tan(1/2*d*x+1/2*c
)^2)^4*tan(1/2*d*x+1/2*c)^5*b^2-24/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2
*d*x+1/2*c)^5*b^3+11/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^
5-11/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3+3/d/a^4/(1+tan
(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*b^2+52/3/d/a^3/(1+tan(1/2*d*x+1/2
```

```
*c)^2)^4*tan(1/2*d*x+1/2*c)^3*b-24/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2
*d*x+1/2*c)^3*b^3+4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*b-8
/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*b^3-3/4/d/a^2/(1+tan(1
/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)+3/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^4*ta
n(1/2*d*x+1/2*c)*b^2-9/d/a^4*arctan(tan(1/2*d*x+1/2*c))*b^2+10/d/a^6*arctan
(tan(1/2*d*x+1/2*c))*b^4+3/4/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

maxima [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [**B**] time = 3.86, size = 2804, normalized size = 10.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^5*(33*a^4 - 360*b^4 + 244*a^2*b^2))/(6*a^5) - (tan(c/2
 + (d*x)/2)^3*(60*a*b^3 - 59*a^3*b + 12*a^4 + 240*b^4 - 176*a^2*b^2))/(6*a^
5) - (tan(c/2 + (d*x)/2)^7*(59*a^3*b - 60*a*b^3 + 12*a^4 + 240*b^4 - 176*a^
2*b^2))/(6*a^5) + (tan(c/2 + (d*x)/2)^9*(a - b)*(20*a*b^2 - 16*a^2*b - 3*a^
3 + 40*b^3))/(4*a^5) + (tan(c/2 + (d*x)/2)*(a + b)*(20*a*b^2 + 16*a^2*b - 3
*a^3 - 40*b^3))/(4*a^5))/(d*(a + b - tan(c/2 + (d*x)/2)^10*(a - b) + tan(c/
2 + (d*x)/2)^2*(3*a + 5*b) + tan(c/2 + (d*x)/2)^4*(2*a + 10*b) - tan(c/2 +
(d*x)/2)^8*(3*a - 5*b) - tan(c/2 + (d*x)/2)^6*(2*a - 10*b))) + (atan((((((
76*a^17*b - 12*a^18 - 160*a^12*b^6 + 240*a^13*b^5 + 144*a^14*b^4 - 316*a^15
*b^3 + 28*a^16*b^2)/a^15 - (tan(c/2 + (d*x)/2)*(a^4*3i + b^4*40i - a^2*b^2*
36i)*(128*a^14*b + 128*a^12*b^3 - 256*a^13*b^2))/(16*a^16))*(a^4*3i + b^4*4
0i - a^2*b^2*36i))/(8*a^6) + (tan(c/2 + (d*x)/2)*(6400*a*b^10 - 27*a^10*b +
 9*a^11 - 3200*b^11 + 2560*a^2*b^9 - 11520*a^3*b^8 + 2688*a^4*b^7 + 6144*a^
5*b^6 - 2600*a^6*b^5 - 904*a^7*b^4 + 383*a^8*b^3 + 67*a^9*b^2))/(2*a^10))*(
a^4*3i + b^4*40i - a^2*b^2*36i)*1i)/(8*a^6) - (((((76*a^17*b - 12*a^18 - 16
0*a^12*b^6 + 240*a^13*b^5 + 144*a^14*b^4 - 316*a^15*b^3 + 28*a^16*b^2)/a^15
 + (tan(c/2 + (d*x)/2)*(a^4*3i + b^4*40i - a^2*b^2*36i)*(128*a^14*b + 128*a
^12*b^3 - 256*a^13*b^2))/(16*a^16))*(a^4*3i + b^4*40i - a^2*b^2*36i))/(8*a^
6) - (tan(c/2 + (d*x)/2)*(6400*a*b^10 - 27*a^10*b + 9*a^11 - 3200*b^11 + 25
60*a^2*b^9 - 11520*a^3*b^8 + 2688*a^4*b^7 + 6144*a^5*b^6 - 2600*a^6*b^5 - 9
04*a^7*b^4 + 383*a^8*b^3 + 67*a^9*b^2))/(2*a^10))*(a^4*3i + b^4*40i - a^2*b
^2*36i)*1i)/(8*a^6))/((12000*a*b^13 + 18*a^13*b - 8000*b^14 + 21600*a^2*b^1
2 - 37400*a^3*b^11 - 19240*a^4*b^10 + 43960*a^5*b^9 + 4672*a^6*b^8 - 23963*
a^7*b^7 + 1742*a^8*b^6 + 5958*a^9*b^5 - 834*a^10*b^4 - 573*a^11*b^3 + 60*a^
12*b^2)/a^15 + (((((76*a^17*b - 12*a^18 - 160*a^12*b^6 + 240*a^13*b^5 + 144
*a^14*b^4 - 316*a^15*b^3 + 28*a^16*b^2)/a^15 - (tan(c/2 + (d*x)/2)*(a^4*3i
+ b^4*40i - a^2*b^2*36i)*(128*a^14*b + 128*a^12*b^3 - 256*a^13*b^2))/(16*a^
16))*(a^4*3i + b^4*40i - a^2*b^2*36i))/(8*a^6) + (tan(c/2 + (d*x)/2)*(6400*
a*b^10 - 27*a^10*b + 9*a^11 - 3200*b^11 + 2560*a^2*b^9 - 11520*a^3*b^8 + 26
88*a^4*b^7 + 6144*a^5*b^6 - 2600*a^6*b^5 - 904*a^7*b^4 + 383*a^8*b^3 + 67*a
^9*b^2))/(2*a^10))*(a^4*3i + b^4*40i - a^2*b^2*36i))/(8*a^6) + (((((76*a^17
*b - 12*a^18 - 160*a^12*b^6 + 240*a^13*b^5 + 144*a^14*b^4 - 316*a^15*b^3 +
28*a^16*b^2)/a^15 + (tan(c/2 + (d*x)/2)*(a^4*3i + b^4*40i - a^2*b^2*36i)*(1
28*a^14*b + 128*a^12*b^3 - 256*a^13*b^2))/(16*a^16))*(a^4*3i + b^4*40i - a^
```

```
2*b^2*36i))/(8*a^6) - (tan(c/2 + (d*x)/2)*(6400*a*b^10 - 27*a^10*b + 9*a^11
 - 3200*b^11 + 2560*a^2*b^9 - 11520*a^3*b^8 + 2688*a^4*b^7 + 6144*a^5*b^6 -
 2600*a^6*b^5 - 904*a^7*b^4 + 383*a^8*b^3 + 67*a^9*b^2))/(2*a^10))*(a^4*3i
+ b^4*40i - a^2*b^2*36i))/(8*a^6)))*(a^4*3i + b^4*40i - a^2*b^2*36i)*1i)/(4
*a^6*d) + (b*atan(((b*(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2)*((tan(c/2 + (d*x)/2
)*(6400*a*b^10 - 27*a^10*b + 9*a^11 - 3200*b^11 + 2560*a^2*b^9 - 11520*a^3*
b^8 + 2688*a^4*b^7 + 6144*a^5*b^6 - 2600*a^6*b^5 - 904*a^7*b^4 + 383*a^8*b^
3 + 67*a^9*b^2))/(2*a^10) + (b*((76*a^17*b - 12*a^18 - 160*a^12*b^6 + 240*a
^13*b^5 + 144*a^14*b^4 - 316*a^15*b^3 + 28*a^16*b^2)/a^15 - (b*tan(c/2 + (d
*x)/2)*(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2)*(128*a^14*b + 128*a^12*b^3 - 256*a
^13*b^2))/(2*a^16))*(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2))/a^6)*1i)/a^6 + (b*(a
^2 - b^2)^(1/2)*(2*a^2 - 5*b^2)*((tan(c/2 + (d*x)/2)*(6400*a*b^10 - 27*a^10
*b + 9*a^11 - 3200*b^11 + 2560*a^2*b^9 - 11520*a^3*b^8 + 2688*a^4*b^7 + 614
4*a^5*b^6 - 2600*a^6*b^5 - 904*a^7*b^4 + 383*a^8*b^3 + 67*a^9*b^2))/(2*a^10
) - (b*((76*a^17*b - 12*a^18 - 160*a^12*b^6 + 240*a^13*b^5 + 144*a^14*b^4 -
 316*a^15*b^3 + 28*a^16*b^2)/a^15 + (b*tan(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)
*(2*a^2 - 5*b^2)*(128*a^14*b + 128*a^12*b^3 - 256*a^13*b^2))/(2*a^16))*(a^2
 - b^2)^(1/2)*(2*a^2 - 5*b^2))/a^6)*1i)/a^6)/((12000*a*b^13 + 18*a^13*b - 8
000*b^14 + 21600*a^2*b^12 - 37400*a^3*b^11 - 19240*a^4*b^10 + 43960*a^5*b^9
 + 4672*a^6*b^8 - 23963*a^7*b^7 + 1742*a^8*b^6 + 5958*a^9*b^5 - 834*a^10*b^
4 - 573*a^11*b^3 + 60*a^12*b^2)/a^15 + (b*(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2)
*((tan(c/2 + (d*x)/2)*(6400*a*b^10 - 27*a^10*b + 9*a^11 - 3200*b^11 + 2560*
a^2*b^9 - 11520*a^3*b^8 + 2688*a^4*b^7 + 6144*a^5*b^6 - 2600*a^6*b^5 - 904*
a^7*b^4 + 383*a^8*b^3 + 67*a^9*b^2))/(2*a^10) + (b*((76*a^17*b - 12*a^18 -
160*a^12*b^6 + 240*a^13*b^5 + 144*a^14*b^4 - 316*a^15*b^3 + 28*a^16*b^2)/a^
15 - (b*tan(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2)*(128*a^14*b +
128*a^12*b^3 - 256*a^13*b^2))/(2*a^16))*(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2))/
a^6))/a^6 - (b*(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2)*((tan(c/2 + (d*x)/2)*(6400
*a*b^10 - 27*a^10*b + 9*a^11 - 3200*b^11 + 2560*a^2*b^9 - 11520*a^3*b^8 + 2
688*a^4*b^7 + 6144*a^5*b^6 - 2600*a^6*b^5 - 904*a^7*b^4 + 383*a^8*b^3 + 67*
a^9*b^2))/(2*a^10) - (b*((76*a^17*b - 12*a^18 - 160*a^12*b^6 + 240*a^13*b^5
 + 144*a^14*b^4 - 316*a^15*b^3 + 28*a^16*b^2)/a^15 + (b*tan(c/2 + (d*x)/2)*
(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2)*(128*a^14*b + 128*a^12*b^3 - 256*a^13*b^2
))/(2*a^16))*(a^2 - b^2)^(1/2)*(2*a^2 - 5*b^2))/a^6))/a^6))*(a^2 - b^2)^(1/
2)*(2*a^2 - 5*b^2)*2i)/(a^6*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

3.218 $\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=152

$$\frac{3b\sin(c+dx)}{a^3d}-\frac{3\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{2b\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(a^2-6b^2\right)}{2a^4}+\frac{\sin(c+dx)\cos(c+dx)}{ad(a\cos(c+dx)+b)}$$

```
[Out] 1/2*(a^2-6*b^2)*x/a^4+3*b*sin(d*x+c)/a^3/d-3/2*cos(d*x+c)*sin(d*x+c)/a^2/d+
cos(d*x+c)^2*sin(d*x+c)/a/d/(b+a*cos(d*x+c))-2*b*(2*a^2-3*b^2)*arctanh((a-b
)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2889, 3048, 3050, 3023, 2735, 2659, 208}

$$-\frac{2b\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(a^2-6b^2\right)}{2a^4}+\frac{3b\sin(c+dx)}{a^3d}-\frac{3\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{\sin(c+dx)\cos(c+dx)}{ad(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((a^2 - 6*b^2)*x)/(2*a^4) - (2*b*(2*a^2 - 3*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(
c + d*x)/2])/Sqrt[a + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]*d) + (3*b*Sin[c + d
*x])/(a^3*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + (Cos[c + d*x]^2*Si
n[c + d*x])/(a*d*(b + a*Cos[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
```

```
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(-b-a\cos(c+dx))^2}\,dx \\
&= \int \frac{\cos^2(c+dx)\left(1-\cos^2(c+dx)\right)}{(-b-a\cos(c+dx))^2}\,dx \\
&= \frac{\cos^2(c+dx)\sin(c+dx)}{ad(b+a\cos(c+dx))} - \frac{\int \frac{\cos(c+dx)\left(2\left(a^2-b^2\right)-3\left(a^2-b^2\right)\cos^2(c+dx)\right)}{-b-a\cos(c+dx)}\,dx}{a\left(a^2-b^2\right)} \\
&= -\frac{3\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{ad(b+a\cos(c+dx))} + \frac{\int \frac{3b\left(a^2-b^2\right)-a\left(a^2-b^2\right)\cos(c+dx)-}{-b-a\cos(c+}}{2a^2\left(a^2-\right.} \\
&= \frac{3b\sin(c+dx)}{a^3d} - \frac{3\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{ad(b+a\cos(c+dx))} - \frac{\int \frac{-3ab\left(a^2-b^2\right.}{}}{} \\
&= \frac{\left(a^2-6b^2\right)x}{2a^4} + \frac{3b\sin(c+dx)}{a^3d} - \frac{3\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+dx}{ad(b+a\cos(c+dx))} \\
&= \frac{\left(a^2-6b^2\right)x}{2a^4} + \frac{3b\sin(c+dx)}{a^3d} - \frac{3\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+dx}{ad(b+a\cos(c+dx))} \\
&= \frac{\left(a^2-6b^2\right)x}{2a^4} - \frac{2b\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}\,d} + \frac{3b\sin(c+dx)}{a^3d} - \frac{3\cos(c+}{}
\end{aligned}
$$

Mathematica [A] time = 1.33, size = 178, normalized size = 1.17

$$
\frac{\frac{16b\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{a^3(-\sin(3(c+dx)))-a\left(a^2-24b^2\right)\sin(c+dx)+4a\left(a^2-6b^2\right)(c+dx)\cos(c+dx)+6a^2b\sin(2(c+dx))+4a^2bc+4a^2b}{a\cos(c+dx)+b}}{8a^4d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((16*b*(2*a^2 - 3*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
)/Sqrt[a^2 - b^2] + (4*a^2*b*c - 24*b^3*c + 4*a^2*b*d*x - 24*b^3*d*x + 4*a*
(a^2 - 6*b^2)*(c + d*x)*Cos[c + d*x] - a*(a^2 - 24*b^2)*Sin[c + d*x] + 6*a^
2*b*Sin[2*(c + d*x)] - a^3*Sin[3*(c + d*x)])/(b + a*Cos[c + d*x]))/(8*a^4*d
)
```

fricas [A] time = 0.55, size = 551, normalized size = 3.62

$$
\left[\frac{\left(a^5-7a^3b^2+6ab^4\right)dx\cos(dx+c) + \left(a^4b-7a^2b^3+6b^5\right)dx - \left(2a^2b^2-3b^4+\left(2a^3b-3ab^3\right)\cos(dx+c)\right)\sqrt{a^2}}{}\right.
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^5 - 7*a^3*b^2 + 6*a*b^4)*d*x*cos(d*x + c) + (a^4*b - 7*a^2*b^3 + 6
*b^5)*d*x - (2*a^2*b^2 - 3*b^4 + (2*a^3*b - 3*a*b^3)*cos(d*x + c))*sqrt(a^2
 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2
```

```
- b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^
2 + 2*a*b*cos(d*x + c) + b^2)) + (6*a^3*b^2 - 6*a*b^4 - (a^5 - a^3*b^2)*cos
(d*x + c)^2 + 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^7 - a^5*b
^2)*d*cos(d*x + c) + (a^6*b - a^4*b^3)*d), 1/2*((a^5 - 7*a^3*b^2 + 6*a*b^4)
*d*x*cos(d*x + c) + (a^4*b - 7*a^2*b^3 + 6*b^5)*d*x - 2*(2*a^2*b^2 - 3*b^4
+ (2*a^3*b - 3*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^
2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (6*a^3*b^2 - 6*a*b^4
- (a^5 - a^3*b^2)*cos(d*x + c)^2 + 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*
x + c))/((a^7 - a^5*b^2)*d*cos(d*x + c) + (a^6*b - a^4*b^3)*d)]
```

giac [A] time = 0.26, size = 240, normalized size = 1.58

$$-\frac{\frac{4\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)a^3}-\frac{\left(a^2-6\,b^2\right)(dx+c)}{a^4}+\frac{4\left(2\,a^2b-3\,b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\right.}{\sqrt{-a^2+b^2}}\right.\right.}{\sqrt{-a^2+b^2}\,a^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*b^2*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x
 + 1/2*c)^2 - a - b)*a^3) - (a^2 - 6*b^2)*(d*x + c)/a^4 + 4*(2*a^2*b - 3*b^
3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*
a^4) - 2*(a*tan(1/2*d*x + 1/2*c)^3 + 4*b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2
*d*x + 1/2*c) + 4*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a
^3))/d
```

maple [B] time = 0.44, size = 325, normalized size = 2.14

$$-\frac{2b^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d\,a^3\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)}-\frac{4b\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{d\,a^2\sqrt{(a-b)\,(a+b)}}+\frac{6b^3\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b}{\sqrt{(a-b)(a+b)}}\right.}{d\,a^4\sqrt{(a-b)\,(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/d*b^2/a^3*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^
2*b-a-b)-4/d*b/a^2/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a
-b)*(a+b))^(1/2))+6/d*b^3/a^4/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c
)*(a-b)/((a-b)*(a+b))^(1/2))+1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x
+1/2*c)^3+4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*b+4/d/a^3
/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*b-1/d/a^2/(1+tan(1/2*d*x+1/2
*c)^2)^2*tan(1/2*d*x+1/2*c)-6/d/a^4*arctan(tan(1/2*d*x+1/2*c))*b^2+1/d/a^2*
arctan(tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 3.57, size = 1655, normalized size = 10.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^3*(a^2 + 6*b^2))/a^3 + (tan(c/2 + (d*x)/2)*(3*a*b -
a^2 + 6*b^2))/a^3 - (tan(c/2 + (d*x)/2)^5*(3*a*b + a^2 - 6*b^2))/a^3)/(d*(a
 + b + tan(c/2 + (d*x)/2)^2*(a + 3*b) - tan(c/2 + (d*x)/2)^4*(a - 3*b) - ta
n(c/2 + (d*x)/2)^6*(a - b))) + (atan((8*tan(c/2 + (d*x)/2))/((8*b)/a + (24*
b^2)/a^2 - (24*b^3)/a^3 + (144*b^4)/a^4 - (144*b^5)/a^5 - 8) + (8*b*tan(c/2
 + (d*x)/2))/(8*a - 8*b - (24*b^2)/a + (24*b^3)/a^2 - (144*b^4)/a^3 + (144*
b^5)/a^4) - (24*b^2*tan(c/2 + (d*x)/2))/(8*a*b - 8*a^2 + 24*b^2 - (24*b^3)/
a + (144*b^4)/a^2 - (144*b^5)/a^3) + (24*b^3*tan(c/2 + (d*x)/2))/(24*a*b^2
+ 8*a^2*b - 8*a^3 - 24*b^3 + (144*b^4)/a - (144*b^5)/a^2) + (144*b^4*tan(c/
2 + (d*x)/2))/(24*a*b^3 - 8*a^3*b + 8*a^4 - 144*b^4 - 24*a^2*b^2 + (144*b^5
)/a) + (144*b^5*tan(c/2 + (d*x)/2))/(144*a*b^4 + 8*a^4*b - 8*a^5 - 144*b^5
- 24*a^2*b^3 + 24*a^3*b^2))*(a^2*1i - b^2*6i)*1i)/(a^4*d) - (b*atan(((b*((a
 + b)*(a - b))^(1/2)*(2*a^2 - 3*b^2)*((8*tan(c/2 + (d*x)/2)*(144*a*b^6 - 3*
a^6*b + a^7 - 72*b^7 - 48*a^2*b^5 - 48*a^3*b^4 + 19*a^4*b^3 + 7*a^5*b^2))/a
^6 + (b*((a + b)*(a - b))^(1/2)*((8*(2*a^12 - 10*a^11*b - 12*a^8*b^4 + 18*a
^9*b^3 + 2*a^10*b^2))/a^9 - (8*b*tan(c/2 + (d*x)/2)*((a + b)*(a - b))^(1/2)
*(2*a^2 - 3*b^2)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2))/(a^6*(a^6 - a^4*b^2))
)*(2*a^2 - 3*b^2))/(a^6 - a^4*b^2))*1i)/(a^6 - a^4*b^2) + (b*((a + b)*(a -
b))^(1/2)*(2*a^2 - 3*b^2)*((8*tan(c/2 + (d*x)/2)*(144*a*b^6 - 3*a^6*b + a^7
 - 72*b^7 - 48*a^2*b^5 - 48*a^3*b^4 + 19*a^4*b^3 + 7*a^5*b^2))/a^6 - (b*((a
 + b)*(a - b))^(1/2)*((8*(2*a^12 - 10*a^11*b - 12*a^8*b^4 + 18*a^9*b^3 + 2*
a^10*b^2))/a^9 + (8*b*tan(c/2 + (d*x)/2)*((a + b)*(a - b))^(1/2)*(2*a^2 - 3
*b^2)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2))/(a^6*(a^6 - a^4*b^2)))*(2*a^2 -
3*b^2))/(a^6 - a^4*b^2))*1i)/(a^6 - a^4*b^2))/((16*(162*a*b^7 - 2*a^7*b - 1
08*b^8 + 54*a^2*b^6 - 153*a^3*b^5 + 18*a^4*b^4 + 33*a^5*b^3 - 4*a^6*b^2))/a
^9 + (b*((a + b)*(a - b))^(1/2)*(2*a^2 - 3*b^2)*((8*tan(c/2 + (d*x)/2)*(144
*a*b^6 - 3*a^6*b + a^7 - 72*b^7 - 48*a^2*b^5 - 48*a^3*b^4 + 19*a^4*b^3 + 7*
a^5*b^2))/a^6 + (b*((a + b)*(a - b))^(1/2)*((8*(2*a^12 - 10*a^11*b - 12*a^8
*b^4 + 18*a^9*b^3 + 2*a^10*b^2))/a^9 - (8*b*tan(c/2 + (d*x)/2)*((a + b)*(a
- b))^(1/2)*(2*a^2 - 3*b^2)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2))/(a^6*(a^6
- a^4*b^2)))*(2*a^2 - 3*b^2))/(a^6 - a^4*b^2)))/(a^6 - a^4*b^2) - (b*((a +
b)*(a - b))^(1/2)*(2*a^2 - 3*b^2)*((8*tan(c/2 + (d*x)/2)*(144*a*b^6 - 3*a^6
*b + a^7 - 72*b^7 - 48*a^2*b^5 - 48*a^3*b^4 + 19*a^4*b^3 + 7*a^5*b^2))/a^6
- (b*((a + b)*(a - b))^(1/2)*((8*(2*a^12 - 10*a^11*b - 12*a^8*b^4 + 18*a^9*
b^3 + 2*a^10*b^2))/a^9 + (8*b*tan(c/2 + (d*x)/2)*((a + b)*(a - b))^(1/2)*(2
*a^2 - 3*b^2)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2))/(a^6*(a^6 - a^4*b^2)))*(
2*a^2 - 3*b^2))/(a^6 - a^4*b^2)))/(a^6 - a^4*b^2)))*((a + b)*(a - b))^(1/2)
*(2*a^2 - 3*b^2)*2i)/(d*(a^6 - a^4*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

3.219 $\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=203

$$\frac{ab^2\sin(c+dx)}{d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sin(c}{2d(a+b)^2(1}$$

```
[Out] -4*a^2*b*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/(a
+b)^(5/2)/d-2*b^3*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)
^(5/2)/(a+b)^(5/2)/d-1/2*sin(d*x+c)/(a+b)^2/d/(1-cos(d*x+c))+1/2*sin(d*x+c)
/(a-b)^2/d/(1+cos(d*x+c))+a*b^2*sin(d*x+c)/(a^2-b^2)^2/d/(b+a*cos(d*x+c))
```

Rubi [A] time = 0.44, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2731, 2648, 2664, 12, 2659, 208}

$$\frac{ab^2\sin(c+dx)}{d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sin(c}{2d(a+b)^2(1}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-4*a^2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/
2)*(a + b)^(5/2)*d) - (2*b^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[c + d*x]/(2*(a + b)^2*d*(1 - C
os[c + d*x])) + Sin[c + d*x]/(2*(a - b)^2*d*(1 + Cos[c + d*x])) + (a*b^2*Si
n[c + d*x])/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
```

```
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m)
/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \int \left(-\frac{1}{2(a+b)^2(-1+\cos(c+dx))} + \frac{1}{2(a-b)^2(1+\cos(c+dx))} - \frac{b^2}{\left(-a^2+b^2\right)(b+a\,c$$

$$= \frac{\int \frac{1}{1+\cos(c+dx)}\,dx}{2(a-b)^2} - \frac{\int \frac{1}{-1+\cos(c+dx)}\,dx}{2(a+b)^2} - \frac{\left(2a^2b\right)\int \frac{1}{b+a\cos(c+dx)}\,dx}{\left(a^2-b^2\right)^2} + \frac{b^2\int \frac{1}{(b+a\cos(c+dx))^2}\,c}{a^2-b^2}$$

$$= -\frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} + \frac{ab^2\sin(c+dx}{\left(a^2-b^2\right)^2d(b+a\cos$$

$$= -\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx}{2(a-b)^2d(1+\cos$$

$$= -\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx}{2(a-b)^2d(1+\cos$$

$$= -\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+}{2(a+b)^2d(1-c}$$

Mathematica [A] time = 0.91, size = 128, normalized size = 0.63

$$\frac{\frac{4b\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{\frac{2ab^2\sin(c+dx)}{(a+b)^2(a\cos(c+dx)+b)}+\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((4*b*(2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(5/2) - Cot[(c + d*x)/2]/(a + b)^2 + ((2*a*b^2*Sin[c + d*x])/((a
 + b)^2*(b + a*Cos[c + d*x])) + Tan[(c + d*x)/2])/(a - b)^2)/(2*d)
```

fricas [A]   time = 0.60, size = 526, normalized size = 2.59

$$\left[\frac{6\,a^3b^2 - 6\,ab^4 + \left(2\,a^2b^2 + b^4 + \left(2\,a^3b + ab^3\right)\cos\left(dx+c\right)\right)\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos(dx+c) - \left(a^2-2\,b^2\right)\cos(dx+c)^2 - 2\,\sqrt{a^2-b^2}}{a^2\cos(dx+c)^2 + 2\,ab\cos}\right.}{2\left(\left(a^7 - 3\,a^5b^2 + 3\,a^3b^4 - ab^6\right)d\cos\left(dx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(6*a^3*b^2 - 6*a*b^4 + (2*a^2*b^2 + b^4 + (2*a^3*b + a*b^3)*cos(d*x +
c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2
- 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*c
os(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(a^5 + a^3*b^2
- 2*a*b^4)*cos(d*x + c)^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/(((a^
7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*
a^2*b^5 - b^7)*d)*sin(d*x + c)), (3*a^3*b^2 - 3*a*b^4 - (2*a^2*b^2 + b^4 +
(2*a^3*b + a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(
b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (a^5 + a^3*b
^2 - 2*a*b^4)*cos(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/(((a
^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3
*a^2*b^5 - b^7)*d)*sin(d*x + c))]
```

giac [A]   time = 0.51, size = 289, normalized size = 1.42

$$\frac{\frac{4\left(2\,a^2b+b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^4-2\,a^2b^2+b^4\right)\sqrt{-a^2+b^2}} + \frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2-2\,ab+b^2} - \frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 + 7}{\left(a^4-2\,a^2b^2+b^4\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - b\,t\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(2*a^2*b + b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + a
rctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/
((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^2 - 2*
a*b + b^2) - (a^3*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^2 +
 7*a*b^2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x + 1/2*c)^2 - a^3 + a^2*b
+ a*b^2 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1
/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))))/d
```

maple [A]   time = 0.50, size = 162, normalized size = 0.80

$$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2-4ab+2b^2} + \frac{2b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b} - \frac{\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a-b)^2(a+b)^2} - \frac{1}{2(a+b)^2\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(1/2/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+2*b/(a-b)^2/(a+b)^2*(-a*b*tan(1
/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)-(2*a^2+b^
2)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)
))-1/2/(a+b)^2/tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.79, size = 245, normalized size = 1.21

$$\frac{\frac{a^2-2\,a\,b+b^2}{a+b}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(a^3-3\,a^2\,b+7\,a\,b^2-b^3\right)}{{\left(a+b\right)}^2}}{d\,\left(\left(2\,a^3-6\,a^2\,b+6\,a\,b^2-2\,b^3\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\left(-2\,a^3+2\,a^2\,b+2\,a\,b^2-2\,b^3\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,{\left(a-b\right)}^2}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b/cos(c + d*x))^2),x)
```

```
[Out] ((a^2 - 2*a*b + b^2)/(a + b) - (tan(c/2 + (d*x)/2)^2*(7*a*b^2 - 3*a^2*b + a
^3 - b^3))/(a + b)^2)/(d*(tan(c/2 + (d*x)/2)^3*(6*a*b^2 - 6*a^2*b + 2*a^3 -
 2*b^3) + tan(c/2 + (d*x)/2)*(2*a*b^2 + 2*a^2*b - 2*a^3 - 2*b^3))) + tan(c/
2 + (d*x)/2)/(2*d*(a - b)^2) + (b*atan((a^4*tan(c/2 + (d*x)/2)*1i + b^4*tan
(c/2 + (d*x)/2)*1i - a^2*b^2*tan(c/2 + (d*x)/2)*2i)/((a + b)^(5/2)*(a - b)^
(3/2)))*(2*a^2 + b^2)*2i)/(d*(a + b)^(5/2)*(a - b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

3.220 $\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=343

$$-\frac{2a^2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{4a^2b\left(a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a^3b^2\sin(c+dx)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-$$

```
[Out] -2*a^2*b^3*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/
(a+b)^(7/2)/d-4*a^2*b*(a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b
)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d-1/12*sin(d*x+c)/(a+b)^2/d/(1-cos(d*x+c))
^2-1/4*(a-b)*sin(d*x+c)/(a+b)^3/d/(1-cos(d*x+c))-1/12*sin(d*x+c)/(a+b)^2/d/
(1-cos(d*x+c))+1/12*sin(d*x+c)/(a-b)^2/d/(1+cos(d*x+c))^2+1/12*sin(d*x+c)/(
a-b)^2/d/(1+cos(d*x+c))+1/4*(a+b)*sin(d*x+c)/(a-b)^3/d/(1+cos(d*x+c))+a^3*b
^2*sin(d*x+c)/(a^2-b^2)^3/d/(b+a*cos(d*x+c))
```

Rubi [A] time = 0.55, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2897, 2650, 2648, 2664, 12, 2659, 208}

$$\frac{a^3b^2\sin(c+dx)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{4a^2b\left(a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{2a^2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a^2*b^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(
7/2)*(a + b)^(7/2)*d) - (4*a^2*b*(a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c +
d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - Sin[c + d*x]/(12*(
a + b)^2*d*(1 - Cos[c + d*x])^2) - ((a - b)*Sin[c + d*x])/(4*(a + b)^3*d*(1
 - Cos[c + d*x])) - Sin[c + d*x]/(12*(a + b)^2*d*(1 - Cos[c + d*x])) + Sin[
c + d*x]/(12*(a - b)^2*d*(1 + Cos[c + d*x])^2) + Sin[c + d*x]/(12*(a - b)^2
*d*(1 + Cos[c + d*x])) + ((a + b)*Sin[c + d*x])/(4*(a - b)^3*d*(1 + Cos[c +
 d*x])) + (a^3*b^2*Sin[c + d*x])/((a^2 - b^2)^3*d*(b + a*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
```

```
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$
$$= \int \left(\frac{1}{4(a-b)^2(-1-\cos(c+dx))^2} + \frac{-a-b}{4(a-b)^3(-1-\cos(c+dx))} + \frac{1}{4(a+b)^2(1-c\ldots}\right.$$
$$= \frac{\int \frac{1}{(-1-\cos(c+dx))^2}\,dx}{4(a-b)^2} + \frac{(a-b)\int \frac{1}{1-\cos(c+dx)}\,dx}{4(a+b)^3} + \frac{\int \frac{1}{(1-\cos(c+dx))^2}\,dx}{4(a+b)^2} - \frac{(a+b)\int \frac{1}{-1-c\ldots}}{4(a-\ldots}$$
$$= -\frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} - \frac{(a-b)\sin(c+dx)}{4(a+b)^3d(1-\cos(c+dx))} + \frac{\sin(c+\ldots}{12(a-b)^2d(1+\ldots}$$
$$= -\frac{4a^2b\left(a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} - \frac{\ldots}{4(a\ldots}$$
$$= -\frac{4a^2b\left(a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} - \frac{\ldots}{4(a\ldots}$$
$$= -\frac{2a^2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{4a^2b\left(a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\ldots}{12\ldots}$$

Mathematica [A] time = 1.13, size = 281, normalized size = 0.82

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)\left(\frac{24a^3b^2\sin(c+dx)}{(a-b)^3(a+b)^3} + \frac{48a^2b\left(2a^2+3b^2\right)(a\cos(c+dx)+b)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} + \frac{4(2a+b)\tan\left(\frac{1}{2}(c+\ldots\right.}{(a\ldots}\right.}{24d(a\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*((48*a^2*b*(2*a^2 + 3*b^2)*ArcTanh[((-
a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x]))/(a^2 - b^2)
^(7/2) - (4*(2*a - b)*(b + a*Cos[c + d*x])*Cot[(c + d*x)/2])/(a + b)^3 - ((
b + a*Cos[c + d*x])*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(a + b)^2 + (24*a^
3*b^2*Sin[c + d*x])/((a - b)^3*(a + b)^3) + (4*(2*a + b)*(b + a*Cos[c + d*x
])*Tan[(c + d*x)/2])/(a - b)^3 + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2])/(a - b)^2))/(24*d*(a + b*Sec[c + d*x])^2)
```

fricas [A] time = 0.70, size = 1040, normalized size = 3.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(22*a^5*b^2 - 14*a^3*b^4 - 8*a*b^6 + 2*(2*a^7 + 10*a^5*b^2 - 11*a^3*b
^4 - a*b^6)*cos(d*x + c)^4 - 2*(4*a^6*b - 7*a^4*b^3 + 2*a^2*b^5 + b^7)*cos(
```

```
d*x + c)^3 - 3*(2*a^4*b^2 + 3*a^2*b^4 - (2*a^5*b + 3*a^3*b^3)*cos(d*x + c)^
3 - (2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 + (2*a^5*b + 3*a^3*b^3)*cos(d*x
+ c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^
2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2
*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 6*(a^7 + 6*a^5*
b^2 - 5*a^3*b^4 - 2*a*b^6)*cos(d*x + c)^2 + 10*(a^6*b - 2*a^4*b^3 + a^2*b^5
)*cos(d*x + c))/(((a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d
*x + c)^3 + (a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c
)^2 - (a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c) - (a
^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d)*sin(d*x + c)), -1/3*(11*
a^5*b^2 - 7*a^3*b^4 - 4*a*b^6 + (2*a^7 + 10*a^5*b^2 - 11*a^3*b^4 - a*b^6)*c
os(d*x + c)^4 - (4*a^6*b - 7*a^4*b^3 + 2*a^2*b^5 + b^7)*cos(d*x + c)^3 - 3*
(2*a^4*b^2 + 3*a^2*b^4 - (2*a^5*b + 3*a^3*b^3)*cos(d*x + c)^3 - (2*a^4*b^2
+ 3*a^2*b^4)*cos(d*x + c)^2 + (2*a^5*b + 3*a^3*b^3)*cos(d*x + c))*sqrt(-a^2
 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x
+ c)))*sin(d*x + c) - 3*(a^7 + 6*a^5*b^2 - 5*a^3*b^4 - 2*a*b^6)*cos(d*x + c
)^2 + 5*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c))/(((a^9 - 4*a^7*b^2 + 6*
a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 + (a^8*b - 4*a^6*b^3 + 6*a^4*
b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^2 - (a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*
a^3*b^6 + a*b^8)*d*cos(d*x + c) - (a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^
7 + b^9)*d)*sin(d*x + c))]
```

giac [A] time = 0.36, size = 457, normalized size = 1.33

$$-\frac{\frac{48\,a^{3}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a-b\right)}-\frac{48\left(2\,a^{4}b+3\,a^{2}b^{3}\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\cdots\right.}{\sqrt{-a^{2}+b^{2}}}\right.\cdots}{\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)\sqrt{-a^{2}+b^{2}}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(48*a^3*b^2*tan(1/2*d*x + 1/2*c)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - 48*(2*a^4
*b + 3*a^2*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((
a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6 -
 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - (a^4*tan(1/2*d*x + 1/2*c)
^3 - 4*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*
a*b^3*tan(1/2*d*x + 1/2*c)^3 + b^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*tan(1/2*d
*x + 1/2*c) - 24*a^3*b*tan(1/2*d*x + 1/2*c) + 18*a^2*b^2*tan(1/2*d*x + 1/2*
c) - 3*b^4*tan(1/2*d*x + 1/2*c))/(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 +
 15*a^2*b^4 - 6*a*b^5 + b^6) + (9*a*tan(1/2*d*x + 1/2*c)^2 - 3*b*tan(1/2*d*
x + 1/2*c)^2 + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(1/2*d*x + 1/2*c)
^3))/d
```

maple [A] time = 0.56, size = 242, normalized size = 0.71

$$\frac{\frac{\frac{a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{3}+3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8\left(a^{2}-2ab+b^{2}\right)(a-b)}+\frac{2a^{2}b\left(-\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b}-\frac{\left(2a^{2}+3b^{2}\right)\operatorname{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{\sqrt{(a-b)(a+b)}}\right)}{(a+b)^{3}(a-b)^{3}}-\frac{\cdots}{24(\cdots}\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(1/8/(a^2-2*a*b+b^2)/(a-b)*(1/3*a*tan(1/2*d*x+1/2*c)^3-1/3*tan(1/2*d*x+
1/2*c)^3*b+3*a*tan(1/2*d*x+1/2*c)+tan(1/2*d*x+1/2*c)*b)+2*a^2*b/(a+b)^3/(a-
b)^3*(-a*b*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*
b-a-b)-(2*a^2+3*b^2)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/(
(a-b)*(a+b))^(1/2)))-1/24/(a+b)^2/tan(1/2*d*x+1/2*c)^3-1/8*(3*a-b)/(a+b)^3/
tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.62, size = 403, normalized size = 1.17

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24\,d\,{\left(a-b\right)}^2}+\frac{\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{3\,\left(a+b\right)}+\frac{2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(4\,a^4-13\,a^3\,b+15\,a^2\,b^2-7\,a\,b^3+b^4\right)}{3\,{\left(a+b\right)}^2}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(3\,a^5-13\,a^4\,b+38\,a^3\,b^2-18\,a^2\dots\right.}{{\left(a+b\right)}^3}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(8\,a^4-32\,a^3\,b+48\,a^2\,b^2-32\,a\,b^3+8\,b^4\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(8\,a^4-16\,a^3\,b+\dots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + b/cos(c + d*x))^2),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*d*(a - b)^2) + ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(3
*(a + b)) + (2*tan(c/2 + (d*x)/2)^2*(4*a^4 - 13*a^3*b - 7*a*b^3 + b^4 + 15*
a^2*b^2))/(3*(a + b)^2) - (tan(c/2 + (d*x)/2)^4*(7*a*b^4 - 13*a^4*b + 3*a^5
 - b^5 - 18*a^2*b^3 + 38*a^3*b^2))/(a + b)^3)/(d*(tan(c/2 + (d*x)/2)^5*(8*a
^4 - 32*a^3*b - 32*a*b^3 + 8*b^4 + 48*a^2*b^2) - tan(c/2 + (d*x)/2)^3*(16*a
*b^3 - 16*a^3*b + 8*a^4 - 8*b^4))) + (tan(c/2 + (d*x)/2)*((16*a^2 - 16*b^2)
/(64*(a - b)^4) + 1/(8*(a - b)^2)))/d + (a^2*b*atan((a^6*tan(c/2 + (d*x)/2)
*1i - b^6*tan(c/2 + (d*x)/2)*1i + a^2*b^4*tan(c/2 + (d*x)/2)*3i - a^4*b^2*t
an(c/2 + (d*x)/2)*3i)/((a + b)^(7/2)*(a - b)^(5/2)))*(2*a^2 + 3*b^2)*2i)/(d
*(a + b)^(7/2)*(a - b)^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{4}{\left(c + d x \right)}}{\left(a + b \sec{\left(c + d x \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

3.221 $\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=329

$$-\frac{b\cos^6(c+dx)}{2a^4d}+\frac{\cos^7(c+dx)}{7a^3d}+\frac{3b^2\left(a^2-3b^2\right)\left(a^2-b^2\right)^2}{a^{10}d(a\cos(c+dx)+b)}-\frac{b^3\left(a^2-b^2\right)^3}{2a^{10}d(a\cos(c+dx)+b)^2}+\frac{b\left(9a^2-10b^2\right)\cos^4(c+dx}{4a^6d}$$

```
[Out] -(a^6-18*a^4*b^2+45*a^2*b^4-28*b^6)*cos(d*x+c)/a^9/d-3/2*b*(3*a^4-10*a^2*b^
2+7*b^4)*cos(d*x+c)^2/a^8/d+(a^4-6*a^2*b^2+5*b^4)*cos(d*x+c)^3/a^7/d+1/4*b*
(9*a^2-10*b^2)*cos(d*x+c)^4/a^6/d-3/5*(a^2-2*b^2)*cos(d*x+c)^5/a^5/d-1/2*b*
cos(d*x+c)^6/a^4/d+1/7*cos(d*x+c)^7/a^3/d-1/2*b^3*(a^2-b^2)^3/a^10/d/(b+a*c
os(d*x+c))^2+3*b^2*(a^2-3*b^2)*(a^2-b^2)^2/a^10/d/(b+a*cos(d*x+c))+3*b*(a^2
-b^2)*(a^4-9*a^2*b^2+12*b^4)*ln(b+a*cos(d*x+c))/a^10/d
```

Rubi [A] time = 0.50, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2837, 12, 948}

$$-\frac{3\left(a^2-2b^2\right)\cos^5(c+dx)}{5a^5d}+\frac{b\left(9a^2-10b^2\right)\cos^4(c+dx)}{4a^6d}+\frac{\left(-6a^2b^2+a^4+5b^4\right)\cos^3(c+dx)}{a^7d}-\frac{3b\left(-10a^2b^2+3a^4+\right.}{2a^8}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(((a^6 - 18*a^4*b^2 + 45*a^2*b^4 - 28*b^6)*Cos[c + d*x])/(a^9*d)) - (3*b*(
3*a^4 - 10*a^2*b^2 + 7*b^4)*Cos[c + d*x]^2)/(2*a^8*d) + ((a^4 - 6*a^2*b^2 +
 5*b^4)*Cos[c + d*x]^3)/(a^7*d) + (b*(9*a^2 - 10*b^2)*Cos[c + d*x]^4)/(4*a^
6*d) - (3*(a^2 - 2*b^2)*Cos[c + d*x]^5)/(5*a^5*d) - (b*Cos[c + d*x]^6)/(2*a
^4*d) + Cos[c + d*x]^7/(7*a^3*d) - (b^3*(a^2 - b^2)^3)/(2*a^10*d*(b + a*Cos
[c + d*x])^2) + (3*b^2*(a^2 - 3*b^2)*(a^2 - b^2)^2)/(a^10*d*(b + a*Cos[c +
d*x])) + (3*b*(a^2 - b^2)*(a^4 - 9*a^2*b^2 + 12*b^4)*Log[b + a*Cos[c + d*x]
])/(a^10*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
```

```
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^7(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^3\left(a^2-x^2\right)^3}{a^3(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3\left(a^2-x^2\right)^3}{(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^{10} d}$$
$$= \frac{\text{Subst}\left(\int \left(a^6\left(1+\frac{-18a^4b^2+45a^2b^4-28b^6}{a^6}\right)+\frac{b^3\left(-a^2+b^2\right)^3}{(b-x)^3}+\frac{3b^2\left(a^2-3b^2\right)\left(a^2-b^2\right)^2}{(b-x)^2}+\frac{3b\left(-a^6+10\right.}{}\right.\right.}{}$$
$$= -\frac{\left(a^6-18a^4b^2+45a^2b^4-28b^6\right)\cos(c+dx)}{a^9 d} - \frac{3b\left(3a^4-10a^2b^2+7b^4\right)\cos^2(c+}{2a^8 d}$$

Mathematica [A] time = 5.09, size = 550, normalized size = 1.67

$$\frac{-784a^9\cos(3(c+dx))+152a^9\cos(5(c+dx))-39a^9\cos(7(c+dx))+5a^9\cos(9(c+dx))-1456a^8b\cos(4(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-7945*a^8*b + 164080*a^6*b^3 - 502320*a^4*b^5 + 425600*a^2*b^7 - 76160*b^9
 - 784*a^9*Cos[3*(c + d*x)] + 17528*a^7*b^2*Cos[3*(c + d*x)] - 43680*a^5*b^
4*Cos[3*(c + d*x)] + 26880*a^3*b^6*Cos[3*(c + d*x)] - 1456*a^8*b*Cos[4*(c +
 d*x)] + 4872*a^6*b^3*Cos[4*(c + d*x)] - 3360*a^4*b^5*Cos[4*(c + d*x)] + 15
2*a^9*Cos[5*(c + d*x)] - 840*a^7*b^2*Cos[5*(c + d*x)] + 672*a^5*b^4*Cos[5*(
c + d*x)] + 174*a^8*b*Cos[6*(c + d*x)] - 168*a^6*b^3*Cos[6*(c + d*x)] - 39*
a^9*Cos[7*(c + d*x)] + 48*a^7*b^2*Cos[7*(c + d*x)] - 15*a^8*b*Cos[8*(c + d*
x)] + 5*a^9*Cos[9*(c + d*x)] + 13440*a^8*b*Log[b + a*Cos[c + d*x]] - 107520
*a^6*b^3*Log[b + a*Cos[c + d*x]] + 13440*a^4*b^5*Log[b + a*Cos[c + d*x]] +
403200*a^2*b^7*Log[b + a*Cos[c + d*x]] - 322560*b^9*Log[b + a*Cos[c + d*x]]
 + 70*a^2*b*Cos[2*(c + d*x)]*(-137*a^6 + 1896*a^4*b^2 - 4656*a^2*b^4 + 2912
*b^6 + 192*(a^6 - 10*a^4*b^2 + 21*a^2*b^4 - 12*b^6)*Log[b + a*Cos[c + d*x]]
) - 70*a*Cos[c + d*x]*(49*a^8 - 1472*a^6*b^2 + 3216*a^4*b^4 + 576*a^2*b^6 -
 2432*b^8 - 768*b^2*(a^6 - 10*a^4*b^2 + 21*a^2*b^4 - 12*b^6)*Log[b + a*Cos[
c + d*x]]))/(8960*a^10*d*(b + a*Cos[c + d*x])^2)
```

fricas [A] time = 0.81, size = 447, normalized size = 1.36

$$\frac{80\,a^9\cos\left(dx+c\right)^9-120\,a^8b\cos\left(dx+c\right)^8+2275\,a^6b^3-11235\,a^4b^5+13860\,a^2b^7-4760\,b^9-48\left(7\,a^9-4\,a^7b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/560*(80*a^9*cos(d*x + c)^9 - 120*a^8*b*cos(d*x + c)^8 + 2275*a^6*b^3 - 11
235*a^4*b^5 + 13860*a^2*b^7 - 4760*b^9 - 48*(7*a^9 - 4*a^7*b^2)*cos(d*x + c
```

```
)^7 + 84*(7*a^8*b - 4*a^6*b^3)*cos(d*x + c)^6 + 56*(10*a^9 - 21*a^7*b^2 + 1
2*a^5*b^4)*cos(d*x + c)^5 - 140*(10*a^8*b - 21*a^6*b^3 + 12*a^4*b^5)*cos(d*
x + c)^4 - 560*(a^9 - 10*a^7*b^2 + 21*a^5*b^4 - 12*a^3*b^6)*cos(d*x + c)^3
- 35*(7*a^8*b - 399*a^6*b^3 + 1116*a^4*b^5 - 728*a^2*b^7)*cos(d*x + c)^2 +
70*(41*a^7*b^2 - 81*a^5*b^4 - 108*a^3*b^6 + 152*a*b^8)*cos(d*x + c) + 1680*
(a^6*b^3 - 10*a^4*b^5 + 21*a^2*b^7 - 12*b^9 + (a^8*b - 10*a^6*b^3 + 21*a^4*
b^5 - 12*a^2*b^7)*cos(d*x + c)^2 + 2*(a^7*b^2 - 10*a^5*b^4 + 21*a^3*b^6 - 1
2*a*b^8)*cos(d*x + c))*log(a*cos(d*x + c) + b))/(a^12*d*cos(d*x + c)^2 + 2*
a^11*b*d*cos(d*x + c) + a^10*b^2*d)
```

giac [B] time = 2.60, size = 2150, normalized size = 6.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/140*(420*(a^7*b - a^6*b^2 - 10*a^5*b^3 + 10*a^4*b^4 + 21*a^3*b^5 - 21*a^2
*b^6 - 12*a*b^7 + 12*b^8)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^11 - a^10*b) - 420*(a
^6*b - 10*a^4*b^3 + 21*a^2*b^5 - 12*b^7)*log(abs(-(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 1))/a^10 - 70*(9*a^8*b + 6*a^7*b^2 - 105*a^6*b^3 - 148*a^5*b
^4 + 187*a^4*b^5 + 390*a^3*b^6 + 17*a^2*b^7 - 248*a*b^8 - 108*b^9 + 18*a^8*
b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a^7*b^2*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) - 202*a^6*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 56*a^
5*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 566*a^4*b^5*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) - 76*a^3*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 5
98*a^2*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 32*a*b^8*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + 216*b^9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*
a^8*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^7*b^2*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 - 81*a^6*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 + 180*a^5*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 99*a^4*b^
5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 378*a^3*b^6*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 + 81*a^2*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2 + 216*a*b^8*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 108*b^9*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((a + b + a*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2*a^10) + (128*a^7 -
 1089*a^6*b - 3696*a^5*b^2 + 10890*a^4*b^3 + 11200*a^3*b^4 - 22869*a^2*b^5
- 7840*a*b^6 + 13068*b^7 - 896*a^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
8463*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 24192*a^5*b^2*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) - 81830*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) - 70000*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 165963*a^2*b
^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 47040*a*b^6*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - 91476*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2688*
a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 28749*a^6*b*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 - 64176*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x +
c) + 1)^2 + 262290*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1764
00*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 509649*a^2*b^5*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 117600*a*b^6*(cos(d*x + c) - 1)^2/(c
os(d*x + c) + 1)^2 + 274428*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 -
 4480*a^7*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*a^6*b*(cos(d*x
+ c) - 1)^3/(cos(d*x + c) + 1)^3 + 80640*a^5*b^2*(cos(d*x + c) - 1)^3/(cos(
d*x + c) + 1)^3 - 453950*a^4*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3
- 229600*a^3*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 859215*a^2*b^5
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 156800*a*b^6*(cos(d*x + c) - 1
)^3/(cos(d*x + c) + 1)^3 - 457380*b^7*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
1)^3 - 56035*a^6*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 48720*a^5*b^
2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 453950*a^4*b^3*(cos(d*x + c)
- 1)^4/(cos(d*x + c) + 1)^4 + 162400*a^3*b^4*(cos(d*x + c) - 1)^4/(cos(d*x
+ c) + 1)^4 - 859215*a^2*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 11
```

```
7600*a*b^6*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 457380*b^7*(cos(d*x
+ c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a^6*b*(cos(d*x + c) - 1)^5/(cos(d*
x + c) + 1)^5 + 13440*a^5*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 2
62290*a^4*b^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 58800*a^3*b^4*(co
s(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 509649*a^2*b^5*(cos(d*x + c) - 1)^
5/(cos(d*x + c) + 1)^5 + 47040*a*b^6*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1
)^5 - 274428*b^7*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 8463*a^6*b*(co
s(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 1680*a^5*b^2*(cos(d*x + c) - 1)^6/
(cos(d*x + c) + 1)^6 + 81830*a^4*b^3*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1
)^6 + 8400*a^3*b^4*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 165963*a^2*b
^5*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 7840*a*b^6*(cos(d*x + c) - 1
)^6/(cos(d*x + c) + 1)^6 + 91476*b^7*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1
)^6 + 1089*a^6*b*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 10890*a^4*b^3*
(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 22869*a^2*b^5*(cos(d*x + c) - 1
)^7/(cos(d*x + c) + 1)^7 - 13068*b^7*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1
)^7)/(a^10*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7))/d
```

maple [A] time = 0.59, size = 549, normalized size = 1.67

$$\frac{5\left(\cos^3(dx+c)\right)b^4}{d\,a^7}+\frac{15\left(\cos^2(dx+c)\right)b^3}{d\,a^6}-\frac{21\left(\cos^2(dx+c)\right)b^5}{2d\,a^8}+\frac{3b^5}{2d\,a^6\,(b+a\cos(dx+c))^2}-\frac{3b^7}{2d\,a^8\,(b+a\cos$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 3/2/d*b^5/a^6/(b+a*cos(d*x+c))^2-3/2/d*b^7/a^8/(b+a*cos(d*x+c))^2+1/2/d*b^9
/a^10/(b+a*cos(d*x+c))^2-30/d/a^6*b^3*ln(b+a*cos(d*x+c))+63/d/a^8*b^5*ln(b+
a*cos(d*x+c))-36/d/a^10*b^7*ln(b+a*cos(d*x+c))+5/d/a^7*cos(d*x+c)^3*b^4+15/
d/a^6*cos(d*x+c)^2*b^3-15/d*b^4/a^6/(b+a*cos(d*x+c))-21/2/d/a^8*cos(d*x+c)^
2*b^5+18/d/a^5*cos(d*x+c)*b^2-45/d/a^7*cos(d*x+c)*b^4+28/d/a^9*cos(d*x+c)*b
^6+6/5/d/a^5*cos(d*x+c)^5*b^2-5/2/d/a^6*cos(d*x+c)^4*b^3-6/d/a^5*cos(d*x+c)
^3*b^2-9/d*b^8/a^10/(b+a*cos(d*x+c))+21/d*b^6/a^8/(b+a*cos(d*x+c))+3*b*ln(b
+a*cos(d*x+c))/a^4/d-1/2*b*cos(d*x+c)^6/a^4/d+9/4*b*cos(d*x+c)^4/a^4/d-9/2*
b*cos(d*x+c)^2/a^4/d-1/2*b^3/a^4/d/(b+a*cos(d*x+c))^2+3*b^2/a^4/d/(b+a*cos(
d*x+c))-3/5*cos(d*x+c)^5/a^3/d+1/7*cos(d*x+c)^7/a^3/d-cos(d*x+c)/a^3/d+cos(
d*x+c)^3/a^3/d
```

maxima [A] time = 0.92, size = 326, normalized size = 0.99

$$\frac{\frac{70\left(5\,a^6b^3-27\,a^4b^5+39\,a^2b^7-17\,b^9+6\left(a^7b^2-5\,a^5b^4+7\,a^3b^6-3\,ab^8\right)\cos(dx+c)\right)}{a^{12}\cos(dx+c)^2+2\,a^{11}b\cos(dx+c)+a^{10}b^2}+\frac{20\,a^6\cos(dx+c)^7-70\,a^5b\cos(dx+c)^6-84\left(a^6-2\,a^4b^2\right)\cos(dx+c)^5+}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/140*(70*(5*a^6*b^3 - 27*a^4*b^5 + 39*a^2*b^7 - 17*b^9 + 6*(a^7*b^2 - 5*a^
5*b^4 + 7*a^3*b^6 - 3*a*b^8)*cos(d*x + c))/(a^12*cos(d*x + c)^2 + 2*a^11*b*
cos(d*x + c) + a^10*b^2) + (20*a^6*cos(d*x + c)^7 - 70*a^5*b*cos(d*x + c)^6
 - 84*(a^6 - 2*a^4*b^2)*cos(d*x + c)^5 + 35*(9*a^5*b - 10*a^3*b^3)*cos(d*x
+ c)^4 + 140*(a^6 - 6*a^4*b^2 + 5*a^2*b^4)*cos(d*x + c)^3 - 210*(3*a^5*b -
10*a^3*b^3 + 7*a*b^5)*cos(d*x + c)^2 - 140*(a^6 - 18*a^4*b^2 + 45*a^2*b^4 -
 28*b^6)*cos(d*x + c))/a^9 + 420*(a^6*b - 10*a^4*b^3 + 21*a^2*b^5 - 12*b^7)
*log(a*cos(d*x + c) + b)/a^10)/d
```

mupad [B] time = 0.24, size = 762, normalized size = 2.32

$$\frac{\cos(c+d\,x)^4\left(\frac{2\,b^3}{a^6}+\frac{3\,b\left(\frac{3}{a^3}-\frac{6\,b^2}{a^5}\right)}{4\,a}\right)}{d}-\frac{\frac{-5\,a^6\,b^3+27\,a^4\,b^5-39\,a^2\,b^7+17\,b^9}{2\,a}+\cos(c+d\,x)\,\left(-3\,a^6\,b^2+15\,a^4\,b^4-21\,a^2\,b^6+9\,b^8\right)}{d\,\left(a^{11}\cos(c+d\,x)^2+2\,a^{10}\,b\,\cos(c+d\,x)+a^9\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a + b/cos(c + d*x))^3,x)
```

```
[Out] (cos(c + d*x)^4*((2*b^3)/a^6 + (3*b*(3/a^3 - (6*b^2)/a^5))/(4*a)))/d - ((17
*b^9 - 39*a^2*b^7 + 27*a^4*b^5 - 5*a^6*b^3)/(2*a) + cos(c + d*x)*(9*b^8 - 2
1*a^2*b^6 + 15*a^4*b^4 - 3*a^6*b^2))/(d*(a^11*cos(c + d*x)^2 + a^9*b^2 + 2*
a^10*b*cos(c + d*x))) - (cos(c + d*x)^5*(3/(5*a^3) - (6*b^2)/(5*a^5)))/d -
(cos(c + d*x)^2*((3*b^2*((8*b^3)/a^6 + (3*b*(3/a^3 - (6*b^2)/a^5))/a))/(2*a
^2) - (b^3*(3/a^3 - (6*b^2)/a^5))/(2*a^3) + (3*b*(3/a^3 + (3*b^4)/a^7 + (3*
b^2*(3/a^3 - (6*b^2)/a^5))/a^2 - (3*b*((8*b^3)/a^6 + (3*b*(3/a^3 - (6*b^2)/
a^5))/a))/a))/(2*a)))/d + cos(c + d*x)^7/(7*a^3*d) - (cos(c + d*x)*(1/a^3 +
 (b^3*((8*b^3)/a^6 + (3*b*(3/a^3 - (6*b^2)/a^5))/a))/a^3 + (3*b^2*(3/a^3 +
(3*b^4)/a^7 + (3*b^2*(3/a^3 - (6*b^2)/a^5))/a^2 - (3*b*((8*b^3)/a^6 + (3*b*
(3/a^3 - (6*b^2)/a^5))/a))/a))/a^2 - (3*b*((3*b^2*((8*b^3)/a^6 + (3*b*(3/a^
3 - (6*b^2)/a^5))/a))/a^2 - (b^3*(3/a^3 - (6*b^2)/a^5))/a^3 + (3*b*(3/a^3 +
 (3*b^4)/a^7 + (3*b^2*(3/a^3 - (6*b^2)/a^5))/a^2 - (3*b*((8*b^3)/a^6 + (3*b
*(3/a^3 - (6*b^2)/a^5))/a))/a))/a))/a))/d + (cos(c + d*x)^3*(1/a^3 + b^4/a^
7 + (b^2*(3/a^3 - (6*b^2)/a^5))/a^2 - (b*((8*b^3)/a^6 + (3*b*(3/a^3 - (6*b^
2)/a^5))/a))/a))/d - (b*cos(c + d*x)^6)/(2*a^4*d) + (log(b + a*cos(c + d*x)
)*(3*a^6*b - 36*b^7 + 63*a^2*b^5 - 30*a^4*b^3))/(a^10*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.222 $\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=239

$$\frac{3b\cos^4(c+dx)}{4a^4d}-\frac{\cos^5(c+dx)}{5a^3d}-\frac{b^3\left(a^2-b^2\right)^2}{2a^8d(a\cos(c+dx)+b)^2}-\frac{b\left(3a^2-5b^2\right)\cos^2(c+dx)}{a^6d}+\frac{2\left(a^2-3b^2\right)\cos^3(c+d}{3a^5d}$$

```
[Out] -(a^4-12*a^2*b^2+15*b^4)*cos(d*x+c)/a^7/d-b*(3*a^2-5*b^2)*cos(d*x+c)^2/a^6/
d+2/3*(a^2-3*b^2)*cos(d*x+c)^3/a^5/d+3/4*b*cos(d*x+c)^4/a^4/d-1/5*cos(d*x+c
)^5/a^3/d-1/2*b^3*(a^2-b^2)^2/a^8/d/(b+a*cos(d*x+c))^2+b^2*(3*a^4-10*a^2*b^
2+7*b^4)/a^8/d/(b+a*cos(d*x+c))+b*(3*a^4-20*a^2*b^2+21*b^4)*ln(b+a*cos(d*x+
c))/a^8/d
```

Rubi [A] time = 0.36, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3872, 2837, 12, 948}

$$\frac{2\left(a^2-3b^2\right)\cos^3(c+dx)}{3a^5d}-\frac{b\left(3a^2-5b^2\right)\cos^2(c+dx)}{a^6d}-\frac{\left(-12a^2b^2+a^4+15b^4\right)\cos(c+dx)}{a^7d}+\frac{b^2\left(-10a^2b^2+3a^4}{a^8d(a\cos(c+dx}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(((a^4 - 12*a^2*b^2 + 15*b^4)*Cos[c + d*x])/(a^7*d)) - (b*(3*a^2 - 5*b^2)*
Cos[c + d*x]^2)/(a^6*d) + (2*(a^2 - 3*b^2)*Cos[c + d*x]^3)/(3*a^5*d) + (3*b
*Cos[c + d*x]^4)/(4*a^4*d) - Cos[c + d*x]^5/(5*a^3*d) - (b^3*(a^2 - b^2)^2)
/(2*a^8*d*(b + a*Cos[c + d*x])^2) + (b^2*(3*a^4 - 10*a^2*b^2 + 7*b^4))/(a^8
*d*(b + a*Cos[c + d*x])) + (b*(3*a^4 - 20*a^2*b^2 + 21*b^4)*Log[b + a*Cos[c
 + d*x]])/(a^8*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^5(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^3(a^2-x^2)^2}{a^3(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3(a^2-x^2)^2}{(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^8d}$$
$$= \frac{\text{Subst}\left(\int \left(a^4\left(1+\frac{3b^2(-4a^2+5b^2)}{a^4}\right) - \frac{b^3(-a^2+b^2)^2}{(b-x)^3} + \frac{3a^4b^2-10a^2b^4+7b^6}{(b-x)^2} + \frac{-3a^4b+20a^2b^3-21b^5}{b-x} + \right.\right.}{a^8d}$$
$$= -\frac{(a^4-12a^2b^2+15b^4)\cos(c+dx)}{a^7d} - \frac{b(3a^2-5b^2)\cos^2(c+dx)}{a^6d} + \frac{2(a^2-3b^2)\cos^3}{3a^5d}$$

Mathematica [A] time = 3.00, size = 388, normalized size = 1.62

$$\frac{-206a^7\cos(3(c+dx)) + 38a^7\cos(5(c+dx)) - 6a^7\cos(7(c+dx)) - 274a^6b\cos(4(c+dx)) + 21a^6b\cos(6(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-1740*a^6*b + 26160*a^4*b^3 - 46080*a^2*b^5 + 12480*b^7 - 206*a^7*Cos[3*(c
 + d*x)] + 2780*a^5*b^2*Cos[3*(c + d*x)] - 3360*a^3*b^4*Cos[3*(c + d*x)] -
274*a^6*b*Cos[4*(c + d*x)] + 420*a^4*b^3*Cos[4*(c + d*x)] + 38*a^7*Cos[5*(c
 + d*x)] - 84*a^5*b^2*Cos[5*(c + d*x)] + 21*a^6*b*Cos[6*(c + d*x)] - 6*a^7*
Cos[7*(c + d*x)] + 2880*a^6*b*Log[b + a*Cos[c + d*x]] - 13440*a^4*b^3*Log[b
 + a*Cos[c + d*x]] - 18240*a^2*b^5*Log[b + a*Cos[c + d*x]] + 40320*b^7*Log[
b + a*Cos[c + d*x]] + 5*a^2*b*Cos[2*(c + d*x)]*(-407*a^4 + 3888*a^2*b^2 - 4
800*b^4 + 192*(3*a^4 - 20*a^2*b^2 + 21*b^4)*Log[b + a*Cos[c + d*x]]) - 10*a
*Cos[c + d*x]*(85*a^6 - 1728*a^4*b^2 + 1584*a^2*b^4 + 1536*b^6 - 384*b^2*(3
*a^4 - 20*a^2*b^2 + 21*b^4)*Log[b + a*Cos[c + d*x]]))/(1920*a^8*d*(b + a*Co
s[c + d*x])^2)
```

fricas [A] time = 0.71, size = 331, normalized size = 1.38

$$-\frac{96\,a^7\cos(dx+c)^7 - 168\,a^6b\cos(dx+c)^6 - 1785\,a^4b^3 + 5520\,a^2b^5 - 3120\,b^7 - 16\left(20\,a^7 - 21\,a^5b^2\right)\cos(dx+c)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/480*(96*a^7*cos(d*x + c)^7 - 168*a^6*b*cos(d*x + c)^6 - 1785*a^4*b^3 + 5
520*a^2*b^5 - 3120*b^7 - 16*(20*a^7 - 21*a^5*b^2)*cos(d*x + c)^5 + 40*(20*a
^6*b - 21*a^4*b^3)*cos(d*x + c)^4 + 160*(3*a^7 - 20*a^5*b^2 + 21*a^3*b^4)*c
os(d*x + c)^3 + 15*(25*a^6*b - 592*a^4*b^3 + 800*a^2*b^5)*cos(d*x + c)^2 -
30*(71*a^5*b^2 - 48*a^3*b^4 - 128*a*b^6)*cos(d*x + c) - 480*(3*a^4*b^3 - 20
*a^2*b^5 + 21*b^7 + (3*a^6*b - 20*a^4*b^3 + 21*a^2*b^5)*cos(d*x + c)^2 + 2*
(3*a^5*b^2 - 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c))*log(a*cos(d*x + c) + b))/
(a^10*d*cos(d*x + c)^2 + 2*a^9*b*d*cos(d*x + c) + a^8*b^2*d)
```

giac [B] time = 0.45, size = 1337, normalized size = 5.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(60*(3*a^5*b - 3*a^4*b^2 - 20*a^3*b^3 + 20*a^2*b^4 + 21*a*b^5 - 21*b^6
)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1)))/(a^9 - a^8*b) - 60*(3*a^4*b - 20*a^2*b^3 + 21*b^
5)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^8 - 30*(9*a^6*b +
 6*a^5*b^2 - 75*a^4*b^3 - 108*a^3*b^4 + 51*a^2*b^5 + 150*a*b^6 + 63*b^7 + 1
8*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a^5*b^2*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) - 142*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 36*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 250*a^2*b^5*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 24*a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - 126*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^6*b*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2 - 18*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 - 51*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 120*a^3*b^4*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3*a^2*b^5*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 - 126*a*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
+ 63*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((a + b + a*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2*a^8
) + (64*a^5 - 411*a^4*b - 1200*a^3*b^2 + 2740*a^2*b^3 + 1800*a*b^4 - 2877*b
^5 - 320*a^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2415*a^4*b*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 5280*a^3*b^2*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - 14900*a^2*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 7200*a*b^4*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 14385*b^5*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) + 640*a^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 5910*a^4*b*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 7680*a^3*b^2*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 31000*a^2*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 + 10800*a*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 28770*b^5*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 5910*a^4*b*(cos(d*x + c) - 1)^3
/(cos(d*x + c) + 1)^3 + 4320*a^3*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1
)^3 - 31000*a^2*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 7200*a*b^4*
(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 28770*b^5*(cos(d*x + c) - 1)^3/
(cos(d*x + c) + 1)^3 - 2415*a^4*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4
 - 720*a^3*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 14900*a^2*b^3*(c
os(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1800*a*b^4*(cos(d*x + c) - 1)^4/(
cos(d*x + c) + 1)^4 - 14385*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 +
 411*a^4*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 2740*a^2*b^3*(cos(d*
x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 2877*b^5*(cos(d*x + c) - 1)^5/(cos(d*x
 + c) + 1)^5)/(a^8*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5))/d
```

maple [A] time = 0.52, size = 355, normalized size = 1.49

$$-\frac{\cos^5(dx+c)}{5a^3d}+\frac{3b\left(\cos^4(dx+c)\right)}{4a^4d}+\frac{2\left(\cos^3(dx+c)\right)}{3a^3d}-\frac{2\left(\cos^3(dx+c)\right)b^2}{d\,a^5}-\frac{3b\left(\cos^2(dx+c)\right)}{a^4d}+\frac{5\left(\cos^2(dx+\right.}{d\,a^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -1/5*cos(d*x+c)^5/a^3/d+3/4*b*cos(d*x+c)^4/a^4/d+2/3*cos(d*x+c)^3/a^3/d-2/d
/a^5*cos(d*x+c)^3*b^2-3*b*cos(d*x+c)^2/a^4/d+5/d/a^6*cos(d*x+c)^2*b^3-cos(d
*x+c)/a^3/d+12/d/a^5*cos(d*x+c)*b^2-15/d/a^7*cos(d*x+c)*b^4-1/2*b^3/a^4/d/(
b+a*cos(d*x+c))^2+1/d*b^5/a^6/(b+a*cos(d*x+c))^2-1/2/d*b^7/a^8/(b+a*cos(d*x
+c))^2+3*b*ln(b+a*cos(d*x+c))/a^4/d-20/d/a^6*b^3*ln(b+a*cos(d*x+c))+21/d/a^
8*b^5*ln(b+a*cos(d*x+c))+3*b^2/a^4/d/(b+a*cos(d*x+c))-10/d*b^4/a^6/(b+a*cos
(d*x+c))+7/d*b^6/a^8/(b+a*cos(d*x+c))
```

maxima [A] time = 0.66, size = 234, normalized size = 0.98

$$\frac{\frac{30\left(5a^4b^3-18a^2b^5+13b^7+2\left(3a^5b^2-10a^3b^4+7ab^6\right)\cos(dx+c)\right)}{a^{10}\cos(dx+c)^2+2a^9b\cos(dx+c)+a^8b^2}-\frac{12a^4\cos(dx+c)^5-45a^3b\cos(dx+c)^4-40\left(a^4-3a^2b^2\right)\cos(dx+c)^3+60\left(3a^3b-5ab^3\right)\cdots}{a^7}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(30*(5*a^4*b^3 - 18*a^2*b^5 + 13*b^7 + 2*(3*a^5*b^2 - 10*a^3*b^4 + 7*a
*b^6)*cos(d*x + c))/(a^10*cos(d*x + c)^2 + 2*a^9*b*cos(d*x + c) + a^8*b^2)
- (12*a^4*cos(d*x + c)^5 - 45*a^3*b*cos(d*x + c)^4 - 40*(a^4 - 3*a^2*b^2)*c
os(d*x + c)^3 + 60*(3*a^3*b - 5*a*b^3)*cos(d*x + c)^2 + 60*(a^4 - 12*a^2*b^
2 + 15*b^4)*cos(d*x + c))/a^7 + 60*(3*a^4*b - 20*a^2*b^3 + 21*b^5)*log(a*co
s(d*x + c) + b)/a^8)/d
```

mupad [B] time = 1.11, size = 315, normalized size = 1.32

$$\frac{\cos(c+dx)^3\left(\frac{2}{3a^3}-\frac{2b^2}{a^5}\right)}{d}-\frac{\cos(c+dx)^2\left(\frac{4b^3}{a^6}+\frac{3b\left(\frac{2}{a^3}-\frac{6b^2}{a^5}\right)}{2a}\right)}{d}+\frac{\cos(c+dx)\left(3a^4b^2-10a^2b^4+7b^6\right)+\frac{5a^4b^3-18\cdots}{2\cdots}}{d\left(a^9\cos(c+dx)^2+2a^8b\cos(c+dx)+a^7\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + b/cos(c + d*x))^3,x)
```

```
[Out] (cos(c + d*x)^3*(2/(3*a^3) - (2*b^2)/a^5))/d - (cos(c + d*x)^2*((4*b^3)/a^6
 + (3*b*(2/a^3 - (6*b^2)/a^5))/(2*a)))/d + (cos(c + d*x)*(7*b^6 - 10*a^2*b^
4 + 3*a^4*b^2) + (13*b^7 - 18*a^2*b^5 + 5*a^4*b^3)/(2*a))/(d*(a^9*cos(c + d
*x)^2 + a^7*b^2 + 2*a^8*b*cos(c + d*x))) - (cos(c + d*x)*(1/a^3 + (3*b^4)/a
^7 + (3*b^2*(2/a^3 - (6*b^2)/a^5))/a^2 - (3*b*((8*b^3)/a^6 + (3*b*(2/a^3 -
(6*b^2)/a^5))/a))/a))/d - cos(c + d*x)^5/(5*a^3*d) + (3*b*cos(c + d*x)^4)/(
4*a^4*d) + (log(b + a*cos(c + d*x))*(3*a^4*b + 21*b^5 - 20*a^2*b^3))/(a^8*d
)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.223 $\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=158

$$-\frac{3b\cos^2(c+dx)}{2a^4d}+\frac{\cos^3(c+dx)}{3a^3d}+\frac{b^2\left(3a^2-5b^2\right)}{a^6d(a\cos(c+dx)+b)}+\frac{b\left(3a^2-10b^2\right)\log(a\cos(c+dx)+b)}{a^6d}-\frac{b^3\left(a^2\right.}{2a^6d(a\cos(c}$$

```
[Out] -(a^2-6*b^2)*cos(d*x+c)/a^5/d-3/2*b*cos(d*x+c)^2/a^4/d+1/3*cos(d*x+c)^3/a^3
/d-1/2*b^3*(a^2-b^2)/a^6/d/(b+a*cos(d*x+c))^2+b^2*(3*a^2-5*b^2)/a^6/d/(b+a*
cos(d*x+c))+b*(3*a^2-10*b^2)*ln(b+a*cos(d*x+c))/a^6/d
```

Rubi [A] time = 0.27, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2837, 12, 894}

$$-\frac{b^3\left(a^2-b^2\right)}{2a^6d(a\cos(c+dx)+b)^2}+\frac{b^2\left(3a^2-5b^2\right)}{a^6d(a\cos(c+dx)+b)}-\frac{\left(a^2-6b^2\right)\cos(c+dx)}{a^5d}+\frac{b\left(3a^2-10b^2\right)\log(a\cos(c+dx)}{a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(((a^2 - 6*b^2)*Cos[c + d*x])/(a^5*d)) - (3*b*Cos[c + d*x]^2)/(2*a^4*d) +
Cos[c + d*x]^3/(3*a^3*d) - (b^3*(a^2 - b^2))/(2*a^6*d*(b + a*Cos[c + d*x])^
2) + (b^2*(3*a^2 - 5*b^2))/(a^6*d*(b + a*Cos[c + d*x])) + (b*(3*a^2 - 10*b^
2)*Log[b + a*Cos[c + d*x]])/(a^6*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^3(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^3(a^2-x^2)}{a^3(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3(a^2-x^2)}{(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^6d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{6b^2}{a^2}\right)+\frac{-a^2b^3+b^5}{(b-x)^3}+\frac{3a^2b^2-5b^4}{(b-x)^2}+\frac{-3a^2b+10b^3}{b-x}-3bx-x^2\right)dx, x, -a\cos(c\right.}{a^6d}$$
$$= -\frac{(a^2-6b^2)\cos(c+dx)}{a^5d} - \frac{3b\cos^2(c+dx)}{2a^4d} + \frac{\cos^3(c+dx)}{3a^3d} - \frac{b^3(a^2-b^2)}{2a^6d(b+a\cos(c+dx)}$$

Mathematica [A] time = 0.78, size = 208, normalized size = 1.32

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b)\left(9a^4(2a\cos(c+dx)+b)-(a\cos(c+dx)+b)^2\left(-8a^3\cos(3(c+dx))+96\left(10b^3-\right.\right.\right.}{96a^6d(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*(9*a^4*(b + 2*a*Cos[c + d*x]) - (b + a*Cos[c + d*x])^
2*(72*a*(a^2 - 8*b^2)*Cos[c + d*x] + (-9*a^4*b + 48*a^2*b^3 - 48*b^5)/(b +
a*Cos[c + d*x])^2 + (6*(3*a^4 - 48*a^2*b^2 + 80*b^4))/(b + a*Cos[c + d*x])
+ 72*a^2*b*Cos[2*(c + d*x)] - 8*a^3*Cos[3*(c + d*x)] + 96*(-3*a^2*b + 10*b^
3)*Log[b + a*Cos[c + d*x]]))*Sec[c + d*x]^3)/(96*a^6*d*(a + b*Sec[c + d*x])
^3)
```

fricas [A] time = 0.55, size = 226, normalized size = 1.43

$$\frac{4\,a^5\cos(dx+c)^5 - 10\,a^4b\cos(dx+c)^4 + 39\,a^2b^3 - 54\,b^5 - 4\left(3\,a^5 - 10\,a^3b^2\right)\cos(dx+c)^3 - 3\left(5\,a^4b - 42\,a^2b^3\right)c}{12\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a^5*cos(d*x + c)^5 - 10*a^4*b*cos(d*x + c)^4 + 39*a^2*b^3 - 54*b^5
- 4*(3*a^5 - 10*a^3*b^2)*cos(d*x + c)^3 - 3*(5*a^4*b - 42*a^2*b^3)*cos(d*x
+ c)^2 + 6*(7*a^3*b^2 + 2*a*b^4)*cos(d*x + c) + 12*(3*a^2*b^3 - 10*b^5 + (3
*a^4*b - 10*a^2*b^3)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - 10*a*b^4)*cos(d*x + c)
)*log(a*cos(d*x + c) + b))/(a^8*d*cos(d*x + c)^2 + 2*a^7*b*d*cos(d*x + c) +
 a^6*b^2*d)
```

giac [A] time = 0.50, size = 170, normalized size = 1.08

$$\frac{\left(3\,a^2b - 10\,b^3\right)\log\left(|-a\cos(dx+c) - b|\right)}{a^6d} + \frac{5\,a^2b^3 - 9\,b^5 + \frac{2\left(3\,a^3b^2d - 5\,ab^4d\right)\cos(dx+c)}{d}}{2\,(a\cos(dx+c)+b)^2a^6d} + \frac{2\,a^6d^8\cos(dx+c)^3 - 9\,a^5bd^8\,c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*a^2*b - 10*b^3)*log(abs(-a*cos(d*x + c) - b))/(a^6*d) + 1/2*(5*a^2*b^3 -
 9*b^5 + 2*(3*a^3*b^2*d - 5*a*b^4*d)*cos(d*x + c)/d)/((a*cos(d*x + c) + b)^
2*a^6*d) + 1/6*(2*a^6*d^8*cos(d*x + c)^3 - 9*a^5*b*d^8*cos(d*x + c)^2 - 6*a
^6*d^8*cos(d*x + c) + 36*a^4*b^2*d^8*cos(d*x + c))/(a^9*d^9)
```

maple [A] time = 0.52, size = 200, normalized size = 1.27

$$\frac{\cos^3(dx+c)}{3a^3d}-\frac{3b\left(\cos^2(dx+c)\right)}{2a^4d}-\frac{\cos(dx+c)}{a^3d}+\frac{6\cos(dx+c)\,b^2}{d\,a^5}-\frac{b^3}{2a^4d\,(b+a\cos(dx+c))^2}+\frac{b^5}{2d\,a^6\,(b+a\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/3*cos(d*x+c)^3/a^3/d-3/2*b*cos(d*x+c)^2/a^4/d-cos(d*x+c)/a^3/d+6/d/a^5*co
s(d*x+c)*b^2-1/2*b^3/a^4/d/(b+a*cos(d*x+c))^2+1/2/d*b^5/a^6/(b+a*cos(d*x+c)
)^2+3*b*ln(b+a*cos(d*x+c))/a^4/d-10/d/a^6*b^3*ln(b+a*cos(d*x+c))+3*b^2/a^4/
d/(b+a*cos(d*x+c))-5/d*b^4/a^6/(b+a*cos(d*x+c))
```

maxima [A] time = 0.87, size = 154, normalized size = 0.97

$$\frac{\frac{3\left(5a^2b^3-9b^5+2\left(3a^3b^2-5ab^4\right)\cos(dx+c)\right)}{a^8\cos(dx+c)^2+2a^7b\cos(dx+c)+a^6b^2}+\frac{2a^2\cos(dx+c)^3-9ab\cos(dx+c)^2-6\left(a^2-6b^2\right)\cos(dx+c)}{a^5}+\frac{6\left(3a^2b-10b^3\right)\log(a\cos(dx+c)+b)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(5*a^2*b^3 - 9*b^5 + 2*(3*a^3*b^2 - 5*a*b^4)*cos(d*x + c))/(a^8*cos(
d*x + c)^2 + 2*a^7*b*cos(d*x + c) + a^6*b^2) + (2*a^2*cos(d*x + c)^3 - 9*a*
b*cos(d*x + c)^2 - 6*(a^2 - 6*b^2)*cos(d*x + c))/a^5 + 6*(3*a^2*b - 10*b^3)
*log(a*cos(d*x + c) + b)/a^6)/d
```

mupad [B] time = 0.10, size = 167, normalized size = 1.06

$$\frac{\cos(c+d\,x)^3}{3\,a^3\,d}-\frac{\cos(c+d\,x)\,\left(5\,b^4-3\,a^2\,b^2\right)+\frac{9\,b^5-5\,a^2\,b^3}{2\,a}}{d\,\left(a^7\cos(c+d\,x)^2+2\,a^6\,b\,\cos(c+d\,x)+a^5\,b^2\right)}-\frac{\cos(c+d\,x)\,\left(\frac{1}{a^3}-\frac{6\,b^2}{a^5}\right)}{d}-\frac{3\,b\cos(c+d\,x)^2}{2\,a^4\,d}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + b/cos(c + d*x))^3,x)
```

```
[Out] cos(c + d*x)^3/(3*a^3*d) - (cos(c + d*x)*(5*b^4 - 3*a^2*b^2) + (9*b^5 - 5*a
^2*b^3)/(2*a))/(d*(a^7*cos(c + d*x)^2 + a^5*b^2 + 2*a^6*b*cos(c + d*x))) -
(cos(c + d*x)*(1/a^3 - (6*b^2)/a^5))/d - (3*b*cos(c + d*x)^2)/(2*a^4*d) + (
log(b + a*cos(c + d*x))*(3*a^2*b - 10*b^3))/(a^6*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.224 $\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=83

$$-\frac{b^3}{2a^4d(a\cos(c+dx)+b)^2}+\frac{3b^2}{a^4d(a\cos(c+dx)+b)}+\frac{3b\log(a\cos(c+dx)+b)}{a^4d}-\frac{\cos(c+dx)}{a^3d}$$

```
[Out] -cos(d*x+c)/a^3/d-1/2*b^3/a^4/d/(b+a*cos(d*x+c))^2+3*b^2/a^4/d/(b+a*cos(d*x
+c))+3*b*ln(b+a*cos(d*x+c))/a^4/d
```

Rubi [A] time = 0.13, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.210$, Rules used = {3872, 2833, 12, 43}

$$-\frac{b^3}{2a^4d(a\cos(c+dx)+b)^2}+\frac{3b^2}{a^4d(a\cos(c+dx)+b)}+\frac{3b\log(a\cos(c+dx)+b)}{a^4d}-\frac{\cos(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(Cos[c + d*x]/(a^3*d)) - b^3/(2*a^4*d*(b + a*Cos[c + d*x])^2) + (3*b^2)/(a
^4*d*(b + a*Cos[c + d*x])) + (3*b*Log[b + a*Cos[c + d*x]])/(a^4*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{a^3(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^4d}$$
$$= \frac{\text{Subst}\left(\int \left(1-\frac{b^3}{(b-x)^3}+\frac{3b^2}{(b-x)^2}-\frac{3b}{b-x}\right)dx, x, -a\cos(c+dx)\right)}{a^4d}$$
$$= -\frac{\cos(c+dx)}{a^3d} - \frac{b^3}{2a^4d(b+a\cos(c+dx))^2} + \frac{3b^2}{a^4d(b+a\cos(c+dx))} + \frac{3b\log(b+a}{a}$$

Mathematica [A] time = 0.46, size = 111, normalized size = 1.34

$$\frac{-2a^3\cos^3(c+dx)+2a^2b\cos^2(c+dx)(3\log(a\cos(c+dx)+b)-2)+b^3(6\log(a\cos(c+dx)+b)+5)+4ab^2}{2a^4d(a\cos(c+dx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-2*a^3*Cos[c + d*x]^3 + 2*a^2*b*Cos[c + d*x]^2*(-2 + 3*Log[b + a*Cos[c + d
*x]]) + 4*a*b^2*Cos[c + d*x]*(1 + 3*Log[b + a*Cos[c + d*x]]) + b^3*(5 + 6*L
og[b + a*Cos[c + d*x]]))/(2*a^4*d*(b + a*Cos[c + d*x])^2)
```

fricas [A] time = 0.59, size = 126, normalized size = 1.52

$$-\frac{2\,a^3\cos(dx+c)^3+4\,a^2b\cos(dx+c)^2-4\,ab^2\cos(dx+c)-5\,b^3-6\left(a^2b\cos(dx+c)^2+2\,ab^2\cos(dx+c)\right.}{2\left(a^6d\cos(dx+c)^2+2\,a^5bd\cos(dx+c)+a^4b^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*cos(d*x + c)^3 + 4*a^2*b*cos(d*x + c)^2 - 4*a*b^2*cos(d*x + c)
- 5*b^3 - 6*(a^2*b*cos(d*x + c)^2 + 2*a*b^2*cos(d*x + c) + b^3)*log(a*cos(d
*x + c) + b))/(a^6*d*cos(d*x + c)^2 + 2*a^5*b*d*cos(d*x + c) + a^4*b^2*d)
```

giac [A] time = 0.35, size = 77, normalized size = 0.93

$$-\frac{\cos(dx+c)}{a^3d}+\frac{3\,b\log(|-a\cos(dx+c)-b|)}{a^4d}+\frac{6\,ab^2\cos(dx+c)+5\,b^3}{2\,(a\cos(dx+c)+b)^2a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -cos(d*x + c)/(a^3*d) + 3*b*log(abs(-a*cos(d*x + c) - b))/(a^4*d) + 1/2*(6*
a*b^2*cos(d*x + c) + 5*b^3)/((a*cos(d*x + c) + b)^2*a^4*d)
```

maple [A] time = 0.14, size = 96, normalized size = 1.16

$$-\frac{b}{2d\,a^2\,(a+b\sec(dx+c))^2}+\frac{3b\ln(a+b\sec(dx+c))}{d\,a^4}-\frac{2b}{d\,a^3\,(a+b\sec(dx+c))}-\frac{1}{d\,a^3\sec(dx+c)}-\frac{3b\ln(\sec(}{d\,a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -1/2/d*b/a^2/(a+b*sec(d*x+c))^2+3/d/a^4*b*ln(a+b*sec(d*x+c))-2/d/a^3*b/(a+b
*sec(d*x+c))-1/d/a^3/sec(d*x+c)-3/d/a^4*b*ln(sec(d*x+c))
```

maxima [A] time = 0.84, size = 87, normalized size = 1.05

$$\frac{\frac{6\,ab^2\cos(dx+c)+5\,b^3}{a^6\cos(dx+c)^2+2\,a^5b\cos(dx+c)+a^4b^2} - \frac{2\cos(dx+c)}{a^3} + \frac{6\,b\log(a\cos(dx+c)+b)}{a^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((6*a*b^2*cos(d*x + c) + 5*b^3)/(a^6*cos(d*x + c)^2 + 2*a^5*b*cos(d*x +
 c) + a^4*b^2) - 2*cos(d*x + c)/a^3 + 6*b*log(a*cos(d*x + c) + b)/a^4)/d
```

mupad [B] time = 1.07, size = 93, normalized size = 1.12

$$\frac{3\,b^2\,\cos\left(c+d\,x\right)+\frac{5\,b^3}{2\,a}}{d\,\left(a^5\,\cos\left(c+d\,x\right)^2+2\,a^4\,b\,\cos\left(c+d\,x\right)+a^3\,b^2\right)} - \frac{\cos\left(c+d\,x\right)}{a^3\,d} + \frac{3\,b\,\ln\left(b+a\,\cos\left(c+d\,x\right)\right)}{a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b/cos(c + d*x))^3,x)
```

```
[Out] (3*b^2*cos(c + d*x) + (5*b^3)/(2*a))/(d*(a^5*cos(c + d*x)^2 + a^3*b^2 + 2*a
^4*b*cos(c + d*x))) - cos(c + d*x)/(a^3*d) + (3*b*log(b + a*cos(c + d*x)))/
(a^4*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

3.225 $\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=163

$$\frac{b^2\left(3a^2-b^2\right)}{a^2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{b\left(3a^2+b^2\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^3}-\frac{b^3}{2a^2d\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}+\text{lo}$$

```
[Out] -1/2*b^3/a^2/(a^2-b^2)/d/(b+a*cos(d*x+c))^2+b^2*(3*a^2-b^2)/a^2/(a^2-b^2)^2
/d/(b+a*cos(d*x+c))+1/2*ln(1-cos(d*x+c))/(a+b)^3/d-1/2*ln(1+cos(d*x+c))/(a-
b)^3/d+b*(3*a^2+b^2)*ln(b+a*cos(d*x+c))/(a^2-b^2)^3/d
```

Rubi [A] time = 0.32, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3872, 2837, 12, 1629}

$$-\frac{b^3}{2a^2d\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}+\frac{b^2\left(3a^2-b^2\right)}{a^2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{b\left(3a^2+b^2\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^3}+$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -b^3/(2*a^2*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + (b^2*(3*a^2 - b^2))/(a^
2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])) + Log[1 - Cos[c + d*x]]/(2*(a + b)^
3*d) - Log[1 + Cos[c + d*x]]/(2*(a - b)^3*d) + (b*(3*a^2 + b^2)*Log[b + a*C
os[c + d*x]])/((a^2 - b^2)^3*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos^2(c+dx)\cot(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$
$$= \frac{a\,\mathrm{Subst}\left(\int \frac{x^3}{a^3(-b+x)^3(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{\mathrm{Subst}\left(\int \frac{x^3}{(-b+x)^3(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{a^2d}$$
$$= \frac{\mathrm{Subst}\left(\int \left(\frac{a^2}{2(a-b)^3(a-x)} - \frac{b^3}{(a-b)(a+b)(b-x)^3} + \frac{3a^2b^2-b^4}{(a-b)^2(a+b)^2(b-x)^2} - \frac{a^2b(3a^2+b^2)}{(a-b)^3(a+b)^3(b-x)} + \frac{a^2}{2(a+b)^3(a+\cdots}\right.\right.}{a^2d}$$
$$= -\frac{b^3}{2a^2(a^2-b^2)\,d(b+a\cos(c+dx))^2} + \frac{b^2(3a^2-b^2)}{a^2(a^2-b^2)^2\,d(b+a\cos(c+dx))} + \frac{\log(1-\cdots}{2(a\cdots}$$

Mathematica [A] time = 0.61, size = 203, normalized size = 1.25

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b)\left(-\frac{2b^2(b^2-3a^2)(a\cos(c+dx)+b)}{a^2(a-b)^2(a+b)^2} + \frac{2b(3a^2+b^2)(a\cos(c+dx)+b)^2\log(a\cos(c+dx)+b)}{(a^2-b^2)^3} + \frac{b^3}{a^2(b^2-a^2)} + \frac{2\log\cdots}{\cdots}\right.}{2d(a+b\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*(b^3/(a^2*(-a^2 + b^2)) - (2*b^2*(-3*a^2 + b^2)*(b +
a*Cos[c + d*x]))/(a^2*(a - b)^2*(a + b)^2) + (2*(b + a*Cos[c + d*x])^2*Log[
Cos[(c + d*x)/2]])/(-a + b)^3 + (2*b*(3*a^2 + b^2)*(b + a*Cos[c + d*x])^2*L
og[b + a*Cos[c + d*x]])/(a^2 - b^2)^3 + (2*(b + a*Cos[c + d*x])^2*Log[Sin[(
c + d*x)/2]])/(a + b)^3)*Sec[c + d*x]^3)/(2*d*(a + b*Sec[c + d*x])^3)
```

fricas [B] time = 0.60, size = 474, normalized size = 2.91

$$\frac{5\,a^4b^3 - 6\,a^2b^5 + b^7 + 2\left(3\,a^5b^2 - 4\,a^3b^4 + ab^6\right)\cos(dx+c) + 2\left(3\,a^4b^3 + a^2b^5 + \left(3\,a^6b + a^4b^3\right)\cos(dx+c)^2 + 2\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(5*a^4*b^3 - 6*a^2*b^5 + b^7 + 2*(3*a^5*b^2 - 4*a^3*b^4 + a*b^6)*cos(d*
x + c) + 2*(3*a^4*b^3 + a^2*b^5 + (3*a^6*b + a^4*b^3)*cos(d*x + c)^2 + 2*(3
*a^5*b^2 + a^3*b^4)*cos(d*x + c))*log(a*cos(d*x + c) + b) - (a^5*b^2 + 3*a^
4*b^3 + 3*a^3*b^4 + a^2*b^5 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*cos(d*x
 + c)^2 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*cos(d*x + c))*log(1/2
*cos(d*x + c) + 1/2) + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5 + (a^7 -
3*a^6*b + 3*a^5*b^2 - a^4*b^3)*cos(d*x + c)^2 + 2*(a^6*b - 3*a^5*b^2 + 3*a^
4*b^3 - a^3*b^4)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^10 - 3*a^8
*b^2 + 3*a^6*b^4 - a^4*b^6)*d*cos(d*x + c)^2 + 2*(a^9*b - 3*a^7*b^3 + 3*a^5
*b^5 - a^3*b^7)*d*cos(d*x + c) + (a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8
)*d)
```

giac [B] time = 1.51, size = 452, normalized size = 2.77

$$\frac{\frac{2\left(3\,a^2b+b^3\right)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} + \frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{9\,a^3b+15\,a^2b^2+3\,ab^3-3\,b^4+\frac{18\,a^3b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{6\,a^2b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\cdots}{\left(a^5+a^4b-2\,a\cdots\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(3*a^2*b + b^3)*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6) + log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^3 + 3*a^2*
b + 3*a*b^2 + b^3) - (9*a^3*b + 15*a^2*b^2 + 3*a*b^3 - 3*b^4 + 18*a^3*b*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*a^2*b^2*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) - 10*a*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b^4*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 9*a^3*b*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - 9*a^2*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3*a*b^3*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2)/((a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*(a + b +
 a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1))^2))/d
```

maple [A] time = 0.51, size = 206, normalized size = 1.26

$$-\frac{b^3}{2d\,a^2\,(a+b)\,(a-b)\,(b+a\cos(dx+c))^2}+\frac{3b\ln(b+a\cos(dx+c))\,a^2}{d\,(a+b)^3\,(a-b)^3}+\frac{b^3\ln(b+a\cos(dx+c))}{d\,(a+b)^3\,(a-b)^3}+\frac{}{d\,(a+b)^2\,(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -1/2/d/a^2*b^3/(a+b)/(a-b)/(b+a*cos(d*x+c))^2+3/d*b/(a+b)^3/(a-b)^3*ln(b+a*
cos(d*x+c))*a^2+1/d*b^3/(a+b)^3/(a-b)^3*ln(b+a*cos(d*x+c))+3/d*b^2/(a+b)^2/
(a-b)^2/(b+a*cos(d*x+c))-1/d*b^4/(a+b)^2/(a-b)^2/a^2/(b+a*cos(d*x+c))+1/2/d
/(a+b)^3*ln(-1+cos(d*x+c))-1/2*ln(1+cos(d*x+c))/(a-b)^3/d
```

maxima [A] time = 0.83, size = 241, normalized size = 1.48

$$\frac{\frac{2\left(3a^2b+b^3\right)\log(a\cos(dx+c)+b)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{5a^2b^3-b^5+2\left(3a^3b^2-ab^4\right)\cos(dx+c)}{a^6b^2-2a^4b^4+a^2b^6+\left(a^8-2a^6b^2+a^4b^4\right)\cos(dx+c)^2+2\left(a^7b-2a^5b^3+a^3b^5\right)\cos(dx+c)}-\frac{\log(\cos(dx+c)+1)}{a^3-3a^2b+3ab^2-b^3}+}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(3*a^2*b + b^3)*log(a*cos(d*x + c) + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6) + (5*a^2*b^3 - b^5 + 2*(3*a^3*b^2 - a*b^4)*cos(d*x + c))/(a^6*b^2 -
 2*a^4*b^4 + a^2*b^6 + (a^8 - 2*a^6*b^2 + a^4*b^4)*cos(d*x + c)^2 + 2*(a^7*
b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c)) - log(cos(d*x + c) + 1)/(a^3 - 3*a^2
*b + 3*a*b^2 - b^3) + log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
)/d
```

mupad [B] time = 1.39, size = 182, normalized size = 1.12

$$\frac{\ln(\cos(c+d\,x)-1)}{2\,d\,(a+b)^3}-\frac{\frac{\cos(c+d\,x)\left(b^4-3\,a^2\,b^2\right)}{a\left(a^4-2\,a^2\,b^2+b^4\right)}+\frac{b\left(b^4-5\,a^2\,b^2\right)}{2\,a^2\left(a^4-2\,a^2\,b^2+b^4\right)}}{d\left(a^2\cos(c+d\,x)^2+2\,a\,b\,\cos(c+d\,x)+b^2\right)}-\frac{\ln(\cos(c+d\,x)+1)}{2\,d\,(a-b)^3}-\frac{\ln(b+a\,\cos(c+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + b/cos(c + d*x))^3),x)
```

```
[Out] log(cos(c + d*x) - 1)/(2*d*(a + b)^3) - ((cos(c + d*x)*(b^4 - 3*a^2*b^2))/(
a*(a^4 + b^4 - 2*a^2*b^2)) + (b*(b^4 - 5*a^2*b^2))/(2*a^2*(a^4 + b^4 - 2*a^
2*b^2)))/(d*(b^2 + a^2*cos(c + d*x)^2 + 2*a*b*cos(c + d*x))) - log(cos(c +
```

```
d*x) + 1)/(2*d*(a - b)^3) - (log(b + a*cos(c + d*x))*(1/(2*(a + b)^3) - 1/(
2*(a - b)^3)))/d
```

sympy [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

3.226 $\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=229

$$\frac{b^2\left(3a^2+b^2\right)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+\frac{\csc^2(c+dx)\left(b\left(3a^2+b^2\right)-a\left(a^2+3b^2\right)\cos(c+dx)\right)}{2d\left(a^2-b^2\right)^3}-\frac{b^3}{2d\left(a^2-b^2\right)^2(a\cos(c}$$

```
[Out] -1/2*b^3/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2+b^2*(3*a^2+b^2)/(a^2-b^2)^3/d/(b+
a*cos(d*x+c))+1/2*(b*(3*a^2+b^2)-a*(a^2+3*b^2)*cos(d*x+c))*csc(d*x+c)^2/(a^
2-b^2)^3/d+1/4*(a-2*b)*ln(1-cos(d*x+c))/(a+b)^4/d-1/4*(a+2*b)*ln(1+cos(d*x+
c))/(a-b)^4/d+b*(3*a^4+8*a^2*b^2+b^4)*ln(b+a*cos(d*x+c))/(a^2-b^2)^4/d
```

Rubi [A] time = 0.51, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3872, 2721, 1647, 1629}

$$-\frac{b^3}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+\frac{b^2\left(3a^2+b^2\right)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+\frac{b\left(8a^2b^2+3a^4+b^4\right)\log(a\cos(c+dx)+}{d\left(a^2-b^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -b^3/(2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (b^2*(3*a^2 + b^2))/((a^2
 - b^2)^3*d*(b + a*Cos[c + d*x])) + ((b*(3*a^2 + b^2) - a*(a^2 + 3*b^2)*Cos
[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) + ((a - 2*b)*Log[1 - Cos[c +
 d*x]])/(4*(a + b)^4*d) - ((a + 2*b)*Log[1 + Cos[c + d*x]])/(4*(a - b)^4*d)
 + (b*(3*a^4 + 8*a^2*b^2 + b^4)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cot^3(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{x^3}{(-b+x)^3\left(a^2-x^2\right)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{\left(b\left(3a^2+b^2\right)-a\left(a^2+3b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} + \frac{\text{Subst}\left(\int \frac{\frac{a^4b^3\left(a^2+3b^2\right)}{\left(a^2-b^2\right)^3} - \frac{a^2b^2\left(3a^4+\right.}{\left(a^2\right.}}{}\right.}{}$$

$$= \frac{\left(b\left(3a^2+b^2\right)-a\left(a^2+3b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} + \frac{\text{Subst}\left(\int\left(\frac{a^2(a+2b)}{2(a-b)^4(a-x)} - \frac{}{\left(a^2-\right.}\right.\right.}{}$$

$$= -\frac{b^3}{2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2} + \frac{b^2\left(3a^2+b^2\right)}{\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))} + \frac{\left(b\left(3a^2+b^2\right.\right.}{}$$

Mathematica [C] time = 6.34, size = 332, normalized size = 1.45

$$-\frac{b^2\left(3a^2+b^2\right)}{d(b-a)^3(a+b)^3(a\cos(c+dx)+b)} - \frac{2i\left(3a^4b+8a^2b^3+b^5\right)(c+dx)}{d(a-b)^4(a+b)^4} + \frac{\left(3a^4b+8a^2b^3+b^5\right)\log(a\cos(c+dx)+b)}{d\left(b^2-a^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-2*I)*(3*a^4*b + 8*a^2*b^3 + b^5)*(c + d*x))/((a - b)^4*(a + b)^4*d) - ((
I/2)*(-a - 2*b)*ArcTan[Tan[c + d*x]])/((-a + b)^4*d) - ((I/2)*(a - 2*b)*Arc
Tan[Tan[c + d*x]])/((a + b)^4*d) - b^3/(2*(-a + b)^2*(a + b)^2*d*(b + a*Cos
[c + d*x])^2) - (b^2*(3*a^2 + b^2))/((-a + b)^3*(a + b)^3*d*(b + a*Cos[c +
d*x])) - Csc[(c + d*x)/2]^2/(8*(a + b)^3*d) + ((-a - 2*b)*Log[Cos[(c + d*x)
/2]^2])/(4*(-a + b)^4*d) + ((3*a^4*b + 8*a^2*b^3 + b^5)*Log[b + a*Cos[c + d
*x]])/((-a^2 + b^2)^4*d) + ((a - 2*b)*Log[Sin[(c + d*x)/2]^2])/(4*(a + b)^4
*d) - Sec[(c + d*x)/2]^2/(8*(-a + b)^3*d)
```

fricas [B] time = 0.89, size = 1071, normalized size = 4.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(16*a^4*b^3 - 8*a^2*b^5 - 8*b^7 - 2*(a^7 + 8*a^5*b^2 - 7*a^3*b^4 - 2*a
*b^6)*cos(d*x + c)^3 + 2*(a^6*b - 11*a^4*b^3 + 7*a^2*b^5 + 3*b^7)*cos(d*x +
 c)^2 + 2*(11*a^5*b^2 - 10*a^3*b^4 - a*b^6)*cos(d*x + c) + 4*(3*a^4*b^3 + 8
*a^2*b^5 + b^7 - (3*a^6*b + 8*a^4*b^3 + a^2*b^5)*cos(d*x + c)^4 - 2*(3*a^5*
b^2 + 8*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (3*a^6*b + 5*a^4*b^3 - 7*a^2*b^5
- b^7)*cos(d*x + c)^2 + 2*(3*a^5*b^2 + 8*a^3*b^4 + a*b^6)*cos(d*x + c))*log
(a*cos(d*x + c) + b) - (a^5*b^2 + 6*a^4*b^3 + 14*a^3*b^4 + 16*a^2*b^5 + 9*a
*b^6 + 2*b^7 - (a^7 + 6*a^6*b + 14*a^5*b^2 + 16*a^4*b^3 + 9*a^3*b^4 + 2*a^2
*b^5)*cos(d*x + c)^4 - 2*(a^6*b + 6*a^5*b^2 + 14*a^4*b^3 + 16*a^3*b^4 + 9*a
```

```
^2*b^5 + 2*a*b^6)*cos(d*x + c)^3 + (a^7 + 6*a^6*b + 13*a^5*b^2 + 10*a^4*b^3
 - 5*a^3*b^4 - 14*a^2*b^5 - 9*a*b^6 - 2*b^7)*cos(d*x + c)^2 + 2*(a^6*b + 6*
a^5*b^2 + 14*a^4*b^3 + 16*a^3*b^4 + 9*a^2*b^5 + 2*a*b^6)*cos(d*x + c))*log(
1/2*cos(d*x + c) + 1/2) + (a^5*b^2 - 6*a^4*b^3 + 14*a^3*b^4 - 16*a^2*b^5 +
9*a*b^6 - 2*b^7 - (a^7 - 6*a^6*b + 14*a^5*b^2 - 16*a^4*b^3 + 9*a^3*b^4 - 2*
a^2*b^5)*cos(d*x + c)^4 - 2*(a^6*b - 6*a^5*b^2 + 14*a^4*b^3 - 16*a^3*b^4 +
9*a^2*b^5 - 2*a*b^6)*cos(d*x + c)^3 + (a^7 - 6*a^6*b + 13*a^5*b^2 - 10*a^4*
b^3 - 5*a^3*b^4 + 14*a^2*b^5 - 9*a*b^6 + 2*b^7)*cos(d*x + c)^2 + 2*(a^6*b -
 6*a^5*b^2 + 14*a^4*b^3 - 16*a^3*b^4 + 9*a^2*b^5 - 2*a*b^6)*cos(d*x + c))*l
og(-1/2*cos(d*x + c) + 1/2))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a
^2*b^8)*d*cos(d*x + c)^4 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a
*b^9)*d*cos(d*x + c)^3 - (a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^
2*b^8 - b^10)*d*cos(d*x + c)^2 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b
^7 + a*b^9)*d*cos(d*x + c) - (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 +
 b^10)*d)
```

giac [B] time = 2.65, size = 800, normalized size = 3.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*(a - 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 8*(3*a^4*b + 8*a^2*b^3 + b^5)*log(abs
(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (a + b
 - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*(cos(d*x + c) + 1)) - 4*(9*a^6*b + 6*a^5*b^2 + 9*a^4*b^3 + 28*a^3*b^4 +
11*a^2*b^5 - 2*a*b^6 + 3*b^7 + 18*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) - 12*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 26*a^4*b^3*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 4*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - 38*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*a*b^6*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - 6*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 9*a^6*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^5*b^2*(cos(d*x +
 c) - 1)^2/(cos(d*x + c) + 1)^2 + 33*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 - 48*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 27*a^2
*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*a*b^6*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 3*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/
((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))/d
```

maple [A] time = 0.68, size = 322, normalized size = 1.41

$$-\frac{b^3}{2d\left(a+b\right)^2\left(a-b\right)^2\left(b+a\cos\left(dx+c\right)\right)^2}+\frac{3b\ln\left(b+a\cos\left(dx+c\right)\right)a^4}{d\left(a+b\right)^4\left(a-b\right)^4}+\frac{8b^3\ln\left(b+a\cos\left(dx+c\right)\right)a^2}{d\left(a+b\right)^4\left(a-b\right)^4}+\frac{b^5\ln\left(b\right.}{d\left(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -1/2/d*b^3/(a+b)^2/(a-b)^2/(b+a*cos(d*x+c))^2+3/d*b/(a+b)^4/(a-b)^4*ln(b+a*
cos(d*x+c))*a^4+8/d*b^3/(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))*a^2+1/d*b^5/(a+b
)^4/(a-b)^4*ln(b+a*cos(d*x+c))+3/d*b^2/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))*a^2
+1/d*b^4/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))+1/4/d/(a+b)^3/(-1+cos(d*x+c))+1/4
/d/(a+b)^4*ln(-1+cos(d*x+c))*a-1/2/d/(a+b)^4*ln(-1+cos(d*x+c))*b+1/4/d/(a-b
)^3/(1+cos(d*x+c))-1/4/d/(a-b)^4*ln(1+cos(d*x+c))*a-1/2/d/(a-b)^4*ln(1+cos(
d*x+c))*b
```

maxima [A] time = 0.79, size = 435, normalized size = 1.90

$$\frac{\frac{4\left(3a^4b+8a^2b^3+b^5\right)\log(a\cos(dx+c)+b)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}-\frac{(a+2b)\log(\cos(dx+c)+1)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}+\frac{(a-2b)\log(\cos(dx+c)-1)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}+\frac{2(8\cdots}{a^6b^2-3a^4b^4+3a^2b^6-b^8-\left(a^8-3a^6b^2+3a^4b^4\cdots\right.}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(3*a^4*b + 8*a^2*b^3 + b^5)*log(a*cos(d*x + c) + b)/(a^8 - 4*a^6*b^2
 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - (a + 2*b)*log(cos(d*x + c) + 1)/(a^4 - 4*
a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + (a - 2*b)*log(cos(d*x + c) - 1)/(a^4 +
 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(8*a^2*b^3 + 4*b^5 - (a^5 + 9*a^3
*b^2 + 2*a*b^4)*cos(d*x + c)^3 + (a^4*b - 10*a^2*b^3 - 3*b^5)*cos(d*x + c)^
2 + (11*a^3*b^2 + a*b^4)*cos(d*x + c))/(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b
^8 - (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*cos(d*x + c)^4 - 2*(a^7*b - 3*
a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c)^3 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4
- 4*a^2*b^6 + b^8)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^
7)*cos(d*x + c)))/d
```

mupad [B] time = 1.70, size = 378, normalized size = 1.65

$$\frac{\ln\left(b+a\,\cos\left(c+d\,x\right)\right)\,\left(\frac{3\,b}{4\,{\left(a+b\right)}^4}-\frac{1}{4\,{\left(a+b\right)}^3}+\frac{3\,b}{4\,{\left(a-b\right)}^4}+\frac{1}{4\,{\left(a-b\right)}^3}\right)}{d}-\frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\,\left(\frac{3\,b}{4\,{\left(a+b\right)}^4}-\frac{1}{4\,{\left(a+b\right)}^3}\right)}{d}-\frac{\frac{{\cos\left(c+d\,x\right)}^3\cdots}{2\,\left(a^6-3\,a\cdots\right.}}{d\,\left(\cos\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + b/cos(c + d*x))^3),x)
```

```
[Out] (log(b + a*cos(c + d*x))*((3*b)/(4*(a + b)^4) - 1/(4*(a + b)^3) + (3*b)/(4*
(a - b)^4) + 1/(4*(a - b)^3)))/d - (log(cos(c + d*x) - 1)*((3*b)/(4*(a + b)
^4) - 1/(4*(a + b)^3)))/d - ((cos(c + d*x)^3*(2*a*b^4 + a^5 + 9*a^3*b^2))/(
2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) + (cos(c + d*x)^2*(3*b^5 - a^4*b + 1
0*a^2*b^3))/(2*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)) - (2*b*(b^4 + 2*a^2*b^2
))/((a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (a*cos(c + d*x)*(b^4 + 11*a^2*b^
2))/(2*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)))/(d*(cos(c + d*x)^2*(a^2 - b^2)
 + b^2 - a^2*cos(c + d*x)^4 + 2*a*b*cos(c + d*x) - 2*a*b*cos(c + d*x)^3)) -
 (log(cos(c + d*x) + 1)*(a + 2*b))/(4*d*(a - b)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{3}{\left(c + dx \right)}}{\left(a + b \sec{\left(c + dx \right)}\right)^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**3/(a + b*sec(c + d*x))**3, x)
```

3.227 $\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=313

$$\frac{3a^2b^2\left(a^2+b^2\right)}{d\left(a^2-b^2\right)^4(a\cos(c+dx)+b)}+\frac{\csc^4(c+dx)\left(b\left(3a^2+b^2\right)-a\left(a^2+3b^2\right)\cos(c+dx)\right)}{4d\left(a^2-b^2\right)^3}-\frac{a^2b^3}{2d\left(a^2-b^2\right)^3(a\cos(c$$

```
[Out] -1/2*a^2*b^3/(a^2-b^2)^3/d/(b+a*cos(d*x+c))^2+3*a^2*b^2*(a^2+b^2)/(a^2-b^2)
^4/d/(b+a*cos(d*x+c))+1/8*(4*b*(3*a^4+8*a^2*b^2+b^4)-3*a*(a^4+10*a^2*b^2+5*
b^4)*cos(d*x+c))*csc(d*x+c)^2/(a^2-b^2)^4/d+1/4*(b*(3*a^2+b^2)-a*(a^2+3*b^2
)*cos(d*x+c))*csc(d*x+c)^4/(a^2-b^2)^3/d+3/16*a*(a-3*b)*ln(1-cos(d*x+c))/(a
+b)^5/d-3/16*a*(a+3*b)*ln(1+cos(d*x+c))/(a-b)^5/d+3*a^2*b*(a^4+5*a^2*b^2+2*
b^4)*ln(b+a*cos(d*x+c))/(a^2-b^2)^5/d
```

Rubi [A] time = 1.01, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3872, 2837, 12, 1647, 1629}

$$\frac{3a^2b^2\left(a^2+b^2\right)}{d\left(a^2-b^2\right)^4(a\cos(c+dx)+b)}-\frac{a^2b^3}{2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)^2}+\frac{3a^2b\left(5a^2b^2+a^4+2b^4\right)\log(a\cos(c+dx)}{d\left(a^2-b^2\right)^5}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(a^2*b^3)/(2*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])^2) + (3*a^2*b^2*(a^2 + b
^2))/((a^2 - b^2)^4*d*(b + a*Cos[c + d*x])) + ((4*b*(3*a^4 + 8*a^2*b^2 + b^
4) - 3*a*(a^4 + 10*a^2*b^2 + 5*b^4)*Cos[c + d*x])*Csc[c + d*x]^2)/(8*(a^2 -
 b^2)^4*d) + ((b*(3*a^2 + b^2) - a*(a^2 + 3*b^2)*Cos[c + d*x])*Csc[c + d*x]
^4)/(4*(a^2 - b^2)^3*d) + (3*a*(a - 3*b)*Log[1 - Cos[c + d*x]])/(16*(a + b)
^5*d) - (3*a*(a + 3*b)*Log[1 + Cos[c + d*x]])/(16*(a - b)^5*d) + (3*a^2*b*(
a^4 + 5*a^2*b^2 + 2*b^4)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^5*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
```

```
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cot^3(c+dx)\csc^2(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= \frac{a^5\,\text{Subst}\left(\int \frac{x^3}{a^3(-b+x)^3\left(a^2-x^2\right)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^2\,\text{Subst}\left(\int \frac{x^3}{(-b+x)^3\left(a^2-x^2\right)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{\left(b\left(3a^2+b^2\right)-a\left(a^2+3b^2\right)\cos(c+dx)\right)\csc^4(c+dx)}{4\left(a^2-b^2\right)^3 d} + \frac{\text{Subst}\left(\int \frac{\frac{a^4b^3\left(a^2+3b^2\right)}{\left(a^2-b^2\right)^3} - \frac{a^2b^2\left(3a^4-\right.}{\left(a^2\right.}}{}\right.}{}$$

$$= \frac{\left(4b\left(3a^4+8a^2b^2+b^4\right)-3a\left(a^4+10a^2b^2+5b^4\right)\cos(c+dx)\right)\csc^2(c+dx)}{8\left(a^2-b^2\right)^4 d} + \frac{\left(b\left(3a^2\right.\right.}{}$$

$$= \frac{\left(4b\left(3a^4+8a^2b^2+b^4\right)-3a\left(a^4+10a^2b^2+5b^4\right)\cos(c+dx)\right)\csc^2(c+dx)}{8\left(a^2-b^2\right)^4 d} + \frac{\left(b\left(3a^2\right.\right.}{}$$

$$= -\frac{a^2b^3}{2\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))^2} + \frac{3a^2b^2\left(a^2+b^2\right)}{\left(a^2-b^2\right)^4 d(b+a\cos(c+dx))} + \frac{\left(4b\left(3a^4+8\right.\right.}{}$$

Mathematica [C] time = 4.89, size = 496, normalized size = 1.58

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b)\left(\frac{32a^2b^3}{(b-a)^3(a+b)^3} + \frac{192a^2b^2(a-ib)(a+ib)(a\cos(c+dx)+b)}{(a-b)^4(a+b)^4} - \frac{384ia^2b\left(a^4+5a^2b^2+2b^4\right)(c+dx)(a\cos(c+dx)+b)^2}{(a-b)^5(a+b)^5}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*((32*a^2*b^3)/((-a + b)^3*(a + b)^3) + (192*a^2*(a -
I*b)*(a + I*b)*b^2*(b + a*Cos[c + d*x]))/((a - b)^4*(a + b)^4) - ((384*I)*a
^2*b*(a^4 + 5*a^2*b^2 + 2*b^4)*(c + d*x)*(b + a*Cos[c + d*x])^2)/((a - b)^5
*(a + b)^5) - ((24*I)*a*(a - 3*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])
^2)/(a + b)^5 + ((24*I)*a*(a + 3*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x
```

```
])^2)/(a - b)^5 + (6*(-a + b)*(b + a*Cos[c + d*x])^2*Csc[(c + d*x)/2]^2)/(a
 + b)^4 - ((b + a*Cos[c + d*x])^2*Csc[(c + d*x)/2]^4)/(a + b)^3 - (12*a*(a
+ 3*b)*(b + a*Cos[c + d*x])^2*Log[Cos[(c + d*x)/2]^2])/(a - b)^5 + (192*a^2
*b*(a^4 + 5*a^2*b^2 + 2*b^4)*(b + a*Cos[c + d*x])^2*Log[b + a*Cos[c + d*x]]
)/(a^2 - b^2)^5 + (12*a*(a - 3*b)*(b + a*Cos[c + d*x])^2*Log[Sin[(c + d*x)/
2]^2])/(a + b)^5 + (6*(a + b)*(b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^2)/(a
 - b)^4 + ((b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^4)/(a - b)^3)*Sec[c + d*
x]^3)/(64*d*(a + b*Sec[c + d*x])^3)
```

fricas [B] time = 1.36, size = 1803, normalized size = 5.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(76*a^6*b^3 + 36*a^4*b^5 - 108*a^2*b^7 - 4*b^9 + 6*(a^9 + 17*a^7*b^2 -
 5*a^5*b^4 - 13*a^3*b^6)*cos(d*x + c)^5 - 12*(a^8*b - 9*a^6*b^3 - a^4*b^5 +
 9*a^2*b^7)*cos(d*x + c)^4 - 2*(5*a^9 + 98*a^7*b^2 - 12*a^5*b^4 - 98*a^3*b^
6 + 7*a*b^8)*cos(d*x + c)^3 + 8*(2*a^8*b - 25*a^6*b^3 - 3*a^4*b^5 + 25*a^2*
b^7 + b^9)*cos(d*x + c)^2 + 2*(55*a^7*b^2 - 9*a^5*b^4 - 51*a^3*b^6 + 5*a*b^
8)*cos(d*x + c) + 48*(a^6*b^3 + 5*a^4*b^5 + 2*a^2*b^7 + (a^8*b + 5*a^6*b^3
+ 2*a^4*b^5)*cos(d*x + c)^6 + 2*(a^7*b^2 + 5*a^5*b^4 + 2*a^3*b^6)*cos(d*x +
 c)^5 - (2*a^8*b + 9*a^6*b^3 - a^4*b^5 - 2*a^2*b^7)*cos(d*x + c)^4 - 4*(a^7
*b^2 + 5*a^5*b^4 + 2*a^3*b^6)*cos(d*x + c)^3 + (a^8*b + 3*a^6*b^3 - 8*a^4*b
^5 - 4*a^2*b^7)*cos(d*x + c)^2 + 2*(a^7*b^2 + 5*a^5*b^4 + 2*a^3*b^6)*cos(d*
x + c))*log(a*cos(d*x + c) + b) - 3*(a^7*b^2 + 8*a^6*b^3 + 25*a^5*b^4 + 40*
a^4*b^5 + 35*a^3*b^6 + 16*a^2*b^7 + 3*a*b^8 + (a^9 + 8*a^8*b + 25*a^7*b^2 +
 40*a^6*b^3 + 35*a^5*b^4 + 16*a^4*b^5 + 3*a^3*b^6)*cos(d*x + c)^6 + 2*(a^8*
b + 8*a^7*b^2 + 25*a^6*b^3 + 40*a^5*b^4 + 35*a^4*b^5 + 16*a^3*b^6 + 3*a^2*b
^7)*cos(d*x + c)^5 - (2*a^9 + 16*a^8*b + 49*a^7*b^2 + 72*a^6*b^3 + 45*a^5*b
^4 - 8*a^4*b^5 - 29*a^3*b^6 - 16*a^2*b^7 - 3*a*b^8)*cos(d*x + c)^4 - 4*(a^8
*b + 8*a^7*b^2 + 25*a^6*b^3 + 40*a^5*b^4 + 35*a^4*b^5 + 16*a^3*b^6 + 3*a^2*
b^7)*cos(d*x + c)^3 + (a^9 + 8*a^8*b + 23*a^7*b^2 + 24*a^6*b^3 - 15*a^5*b^4
 - 64*a^4*b^5 - 67*a^3*b^6 - 32*a^2*b^7 - 6*a*b^8)*cos(d*x + c)^2 + 2*(a^8*
b + 8*a^7*b^2 + 25*a^6*b^3 + 40*a^5*b^4 + 35*a^4*b^5 + 16*a^3*b^6 + 3*a^2*b
^7)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 3*(a^7*b^2 - 8*a^6*b^3 + 25
*a^5*b^4 - 40*a^4*b^5 + 35*a^3*b^6 - 16*a^2*b^7 + 3*a*b^8 + (a^9 - 8*a^8*b
+ 25*a^7*b^2 - 40*a^6*b^3 + 35*a^5*b^4 - 16*a^4*b^5 + 3*a^3*b^6)*cos(d*x +
c)^6 + 2*(a^8*b - 8*a^7*b^2 + 25*a^6*b^3 - 40*a^5*b^4 + 35*a^4*b^5 - 16*a^3
*b^6 + 3*a^2*b^7)*cos(d*x + c)^5 - (2*a^9 - 16*a^8*b + 49*a^7*b^2 - 72*a^6*
b^3 + 45*a^5*b^4 + 8*a^4*b^5 - 29*a^3*b^6 + 16*a^2*b^7 - 3*a*b^8)*cos(d*x +
 c)^4 - 4*(a^8*b - 8*a^7*b^2 + 25*a^6*b^3 - 40*a^5*b^4 + 35*a^4*b^5 - 16*a^
3*b^6 + 3*a^2*b^7)*cos(d*x + c)^3 + (a^9 - 8*a^8*b + 23*a^7*b^2 - 24*a^6*b^
3 - 15*a^5*b^4 + 64*a^4*b^5 - 67*a^3*b^6 + 32*a^2*b^7 - 6*a*b^8)*cos(d*x +
c)^2 + 2*(a^8*b - 8*a^7*b^2 + 25*a^6*b^3 - 40*a^5*b^4 + 35*a^4*b^5 - 16*a^3
*b^6 + 3*a^2*b^7)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^12 - 5*a^
10*b^2 + 10*a^8*b^4 - 10*a^6*b^6 + 5*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^6 +
 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*co
s(d*x + c)^5 - (2*a^12 - 11*a^10*b^2 + 25*a^8*b^4 - 30*a^6*b^6 + 20*a^4*b^8
 - 7*a^2*b^10 + b^12)*d*cos(d*x + c)^4 - 4*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5
 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c)^3 + (a^12 - 7*a^10*b^2 +
 20*a^8*b^4 - 30*a^6*b^6 + 25*a^4*b^8 - 11*a^2*b^10 + 2*b^12)*d*cos(d*x + c
)^2 + 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)
*d*cos(d*x + c) + (a^10*b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*b^8 + 5*a^2*b
^10 - b^12)*d)
```

giac [B] time = 0.54, size = 1551, normalized size = 4.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(12*(a^2 - 3*a*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a
^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) + 192*(a^6*b + 5*a^
4*b^3 + 2*a^2*b^5)*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^10 - 5*a^8*b^2 + 10*a^6*b^4
 - 10*a^4*b^6 + 5*a^2*b^8 - b^10) - (8*a^3*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - 12*a^2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*b^3*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) - a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
 3*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3*a*b^2*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 + b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
)/(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6) -
(a^8 - 2*a^7*b - 2*a^6*b^2 + 6*a^5*b^3 - 6*a^3*b^5 + 2*a^2*b^6 + 2*a*b^7 -
b^8 - 6*a^8*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 20*a^7*b*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - 12*a^6*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
- 28*a^5*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 40*a^4*b^4*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 4*a^3*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) - 20*a^2*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12*a*b^7*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 2*b^8*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
6*a^8*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 163*a^7*b*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 - 257*a^6*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 + 339*a^5*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 203*a^4*b
^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 223*a^3*b^5*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2 + 309*a^2*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - 23*a*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 7*b^8*(cos(d*
x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 10*a^8*(cos(d*x + c) - 1)^3/(cos(d*x +
 c) + 1)^3 + 186*a^7*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 274*a^6*
b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 890*a^5*b^3*(cos(d*x + c) -
 1)^3/(cos(d*x + c) + 1)^3 - 894*a^4*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c)
 + 1)^3 + 478*a^3*b^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 374*a^2*b
^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 18*a*b^7*(cos(d*x + c) - 1)^
3/(cos(d*x + c) + 1)^3 - 4*b^8*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 +
9*a^8*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 45*a^7*b*(cos(d*x + c) -
1)^4/(cos(d*x + c) + 1)^4 + 45*a^6*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) +
 1)^4 - 63*a^5*b^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 117*a^4*b^4*
(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 9*a^3*b^5*(cos(d*x + c) - 1)^4/
(cos(d*x + c) + 1)^4 + 63*a^2*b^6*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4
 + 27*a*b^7*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/((a^9 - a^8*b - 4*a^
7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 -
 b^9)*(a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2)^2))/d
```

maple [A] time = 0.64, size = 427, normalized size = 1.36

$$-\frac{b^{3} a^{2}}{2 d \left(a+b\right)^{3} \left(a-b\right)^{3} \left(b+a \cos \left(d x+c\right)\right)^{2}}+\frac{3 a^{6} b \ln \left(b+a \cos \left(d x+c\right)\right)}{d \left(a+b\right)^{5} \left(a-b\right)^{5}}+\frac{15 a^{4} b^{3} \ln \left(b+a \cos \left(d x+c\right)\right)}{d \left(a+b\right)^{5} \left(a-b\right)^{5}}+\frac{6 a^{2} b^{5} \ln (}{d (a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -1/2/d*b^3/(a+b)^3*a^2/(a-b)^3/(b+a*cos(d*x+c))^2+3/d*a^6*b/(a+b)^5/(a-b)^5
*ln(b+a*cos(d*x+c))+15/d*a^4*b^3/(a+b)^5/(a-b)^5*ln(b+a*cos(d*x+c))+6/d*a^2
*b^5/(a+b)^5/(a-b)^5*ln(b+a*cos(d*x+c))+3/d*a^4*b^2/(a+b)^4/(a-b)^4/(b+a*co
s(d*x+c))+3/d*a^2*b^4/(a+b)^4/(a-b)^4/(b+a*cos(d*x+c))-1/16/d/(a+b)^3/(-1+c
os(d*x+c))^2+3/16/d/(a+b)^4/(-1+cos(d*x+c))*a-3/16/d/(a+b)^4/(-1+cos(d*x+c)
)*b+3/16/d*a^2/(a+b)^5*ln(-1+cos(d*x+c))-9/16/d*a/(a+b)^5*ln(-1+cos(d*x+c))
*b+1/16/d/(a-b)^3/(1+cos(d*x+c))^2+3/16/d/(a-b)^4/(1+cos(d*x+c))*a+3/16/d/(
```

```
a-b)^4/(1+cos(d*x+c))*b-3/16/d*a^2/(a-b)^5*ln(1+cos(d*x+c))-9/16/d*a/(a-b)^
5*ln(1+cos(d*x+c))*b
```

maxima [B] time = 0.63, size = 707, normalized size = 2.26

$$\frac{48\left(a^{6}b+5\,a^{4}b^{3}+2\,a^{2}b^{5}\right)\log(a\cos(dx+c)+b)}{a^{10}-5\,a^{8}b^{2}+10\,a^{6}b^{4}-10\,a^{4}b^{6}+5\,a^{2}b^{8}-b^{10}}-\frac{3\left(a^{2}+3\,ab\right)\log(\cos(dx+c)+1)}{a^{5}-5\,a^{4}b+10\,a^{3}b^{2}-10\,a^{2}b^{3}+5\,ab^{4}-b^{5}}+\frac{3\left(a^{2}-3\,ab\right)\log(\cos(dx+c)-1)}{a^{5}+5\,a^{4}b+10\,a^{3}b^{2}+10\,a^{2}b^{3}+5\,ab^{4}+b^{5}}+\frac{\cdots}{a^{8}b^{2}-4\,a^{6}b^{4}+6\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(48*(a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*log(a*cos(d*x + c) + b)/(a^10 - 5*
a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10) - 3*(a^2 + 3*a*b)*log
(cos(d*x + c) + 1)/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5
) + 3*(a^2 - 3*a*b)*log(cos(d*x + c) - 1)/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*
a^2*b^3 + 5*a*b^4 + b^5) + 2*(38*a^4*b^3 + 56*a^2*b^5 + 2*b^7 + 3*(a^7 + 18
*a^5*b^2 + 13*a^3*b^4)*cos(d*x + c)^5 - 6*(a^6*b - 8*a^4*b^3 - 9*a^2*b^5)*c
os(d*x + c)^4 - (5*a^7 + 103*a^5*b^2 + 91*a^3*b^4 - 7*a*b^6)*cos(d*x + c)^3
 + 4*(2*a^6*b - 23*a^4*b^3 - 26*a^2*b^5 - b^7)*cos(d*x + c)^2 + (55*a^5*b^2
 + 46*a^3*b^4 - 5*a*b^6)*cos(d*x + c))/(a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4
*a^2*b^8 + b^10 + (a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*cos(
d*x + c)^6 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*cos(d*x
+ c)^5 - (2*a^10 - 9*a^8*b^2 + 16*a^6*b^4 - 14*a^4*b^6 + 6*a^2*b^8 - b^10)*
cos(d*x + c)^4 - 4*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*cos(
d*x + c)^3 + (a^10 - 6*a^8*b^2 + 14*a^6*b^4 - 16*a^4*b^6 + 9*a^2*b^8 - 2*b^
10)*cos(d*x + c)^2 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*
cos(d*x + c)))/d
```

mupad [B] time = 2.55, size = 673, normalized size = 2.15

$$\frac{\ln\left(\cos\left(c+d\,x\right)-1\right)\left(\frac{3}{16\,{\left(a+b\right)}^3}-\frac{15\,b}{16\,{\left(a+b\right)}^4}+\frac{3\,b^2}{4\,{\left(a+b\right)}^5}\right)}{d}+\frac{\frac{19\,a^4\,b^3+28\,a^2\,b^5+b^7}{4\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}-\frac{{\cos(c+d\,x)}^2\,\left(-2\,a^6\,b+23\,a^4\,b^3+26\,a^2\,b^{\cdots}\right)}{2\,\left(a^8-4\,a^6\,b^2+6\,a^4\,b^4-4\,a^2\,b^6+b^8\right)}\cdots}{d\,\left({\cos\left(c+d\,x\right)}^2\,\left(a^2-2\,b^2\right)-\cdots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^5*(a + b/cos(c + d*x))^3),x)
```

```
[Out] (log(cos(c + d*x) - 1)*(3/(16*(a + b)^3) - (15*b)/(16*(a + b)^4) + (3*b^2)/
(4*(a + b)^5)))/d + ((b^7 + 28*a^2*b^5 + 19*a^4*b^3)/(4*(a^8 + b^8 - 4*a^2*
b^6 + 6*a^4*b^4 - 4*a^6*b^2)) - (cos(c + d*x)^2*(b^7 - 2*a^6*b + 26*a^2*b^5
 + 23*a^4*b^3))/(2*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (3*co
s(c + d*x)^4*(9*a^2*b^5 - a^6*b + 8*a^4*b^3))/(4*(a^8 + b^8 - 4*a^2*b^6 + 6
*a^4*b^4 - 4*a^6*b^2)) + (3*cos(c + d*x)^5*(a^7 + 13*a^3*b^4 + 18*a^5*b^2))
/(8*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)) + (a*cos(c + d*x)*(46*
a^2*b^4 - 5*b^6 + 55*a^4*b^2))/(8*(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^
6*b^2)) - (a*cos(c + d*x)^3*(5*a^6 - 7*b^6 + 91*a^2*b^4 + 103*a^4*b^2))/(8*
(a^8 + b^8 - 4*a^2*b^6 + 6*a^4*b^4 - 4*a^6*b^2)))/(d*(cos(c + d*x)^2*(a^2 -
 2*b^2) - cos(c + d*x)^4*(2*a^2 - b^2) + b^2 + a^2*cos(c + d*x)^6 + 2*a*b*c
os(c + d*x) - 4*a*b*cos(c + d*x)^3 + 2*a*b*cos(c + d*x)^5)) - (log(cos(c +
d*x) + 1)*((3*b^2)/(4*(a - b)^5) + (15*b)/(16*(a - b)^4) + 3/(16*(a - b)^3)
))/d + (log(b + a*cos(c + d*x))*(3*a^6*b + 6*a^2*b^5 + 15*a^4*b^3))/(d*(a^1
0 - b^10 + 5*a^2*b^8 - 10*a^4*b^6 + 10*a^6*b^4 - 5*a^8*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{5}\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+b*sec(d*x+c))**3,x)

[Out] Integral(csc(c + d*x)**5/(a + b*sec(c + d*x))**3, x)
```

3.228 $\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=539

$$\frac{4b\sin(c+dx)\cos^6(c+dx)}{15a^2d(a\cos(c+dx)+b)^2}+\frac{\left(15a^4-110a^2b^2+112b^4\right)\sin(c+dx)\cos^4(c+dx)}{20a^4b^2d(a\cos(c+dx)+b)}-\frac{b\sqrt{a-b}\,\sqrt{a+b}\,\left(6a^4-47a^2b\right.}{}$$

```
[Out] 1/16*(5*a^6-180*a^4*b^2+600*a^2*b^4-448*b^6)*x/a^9+1/30*b*(213*a^4-985*a^2*
b^2+840*b^4)*sin(d*x+c)/a^8/d-1/16*(43*a^4-244*a^2*b^2+224*b^4)*cos(d*x+c)*
sin(d*x+c)/a^7/d+1/30*(45*a^4-291*a^2*b^2+280*b^4)*cos(d*x+c)^2*sin(d*x+c)/
a^6/b/d-1/24*(24*a^4-169*a^2*b^2+168*b^4)*cos(d*x+c)^3*sin(d*x+c)/a^5/b^2/d
-1/4*cos(d*x+c)^4*sin(d*x+c)/b/d/(b+a*cos(d*x+c))^2+1/10*a*cos(d*x+c)^5*sin
(d*x+c)/b^2/d/(b+a*cos(d*x+c))^2+1/60*(9*a^4-60*a^2*b^2+56*b^4)*cos(d*x+c)^
5*sin(d*x+c)/a^3/b^2/d/(b+a*cos(d*x+c))^2+4/15*b*cos(d*x+c)^6*sin(d*x+c)/a^
2/d/(b+a*cos(d*x+c))^2-1/6*cos(d*x+c)^7*sin(d*x+c)/a/d/(b+a*cos(d*x+c))^2+1
/20*(15*a^4-110*a^2*b^2+112*b^4)*cos(d*x+c)^4*sin(d*x+c)/a^4/b^2/d/(b+a*cos
(d*x+c))-b*(6*a^4-47*a^2*b^2+56*b^4)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)
/(a+b)^(1/2))*(a-b)^(1/2)*(a+b)^(1/2)/a^9/d
```

Rubi [A] time = 2.44, antiderivative size = 539, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2896, 3047, 3049, 3023, 2735, 2659, 208}

$$\frac{b\left(-985a^2b^2+213a^4+840b^4\right)\sin(c+dx)}{30a^8d}+\frac{\left(-60a^2b^2+9a^4+56b^4\right)\sin(c+dx)\cos^5(c+dx)}{60a^3b^2d(a\cos(c+dx)+b)^2}+\frac{\left(-110a^2b^2+15\right.}{20}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((5*a^6 - 180*a^4*b^2 + 600*a^2*b^4 - 448*b^6)*x)/(16*a^9) - (Sqrt[a - b]*b
*Sqrt[a + b]*(6*a^4 - 47*a^2*b^2 + 56*b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/(a^9*d) + (b*(213*a^4 - 985*a^2*b^2 + 840*b^4)*Sin[c +
 d*x])/(30*a^8*d) - ((43*a^4 - 244*a^2*b^2 + 224*b^4)*Cos[c + d*x]*Sin[c +
d*x])/(16*a^7*d) + ((45*a^4 - 291*a^2*b^2 + 280*b^4)*Cos[c + d*x]^2*Sin[c +
 d*x])/(30*a^6*b*d) - ((24*a^4 - 169*a^2*b^2 + 168*b^4)*Cos[c + d*x]^3*Sin[
c + d*x])/(24*a^5*b^2*d) - (Cos[c + d*x]^4*Sin[c + d*x])/(4*b*d*(b + a*Cos[
c + d*x])^2) + (a*Cos[c + d*x]^5*Sin[c + d*x])/(10*b^2*d*(b + a*Cos[c + d*x
])^2) + ((9*a^4 - 60*a^2*b^2 + 56*b^4)*Cos[c + d*x]^5*Sin[c + d*x])/(60*a^3
*b^2*d*(b + a*Cos[c + d*x])^2) + (4*b*Cos[c + d*x]^6*Sin[c + d*x])/(15*a^2*
d*(b + a*Cos[c + d*x])^2) - (Cos[c + d*x]^7*Sin[c + d*x])/(6*a*d*(b + a*Cos
[c + d*x])^2) + ((15*a^4 - 110*a^2*b^2 + 112*b^4)*Cos[c + d*x]^4*Sin[c + d*
x])/(20*a^4*b^2*d*(b + a*Cos[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
```

```
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^6(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2} + \frac{a\cos^5(c+dx)\sin(c+dx)}{10b^2d(b+a\cos(c+dx))^2} + \frac{4b\cos^6(c+dx)\sin(c+}{15a^2d(b+a\cos(c+d}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2} + \frac{a\cos^5(c+dx)\sin(c+dx)}{10b^2d(b+a\cos(c+dx))^2} + \frac{\left(9a^4-60a^2b^2+56b^4\right)}{60a^3b^2d(b+}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2} + \frac{a\cos^5(c+dx)\sin(c+dx)}{10b^2d(b+a\cos(c+dx))^2} + \frac{\left(9a^4-60a^2b^2+56b^4\right)}{60a^3b^2d(b+}$$

$$= -\frac{\left(24a^4-169a^2b^2+168b^4\right)\cos^3(c+dx)\sin(c+dx)}{24a^5b^2d} - \frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2} +$$

$$= \frac{\left(45a^4-291a^2b^2+280b^4\right)\cos^2(c+dx)\sin(c+dx)}{30a^6bd} - \frac{\left(24a^4-169a^2b^2+168b^4\right)c}{24a^5b}$$

$$= -\frac{\left(43a^4-244a^2b^2+224b^4\right)\cos(c+dx)\sin(c+dx)}{16a^7d} + \frac{\left(45a^4-291a^2b^2+280b^4\right)}{30a^6}$$

$$= \frac{b\left(213a^4-985a^2b^2+840b^4\right)\sin(c+dx)}{30a^8d} - \frac{\left(43a^4-244a^2b^2+224b^4\right)\cos(c+dx}{16a^7d}$$

$$= \frac{\left(5a^6-180a^4b^2+600a^2b^4-448b^6\right)x}{16a^9} + \frac{b\left(213a^4-985a^2b^2+840b^4\right)\sin(c+dx)}{30a^8d}$$

$$= \frac{\left(5a^6-180a^4b^2+600a^2b^4-448b^6\right)x}{16a^9} + \frac{b\left(213a^4-985a^2b^2+840b^4\right)\sin(c+dx)}{30a^8d}$$

$$= \frac{\left(5a^6-180a^4b^2+600a^2b^4-448b^6\right)x}{16a^9} - \frac{\sqrt{a-b}\,b\sqrt{a+b}\,\left(6a^4-47a^2b^2+56b^4\right)\mathrm{ta}}{a^9d}$$

Mathematica [A] time = 12.70, size = 599, normalized size = 1.11

$$\frac{2\left(a^2-b^2\right)^{5/2}\left(-405a^8\sin(2(c+dx))-140a^8\sin(4(c+dx))+35a^8\sin(6(c+dx))-5a^8\sin(8(c+dx))+600a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-7680*b*(-a^2 + b^2)^3*(6*a^4 - 47*a^2*b^2 + 56*b^4)*ArcTanh[((-a + b)*Tan
[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2 + 2*(a^2 - b^2)^(5/2
)*(600*a^8*c - 20400*a^6*b^2*c + 28800*a^4*b^4*c + 90240*a^2*b^6*c - 107520
*b^8*c + 600*a^8*d*x - 20400*a^6*b^2*d*x + 28800*a^4*b^4*d*x + 90240*a^2*b^
6*d*x - 107520*b^8*d*x + 480*a*b*(5*a^6 - 180*a^4*b^2 + 600*a^2*b^4 - 448*b
^6)*(c + d*x)*Cos[c + d*x] + 120*a^2*(5*a^6 - 180*a^4*b^2 + 600*a^2*b^4 - 4
48*b^6)*(c + d*x)*Cos[2*(c + d*x)] + 2640*a^7*b*Sin[c + d*x] + 16160*a^5*b^
3*Sin[c + d*x] - 117120*a^3*b^5*Sin[c + d*x] + 107520*a*b^7*Sin[c + d*x] -
405*a^8*Sin[2*(c + d*x)] + 24600*a^6*b^2*Sin[2*(c + d*x)] - 99040*a^4*b^4*S
in[2*(c + d*x)] + 80640*a^2*b^6*Sin[2*(c + d*x)] + 2436*a^7*b*Sin[3*(c + d*
x)] - 10880*a^5*b^3*Sin[3*(c + d*x)] + 8960*a^3*b^5*Sin[3*(c + d*x)] - 140*
a^8*Sin[4*(c + d*x)] + 1164*a^6*b^2*Sin[4*(c + d*x)] - 1120*a^4*b^4*Sin[4*(
c + d*x)] - 188*a^7*b*Sin[5*(c + d*x)] + 224*a^5*b^3*Sin[5*(c + d*x)] + 35*
a^8*Sin[6*(c + d*x)] - 56*a^6*b^2*Sin[6*(c + d*x)] + 16*a^7*b*Sin[7*(c + d*
x)] - 5*a^8*Sin[8*(c + d*x)]))/(7680*a^9*(a - b)^2*(a + b)^2*Sqrt[a^2 - b^2
]*d*(b + a*Cos[c + d*x])^2)
```

fricas [A] time = 0.70, size = 1057, normalized size = 1.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/240*(15*(5*a^8 - 180*a^6*b^2 + 600*a^4*b^4 - 448*a^2*b^6)*d*x*cos(d*x +
c)^2 + 30*(5*a^7*b - 180*a^5*b^3 + 600*a^3*b^5 - 448*a*b^7)*d*x*cos(d*x + c
) + 15*(5*a^6*b^2 - 180*a^4*b^4 + 600*a^2*b^6 - 448*b^8)*d*x + 60*(6*a^4*b^
3 - 47*a^2*b^5 + 56*b^7 + (6*a^6*b - 47*a^4*b^3 + 56*a^2*b^5)*cos(d*x + c)^
2 + 2*(6*a^5*b^2 - 47*a^3*b^4 + 56*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log
((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*
cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*c
os(d*x + c) + b^2)) - (40*a^8*cos(d*x + c)^7 - 64*a^7*b*cos(d*x + c)^6 - 17
04*a^5*b^3 + 7880*a^3*b^5 - 6720*a*b^7 - 2*(65*a^8 - 56*a^6*b^2)*cos(d*x +
c)^5 + 4*(67*a^7*b - 56*a^5*b^3)*cos(d*x + c)^4 + (165*a^8 - 694*a^6*b^2 +
560*a^4*b^4)*cos(d*x + c)^3 - 2*(387*a^7*b - 1444*a^5*b^3 + 1120*a^3*b^5)*c
os(d*x + c)^2 - (2763*a^6*b^2 - 12100*a^4*b^4 + 10080*a^2*b^6)*cos(d*x + c)
)*sin(d*x + c))/(a^11*d*cos(d*x + c)^2 + 2*a^10*b*d*cos(d*x + c) + a^9*b^2*
d), 1/240*(15*(5*a^8 - 180*a^6*b^2 + 600*a^4*b^4 - 448*a^2*b^6)*d*x*cos(d*x
 + c)^2 + 30*(5*a^7*b - 180*a^5*b^3 + 600*a^3*b^5 - 448*a*b^7)*d*x*cos(d*x
+ c) + 15*(5*a^6*b^2 - 180*a^4*b^4 + 600*a^2*b^6 - 448*b^8)*d*x - 120*(6*a^
4*b^3 - 47*a^2*b^5 + 56*b^7 + (6*a^6*b - 47*a^4*b^3 + 56*a^2*b^5)*cos(d*x +
 c)^2 + 2*(6*a^5*b^2 - 47*a^3*b^4 + 56*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2
)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))
 - (40*a^8*cos(d*x + c)^7 - 64*a^7*b*cos(d*x + c)^6 - 1704*a^5*b^3 + 7880*a
^3*b^5 - 6720*a*b^7 - 2*(65*a^8 - 56*a^6*b^2)*cos(d*x + c)^5 + 4*(67*a^7*b
- 56*a^5*b^3)*cos(d*x + c)^4 + (165*a^8 - 694*a^6*b^2 + 560*a^4*b^4)*cos(d*
x + c)^3 - 2*(387*a^7*b - 1444*a^5*b^3 + 1120*a^3*b^5)*cos(d*x + c)^2 - (27
63*a^6*b^2 - 12100*a^4*b^4 + 10080*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/(a^
11*d*cos(d*x + c)^2 + 2*a^10*b*d*cos(d*x + c) + a^9*b^2*d)]
```

giac [B] time = 0.81, size = 1030, normalized size = 1.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/240*(15*(5*a^6 - 180*a^4*b^2 + 600*a^2*b^4 - 448*b^6)*(d*x + c)/a^9 - 240
*(6*a^6*b - 53*a^4*b^3 + 103*a^2*b^5 - 56*b^7)*(pi*floor(1/2*(d*x + c)/pi +
```

```
 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1
/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^9) - 240*(6*a^5*b^2*tan(1/2*d
*x + 1/2*c)^3 - 5*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 21*a^3*b^4*tan(1/2*d*x +
 1/2*c)^3 + 19*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 15*a*b^6*tan(1/2*d*x + 1/2*
c)^3 - 14*b^7*tan(1/2*d*x + 1/2*c)^3 - 6*a^5*b^2*tan(1/2*d*x + 1/2*c) - 5*a
^4*b^3*tan(1/2*d*x + 1/2*c) + 21*a^3*b^4*tan(1/2*d*x + 1/2*c) + 19*a^2*b^5*
tan(1/2*d*x + 1/2*c) - 15*a*b^6*tan(1/2*d*x + 1/2*c) - 14*b^7*tan(1/2*d*x +
 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2*a
^8) + 2*(75*a^5*tan(1/2*d*x + 1/2*c)^11 + 720*a^4*b*tan(1/2*d*x + 1/2*c)^11
 - 1260*a^3*b^2*tan(1/2*d*x + 1/2*c)^11 - 4800*a^2*b^3*tan(1/2*d*x + 1/2*c)
^11 + 1800*a*b^4*tan(1/2*d*x + 1/2*c)^11 + 5040*b^5*tan(1/2*d*x + 1/2*c)^11
 + 425*a^5*tan(1/2*d*x + 1/2*c)^9 + 4560*a^4*b*tan(1/2*d*x + 1/2*c)^9 - 522
0*a^3*b^2*tan(1/2*d*x + 1/2*c)^9 - 27200*a^2*b^3*tan(1/2*d*x + 1/2*c)^9 + 5
400*a*b^4*tan(1/2*d*x + 1/2*c)^9 + 25200*b^5*tan(1/2*d*x + 1/2*c)^9 + 990*a
^5*tan(1/2*d*x + 1/2*c)^7 + 12384*a^4*b*tan(1/2*d*x + 1/2*c)^7 - 3960*a^3*b
^2*tan(1/2*d*x + 1/2*c)^7 - 57600*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 3600*a*b
^4*tan(1/2*d*x + 1/2*c)^7 + 50400*b^5*tan(1/2*d*x + 1/2*c)^7 - 990*a^5*tan(
1/2*d*x + 1/2*c)^5 + 12384*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 3960*a^3*b^2*tan(
1/2*d*x + 1/2*c)^5 - 57600*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 3600*a*b^4*tan(
1/2*d*x + 1/2*c)^5 + 50400*b^5*tan(1/2*d*x + 1/2*c)^5 - 425*a^5*tan(1/2*d*x
 + 1/2*c)^3 + 4560*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 5220*a^3*b^2*tan(1/2*d*x
+ 1/2*c)^3 - 27200*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 5400*a*b^4*tan(1/2*d*x
+ 1/2*c)^3 + 25200*b^5*tan(1/2*d*x + 1/2*c)^3 - 75*a^5*tan(1/2*d*x + 1/2*c)
 + 720*a^4*b*tan(1/2*d*x + 1/2*c) + 1260*a^3*b^2*tan(1/2*d*x + 1/2*c) - 480
0*a^2*b^3*tan(1/2*d*x + 1/2*c) - 1800*a*b^4*tan(1/2*d*x + 1/2*c) + 5040*b^5
*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^6*a^8))/d
```

maple [B] time = 0.59, size = 2251, normalized size = 4.18

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -45/2/d/a^5*arctan(tan(1/2*d*x+1/2*c))*b^2+75/d/a^7*arctan(tan(1/2*d*x+1/2*
c))*b^4-56/d/a^9*arctan(tan(1/2*d*x+1/2*c))*b^6+5/8/d/a^3*arctan(tan(1/2*d*
x+1/2*c))-5/8/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)-480/d/a^6
/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*b^3+30/d/a^7/(1+tan(1/2*d*
x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*b^4+420/d/a^8/(1+tan(1/2*d*x+1/2*c)^2)^6
*tan(1/2*d*x+1/2*c)^7*b^5-30/d/a^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1
/2*c)^5*b^4+516/5/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*b-4
80/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*b^3+420/d/a^8/(1+t
an(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*b^5+38/d/a^4/(1+tan(1/2*d*x+1/2
*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b+14/d*b^7/a^8/(a*tan(1/2*d*x+1/2*c)^2-tan(1/
2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)^3+33/d/a^5/(1+tan(1/2*d*x+1/2*c)
^2)^6*tan(1/2*d*x+1/2*c)^5*b^2-40/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*
d*x+1/2*c)^11*b^3+15/d/a^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11
*b^4+87/2/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b^2+5/d*b^3
/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*
c)-19/d*b^5/a^6/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1
/2*d*x+1/2*c)+14/d*b^7/a^8/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a
-b)^2*tan(1/2*d*x+1/2*c)+85/24/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x
+1/2*c)^9+33/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7-33/4/d
/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5-85/24/d/a^3/(1+tan(1/2
*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3+5/8/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*
tan(1/2*d*x+1/2*c)^11+56/d*b^7/a^9/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+
1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+21/d*b^4/a^5/(a*tan(1/2*d*x+1/2*c)^2-tan(
1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)^3-6/d*b/a^3/((a-b)*(a+b))^(1/2
)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+53/d*b^3/a^5/((a-b)
*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+5/d*b^3
```

```
/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*
c)^3-21/d*b^4/a^5/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan
(1/2*d*x+1/2*c)-19/d*b^5/a^6/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b
-a-b)^2*tan(1/2*d*x+1/2*c)^3+42/d/a^8/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*
x+1/2*c)^11*b^5+21/2/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*b^
2-680/3/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b^3-45/d/a^7/
(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b^4+210/d/a^8/(1+tan(1/2*d*
x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b^5-680/3/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)
^6*tan(1/2*d*x+1/2*c)^9*b^3+516/5/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*
d*x+1/2*c)^7*b-33/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*b^2
-15/d/a^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*b^4+6/d*b^2/a^3/(a*
tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)-15/d*
b^6/a^7/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1
/2*c)^3-6/d*b^2/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*t
an(1/2*d*x+1/2*c)^3+6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^1
1*b+42/d/a^8/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*b^5-21/2/d/a^5/(
1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*b^2-40/d/a^6/(1+tan(1/2*d*x
+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*b^3+6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(
1/2*d*x+1/2*c)*b+38/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*b
-87/2/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*b^2+45/d/a^7/(1
+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*b^4+210/d/a^8/(1+tan(1/2*d*x+
1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*b^5-103/d*b^5/a^7/((a-b)*(a+b))^(1/2)*arct
anh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+15/d*b^6/a^7/(a*tan(1/2*d
*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 5.71, size = 3975, normalized size = 7.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + b/cos(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(10080*a*b^6 + 454*a^6*b - 55*a^7 + 9408*b^7 - 9688*
a^2*b^5 - 12212*a^3*b^4 + 608*a^4*b^3 + 2969*a^5*b^2))/(24*a^8) + (tan(c/2
+ (d*x)/2)^13*(454*a^6*b - 10080*a*b^6 + 55*a^7 + 9408*b^7 - 9688*a^2*b^5 +
 12212*a^3*b^4 + 608*a^4*b^3 - 2969*a^5*b^2))/(24*a^8) + (tan(c/2 + (d*x)/2
)^5*(90720*a*b^6 + 2154*a^6*b - 215*a^7 + 141120*b^7 - 163240*a^2*b^5 - 107
220*a^3*b^4 + 32224*a^4*b^3 + 22673*a^5*b^2))/(120*a^8) + (tan(c/2 + (d*x)/
2)^11*(2154*a^6*b - 90720*a*b^6 + 215*a^7 + 141120*b^7 - 163240*a^2*b^5 + 1
07220*a^3*b^4 + 32224*a^4*b^3 - 22673*a^5*b^2))/(120*a^8) + (tan(c/2 + (d*x
)/2)^7*(50400*a*b^6 - 4994*a^6*b + 2545*a^7 + 235200*b^7 - 287000*a^2*b^5 -
 58820*a^3*b^4 + 74752*a^4*b^3 + 11173*a^5*b^2))/(120*a^8) - (tan(c/2 + (d*
x)/2)^9*(50400*a*b^6 + 4994*a^6*b + 2545*a^7 - 235200*b^7 + 287000*a^2*b^5
- 58820*a^3*b^4 - 74752*a^4*b^3 + 11173*a^5*b^2))/(120*a^8) + (tan(c/2 + (d
*x)/2)^15*(a - b)*(224*a*b^5 + 43*a^5*b + 5*a^6 - 448*b^6 + 600*a^2*b^4 - 2
44*a^3*b^3 - 180*a^4*b^2))/(8*a^8) - (tan(c/2 + (d*x)/2)*(2*a*b + a^2 + b^2
)*(224*a*b^4 - 48*a^4*b + 5*a^5 - 448*b^5 + 376*a^2*b^3 - 132*a^3*b^2))/(8*
a^8))/(d*(2*a*b - tan(c/2 + (d*x)/2)^8*(10*a^2 - 70*b^2) + tan(c/2 + (d*x)/
```

2)^2*(12*a*b + 4*a^2 + 8*b^2) + tan(c/2 + (d*x)/2)^14*(4*a^2 - 12*a*b + 8*b
^2) + tan(c/2 + (d*x)/2)^4*(28*a*b + 4*a^2 + 28*b^2) + tan(c/2 + (d*x)/2)^1
2*(4*a^2 - 28*a*b + 28*b^2) + tan(c/2 + (d*x)/2)^6*(28*a*b - 4*a^2 + 56*b^2
) - tan(c/2 + (d*x)/2)^10*(28*a*b + 4*a^2 - 56*b^2) + tan(c/2 + (d*x)/2)^16
*(a^2 - 2*a*b + b^2) + a^2 + b^2)) + (atan(((((((106*a^25*b - 10*a^26 + 896
*a^18*b^8 - 1344*a^19*b^7 - 1200*a^20*b^6 + 2360*a^21*b^5 + 136*a^22*b^4 -
1122*a^23*b^3 + 178*a^24*b^2)/a^24 - (tan(c/2 + (d*x)/2)*(512*a^20*b + 512*
a^18*b^3 - 1024*a^19*b^2)*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i)
)/(128*a^25))*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i))/(16*a^9) +
(tan(c/2 + (d*x)/2)*(802816*a*b^14 - 75*a^14*b + 25*a^15 - 401408*b^15 + 6
73792*a^2*b^13 - 2150400*a^3*b^12 + 32640*a^4*b^11 + 2085120*a^5*b^10 - 601
600*a^6*b^9 - 881920*a^7*b^8 + 364160*a^8*b^7 + 153600*a^9*b^6 - 72696*a^10
*b^5 - 7704*a^11*b^4 + 3071*a^12*b^3 + 579*a^13*b^2))/(8*a^16))*(a^6*5i - b
^6*448i + a^2*b^4*600i - a^4*b^2*180i)*1i)/(16*a^9) - (((((106*a^25*b - 10*
a^26 + 896*a^18*b^8 - 1344*a^19*b^7 - 1200*a^20*b^6 + 2360*a^21*b^5 + 136*a
^22*b^4 - 1122*a^23*b^3 + 178*a^24*b^2)/a^24 + (tan(c/2 + (d*x)/2)*(512*a^2
0*b + 512*a^18*b^3 - 1024*a^19*b^2)*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4
*b^2*180i))/(128*a^25))*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i))/
(16*a^9) - (tan(c/2 + (d*x)/2)*(802816*a*b^14 - 75*a^14*b + 25*a^15 - 40140
8*b^15 + 673792*a^2*b^13 - 2150400*a^3*b^12 + 32640*a^4*b^11 + 2085120*a^5*
b^10 - 601600*a^6*b^9 - 881920*a^7*b^8 + 364160*a^8*b^7 + 153600*a^9*b^6 -
72696*a^10*b^5 - 7704*a^11*b^4 + 3071*a^12*b^3 + 579*a^13*b^2))/(8*a^16))*(
a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i)*1i)/(16*a^9))/(((75*a^19*b
)/4 - 2107392*a*b^19 + 1404928*b^20 - 5644800*a^2*b^18 + 9345280*a^3*b^17 +
8902208*a^4*b^16 - 17144736*a^5*b^15 - 6722456*a^6*b^14 + 16804748*a^7*b^1
3 + 2126380*a^8*b^12 - 9486373*a^9*b^11 + 163573*a^10*b^10 + 3099308*a^11*b
^9 - 297558*a^12*b^8 - (4466945*a^13*b^7)/8 + (296845*a^14*b^6)/4 + (196765
*a^15*b^5)/4 - (26515*a^16*b^4)/4 - (13415*a^17*b^3)/8 + (285*a^18*b^2)/2)/
a^24 + (((((106*a^25*b - 10*a^26 + 896*a^18*b^8 - 1344*a^19*b^7 - 1200*a^20
*b^6 + 2360*a^21*b^5 + 136*a^22*b^4 - 1122*a^23*b^3 + 178*a^24*b^2)/a^24 -
(tan(c/2 + (d*x)/2)*(512*a^20*b + 512*a^18*b^3 - 1024*a^19*b^2)*(a^6*5i - b
^6*448i + a^2*b^4*600i - a^4*b^2*180i))/(128*a^25))*(a^6*5i - b^6*448i + a^
2*b^4*600i - a^4*b^2*180i))/(16*a^9) + (tan(c/2 + (d*x)/2)*(802816*a*b^14 -
75*a^14*b + 25*a^15 - 401408*b^15 + 673792*a^2*b^13 - 2150400*a^3*b^12 + 3
2640*a^4*b^11 + 2085120*a^5*b^10 - 601600*a^6*b^9 - 881920*a^7*b^8 + 364160
*a^8*b^7 + 153600*a^9*b^6 - 72696*a^10*b^5 - 7704*a^11*b^4 + 3071*a^12*b^3
+ 579*a^13*b^2))/(8*a^16))*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i
))/(16*a^9) + (((((106*a^25*b - 10*a^26 + 896*a^18*b^8 - 1344*a^19*b^7 - 12
00*a^20*b^6 + 2360*a^21*b^5 + 136*a^22*b^4 - 1122*a^23*b^3 + 178*a^24*b^2)/
a^24 + (tan(c/2 + (d*x)/2)*(512*a^20*b + 512*a^18*b^3 - 1024*a^19*b^2)*(a^6
*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i))/(128*a^25))*(a^6*5i - b^6*44
8i + a^2*b^4*600i - a^4*b^2*180i))/(16*a^9) - (tan(c/2 + (d*x)/2)*(802816*a
*b^14 - 75*a^14*b + 25*a^15 - 401408*b^15 + 673792*a^2*b^13 - 2150400*a^3*b
^12 + 32640*a^4*b^11 + 2085120*a^5*b^10 - 601600*a^6*b^9 - 881920*a^7*b^8 +
364160*a^8*b^7 + 153600*a^9*b^6 - 72696*a^10*b^5 - 7704*a^11*b^4 + 3071*a^
12*b^3 + 579*a^13*b^2))/(8*a^16))*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b
^2*180i))/(16*a^9)))*(a^6*5i - b^6*448i + a^2*b^4*600i - a^4*b^2*180i)*1i)/
(8*a^9*d) + (b*atan(((b*((a + b)*(a - b))^(1/2)*((tan(c/2 + (d*x)/2)*(80281
6*a*b^14 - 75*a^14*b + 25*a^15 - 401408*b^15 + 673792*a^2*b^13 - 2150400*a^
3*b^12 + 32640*a^4*b^11 + 2085120*a^5*b^10 - 601600*a^6*b^9 - 881920*a^7*b^
8 + 364160*a^8*b^7 + 153600*a^9*b^6 - 72696*a^10*b^5 - 7704*a^11*b^4 + 3071
*a^12*b^3 + 579*a^13*b^2))/(8*a^16) + (b*((a + b)*(a - b))^(1/2)*((106*a^25
*b - 10*a^26 + 896*a^18*b^8 - 1344*a^19*b^7 - 1200*a^20*b^6 + 2360*a^21*b^5
+ 136*a^22*b^4 - 1122*a^23*b^3 + 178*a^24*b^2)/a^24 - (b*tan(c/2 + (d*x)/2
)*((a + b)*(a - b))^(1/2)*(6*a^4 + 56*b^4 - 47*a^2*b^2)*(512*a^20*b + 512*a
^18*b^3 - 1024*a^19*b^2))/(16*a^25))*(6*a^4 + 56*b^4 - 47*a^2*b^2))/(2*a^9)
)*(6*a^4 + 56*b^4 - 47*a^2*b^2)*1i)/(2*a^9) + (b*((a + b)*(a - b))^(1/2)*((
tan(c/2 + (d*x)/2)*(802816*a*b^14 - 75*a^14*b + 25*a^15 - 401408*b^15 + 673
792*a^2*b^13 - 2150400*a^3*b^12 + 32640*a^4*b^11 + 2085120*a^5*b^10 - 60160

```
0*a^6*b^9 - 881920*a^7*b^8 + 364160*a^8*b^7 + 153600*a^9*b^6 - 72696*a^10*b
^5 - 7704*a^11*b^4 + 3071*a^12*b^3 + 579*a^13*b^2))/(8*a^16) - (b*((a + b)*
(a - b))^(1/2)*((106*a^25*b - 10*a^26 + 896*a^18*b^8 - 1344*a^19*b^7 - 1200
*a^20*b^6 + 2360*a^21*b^5 + 136*a^22*b^4 - 1122*a^23*b^3 + 178*a^24*b^2)/a^
24 + (b*tan(c/2 + (d*x)/2)*((a + b)*(a - b))^(1/2)*(6*a^4 + 56*b^4 - 47*a^2
*b^2)*(512*a^20*b + 512*a^18*b^3 - 1024*a^19*b^2))/(16*a^25))*(6*a^4 + 56*b
^4 - 47*a^2*b^2))/(2*a^9))*(6*a^4 + 56*b^4 - 47*a^2*b^2)*1i)/(2*a^9))/(((75
*a^19*b)/4 - 2107392*a*b^19 + 1404928*b^20 - 5644800*a^2*b^18 + 9345280*a^3
*b^17 + 8902208*a^4*b^16 - 17144736*a^5*b^15 - 6722456*a^6*b^14 + 16804748*
a^7*b^13 + 2126380*a^8*b^12 - 9486373*a^9*b^11 + 163573*a^10*b^10 + 3099308
*a^11*b^9 - 297558*a^12*b^8 - (4466945*a^13*b^7)/8 + (296845*a^14*b^6)/4 +
(196765*a^15*b^5)/4 - (26515*a^16*b^4)/4 - (13415*a^17*b^3)/8 + (285*a^18*b
^2)/2)/a^24 + (b*((a + b)*(a - b))^(1/2)*((tan(c/2 + (d*x)/2)*(802816*a*b^1
4 - 75*a^14*b + 25*a^15 - 401408*b^15 + 673792*a^2*b^13 - 2150400*a^3*b^12
+ 32640*a^4*b^11 + 2085120*a^5*b^10 - 601600*a^6*b^9 - 881920*a^7*b^8 + 364
160*a^8*b^7 + 153600*a^9*b^6 - 72696*a^10*b^5 - 7704*a^11*b^4 + 3071*a^12*b
^3 + 579*a^13*b^2))/(8*a^16) + (b*((a + b)*(a - b))^(1/2)*((106*a^25*b - 10
*a^26 + 896*a^18*b^8 - 1344*a^19*b^7 - 1200*a^20*b^6 + 2360*a^21*b^5 + 136*
a^22*b^4 - 1122*a^23*b^3 + 178*a^24*b^2)/a^24 - (b*tan(c/2 + (d*x)/2)*((a +
 b)*(a - b))^(1/2)*(6*a^4 + 56*b^4 - 47*a^2*b^2)*(512*a^20*b + 512*a^18*b^3
 - 1024*a^19*b^2))/(16*a^25))*(6*a^4 + 56*b^4 - 47*a^2*b^2))/(2*a^9))*(6*a^
4 + 56*b^4 - 47*a^2*b^2))/(2*a^9) - (b*((a + b)*(a - b))^(1/2)*((tan(c/2 +
(d*x)/2)*(802816*a*b^14 - 75*a^14*b + 25*a^15 - 401408*b^15 + 673792*a^2*b^
13 - 2150400*a^3*b^12 + 32640*a^4*b^11 + 2085120*a^5*b^10 - 601600*a^6*b^9
- 881920*a^7*b^8 + 364160*a^8*b^7 + 153600*a^9*b^6 - 72696*a^10*b^5 - 7704*
a^11*b^4 + 3071*a^12*b^3 + 579*a^13*b^2))/(8*a^16) - (b*((a + b)*(a - b))^(
1/2)*((106*a^25*b - 10*a^26 + 896*a^18*b^8 - 1344*a^19*b^7 - 1200*a^20*b^6
+ 2360*a^21*b^5 + 136*a^22*b^4 - 1122*a^23*b^3 + 178*a^24*b^2)/a^24 + (b*ta
n(c/2 + (d*x)/2)*((a + b)*(a - b))^(1/2)*(6*a^4 + 56*b^4 - 47*a^2*b^2)*(512
*a^20*b + 512*a^18*b^3 - 1024*a^19*b^2))/(16*a^25))*(6*a^4 + 56*b^4 - 47*a^
2*b^2))/(2*a^9))*(6*a^4 + 56*b^4 - 47*a^2*b^2))/(2*a^9)))*((a + b)*(a - b))
^(1/2)*(6*a^4 + 56*b^4 - 47*a^2*b^2)*1i)/(a^9*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

3.229 $\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=333

$$\frac{\left(2a^2-7b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}+\frac{b\left(13a^2-30b^2\right)\sin(c+dx)}{2a^6d}-\frac{3\left(7a^2\right.}{}$$

```
[Out] 3/8*(a^4-24*a^2*b^2+40*b^4)*x/a^7+1/2*b*(13*a^2-30*b^2)*sin(d*x+c)/a^6/d-3/
8*(7*a^2-20*b^2)*cos(d*x+c)*sin(d*x+c)/a^5/d+1/2*(3*a^2-10*b^2)*cos(d*x+c)^
2*sin(d*x+c)/a^4/b/d-1/4*(4*a^2-15*b^2)*cos(d*x+c)^3*sin(d*x+c)/a^3/b^2/d-1
/2*(a^2-b^2)*cos(d*x+c)^4*sin(d*x+c)/a^2/b/d/(b+a*cos(d*x+c))^2+1/2*(2*a^2-
7*b^2)*cos(d*x+c)^4*sin(d*x+c)/a^2/b^2/d/(b+a*cos(d*x+c))-3*b*(2*a^4-11*a^2
*b^2+10*b^4)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^7/d/(a-b
)^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 1.14, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2891, 3049, 3023, 2735, 2659, 208}

$$\frac{b\left(13a^2-30b^2\right)\sin(c+dx)}{2a^6d}+\frac{\left(2a^2-7b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}-\frac{\left(4a^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*(a^4 - 24*a^2*b^2 + 40*b^4)*x)/(8*a^7) - (3*b*(2*a^4 - 11*a^2*b^2 + 10*b
^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^7*Sqrt[a - b]*S
qrt[a + b]*d) + (b*(13*a^2 - 30*b^2)*Sin[c + d*x])/(2*a^6*d) - (3*(7*a^2 -
20*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*a^5*d) + ((3*a^2 - 10*b^2)*Cos[c + d*
x]^2*Sin[c + d*x])/(2*a^4*b*d) - ((4*a^2 - 15*b^2)*Cos[c + d*x]^3*Sin[c + d
*x])/(4*a^3*b^2*d) - ((a^2 - b^2)*Cos[c + d*x]^4*Sin[c + d*x])/(2*a^2*b*d*(
b + a*Cos[c + d*x])^2) + ((2*a^2 - 7*b^2)*Cos[c + d*x]^4*Sin[c + d*x])/(2*a
^2*b^2*d*(b + a*Cos[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2891

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
```

```
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[((a^2*(n - m + 1) - b^2*(m + n
 + 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/
(a^2*b^2*d*f*(m + 1)*(m + 2)), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^4(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= -\frac{\left(a^2-b^2\right)\cos^4(c+dx)\sin(c+dx)}{2a^2bd(b+a\cos(c+dx))^2} + \frac{\left(2a^2-7b^2\right)\cos^4(c+dx)\sin(c+dx)}{2a^2b^2d(b+a\cos(c+dx))} + \int$$

$$= -\frac{\left(4a^2-15b^2\right)\cos^3(c+dx)\sin(c+dx)}{4a^3b^2d} - \frac{\left(a^2-b^2\right)\cos^4(c+dx)\sin(c+dx)}{2a^2bd(b+a\cos(c+dx))^2} + \frac{(2}{}$$

$$= \frac{\left(3a^2-10b^2\right)\cos^2(c+dx)\sin(c+dx)}{2a^4bd} - \frac{\left(4a^2-15b^2\right)\cos^3(c+dx)\sin(c+dx)}{4a^3b^2d} -$$

$$= -\frac{3\left(7a^2-20b^2\right)\cos(c+dx)\sin(c+dx)}{8a^5d} + \frac{\left(3a^2-10b^2\right)\cos^2(c+dx)\sin(c+dx)}{2a^4bd}$$

$$= \frac{b\left(13a^2-30b^2\right)\sin(c+dx)}{2a^6d} - \frac{3\left(7a^2-20b^2\right)\cos(c+dx)\sin(c+dx)}{8a^5d} + \frac{\left(3a^2-10\right.}{}$$

$$= \frac{3\left(a^4-24a^2b^2+40b^4\right)x}{8a^7} + \frac{b\left(13a^2-30b^2\right)\sin(c+dx)}{2a^6d} - \frac{3\left(7a^2-20b^2\right)\cos(c+}{8a^5d}$$

$$= \frac{3\left(a^4-24a^2b^2+40b^4\right)x}{8a^7} + \frac{b\left(13a^2-30b^2\right)\sin(c+dx)}{2a^6d} - \frac{3\left(7a^2-20b^2\right)\cos(c+}{8a^5d}$$

$$= \frac{3\left(a^4-24a^2b^2+40b^4\right)x}{8a^7} - \frac{3b\left(2a^4-11a^2b^2+10b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^7\sqrt{a-b}\sqrt{a+b}\,d} +$$

Mathematica [B] time = 9.05, size = 1178, normalized size = 3.54

$$-\frac{6\left(8(c+dx)+\frac{2b\left(15a^4-20b^2a^2+8b^4\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}}-\frac{3a\left(2a^4-7b^2a^2+4b^4\right)\sin(c+dx)}{(a-b)^2(a+b)^2(b+a\cos(c+dx))}+\frac{ab\left(3a^2-4b^2\right)\sin(c+dx)}{(a-b)(a+b)(b+a\cos(c+dx))^2}\right)}{a^3}+\frac{6\left(\frac{6ab\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx\right.}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-6*(8*(c + d*x) + (2*b*(15*a^4 - 20*a^2*b^2 + 8*b^4)*ArcTanh[((-a + b)*Ta
n[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (a*b*(3*a^2 - 4*b^2)*
Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])^2) - (3*a*(2*a^4 - 7*a^
2*b^2 + 4*b^4)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*Cos[c + d*x]))))/a
^3 + (6*((6*a*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[
a^2 - b^2] + ((b*(a^2 + 2*b^2) + a*(2*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x]
)/(b + a*Cos[c + d*x])^2))/((a - b)^2*(a + b)^2) - (2*(-24*(a^2 - 8*b^2)*(c
 + d*x) + (6*b*(-35*a^6 + 140*a^4*b^2 - 168*a^2*b^4 + 64*b^6)*ArcTanh[((-a
+ b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - 96*a*b*Sin[c +
 d*x] + (a*b*(-5*a^4 + 20*a^2*b^2 - 16*b^4)*Sin[c + d*x])/((a - b)*(a + b)*
(b + a*Cos[c + d*x])^2) + (a*(10*a^6 - 115*a^4*b^2 + 220*a^2*b^4 - 112*b^6)
*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])) + 8*a^2*Sin[2*(c
+ d*x)]))/a^5 + ((12*b*(105*a^8 - 840*a^6*b^2 + 2016*a^4*b^4 - 1920*a^2*b^6
 + 640*b^8)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^
2)^(5/2) + (48*a^10*c - 960*a^8*b^2*c + 1776*a^6*b^4*c + 2976*a^4*b^6*c - 7
```

```
680*a^2*b^8*c + 3840*b^10*c + 48*a^10*d*x - 960*a^8*b^2*d*x + 1776*a^6*b^4*
d*x + 2976*a^4*b^6*d*x - 7680*a^2*b^8*d*x + 3840*b^10*d*x + 192*a*b*(a^2 -
b^2)^2*(a^4 - 20*a^2*b^2 + 40*b^4)*(c + d*x)*Cos[c + d*x] + 48*(a^3 - a*b^2
)^2*(a^4 - 20*a^2*b^2 + 40*b^4)*(c + d*x)*Cos[2*(c + d*x)] + 114*a^9*b*Sin[
c + d*x] + 788*a^7*b^3*Sin[c + d*x] - 5696*a^5*b^5*Sin[c + d*x] + 8640*a^3*
b^7*Sin[c + d*x] - 3840*a*b^9*Sin[c + d*x] - 36*a^10*Sin[2*(c + d*x)] + 122
1*a^8*b^2*Sin[2*(c + d*x)] - 5182*a^6*b^4*Sin[2*(c + d*x)] + 6880*a^4*b^6*S
in[2*(c + d*x)] - 2880*a^2*b^8*Sin[2*(c + d*x)] + 120*a^9*b*Sin[3*(c + d*x)
] - 560*a^7*b^3*Sin[3*(c + d*x)] + 760*a^5*b^5*Sin[3*(c + d*x)] - 320*a^3*b
^7*Sin[3*(c + d*x)] - 8*a^10*Sin[4*(c + d*x)] + 56*a^8*b^2*Sin[4*(c + d*x)]
 - 88*a^6*b^4*Sin[4*(c + d*x)] + 40*a^4*b^6*Sin[4*(c + d*x)] - 8*a^9*b*Sin[
5*(c + d*x)] + 16*a^7*b^3*Sin[5*(c + d*x)] - 8*a^5*b^5*Sin[5*(c + d*x)] + 2
*a^10*Sin[6*(c + d*x)] - 4*a^8*b^2*Sin[6*(c + d*x)] + 2*a^6*b^4*Sin[6*(c +
d*x)])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2))/a^7)/(256*d)
```

fricas [A] time = 0.69, size = 1041, normalized size = 3.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a^8 - 25*a^6*b^2 + 64*a^4*b^4 - 40*a^2*b^6)*d*x*cos(d*x + c)^2 + 6
*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*d*x*cos(d*x + c) + 3*(a^6*b^2
 - 25*a^4*b^4 + 64*a^2*b^6 - 40*b^8)*d*x + 6*(2*a^4*b^3 - 11*a^2*b^5 + 10*b
^7 + (2*a^6*b - 11*a^4*b^3 + 10*a^2*b^5)*cos(d*x + c)^2 + 2*(2*a^5*b^2 - 11
*a^3*b^4 + 10*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c)
- (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin
(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) +
 (52*a^5*b^3 - 172*a^3*b^5 + 120*a*b^7 + 2*(a^8 - a^6*b^2)*cos(d*x + c)^5 -
 4*(a^7*b - a^5*b^3)*cos(d*x + c)^4 - 5*(a^8 - 3*a^6*b^2 + 2*a^4*b^4)*cos(d
*x + c)^3 + 2*(11*a^7*b - 31*a^5*b^3 + 20*a^3*b^5)*cos(d*x + c)^2 + (83*a^6
*b^2 - 263*a^4*b^4 + 180*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - a^9*
b^2)*d*cos(d*x + c)^2 + 2*(a^10*b - a^8*b^3)*d*cos(d*x + c) + (a^9*b^2 - a^
7*b^4)*d), 1/8*(3*(a^8 - 25*a^6*b^2 + 64*a^4*b^4 - 40*a^2*b^6)*d*x*cos(d*x
+ c)^2 + 6*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*d*x*cos(d*x + c) +
3*(a^6*b^2 - 25*a^4*b^4 + 64*a^2*b^6 - 40*b^8)*d*x - 12*(2*a^4*b^3 - 11*a^2
*b^5 + 10*b^7 + (2*a^6*b - 11*a^4*b^3 + 10*a^2*b^5)*cos(d*x + c)^2 + 2*(2*a
^5*b^2 - 11*a^3*b^4 + 10*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt
(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (52*a^5*b^3
 - 172*a^3*b^5 + 120*a*b^7 + 2*(a^8 - a^6*b^2)*cos(d*x + c)^5 - 4*(a^7*b -
a^5*b^3)*cos(d*x + c)^4 - 5*(a^8 - 3*a^6*b^2 + 2*a^4*b^4)*cos(d*x + c)^3 +
2*(11*a^7*b - 31*a^5*b^3 + 20*a^3*b^5)*cos(d*x + c)^2 + (83*a^6*b^2 - 263*a
^4*b^4 + 180*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - a^9*b^2)*d*cos(d
*x + c)^2 + 2*(a^10*b - a^8*b^3)*d*cos(d*x + c) + (a^9*b^2 - a^7*b^4)*d)]
```

giac [A] time = 3.10, size = 584, normalized size = 1.75

$$\frac{\frac{3\left(a^4-24a^2b^2+40b^4\right)(dx+c)}{a^7}-\frac{24\left(2a^4b-11a^2b^3+10b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^7}-\frac{8\left(6a^3b^2\tan\left(\frac{1}{2}dx+\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^4 - 24*a^2*b^2 + 40*b^4)*(d*x + c)/a^7 - 24*(2*a^4*b - 11*a^2*b^3
 + 10*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*t
```

```
an(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2
 + b^2)*a^7) - 8*(6*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*a^2*b^3*tan(1/2*d*x
+ 1/2*c)^3 - 11*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 10*b^5*tan(1/2*d*x + 1/2*c)^
3 - 6*a^3*b^2*tan(1/2*d*x + 1/2*c) - 5*a^2*b^3*tan(1/2*d*x + 1/2*c) + 11*a*
b^4*tan(1/2*d*x + 1/2*c) + 10*b^5*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1
/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2*a^6) + 2*(3*a^3*tan(1/2*d*x +
 1/2*c)^7 + 24*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 24*a*b^2*tan(1/2*d*x + 1/2*c)
^7 - 80*b^3*tan(1/2*d*x + 1/2*c)^7 + 11*a^3*tan(1/2*d*x + 1/2*c)^5 + 104*a^
2*b*tan(1/2*d*x + 1/2*c)^5 - 24*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 240*b^3*tan(
1/2*d*x + 1/2*c)^5 - 11*a^3*tan(1/2*d*x + 1/2*c)^3 + 104*a^2*b*tan(1/2*d*x
+ 1/2*c)^3 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 240*b^3*tan(1/2*d*x + 1/2*c)
^3 - 3*a^3*tan(1/2*d*x + 1/2*c) + 24*a^2*b*tan(1/2*d*x + 1/2*c) + 24*a*b^2*
tan(1/2*d*x + 1/2*c) - 80*b^3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^
2 + 1)^4*a^6))/d
```

maple [B] time = 0.53, size = 1227, normalized size = 3.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -18/d/a^5*arctan(tan(1/2*d*x+1/2*c))*b^2+30/d/a^7*arctan(tan(1/2*d*x+1/2*c)
)*b^4+3/4/d/a^3*arctan(tan(1/2*d*x+1/2*c))+5/d*b^3/a^4/(a*tan(1/2*d*x+1/2*c
)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)-10/d*b^5/a^6/(a*tan(1/
2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)+3/4/d/a^3/(
1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+11/4/d/a^3/(1+tan(1/2*d*x+1/
2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-11/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/
2*d*x+1/2*c)^3-3/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)-60/d
/a^6/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*b^3+6/d/a^4/(1+tan(1/2
*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*b-20/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^4*t
an(1/2*d*x+1/2*c)*b^3+6/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)
*b^2+11/d*b^4/a^5/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan
(1/2*d*x+1/2*c)^3-6/d*b/a^3/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*
(a-b)/((a-b)*(a+b))^(1/2))+33/d*b^3/a^5/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2
*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+5/d*b^3/a^4/(a*tan(1/2*d*x+1/2*c)^2-
tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)^3-11/d*b^4/a^5/(a*tan(1/2*
d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)-10/d*b^5/a^6/
(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)^3-
20/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*b^3+26/d/a^4/(1+ta
n(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*b-6/d/a^5/(1+tan(1/2*d*x+1/2*c)^
2)^4*tan(1/2*d*x+1/2*c)^5*b^2-60/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d
*x+1/2*c)^5*b^3+6/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*b^2
+26/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*b-6/d/a^5/(1+tan(
1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*b^2+6/d/a^4/(1+tan(1/2*d*x+1/2*c)^
2)^4*tan(1/2*d*x+1/2*c)^7*b+6/d*b^2/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x
+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)-6/d*b^2/a^3/(a*tan(1/2*d*x+1/2*c)^2-t
an(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)^3-30/d*b^5/a^7/((a-b)*(a+b)
)^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 5.76, size = 3255, normalized size = 9.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + b/cos(c + d*x))^3,x)
```

```
[Out] (atan(((((3*((108*a^19*b - 12*a^20 - 480*a^14*b^6 + 720*a^15*b^5 + 288*a^16
*b^4 - 732*a^17*b^3 + 108*a^18*b^2)/a^18 - (3*tan(c/2 + (d*x)/2)*(a^4*1i +
b^4*40i - a^2*b^2*24i)*(128*a^16*b + 128*a^14*b^3 - 256*a^15*b^2))/(16*a^19
))*(a^4*1i + b^4*40i - a^2*b^2*24i))/(8*a^7) + (tan(c/2 + (d*x)/2)*(57600*a
*b^10 - 27*a^10*b + 9*a^11 - 28800*b^11 + 5760*a^2*b^9 - 69120*a^3*b^8 + 22
752*a^4*b^7 + 23616*a^5*b^6 - 10944*a^6*b^5 - 1728*a^7*b^4 + 711*a^8*b^3 +
171*a^9*b^2))/(2*a^12))*(a^4*1i + b^4*40i - a^2*b^2*24i)*3i)/(8*a^7) - (((3
*((108*a^19*b - 12*a^20 - 480*a^14*b^6 + 720*a^15*b^5 + 288*a^16*b^4 - 732*
a^17*b^3 + 108*a^18*b^2)/a^18 + (3*tan(c/2 + (d*x)/2)*(a^4*1i + b^4*40i - a
^2*b^2*24i)*(128*a^16*b + 128*a^14*b^3 - 256*a^15*b^2))/(16*a^19))*(a^4*1i
+ b^4*40i - a^2*b^2*24i))/(8*a^7) - (tan(c/2 + (d*x)/2)*(57600*a*b^10 - 27*
a^10*b + 9*a^11 - 28800*b^11 + 5760*a^2*b^9 - 69120*a^3*b^8 + 22752*a^4*b^7
 + 23616*a^5*b^6 - 10944*a^6*b^5 - 1728*a^7*b^4 + 711*a^8*b^3 + 171*a^9*b^2
))/(2*a^12))*(a^4*1i + b^4*40i - a^2*b^2*24i)*3i)/(8*a^7))/((324000*a*b^13
+ 27*a^13*b - 216000*b^14 + 388800*a^2*b^12 - 718200*a^3*b^11 - 195480*a^4*
b^10 + 576720*a^5*b^9 - 4104*a^6*b^8 - 205119*a^7*b^7 + 24408*a^8*b^6 + (62
181*a^9*b^5)/2 - 4671*a^10*b^4 - (3267*a^11*b^3)/2 + 162*a^12*b^2)/a^18 + (
3*((3*((108*a^19*b - 12*a^20 - 480*a^14*b^6 + 720*a^15*b^5 + 288*a^16*b^4 -
 732*a^17*b^3 + 108*a^18*b^2)/a^18 - (3*tan(c/2 + (d*x)/2)*(a^4*1i + b^4*40
i - a^2*b^2*24i)*(128*a^16*b + 128*a^14*b^3 - 256*a^15*b^2))/(16*a^19))*(a^
4*1i + b^4*40i - a^2*b^2*24i))/(8*a^7) + (tan(c/2 + (d*x)/2)*(57600*a*b^10
- 27*a^10*b + 9*a^11 - 28800*b^11 + 5760*a^2*b^9 - 69120*a^3*b^8 + 22752*a^
4*b^7 + 23616*a^5*b^6 - 10944*a^6*b^5 - 1728*a^7*b^4 + 711*a^8*b^3 + 171*a^
9*b^2))/(2*a^12))*(a^4*1i + b^4*40i - a^2*b^2*24i))/(8*a^7) + (3*((3*((108*
a^19*b - 12*a^20 - 480*a^14*b^6 + 720*a^15*b^5 + 288*a^16*b^4 - 732*a^17*b^
3 + 108*a^18*b^2)/a^18 + (3*tan(c/2 + (d*x)/2)*(a^4*1i + b^4*40i - a^2*b^2*
24i)*(128*a^16*b + 128*a^14*b^3 - 256*a^15*b^2))/(16*a^19))*(a^4*1i + b^4*4
0i - a^2*b^2*24i))/(8*a^7) - (tan(c/2 + (d*x)/2)*(57600*a*b^10 - 27*a^10*b
+ 9*a^11 - 28800*b^11 + 5760*a^2*b^9 - 69120*a^3*b^8 + 22752*a^4*b^7 + 2361
6*a^5*b^6 - 10944*a^6*b^5 - 1728*a^7*b^4 + 711*a^8*b^3 + 171*a^9*b^2))/(2*a
^12))*(a^4*1i + b^4*40i - a^2*b^2*24i))/(8*a^7)))*(a^4*1i + b^4*40i - a^2*b
^2*24i)*3i)/(4*a^7*d) - ((tan(c/2 + (d*x)/2)^5*(180*a*b^4 + 26*a^4*b - 15*a
^5 + 600*b^5 - 300*a^2*b^3 - 73*a^3*b^2))/(2*a^6) - (3*tan(c/2 + (d*x)/2)^1
1*(60*a*b^4 + 6*a^4*b + a^5 - 40*b^5 + 4*a^2*b^3 - 31*a^3*b^2))/(4*a^6) + (
tan(c/2 + (d*x)/2)^7*(26*a^4*b - 180*a*b^4 + 15*a^5 + 600*b^5 - 300*a^2*b^3
 + 73*a^3*b^2))/(2*a^6) + (tan(c/2 + (d*x)/2)^3*(540*a*b^4 - 34*a^4*b + 5*a
^5 + 600*b^5 - 220*a^2*b^3 - 239*a^3*b^2))/(4*a^6) - (tan(c/2 + (d*x)/2)^9*
(540*a*b^4 + 34*a^4*b + 5*a^5 - 600*b^5 + 220*a^2*b^3 - 239*a^3*b^2))/(4*a^
6) + (3*tan(c/2 + (d*x)/2)*(a + b)*(20*a*b^3 - 7*a^3*b + a^4 + 40*b^4 - 24*
a^2*b^2))/(4*a^6))/(d*(2*a*b - tan(c/2 + (d*x)/2)^6*(4*a^2 - 20*b^2) + tan(
c/2 + (d*x)/2)^2*(8*a*b + 2*a^2 + 6*b^2) + tan(c/2 + (d*x)/2)^10*(2*a^2 - 8
*a*b + 6*b^2) + tan(c/2 + (d*x)/2)^4*(10*a*b - a^2 + 15*b^2) + tan(c/2 + (d
*x)/2)^12*(a^2 - 2*a*b + b^2) + a^2 + b^2 - tan(c/2 + (d*x)/2)^8*(10*a*b +
a^2 - 15*b^2))) + (b*atan(((b*((a + b)*(a - b))^(1/2)*((tan(c/2 + (d*x)/2)*
(57600*a*b^10 - 27*a^10*b + 9*a^11 - 28800*b^11 + 5760*a^2*b^9 - 69120*a^3*
b^8 + 22752*a^4*b^7 + 23616*a^5*b^6 - 10944*a^6*b^5 - 1728*a^7*b^4 + 711*a^
8*b^3 + 171*a^9*b^2))/(2*a^12) + (3*b*((a + b)*(a - b))^(1/2)*((108*a^19*b
- 12*a^20 - 480*a^14*b^6 + 720*a^15*b^5 + 288*a^16*b^4 - 732*a^17*b^3 + 108
*a^18*b^2)/a^18 - (3*b*tan(c/2 + (d*x)/2)*((a + b)*(a - b))^(1/2)*(2*a^4 +
10*b^4 - 11*a^2*b^2)*(128*a^16*b + 128*a^14*b^3 - 256*a^15*b^2))/(4*a^12*(a
```

```
^9 - a^7*b^2)))*(2*a^4 + 10*b^4 - 11*a^2*b^2))/(2*(a^9 - a^7*b^2)))*(2*a^4
+ 10*b^4 - 11*a^2*b^2)*3i)/(2*(a^9 - a^7*b^2)) + (b*((a + b)*(a - b))^(1/2)
*((tan(c/2 + (d*x)/2)*(57600*a*b^10 - 27*a^10*b + 9*a^11 - 28800*b^11 + 576
0*a^2*b^9 - 69120*a^3*b^8 + 22752*a^4*b^7 + 23616*a^5*b^6 - 10944*a^6*b^5 -
 1728*a^7*b^4 + 711*a^8*b^3 + 171*a^9*b^2))/(2*a^12) - (3*b*((a + b)*(a - b
))^(1/2)*((108*a^19*b - 12*a^20 - 480*a^14*b^6 + 720*a^15*b^5 + 288*a^16*b^
4 - 732*a^17*b^3 + 108*a^18*b^2)/a^18 + (3*b*tan(c/2 + (d*x)/2)*((a + b)*(a
 - b))^(1/2)*(2*a^4 + 10*b^4 - 11*a^2*b^2)*(128*a^16*b + 128*a^14*b^3 - 256
*a^15*b^2))/(4*a^12*(a^9 - a^7*b^2)))*(2*a^4 + 10*b^4 - 11*a^2*b^2))/(2*(a^
9 - a^7*b^2)))*(2*a^4 + 10*b^4 - 11*a^2*b^2)*3i)/(2*(a^9 - a^7*b^2)))/((324
000*a*b^13 + 27*a^13*b - 216000*b^14 + 388800*a^2*b^12 - 718200*a^3*b^11 -
195480*a^4*b^10 + 576720*a^5*b^9 - 4104*a^6*b^8 - 205119*a^7*b^7 + 24408*a^
8*b^6 + (62181*a^9*b^5)/2 - 4671*a^10*b^4 - (3267*a^11*b^3)/2 + 162*a^12*b^
2)/a^18 + (3*b*((a + b)*(a - b))^(1/2)*((tan(c/2 + (d*x)/2)*(57600*a*b^10 -
 27*a^10*b + 9*a^11 - 28800*b^11 + 5760*a^2*b^9 - 69120*a^3*b^8 + 22752*a^4
*b^7 + 23616*a^5*b^6 - 10944*a^6*b^5 - 1728*a^7*b^4 + 711*a^8*b^3 + 171*a^9
*b^2))/(2*a^12) + (3*b*((a + b)*(a - b))^(1/2)*((108*a^19*b - 12*a^20 - 480
*a^14*b^6 + 720*a^15*b^5 + 288*a^16*b^4 - 732*a^17*b^3 + 108*a^18*b^2)/a^18
 - (3*b*tan(c/2 + (d*x)/2)*((a + b)*(a - b))^(1/2)*(2*a^4 + 10*b^4 - 11*a^2
*b^2)*(128*a^16*b + 128*a^14*b^3 - 256*a^15*b^2))/(4*a^12*(a^9 - a^7*b^2)))
*(2*a^4 + 10*b^4 - 11*a^2*b^2))/(2*(a^9 - a^7*b^2)))*(2*a^4 + 10*b^4 - 11*a
^2*b^2))/(2*(a^9 - a^7*b^2)) - (3*b*((a + b)*(a - b))^(1/2)*((tan(c/2 + (d*
x)/2)*(57600*a*b^10 - 27*a^10*b + 9*a^11 - 28800*b^11 + 5760*a^2*b^9 - 6912
0*a^3*b^8 + 22752*a^4*b^7 + 23616*a^5*b^6 - 10944*a^6*b^5 - 1728*a^7*b^4 +
711*a^8*b^3 + 171*a^9*b^2))/(2*a^12) - (3*b*((a + b)*(a - b))^(1/2)*((108*a
^19*b - 12*a^20 - 480*a^14*b^6 + 720*a^15*b^5 + 288*a^16*b^4 - 732*a^17*b^3
 + 108*a^18*b^2)/a^18 + (3*b*tan(c/2 + (d*x)/2)*((a + b)*(a - b))^(1/2)*(2*
a^4 + 10*b^4 - 11*a^2*b^2)*(128*a^16*b + 128*a^14*b^3 - 256*a^15*b^2))/(4*a
^12*(a^9 - a^7*b^2)))*(2*a^4 + 10*b^4 - 11*a^2*b^2))/(2*(a^9 - a^7*b^2)))*(
2*a^4 + 10*b^4 - 11*a^2*b^2))/(2*(a^9 - a^7*b^2))))*((a + b)*(a - b))^(1/2)
*(2*a^4 + 10*b^4 - 11*a^2*b^2)*3i)/(d*(a^9 - a^7*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)**4/(a + b*sec(c + d*x))**3, x)
```

3.230 $\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=267

$$\frac{(3a^2-4b^2)\sin(c+dx)\cos^2(c+dx)}{2a^2d(a^2-b^2)(a\cos(c+dx)+b)}+\frac{x(a^2-12b^2)}{2a^5}+\frac{b(11a^2-12b^2)\sin(c+dx)}{2a^4d(a^2-b^2)}-\frac{(5a^2-6b^2)\sin(c+dx)\cos(c+}{2a^3d(a^2-b^2)}$$

```
[Out] 1/2*(a^2-12*b^2)*x/a^5-b*(6*a^4-19*a^2*b^2+12*b^4)*arctanh((a-b)^(1/2)*tan(
1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/(a-b)^(3/2)/(a+b)^(3/2)/d+1/2*b*(11*a^2-12*
b^2)*sin(d*x+c)/a^4/(a^2-b^2)/d-1/2*(5*a^2-6*b^2)*cos(d*x+c)*sin(d*x+c)/a^3
/(a^2-b^2)/d+1/2*cos(d*x+c)^3*sin(d*x+c)/a/d/(b+a*cos(d*x+c))^2+1/2*(3*a^2-
4*b^2)*cos(d*x+c)^2*sin(d*x+c)/a^2/(a^2-b^2)/d/(b+a*cos(d*x+c))
```

Rubi [A] time = 0.94, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3872, 2889, 3048, 3049, 3023, 2735, 2659, 208}

$$\frac{b(11a^2-12b^2)\sin(c+dx)}{2a^4d(a^2-b^2)}+\frac{(3a^2-4b^2)\sin(c+dx)\cos^2(c+dx)}{2a^2d(a^2-b^2)(a\cos(c+dx)+b)}-\frac{(5a^2-6b^2)\sin(c+dx)\cos(c+dx)}{2a^3d(a^2-b^2)}-\frac{b(-19a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((a^2 - 12*b^2)*x)/(2*a^5) - (b*(6*a^4 - 19*a^2*b^2 + 12*b^4)*ArcTanh[(Sqrt
[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(3/2)*(a + b)^(3/2)*d)
 + (b*(11*a^2 - 12*b^2)*Sin[c + d*x])/(2*a^4*(a^2 - b^2)*d) - ((5*a^2 - 6*b
^2)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*(a^2 - b^2)*d) + (Cos[c + d*x]^3*Sin[
c + d*x])/(2*a*d*(b + a*Cos[c + d*x])^2) + ((3*a^2 - 4*b^2)*Cos[c + d*x]^2*
Sin[c + d*x])/(2*a^2*(a^2 - b^2)*d*(b + a*Cos[c + d*x]))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^2(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= -\int \frac{\cos^3(c+dx)\left(1-\cos^2(c+dx)\right)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)}{2ad(b+a\cos(c+dx))^2} + \frac{\int \frac{\cos^2(c+dx)\left(3\left(a^2-b^2\right)-4\left(a^2-b^2\right)\cos^2(c+dx)\right)}{(-b-a\cos(c+dx))^2}\,dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)}{2ad(b+a\cos(c+dx))^2} + \frac{\left(3a^2-4b^2\right)\cos^2(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)d(b+a\cos(c+dx))} - \frac{\int \frac{\cos(c+dx)\left(2\left(3a^4-7a\right.\right.}{}}{}$$

$$= -\frac{\left(5a^2-6b^2\right)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cos^3(c+dx)\sin(c+dx)}{2ad(b+a\cos(c+dx))^2} + \frac{\left(3a^2-4b^2\right)\cos^2}{2a^2\left(a^2-b^2\right)d(}$$

$$= \frac{b\left(11a^2-12b^2\right)\sin(c+dx)}{2a^4\left(a^2-b^2\right)d} - \frac{\left(5a^2-6b^2\right)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cos^3(c+dx)\sin}{2ad(b+a\cos(c}$$

$$= \frac{\left(a^2-12b^2\right)x}{2a^5} + \frac{b\left(11a^2-12b^2\right)\sin(c+dx)}{2a^4\left(a^2-b^2\right)d} - \frac{\left(5a^2-6b^2\right)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)d} +$$

$$= \frac{\left(a^2-12b^2\right)x}{2a^5} + \frac{b\left(11a^2-12b^2\right)\sin(c+dx)}{2a^4\left(a^2-b^2\right)d} - \frac{\left(5a^2-6b^2\right)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)d} +$$

$$= \frac{\left(a^2-12b^2\right)x}{2a^5} - \frac{b\left(6a^4-19a^2b^2+12b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b\left(11a^2-12b^2\right)}{2a^4\left(a^2-\right.}$$

Mathematica [A] time = 4.20, size = 282, normalized size = 1.06

$$\frac{\frac{4b\left(6a^4-19a^2b^2+12b^4\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{2a^2\left(a^2-b^2\right)\cos^2(c+dx)\left(\left(a^2-12b^2\right)(c+dx)+4ab\sin(c+dx)\right)-2a^4\left(a^2-b^2\right)\sin(c+dx)\cos^3(c+dx)+4ab}{}}{4a^5d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((4*b*(6*a^4 - 19*a^2*b^2 + 12*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqr
t[a^2 - b^2]])/Sqrt[a^2 - b^2] + (4*a*b*(a^4 - 13*a^2*b^2 + 12*b^4)*(c + d*
x)*Cos[c + d*x] - 2*a^4*(a^2 - b^2)*Cos[c + d*x]^3*Sin[c + d*x] + 2*a^2*(a^
2 - b^2)*Cos[c + d*x]^2*((a^2 - 12*b^2)*(c + d*x) + 4*a*b*Sin[c + d*x]) + b
^2*(2*(a^4 - 13*a^2*b^2 + 12*b^4)*(c + d*x) + (22*a^3*b - 24*a*b^3)*Sin[c +
 d*x] + (17*a^4 - 18*a^2*b^2)*Sin[2*(c + d*x)]))/(b + a*Cos[c + d*x])^2)/(4
*a^5*(a - b)*(a + b)*d)
```

fricas [A] time = 0.61, size = 984, normalized size = 3.69

$$\left[\frac{2\left(a^8-14a^6b^2+25a^4b^4-12a^2b^6\right)dx\cos(dx+c)^2+4\left(a^7b-14a^5b^3+25a^3b^5-12ab^7\right)dx\cos(dx+c)+2\left(a^6\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^8 - 14*a^6*b^2 + 25*a^4*b^4 - 12*a^2*b^6)*d*x*cos(d*x + c)^2 + 4
*(a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^7)*d*x*cos(d*x + c) + 2*(a^6*b^2
 - 14*a^4*b^4 + 25*a^2*b^6 - 12*b^8)*d*x - (6*a^4*b^3 - 19*a^2*b^5 + 12*b^7
 + (6*a^6*b - 19*a^4*b^3 + 12*a^2*b^5)*cos(d*x + c)^2 + 2*(6*a^5*b^2 - 19*a
^3*b^4 + 12*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) -
(a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d
*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2
*(11*a^5*b^3 - 23*a^3*b^5 + 12*a*b^7 - (a^8 - 2*a^6*b^2 + a^4*b^4)*cos(d*x
+ c)^3 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c)^2 + (17*a^6*b^2 - 35*
a^4*b^4 + 18*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 2*a^9*b^2 + a^7*
b^4)*d*cos(d*x + c)^2 + 2*(a^10*b - 2*a^8*b^3 + a^6*b^5)*d*cos(d*x + c) + (
a^9*b^2 - 2*a^7*b^4 + a^5*b^6)*d), 1/2*((a^8 - 14*a^6*b^2 + 25*a^4*b^4 - 12
*a^2*b^6)*d*x*cos(d*x + c)^2 + 2*(a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^
7)*d*x*cos(d*x + c) + (a^6*b^2 - 14*a^4*b^4 + 25*a^2*b^6 - 12*b^8)*d*x - (6
*a^4*b^3 - 19*a^2*b^5 + 12*b^7 + (6*a^6*b - 19*a^4*b^3 + 12*a^2*b^5)*cos(d*
x + c)^2 + 2*(6*a^5*b^2 - 19*a^3*b^4 + 12*a*b^6)*cos(d*x + c))*sqrt(-a^2 +
b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c
))) + (11*a^5*b^3 - 23*a^3*b^5 + 12*a*b^7 - (a^8 - 2*a^6*b^2 + a^4*b^4)*cos
(d*x + c)^3 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c)^2 + (17*a^6*b^2
- 35*a^4*b^4 + 18*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 2*a^9*b^2 +
 a^7*b^4)*d*cos(d*x + c)^2 + 2*(a^10*b - 2*a^8*b^3 + a^6*b^5)*d*cos(d*x + c
) + (a^9*b^2 - 2*a^7*b^4 + a^5*b^6)*d)]
```

giac [B] time = 0.65, size = 1193, normalized size = 4.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*((a^11 - 7*a^10*b - 14*a^9*b^2 + 39*a^8*b^3 + 25*a^7*b^4 - 56*a^6*b^5 -
 12*a^5*b^6 + 24*a^4*b^7 - a^4*abs(-a^7 + a^5*b^2) - 5*a^3*b*abs(-a^7 + a^5
*b^2) + 13*a^2*b^2*abs(-a^7 + a^5*b^2) + 6*a*b^3*abs(-a^7 + a^5*b^2) - 12*b
^4*abs(-a^7 + a^5*b^2))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*
d*x + 1/2*c)/sqrt(-(a^6*b - a^4*b^3 + sqrt((a^7 + a^6*b - a^5*b^2 - a^4*b^3
)*(a^7 - a^6*b - a^5*b^2 + a^4*b^3) + (a^6*b - a^4*b^3)^2))/(a^7 - a^6*b -
a^5*b^2 + a^4*b^3))))/(a^6*b*abs(-a^7 + a^5*b^2) - a^4*b^3*abs(-a^7 + a^5*b
^2) + (a^7 - a^5*b^2)^2) + ((a^4 + 5*a^3*b - 13*a^2*b^2 - 6*a*b^3 + 12*b^4)
*sqrt(-a^2 + b^2)*abs(-a^7 + a^5*b^2)*abs(-a + b) + (a^11 - 7*a^10*b - 14*a
^9*b^2 + 39*a^8*b^3 + 25*a^7*b^4 - 56*a^6*b^5 - 12*a^5*b^6 + 24*a^4*b^7)*sq
rt(-a^2 + b^2)*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(
1/2*d*x + 1/2*c)/sqrt(-(a^6*b - a^4*b^3 - sqrt((a^7 + a^6*b - a^5*b^2 - a^4
*b^3)*(a^7 - a^6*b - a^5*b^2 + a^4*b^3) + (a^6*b - a^4*b^3)^2))/(a^7 - a^6*
b - a^5*b^2 + a^4*b^3))))/((a^7 - a^5*b^2)^2*(a^2 - 2*a*b + b^2) - (a^8*b -
 2*a^7*b^2 + 2*a^5*b^4 - a^4*b^5)*abs(-a^7 + a^5*b^2)) + 2*(a^5*tan(1/2*d*x
 + 1/2*c)^7 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^7 - 18*a^3*b^2*tan(1/2*d*x + 1/2
*c)^7 + 7*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 18*a*b^4*tan(1/2*d*x + 1/2*c)^7
- 12*b^5*tan(1/2*d*x + 1/2*c)^7 - 3*a^5*tan(1/2*d*x + 1/2*c)^5 - 4*a^4*b*ta
n(1/2*d*x + 1/2*c)^5 - 14*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 37*a^2*b^3*tan(1
/2*d*x + 1/2*c)^5 + 18*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 36*b^5*tan(1/2*d*x +
1/2*c)^5 + 3*a^5*tan(1/2*d*x + 1/2*c)^3 - 4*a^4*b*tan(1/2*d*x + 1/2*c)^3 +
14*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 37*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 18*
a*b^4*tan(1/2*d*x + 1/2*c)^3 - 36*b^5*tan(1/2*d*x + 1/2*c)^3 - a^5*tan(1/2*
d*x + 1/2*c) + 4*a^4*b*tan(1/2*d*x + 1/2*c) + 18*a^3*b^2*tan(1/2*d*x + 1/2*
c) + 7*a^2*b^3*tan(1/2*d*x + 1/2*c) - 18*a*b^4*tan(1/2*d*x + 1/2*c) - 12*b^
5*tan(1/2*d*x + 1/2*c))/((a^6 - a^4*b^2)*(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(
1/2*d*x + 1/2*c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

maple [B] time = 0.46, size = 729, normalized size = 2.73

$$-\frac{6b^{2}\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{2}\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)^{2}(a+b)}-\frac{b^{3}\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^{3}\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -6/d*b^2/a^2/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)*ta
n(1/2*d*x+1/2*c)^3-1/d*b^3/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2
*b-a-b)^2/(a+b)*tan(1/2*d*x+1/2*c)^3+6/d*b^4/a^4/(a*tan(1/2*d*x+1/2*c)^2-ta
n(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)*tan(1/2*d*x+1/2*c)^3+6/d*b^2/a^2/(a*tan(1
/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)*tan(1/2*d*x+1/2*c)-1/d*
b^3/a^3/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)*tan(1/2
*d*x+1/2*c)-6/d*b^4/a^4/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)
^2/(a-b)*tan(1/2*d*x+1/2*c)-6/d*b/a/(a^2-b^2)/((a-b)*(a+b))^(1/2)*arctanh(t
an(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))+19/d*b^3/a^3/(a^2-b^2)/((a-b)*
(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-12/d*b^5
/a^5/(a^2-b^2)/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*
(a+b))^(1/2))+1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+6/d/a
^4/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*b+6/d/a^4/(1+tan(1/2*d*x
+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)*b-1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/
2*d*x+1/2*c)-12/d/a^5*arctan(tan(1/2*d*x+1/2*c))*b^2+1/d/a^3*arctan(tan(1/2
*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 9.10, size = 4026, normalized size = 15.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b/cos(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(6*a*b^3 - 5*a^3*b + a^4 + 12*b^4 - 13*a^2*b^2))/(a^4*
b - a^5) + (tan(c/2 + (d*x)/2)^3*(18*a*b^4 + 4*a^4*b - 3*a^5 + 36*b^5 - 37*
a^2*b^3 - 14*a^3*b^2))/((a^4*b - a^5)*(a + b)) + (tan(c/2 + (d*x)/2)^5*(4*a
^4*b - 18*a*b^4 + 3*a^5 + 36*b^5 - 37*a^2*b^3 + 14*a^3*b^2))/((a^4*b - a^5)
*(a + b)) + (tan(c/2 + (d*x)/2)^7*(5*a^3*b - 6*a*b^3 + a^4 + 12*b^4 - 13*a^
2*b^2))/(a^4*(a + b)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^4*(2*a^2 - 6*b^2) + t
an(c/2 + (d*x)/2)^2*(4*a*b + 4*b^2) - tan(c/2 + (d*x)/2)^6*(4*a*b - 4*b^2)
+ tan(c/2 + (d*x)/2)^8*(a^2 - 2*a*b + b^2) + a^2 + b^2)) + (atan((((a^2*1i
- b^2*12i)*((((4*(24*a^16*b - 4*a^17 - 48*a^10*b^7 + 24*a^11*b^6 + 124*a^12
*b^5 - 56*a^13*b^4 - 100*a^14*b^3 + 36*a^15*b^2))/(a^14*b + a^15 - a^12*b^3
 - a^13*b^2) - (4*tan(c/2 + (d*x)/2)*(a^2*1i - b^2*12i)*(8*a^15*b - 8*a^10*
b^6 + 8*a^11*b^5 + 16*a^12*b^4 - 16*a^13*b^3 - 8*a^14*b^2))/(a^5*(a^10*b +
```

```
a^11 - a^8*b^3 - a^9*b^2)))*(a^2*1i - b^2*12i))/(2*a^5) + (8*tan(c/2 + (d*x
)/2)*(a^10 - 2*a^9*b - 288*a*b^9 + 288*b^10 - 624*a^2*b^8 + 624*a^3*b^7 + 3
86*a^4*b^6 - 386*a^5*b^5 - 61*a^6*b^4 + 52*a^7*b^3 + 11*a^8*b^2))/(a^10*b +
 a^11 - a^8*b^3 - a^9*b^2))*1i)/(2*a^5) - ((a^2*1i - b^2*12i)*((((4*(24*a^1
6*b - 4*a^17 - 48*a^10*b^7 + 24*a^11*b^6 + 124*a^12*b^5 - 56*a^13*b^4 - 100
*a^14*b^3 + 36*a^15*b^2))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (4*tan(c/
2 + (d*x)/2)*(a^2*1i - b^2*12i)*(8*a^15*b - 8*a^10*b^6 + 8*a^11*b^5 + 16*a^
12*b^4 - 16*a^13*b^3 - 8*a^14*b^2))/(a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2
)))*(a^2*1i - b^2*12i))/(2*a^5) - (8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 2
88*a*b^9 + 288*b^10 - 624*a^2*b^8 + 624*a^3*b^7 + 386*a^4*b^6 - 386*a^5*b^5
 - 61*a^6*b^4 + 52*a^7*b^3 + 11*a^8*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^
2))*1i)/(2*a^5))/((8*(864*a*b^10 + 6*a^10*b - 1728*b^11 + 4752*a^2*b^9 - 21
60*a^3*b^8 - 4356*a^4*b^7 + 1746*a^5*b^6 + 1495*a^6*b^5 - 491*a^7*b^4 - 169
*a^8*b^3 + 30*a^9*b^2))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + ((a^2*1i -
b^2*12i)*((((4*(24*a^16*b - 4*a^17 - 48*a^10*b^7 + 24*a^11*b^6 + 124*a^12*b
^5 - 56*a^13*b^4 - 100*a^14*b^3 + 36*a^15*b^2))/(a^14*b + a^15 - a^12*b^3 -
 a^13*b^2) - (4*tan(c/2 + (d*x)/2)*(a^2*1i - b^2*12i)*(8*a^15*b - 8*a^10*b^
6 + 8*a^11*b^5 + 16*a^12*b^4 - 16*a^13*b^3 - 8*a^14*b^2))/(a^5*(a^10*b + a^
11 - a^8*b^3 - a^9*b^2)))*(a^2*1i - b^2*12i))/(2*a^5) + (8*tan(c/2 + (d*x)/
2)*(a^10 - 2*a^9*b - 288*a*b^9 + 288*b^10 - 624*a^2*b^8 + 624*a^3*b^7 + 386
*a^4*b^6 - 386*a^5*b^5 - 61*a^6*b^4 + 52*a^7*b^3 + 11*a^8*b^2))/(a^10*b + a
^11 - a^8*b^3 - a^9*b^2)))/(2*a^5) + ((a^2*1i - b^2*12i)*((((4*(24*a^16*b -
 4*a^17 - 48*a^10*b^7 + 24*a^11*b^6 + 124*a^12*b^5 - 56*a^13*b^4 - 100*a^14
*b^3 + 36*a^15*b^2))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (4*tan(c/2 + (
d*x)/2)*(a^2*1i - b^2*12i)*(8*a^15*b - 8*a^10*b^6 + 8*a^11*b^5 + 16*a^12*b^
4 - 16*a^13*b^3 - 8*a^14*b^2))/(a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2)))*(
a^2*1i - b^2*12i))/(2*a^5) - (8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 288*a*
b^9 + 288*b^10 - 624*a^2*b^8 + 624*a^3*b^7 + 386*a^4*b^6 - 386*a^5*b^5 - 61
*a^6*b^4 + 52*a^7*b^3 + 11*a^8*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2)))/
(2*a^5)))*(a^2*1i - b^2*12i)*1i)/(a^5*d) + (b*atan(((b*((8*tan(c/2 + (d*x)/
2)*(a^10 - 2*a^9*b - 288*a*b^9 + 288*b^10 - 624*a^2*b^8 + 624*a^3*b^7 + 386
*a^4*b^6 - 386*a^5*b^5 - 61*a^6*b^4 + 52*a^7*b^3 + 11*a^8*b^2))/(a^10*b + a
^11 - a^8*b^3 - a^9*b^2) + (b*((4*(24*a^16*b - 4*a^17 - 48*a^10*b^7 + 24*a^
11*b^6 + 124*a^12*b^5 - 56*a^13*b^4 - 100*a^14*b^3 + 36*a^15*b^2))/(a^14*b
+ a^15 - a^12*b^3 - a^13*b^2) - (4*b*tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^
3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2)*(8*a^15*b - 8*a^10*b^6 + 8*a^11*b^5
+ 16*a^12*b^4 - 16*a^13*b^3 - 8*a^14*b^2))/((a^10*b + a^11 - a^8*b^3 - a^9*
b^2)*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*((a + b)^3*(a - b)^3)^(1/2)
*(6*a^4 + 12*b^4 - 19*a^2*b^2))/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)
))*((a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2)*1i)/(2*(a^11 -
 a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)) + (b*((8*tan(c/2 + (d*x)/2)*(a^10 - 2*a^
9*b - 288*a*b^9 + 288*b^10 - 624*a^2*b^8 + 624*a^3*b^7 + 386*a^4*b^6 - 386*
a^5*b^5 - 61*a^6*b^4 + 52*a^7*b^3 + 11*a^8*b^2))/(a^10*b + a^11 - a^8*b^3 -
 a^9*b^2) - (b*((4*(24*a^16*b - 4*a^17 - 48*a^10*b^7 + 24*a^11*b^6 + 124*a^
12*b^5 - 56*a^13*b^4 - 100*a^14*b^3 + 36*a^15*b^2))/(a^14*b + a^15 - a^12*b
^3 - a^13*b^2) + (4*b*tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(6*a^4
 + 12*b^4 - 19*a^2*b^2)*(8*a^15*b - 8*a^10*b^6 + 8*a^11*b^5 + 16*a^12*b^4 -
 16*a^13*b^3 - 8*a^14*b^2))/((a^10*b + a^11 - a^8*b^3 - a^9*b^2)*(a^11 - a^
5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*((a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^
4 - 19*a^2*b^2))/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*((a + b)^3*(
a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2)*1i)/(2*(a^11 - a^5*b^6 + 3*a^
7*b^4 - 3*a^9*b^2)))/((8*(864*a*b^10 + 6*a^10*b - 1728*b^11 + 4752*a^2*b^9
- 2160*a^3*b^8 - 4356*a^4*b^7 + 1746*a^5*b^6 + 1495*a^6*b^5 - 491*a^7*b^4 -
 169*a^8*b^3 + 30*a^9*b^2))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (b*((8*
tan(c/2 + (d*x)/2)*(a^10 - 2*a^9*b - 288*a*b^9 + 288*b^10 - 624*a^2*b^8 + 6
24*a^3*b^7 + 386*a^4*b^6 - 386*a^5*b^5 - 61*a^6*b^4 + 52*a^7*b^3 + 11*a^8*b
^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2) + (b*((4*(24*a^16*b - 4*a^17 - 48*
a^10*b^7 + 24*a^11*b^6 + 124*a^12*b^5 - 56*a^13*b^4 - 100*a^14*b^3 + 36*a^1
5*b^2))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) - (4*b*tan(c/2 + (d*x)/2)*((a
```

```
 + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2)*(8*a^15*b - 8*a^10*b
^6 + 8*a^11*b^5 + 16*a^12*b^4 - 16*a^13*b^3 - 8*a^14*b^2))/((a^10*b + a^11
- a^8*b^3 - a^9*b^2)*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*((a + b)^3*
(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2))/(2*(a^11 - a^5*b^6 + 3*a^7*
b^4 - 3*a^9*b^2)))*((a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2
))/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)) - (b*((8*tan(c/2 + (d*x)/2)
*(a^10 - 2*a^9*b - 288*a*b^9 + 288*b^10 - 624*a^2*b^8 + 624*a^3*b^7 + 386*a
^4*b^6 - 386*a^5*b^5 - 61*a^6*b^4 + 52*a^7*b^3 + 11*a^8*b^2))/(a^10*b + a^1
1 - a^8*b^3 - a^9*b^2) - (b*((4*(24*a^16*b - 4*a^17 - 48*a^10*b^7 + 24*a^11
*b^6 + 124*a^12*b^5 - 56*a^13*b^4 - 100*a^14*b^3 + 36*a^15*b^2))/(a^14*b +
a^15 - a^12*b^3 - a^13*b^2) + (4*b*tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)
^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2)*(8*a^15*b - 8*a^10*b^6 + 8*a^11*b^5 +
16*a^12*b^4 - 16*a^13*b^3 - 8*a^14*b^2))/((a^10*b + a^11 - a^8*b^3 - a^9*b^
2)*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))*((a + b)^3*(a - b)^3)^(1/2)*(
6*a^4 + 12*b^4 - 19*a^2*b^2))/(2*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2)))
*((a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4 - 19*a^2*b^2))/(2*(a^11 - a^5*
b^6 + 3*a^7*b^4 - 3*a^9*b^2))))*((a + b)^3*(a - b)^3)^(1/2)*(6*a^4 + 12*b^4
 - 19*a^2*b^2)*1i)/(d*(a^11 - a^5*b^6 + 3*a^7*b^4 - 3*a^9*b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

3.231 $\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=376

$$\frac{b^2\left(3a^2-b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{2ab\left(3a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{3b^4\sin(c+dx)}{2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\cdots$$

```
[Out] -2*b^3*(3*a^2-b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/(a
-b)^(7/2)/(a+b)^(7/2)/d-2*a*b*(3*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1
/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d-b^3*(a^2+2*b^2)*arctanh((a-b)^
(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/(a-b)^(7/2)/(a+b)^(7/2)/d-1/2*sin(d
*x+c)/(a+b)^3/d/(1-cos(d*x+c))+1/2*sin(d*x+c)/(a-b)^3/d/(1+cos(d*x+c))-1/2*
b^3*sin(d*x+c)/(a^2-b^2)^2/d/(b+a*cos(d*x+c))^2+3/2*b^4*sin(d*x+c)/(a^2-b^2
)^3/d/(b+a*cos(d*x+c))+b^2*(3*a^2-b^2)*sin(d*x+c)/(a^2-b^2)^3/d/(b+a*cos(d*
x+c))
```

Rubi [A] time = 0.66, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2897, 2648, 2664, 12, 2659, 208, 2754}

$$\frac{3b^4\sin(c+dx)}{2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{b^3\sin(c+dx)}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+\frac{b^2\left(3a^2-b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{2b^3\left(3a^2\cdots\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-2*b^3*(3*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(a*(a - b)^(7/2)*(a + b)^(7/2)*d) - (2*a*b*(3*a^2 + b^2)*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - (b^3*(
a^2 + 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*(a - b
)^(7/2)*(a + b)^(7/2)*d) - Sin[c + d*x]/(2*(a + b)^3*d*(1 - Cos[c + d*x]))
+ Sin[c + d*x]/(2*(a - b)^3*d*(1 + Cos[c + d*x])) - (b^3*Sin[c + d*x])/(2*(
a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (3*b^4*Sin[c + d*x])/(2*(a^2 - b^2
)^3*d*(b + a*Cos[c + d*x])) + (b^2*(3*a^2 - b^2)*Sin[c + d*x])/((a^2 - b^2)
^3*d*(b + a*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos(c+dx)\cot^2(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= \int \left(-\frac{1}{2(a-b)^3(-1-\cos(c+dx))} + \frac{1}{2(a+b)^3(1-\cos(c+dx))} + \frac{3a^2b}{a\left(a^2-b^2\right)^2(-b}\right.$$

$$= -\frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{2(a-b)^3} + \frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{2(a+b)^3} - \frac{b^3\int \frac{1}{(b+a\cos(c+dx))^3}\,dx}{a\left(a^2-b^2\right)} + \frac{\left(b^2\left(3a^2-b^2\right)\right)\int}{a\left(a^2\right.}$$

$$= -\frac{\sin(c+dx)}{2(a+b)^3d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^3d(1+\cos(c+dx))} - \frac{b^3\sin(c}{2\left(a^2-b^2\right)^2d(b+}$$

$$= -\frac{2ab\left(3a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{2(a+b)^3d(1-\cos(c+dx))} + \frac{}{2(a-}$$

$$= -\frac{2ab\left(3a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{2(a+b)^3d(1-\cos(c+dx))} + \frac{}{2(a-}$$

$$= -\frac{2b^3\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{7/2}(a+b)^{7/2}d} - \frac{2ab\left(3a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+\right.}{\sqrt{a+b}}\right.}{(a-b)^{7/2}(a+b)^{7/2}d}$$

$$= -\frac{2b^3\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{7/2}(a+b)^{7/2}d} - \frac{2ab\left(3a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+\right.}{\sqrt{a+b}}\right.}{(a-b)^{7/2}(a+b)^{7/2}d}$$

Mathematica [A] time = 1.11, size = 231, normalized size = 0.61

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b)\left(\frac{b^2\left(6a^2+b^2\right)\sin(c+dx)(a\cos(c+dx)+b)}{(a-b)^3(a+b)^3} + \frac{6ab\left(2a^2+3b^2\right)(a\cos(c+dx)+b)^2\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} - \right.}{2d(a+b\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^3*((6*a*b*(2*a^2 + 3*b^2)*ArcTanh[((-a +
 b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^
(7/2) - ((b + a*Cos[c + d*x])^2*Cot[(c + d*x)/2])/(a + b)^3 - (b^3*Sin[c +
d*x])/((a - b)^2*(a + b)^2) + (b^2*(6*a^2 + b^2)*(b + a*Cos[c + d*x])*Sin[c
 + d*x])/((a - b)^3*(a + b)^3) + ((b + a*Cos[c + d*x])^2*Tan[(c + d*x)/2])/
(a - b)^3))/(2*d*(a + b*Sec[c + d*x])^3)
```

fricas [A] time = 0.74, size = 841, normalized size = 2.24

$$\left[\frac{22\,a^4b^3 - 14\,a^2b^5 - 8\,b^7 - 2\left(2\,a^7 + 10\,a^5b^2 - 11\,a^3b^4 - ab^6\right)\cos(dx+c)^3 - 3\left(2\,a^3b^3 + 3\,ab^5 + \left(2\,a^5b + 3\,a^3b\right.\right.}{4\left(\left(a^{10} - 4\,a^8b^2 + 6\,a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(22*a^4*b^3 - 14*a^2*b^5 - 8*b^7 - 2*(2*a^7 + 10*a^5*b^2 - 11*a^3*b^4
- a*b^6)*cos(d*x + c)^3 - 3*(2*a^3*b^3 + 3*a*b^5 + (2*a^5*b + 3*a^3*b^3)*co
s(d*x + c)^2 + 2*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*log(
(2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*c
os(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*co
s(d*x + c) + b^2))*sin(d*x + c) + 2*(2*a^6*b - 17*a^4*b^3 + 13*a^2*b^5 + 2*
b^7)*cos(d*x + c)^2 + 2*(16*a^5*b^2 - 17*a^3*b^4 + a*b^6)*cos(d*x + c))/(((
a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*d*cos(d*x + c)^2 + 2*(a
^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c) + (a^8*b^2
 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d)*sin(d*x + c)), 1/2*(11*a^4*
b^3 - 7*a^2*b^5 - 4*b^7 - (2*a^7 + 10*a^5*b^2 - 11*a^3*b^4 - a*b^6)*cos(d*x
 + c)^3 - 3*(2*a^3*b^3 + 3*a*b^5 + (2*a^5*b + 3*a^3*b^3)*cos(d*x + c)^2 + 2
*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 +
 b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) + (2*a^
6*b - 17*a^4*b^3 + 13*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + (16*a^5*b^2 - 17*a^
3*b^4 + a*b^6)*cos(d*x + c))/(((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 +
a^2*b^8)*d*cos(d*x + c)^2 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 +
a*b^9)*d*cos(d*x + c) + (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10
)*d)*sin(d*x + c))]
```

giac [A] time = 3.77, size = 386, normalized size = 1.03

$$\frac{\frac{6\left(2a^3b+3ab^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3-3a^2b+3ab^2-b^3}-\frac{2\left(6a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5a^2b^3\tan\left(\frac{1}{2}d\cdots\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*(2*a^3*b + 3*a*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b)
 + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2
)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + tan(1/2*d*x +
1/2*c)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 2*(6*a^3*b^2*tan(1/2*d*x + 1/2*c)^
3 - 5*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + a*b^4*tan(1/2*d*x + 1/2*c)^3 - 2*b^5
*tan(1/2*d*x + 1/2*c)^3 - 6*a^3*b^2*tan(1/2*d*x + 1/2*c) - 5*a^2*b^3*tan(1/
2*d*x + 1/2*c) - a*b^4*tan(1/2*d*x + 1/2*c) - 2*b^5*tan(1/2*d*x + 1/2*c))/(
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d
*x + 1/2*c)^2 - a - b)^2) - 1/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(1/2*d*x
+ 1/2*c)))/d
```

maple [A] time = 0.58, size = 234, normalized size = 0.62

$$\frac{\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3-6a^2b+6b^2a-2b^3}+\frac{2b\left(\frac{\left(-3a^3b+\frac{5}{2}a^2b^2-\frac{1}{2}ab^3+b^4\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(3a^3b+\frac{5}{2}a^2b^2+\frac{1}{2}ab^3+b^4\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)^2}-\frac{3\left(2a^2+3b^2\right)a\,\mathrm{arctanh}\left(\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)(a-b)}{\sqrt{(a-b)(a+b)}}\right)}{2\sqrt{(a-b)(a+b)}}\right)}{(a-b)^3(a+b)^3}-\frac{\cdots}{2(a+b)^3\cdots}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(1/2/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+2*b/(a-b)^3/(a+b)^3*(
((-3*a^3*b+5/2*a^2*b^2-1/2*a*b^3+b^4)*tan(1/2*d*x+1/2*c)^3+(3*a^3*b+5/2*a^2
```

```
*b^2+1/2*a*b^3+b^4)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-tan(1/2*d*x
+1/2*c)^2*b-a-b)^2-3/2*(2*a^2+3*b^2)*a/((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*
d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2))-1/2/(a+b)^3/tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 2.86, size = 423, normalized size = 1.12

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{2\,d\,{\left(a-b\right)}^3}-\frac{\frac{a^3-3\,a^2\,b+3\,a\,b^2-b^3}{a+b}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(a^5-5\,a^4\,b+22\,a^3\,b^2\cdots\right.}{{\left(a+b\right)}^3}}{d\,\left(\left(2\,a^5-10\,a^4\,b+20\,a^3\,b^2-20\,a^2\,b^3+10\,a\,b^4-2\,b^5\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+\left(-4\,a^5+12\,a^4\,b-8\,a^3\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b/cos(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)/(2*d*(a - b)^3) - ((3*a*b^2 - 3*a^2*b + a^3 - b^3)/(a +
b) + (tan(c/2 + (d*x)/2)^4*(7*a*b^4 - 5*a^4*b + a^5 - 5*b^5 - 20*a^2*b^3 +
22*a^3*b^2))/(a + b)^3 - (2*tan(c/2 + (d*x)/2)^2*(a^4 - 4*a^3*b - 5*a*b^3 +
 3*b^4 + 12*a^2*b^2))/(a + b)^2)/(d*(tan(c/2 + (d*x)/2)*(2*a*b^4 - 2*a^4*b
+ 2*a^5 - 2*b^5 + 4*a^2*b^3 - 4*a^3*b^2) - tan(c/2 + (d*x)/2)^3*(4*a^5 - 12
*a^4*b - 12*a*b^4 + 4*b^5 + 8*a^2*b^3 + 8*a^3*b^2) + tan(c/2 + (d*x)/2)^5*(
10*a*b^4 - 10*a^4*b + 2*a^5 - 2*b^5 - 20*a^2*b^3 + 20*a^3*b^2))) + (a*b*ata
n((a^6*tan(c/2 + (d*x)/2)*1i - b^6*tan(c/2 + (d*x)/2)*1i + a^2*b^4*tan(c/2
+ (d*x)/2)*3i - a^4*b^2*tan(c/2 + (d*x)/2)*3i)/((a + b)^(7/2)*(a - b)^(5/2)
))*(2*a^2 + 3*b^2)*3i)/(d*(a + b)^(7/2)*(a - b)^(7/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

3.232 $\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=515

$$\frac{a^2b^2\left(3a^2+b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)^4(a\cos(c+dx)+b)}+\frac{3a^2b^4\sin(c+dx)}{2d\left(a^2-b^2\right)^4(a\cos(c+dx)+b)}-\frac{a^2b^3\sin(c+dx)}{2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)^2}-\frac{2ab^3\left(3a^2\right.}{}$$

```
[Out] -2*a*b^3*(3*a^2+b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a
-b)^(9/2)/(a+b)^(9/2)/d-a*b^3*(a^2+2*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1
/2*c)/(a+b)^(1/2))/(a-b)^(9/2)/(a+b)^(9/2)/d-2*a*b*(3*a^4+8*a^2*b^2+b^4)*ar
ctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(9/2)/(a+b)^(9/2)/d
-1/12*sin(d*x+c)/(a+b)^3/d/(1-cos(d*x+c))^2-1/4*(a-2*b)*sin(d*x+c)/(a+b)^4/
d/(1-cos(d*x+c))-1/12*sin(d*x+c)/(a+b)^3/d/(1-cos(d*x+c))+1/12*sin(d*x+c)/(
a-b)^3/d/(1+cos(d*x+c))^2+1/12*sin(d*x+c)/(a-b)^3/d/(1+cos(d*x+c))+1/4*(a+2
*b)*sin(d*x+c)/(a-b)^4/d/(1+cos(d*x+c))-1/2*a^2*b^3*sin(d*x+c)/(a^2-b^2)^3/
d/(b+a*cos(d*x+c))^2+3/2*a^2*b^4*sin(d*x+c)/(a^2-b^2)^4/d/(b+a*cos(d*x+c))+
a^2*b^2*(3*a^2+b^2)*sin(d*x+c)/(a^2-b^2)^4/d/(b+a*cos(d*x+c))
```

Rubi [A] time = 0.77, antiderivative size = 515, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3872, 2897, 2650, 2648, 2664, 2754, 12, 2659, 208}

$$\frac{3a^2b^4\sin(c+dx)}{2d\left(a^2-b^2\right)^4(a\cos(c+dx)+b)}-\frac{a^2b^3\sin(c+dx)}{2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)^2}+\frac{a^2b^2\left(3a^2+b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)^4(a\cos(c+dx)+b)}-\frac{2ab^3\left(3a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-2*a*b^3*(3*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]]
)/((a - b)^(9/2)*(a + b)^(9/2)*d) - (a*b^3*(a^2 + 2*b^2)*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(9/2)*(a + b)^(9/2)*d) - (2*a*b
*(3*a^4 + 8*a^2*b^2 + b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
b]])/((a - b)^(9/2)*(a + b)^(9/2)*d) - Sin[c + d*x]/(12*(a + b)^3*d*(1 - Co
s[c + d*x])^2) - ((a - 2*b)*Sin[c + d*x])/(4*(a + b)^4*d*(1 - Cos[c + d*x])
) - Sin[c + d*x]/(12*(a + b)^3*d*(1 - Cos[c + d*x])) + Sin[c + d*x]/(12*(a
- b)^3*d*(1 + Cos[c + d*x])^2) + Sin[c + d*x]/(12*(a - b)^3*d*(1 + Cos[c +
d*x])) + ((a + 2*b)*Sin[c + d*x])/(4*(a - b)^4*d*(1 + Cos[c + d*x])) - (a^2
*b^3*Sin[c + d*x])/(2*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])^2) + (3*a^2*b^4*
Sin[c + d*x])/(2*(a^2 - b^2)^4*d*(b + a*Cos[c + d*x])) + (a^2*b^2*(3*a^2 +
b^2)*Sin[c + d*x])/((a^2 - b^2)^4*d*(b + a*Cos[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
```

```
^2, 0]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cot^3(c+dx)\csc(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= \int \left(\frac{1}{4(a-b)^3(-1-\cos(c+dx))^2} + \frac{-a-2b}{4(a-b)^4(-1-\cos(c+dx))} + \frac{1}{4(a+b)^3(1-\cos(\right.$$

$$= \frac{\int \frac{1}{(-1-\cos(c+dx))^2}\,dx}{4(a-b)^3} + \frac{(a-2b)\int \frac{1}{1-\cos(c+dx)}\,dx}{4(a+b)^4} + \frac{\int \frac{1}{(1-\cos(c+dx))^2}\,dx}{4(a+b)^3} - \frac{(a+2b)\int \frac{}{-1-\cos}}{4(a-b}$$

$$= -\frac{\sin(c+dx)}{12(a+b)^3d(1-\cos(c+dx))^2} - \frac{(a-2b)\sin(c+dx)}{4(a+b)^4d(1-\cos(c+dx))} + \frac{\sin(c+dx}{12(a-b)^3d(1+\cos}$$

$$= -\frac{2ab\left(3a^4+8a^2b^2+b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d} - \frac{\sin(c+dx)}{12(a+b)^3d(1-\cos(c+dx))^2}$$

$$= -\frac{2ab\left(3a^4+8a^2b^2+b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d} - \frac{\sin(c+dx)}{12(a+b)^3d(1-\cos(c+dx))^2}$$

$$= -\frac{2ab^3\left(3a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d} - \frac{2ab\left(3a^4+8a^2b^2+b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\mathrm{ta}}{}\right.}{(a-b)^{9/2}(a+b)^{9/2}d}$$

$$= -\frac{2ab^3\left(3a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d} - \frac{ab^3\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right.}{(a-b)^{9/2}(a+b)^{9/2}d}$$

Mathematica [A] time = 1.08, size = 388, normalized size = 0.75

$$\sec^3(c+dx)(a\cos(c+dx)+b)\left(\frac{96ab\left(6a^4+23a^2b^2+6b^4\right)(a\cos(c+dx)+b)^2\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \csc^3(c+dx)\left(-4a^7\cos(3(c\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*((96*a*b*(6*a^4 + 23*a^2*b^2 + 6*b^4)*ArcTanh[((-a +
b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/Sqrt[a^2 - b^
2] + (36*a^6*b + 154*a^4*b^3 + 424*a^2*b^5 + 16*b^7 - 2*a*(16*a^6 - 94*a^4*
b^2 - 35*a^2*b^4 + 8*b^6)*Cos[c + d*x] + 8*(2*a^6*b - 45*a^4*b^3 - 56*a^2*b
^5 - 6*b^7)*Cos[2*(c + d*x)] - 4*a^7*Cos[3*(c + d*x)] - 154*a^5*b^2*Cos[3*(
c + d*x)] - 205*a^3*b^4*Cos[3*(c + d*x)] + 48*a*b^6*Cos[3*(c + d*x)] - 20*a
^6*b*Cos[4*(c + d*x)] + 110*a^4*b^3*Cos[4*(c + d*x)] + 120*a^2*b^5*Cos[4*(c
 + d*x)] + 4*a^7*Cos[5*(c + d*x)] + 62*a^5*b^2*Cos[5*(c + d*x)] + 39*a^3*b^
4*Cos[5*(c + d*x)])*Csc[c + d*x]^3)*Sec[c + d*x]^3)/(96*(a^2 - b^2)^4*d*(a
+ b*Sec[c + d*x])^3)
```

fricas [A] time = 0.83, size = 1550, normalized size = 3.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/12*(78*a^6*b^3 + 46*a^4*b^5 - 116*a^2*b^7 - 8*b^9 + 2*(4*a^9 + 58*a^7*b
^2 - 23*a^5*b^4 - 39*a^3*b^6)*cos(d*x + c)^5 - 10*(2*a^8*b - 13*a^6*b^3 - a
^4*b^5 + 12*a^2*b^7)*cos(d*x + c)^4 - 4*(3*a^9 + 55*a^7*b^2 - 8*a^5*b^4 - 5
6*a^3*b^6 + 6*a*b^8)*cos(d*x + c)^3 + 3*(6*a^5*b^3 + 23*a^3*b^5 + 6*a*b^7 -
 (6*a^7*b + 23*a^5*b^3 + 6*a^3*b^5)*cos(d*x + c)^4 - 2*(6*a^6*b^2 + 23*a^4*
b^4 + 6*a^2*b^6)*cos(d*x + c)^3 + (6*a^7*b + 17*a^5*b^3 - 17*a^3*b^5 - 6*a*
b^7)*cos(d*x + c)^2 + 2*(6*a^6*b^2 + 23*a^4*b^4 + 6*a^2*b^6)*cos(d*x + c))*
sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*
sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d
*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) + 4*(6*a^8*b - 56*a^6*b
^3 - 8*a^4*b^5 + 55*a^2*b^7 + 3*b^9)*cos(d*x + c)^2 + 10*(12*a^7*b^2 - a^5*
b^4 - 13*a^3*b^6 + 2*a*b^8)*cos(d*x + c))/(((a^12 - 5*a^10*b^2 + 10*a^8*b^4
 - 10*a^6*b^6 + 5*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^4 + 2*(a^11*b - 5*a^9*
b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c)^3 - (a^1
2 - 6*a^10*b^2 + 15*a^8*b^4 - 20*a^6*b^6 + 15*a^4*b^8 - 6*a^2*b^10 + b^12)*
d*cos(d*x + c)^2 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*
b^9 - a*b^11)*d*cos(d*x + c) - (a^10*b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*
b^8 + 5*a^2*b^10 - b^12)*d)*sin(d*x + c)), -1/6*(39*a^6*b^3 + 23*a^4*b^5 -
58*a^2*b^7 - 4*b^9 + (4*a^9 + 58*a^7*b^2 - 23*a^5*b^4 - 39*a^3*b^6)*cos(d*x
 + c)^5 - 5*(2*a^8*b - 13*a^6*b^3 - a^4*b^5 + 12*a^2*b^7)*cos(d*x + c)^4 -
2*(3*a^9 + 55*a^7*b^2 - 8*a^5*b^4 - 56*a^3*b^6 + 6*a*b^8)*cos(d*x + c)^3 -
3*(6*a^5*b^3 + 23*a^3*b^5 + 6*a*b^7 - (6*a^7*b + 23*a^5*b^3 + 6*a^3*b^5)*co
s(d*x + c)^4 - 2*(6*a^6*b^2 + 23*a^4*b^4 + 6*a^2*b^6)*cos(d*x + c)^3 + (6*a
^7*b + 17*a^5*b^3 - 17*a^3*b^5 - 6*a*b^7)*cos(d*x + c)^2 + 2*(6*a^6*b^2 + 2
3*a^4*b^4 + 6*a^2*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b
^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) + 2*(6*a^
8*b - 56*a^6*b^3 - 8*a^4*b^5 + 55*a^2*b^7 + 3*b^9)*cos(d*x + c)^2 + 5*(12*a
^7*b^2 - a^5*b^4 - 13*a^3*b^6 + 2*a*b^8)*cos(d*x + c))/(((a^12 - 5*a^10*b^2
 + 10*a^8*b^4 - 10*a^6*b^6 + 5*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^4 + 2*(a^
11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x
+ c)^3 - (a^12 - 6*a^10*b^2 + 15*a^8*b^4 - 20*a^6*b^6 + 15*a^4*b^8 - 6*a^2*
b^10 + b^12)*d*cos(d*x + c)^2 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5
*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c) - (a^10*b^2 - 5*a^8*b^4 + 10*a^6*
b^6 - 10*a^4*b^8 + 5*a^2*b^10 - b^12)*d)*sin(d*x + c))]
```

giac [A] time = 0.50, size = 709, normalized size = 1.38

$$\frac{24\left(6a^5b+23a^3b^3+6ab^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8\right)\sqrt{-a^2+b^2}}+\frac{a^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-6a^5b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+15a^4b^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(24*(6*a^5*b + 23*a^3*b^3 + 6*a*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/
sqrt(-a^2 + b^2)))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt(-a
^2 + b^2)) + (a^6*tan(1/2*d*x + 1/2*c)^3 - 6*a^5*b*tan(1/2*d*x + 1/2*c)^3 +
 15*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 - 20*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 15
*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^5*tan(1/2*d*x + 1/2*c)^3 + b^6*tan(
1/2*d*x + 1/2*c)^3 + 9*a^6*tan(1/2*d*x + 1/2*c) - 36*a^5*b*tan(1/2*d*x + 1/
2*c) + 45*a^4*b^2*tan(1/2*d*x + 1/2*c) - 45*a^2*b^4*tan(1/2*d*x + 1/2*c) +
36*a*b^5*tan(1/2*d*x + 1/2*c) - 9*b^6*tan(1/2*d*x + 1/2*c))/(a^9 - 9*a^8*b
+ 36*a^7*b^2 - 84*a^6*b^3 + 126*a^5*b^4 - 126*a^4*b^5 + 84*a^3*b^6 - 36*a^2
```

```
*b^7 + 9*a*b^8 - b^9) - 24*(6*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*a^4*b^3*ta
n(1/2*d*x + 1/2*c)^3 + 5*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 6*a^2*b^5*tan(1/2
*d*x + 1/2*c)^3 - 6*a^5*b^2*tan(1/2*d*x + 1/2*c) - 5*a^4*b^3*tan(1/2*d*x +
1/2*c) - 5*a^3*b^4*tan(1/2*d*x + 1/2*c) - 6*a^2*b^5*tan(1/2*d*x + 1/2*c))/(
(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a*tan(1/2*d*x + 1/2*c)^2 -
 b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) - (9*a*tan(1/2*d*x + 1/2*c)^2 - 9*b*t
an(1/2*d*x + 1/2*c)^2 + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)
*tan(1/2*d*x + 1/2*c)^3))/d
```

maple [A] time = 0.62, size = 328, normalized size = 0.64

$$\frac{\frac{\frac{a\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b}{3}+3a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{8\left(a^{3}-3a^{2}b+3b^{2}a-b^{3}\right)(a-b)}+\frac{2ab\left(\frac{\left(\frac{5}{2}b^{2}a^{3}+3ab^{4}-3ba^{4}-\frac{5}{2}a^{2}b^{3}\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{5}{2}b^{2}a^{3}+3ab^{4}+3ba^{4}+\frac{5}{2}a^{2}b^{3}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)b-a-b\right)^{2}}-\cdots\right)}{(a-b)^{4}(a+b)^{4}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(1/8/(a^3-3*a^2*b+3*a*b^2-b^3)/(a-b)*(1/3*a*tan(1/2*d*x+1/2*c)^3-1/3*ta
n(1/2*d*x+1/2*c)^3*b+3*a*tan(1/2*d*x+1/2*c)+3*tan(1/2*d*x+1/2*c)*b)+2*a*b/(
a-b)^4/(a+b)^4*(((5/2*b^2*a^3+3*a*b^4-3*b*a^4-5/2*a^2*b^3)*tan(1/2*d*x+1/2*
c)^3+(5/2*b^2*a^3+3*a*b^4+3*b*a^4+5/2*a^2*b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1
/2*d*x+1/2*c)^2-tan(1/2*d*x+1/2*c)^2*b-a-b)^2-1/2*(6*a^4+23*a^2*b^2+6*b^4)/
((a-b)*(a+b))^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*(a-b)/((a-b)*(a+b))^(1/2)))-
1/24/(a+b)^3/tan(1/2*d*x+1/2*c)^3-1/8/(a+b)^4*(3*a-3*b)/tan(1/2*d*x+1/2*c))
```

maxima [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more details)Is 4*a^2-4*b^2 positive or negative?
```

mupad [B] time = 1.78, size = 588, normalized size = 1.14

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}}{24\,d\,(a-b)^{3}}+\frac{\frac{a^{4}-4\,a^{3}\,b+6\,a^{2}\,b^{2}-4\,a\,b^{3}+b^{4}}{3\,(a+b)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{6}\left(3\,a^{7}-21\,a^{6}\,b+111\,a^{5}\,b^{2}-145\,a^{4}\,b^{3}+145\,a^{3}\,b^{4}-111\,a\cdots\right)}{(a+b)^{4}}\cdots}{d\left(\left(-8\,a^{6}+48\,a^{5}\,b-120\,a^{4}\,b^{2}+160\,a^{3}\,b^{3}-120\,a^{2}\,b^{4}+48\,a\,b^{5}-8\,b^{6}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}+\left(16\,a^{6}-\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + b/cos(c + d*x))^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(24*d*(a - b)^3) + ((a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6
*a^2*b^2)/(3*(a + b)) + (tan(c/2 + (d*x)/2)^6*(21*a*b^6 - 21*a^6*b + 3*a^7
- 3*b^7 - 111*a^2*b^5 + 145*a^3*b^4 - 145*a^4*b^3 + 111*a^5*b^2))/(a + b)^4
 - (tan(c/2 + (d*x)/2)^4*(17*a^6 - 102*a^5*b - 102*a*b^5 + 17*b^6 + 399*a^2
*b^4 - 364*a^3*b^3 + 399*a^4*b^2))/(3*(a + b)^3) + (7*tan(c/2 + (d*x)/2)^2*
(a - b)*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))/(3*(a + b)^2))/(d*(tan
```

```
(c/2 + (d*x)/2)^3*(16*a*b^5 + 16*a^5*b - 8*a^6 - 8*b^6 + 8*a^2*b^4 - 32*a^3
*b^3 + 8*a^4*b^2) - tan(c/2 + (d*x)/2)^7*(8*a^6 - 48*a^5*b - 48*a*b^5 + 8*b
^6 + 120*a^2*b^4 - 160*a^3*b^3 + 120*a^4*b^2) + tan(c/2 + (d*x)/2)^5*(64*a*
b^5 - 64*a^5*b + 16*a^6 - 16*b^6 - 80*a^2*b^4 + 80*a^4*b^2))) + (3*tan(c/2
+ (d*x)/2)*(a + b))/(8*d*(a - b)^4) + (a*b*atan((a^8*tan(c/2 + (d*x)/2)*1i
+ b^8*tan(c/2 + (d*x)/2)*1i - a^2*b^6*tan(c/2 + (d*x)/2)*4i + a^4*b^4*tan(c
/2 + (d*x)/2)*6i - a^6*b^2*tan(c/2 + (d*x)/2)*4i)/((a + b)^(9/2)*(a - b)^(7
/2)))*(6*a^4 + 6*b^4 + 23*a^2*b^2)*1i)/(d*(a + b)^(9/2)*(a - b)^(9/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(c + dx)}{(a + b\sec(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**4/(a + b*sec(c + d*x))**3, x)
```

3.233 $\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=516

$$\frac{2e(e\sin(c+dx))^{5/2}(7b-5a\cos(c+dx))}{35a^2d}-\frac{be^{7/2}\left(a^2-b^2\right)^{5/4}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{9/2}d}-\frac{be^{7/2}\left(a^2-b^2\right)^{5/4}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e}}{\sqrt{e}}\right.}{a^{9/2}d}$$

```
[Out] -b*(a^2-b^2)^(5/4)*e^(7/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1
/4)/e^(1/2))/a^(9/2)/d-b*(a^2-b^2)^(5/4)*e^(7/2)*arctanh(a^(1/2)*(e*sin(d*x
+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(9/2)/d+2/35*e*(7*b-5*a*cos(d*x+c))*(
e*sin(d*x+c))^(5/2)/a^2/d-2/21*(5*a^4-28*a^2*b^2+21*b^4)*e^4*(sin(1/2*c+1/4
*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+
1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^5/d/(e*sin(d*x+c))^(1/2)-b^2*(a^2-b^2)
^2*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipt
icPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^
(1/2)/a^5/d/(a^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-b^2*(a^2-b^2)^
2*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipti
cPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(
1/2)/a^5/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2/21*e^3*(21*b*
(a^2-b^2)-a*(5*a^2-7*b^2)*cos(d*x+c))*(e*sin(d*x+c))^(1/2)/a^4/d
```

Rubi [A] time = 1.70, antiderivative size = 516, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3872, 2865, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{be^{7/2}\left(a^2-b^2\right)^{5/4}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{9/2}d}-\frac{be^{7/2}\left(a^2-b^2\right)^{5/4}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{9/2}d}+\frac{2e^3\sqrt{e\sin(c+dx)}\left(21b\left(a^2-\right.\right.}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(7/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*(a^2 - b^2)^(5/4)*e^(7/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2
- b^2)^(1/4)*Sqrt[e])])/(a^(9/2)*d)) - (b*(a^2 - b^2)^(5/4)*e^(7/2)*ArcTanh
[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(9/2)*d) +
 (2*(5*a^4 - 28*a^2*b^2 + 21*b^4)*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt
[Sin[c + d*x]])/(21*a^5*d*Sqrt[e*Sin[c + d*x]]) + (b^2*(a^2 - b^2)^2*e^4*El
lipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d
*x]])/(a^5*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (b^2*(
a^2 - b^2)^2*e^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2
, 2]*Sqrt[Sin[c + d*x]])/(a^5*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[
c + d*x]]) + (2*e^3*(21*b*(a^2 - b^2) - a*(5*a^2 - 7*b^2)*Cos[c + d*x])*Sqr
t[e*Sin[c + d*x]])/(21*a^4*d) + (2*e*(7*b - 5*a*Cos[c + d*x])*(e*Sin[c + d*
x])^(5/2))/(35*a^2*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
```

```
0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{2e(7b-5a\cos(c+dx))(e\sin(c+dx))^{5/2}}{35a^2d} - \frac{\left(2e^2\right)\int \frac{\left(-ab+\frac{1}{2}\left(5a^2-7b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{3/2}}{-b-a\cos(c+dx)}\,c}{7a^2}$$

$$= \frac{2e^3\left(21b\left(a^2-b^2\right)-a\left(5a^2-7b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{21a^4d} + \frac{2e(7b-5a\cos(c+dx}{35a^2}$$

$$= \frac{2e^3\left(21b\left(a^2-b^2\right)-a\left(5a^2-7b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{21a^4d} + \frac{2e(7b-5a\cos(c+dx}{35a^2}$$

$$= \frac{2e^3\left(21b\left(a^2-b^2\right)-a\left(5a^2-7b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{21a^4d} + \frac{2e(7b-5a\cos(c+dx}{35a^2}$$

$$= \frac{2\left(5a^4-28a^2b^2+21b^4\right)e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21a^5d\sqrt{e\sin(c+dx)}} + \frac{2e^3\left(21b\left(a^2-b^2\right)-a\right.}{}$$

$$= \frac{2\left(5a^4-28a^2b^2+21b^4\right)e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21a^5d\sqrt{e\sin(c+dx)}} - \frac{b^2\left(a^2-b^2\right)^{3/2}e^4\Pi\left(\frac{}{a-}\right.}{a^5\left(a-\right.}$$

$$= -\frac{b\left(a^2-b^2\right)^{5/4}e^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{9/2}d} - \frac{b\left(a^2-b^2\right)^{5/4}e^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{9/2}d} +$$

Mathematica [C] time = 17.57, size = 2049, normalized size = 3.97

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(7/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((b + a*Cos[c + d*x])*(-1/42*((23*a^2 - 28*b^2)*Cos[c + d*x])/a^3 - (b*Cos[
2*(c + d*x)])/(5*a^2) + Cos[3*(c + d*x)]/(14*a))*Csc[c + d*x]^3*Sec[c + d*x
]*(e*Sin[c + d*x])^(7/2))/(d*(a + b*Sec[c + d*x])) - ((b + a*Cos[c + d*x])*
```

```
Sec[c + d*x]*(e*Sin[c + d*x])^(7/2)*((2*(-100*a^3 + 98*a*b^2)*Cos[c + d*x]^
2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt
[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Si
n[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(
-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^
2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]
]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^2)*AppellF1[1/4,
 -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c
 + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5
/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5
/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (-a^2
 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos
[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(89*a^2*b - 70*b^3)*Cos[c + d*x]*(b +
 a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 + I)*
Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt
[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I)*
Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] - Log[Sqrt
[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Si
n[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/
4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/(S
qrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d
*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, 1/2, 2, 9
/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*AppellF
1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[
c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*Sqrt
[1 - Sin[c + d*x]^2]) + ((-231*a^2*b + 210*b^3)*Cos[c + d*x]*Cos[2*(c + d*x
)]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[1 -
((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2 - b^
2)^(3/4)) - ((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin
[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2 - b^2)^(3/4)) + ((1/4 - I/4)*
(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[
Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) - ((1/4 - I/
4)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sq
rt[Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) + (4*Sqrt
[Sin[c + d*x]])/a + (4*b*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Si
n[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*b*(a^2
 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*Appe
llF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] +
2*(2*a^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a
^2 - b^2)] + (a^2 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Si
n[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2
)))))/((b + a*Cos[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1 - Sin[c + d*x]^2]
)))/(420*a^3*d*(a + b*Sec[c + d*x])*Sin[c + d*x]^(7/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(7/2)/(b*sec(d*x + c) + a), x)
```

maple [B] time = 12.59, size = 1776, normalized size = 3.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/5/d*b*e/a^2*(e*sin(d*x+c))^(5/2)+2/d*b*e^3/a^2*(e*sin(d*x+c))^(1/2)-2/d*b
^3*e^3/a^4*(e*sin(d*x+c))^(1/2)+1/d*b*e^5*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e
^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-2/d*b^3*
e^5/a^2*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c))^
(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+1/d*b^5*e^5/a^4*(e^2*(a^2-b^2)/a^2)^(1/4)/
(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+1
/2/d*b*e^5*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^
(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^
(1/4)))-1/d*b^3*e^5/a^2*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e
*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a
^2-b^2)/a^2)^(1/4)))+1/2/d*b^5*e^5/a^4*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+
b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c)
)^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+2/7/d*e^4/a*cos(d*x+c)^3/(e*sin(d*x+c))
^(1/2)*sin(d*x+c)-5/21/d*e^4/a/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)
+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)
^(1/2),1/2*2^(1/2))+4/3/d*e^4/a^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2*(-sin
(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*
x+c)+1)^(1/2),1/2*2^(1/2))-1/d*e^4/a^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4*
(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-s
in(d*x+c)+1)^(1/2),1/2*2^(1/2))-16/21/d*e^4/a*cos(d*x+c)/(e*sin(d*x+c))^(1/
2)*sin(d*x+c)+2/3/d*e^4/a^3*cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2*sin(d*x+c)+
1/2/d*e^4/a^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a^2-b^2)^(1/2)*(-sin(d*x
+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*
EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/d*e
^4/a^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*Ellipti
cPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/2/d*e^4/a^
6/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^6/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)
*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((
-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/d*e^4/a^2/cos
(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*s
in(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(
d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/d*e^4/a^4/cos(d*x+c)
/(e*sin(d*x+c))^(1/2)*b^4/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+
1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/d*e^4/a^6/cos(d*x+c)/(e*s
in(d*x+c))^(1/2)*b^6/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)
^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1
/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^(7/2)/(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)\,{\left(e\,\sin\left(c+d\,x\right)\right)}^{7/2}}{b+a\,\cos\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(7/2)/(a + b/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(7/2))/(b + a*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(7/2)/(a+b*sec(d*x+c)),x)
[Out] Timed out
```

3.234 $\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=430

$$\frac{2e(e\sin(c+dx))^{3/2}(5b-3a\cos(c+dx))}{15a^2d}+\frac{be^{5/2}\left(a^2-b^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{7/2}d}-\frac{be^{5/2}\left(a^2-b^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\ldots}}{\sqrt{e}\ldots}\right)}{a^{7/2}d}$$

```
[Out] b*(a^2-b^2)^(3/4)*e^(5/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/
4)/e^(1/2))/a^(7/2)/d-b*(a^2-b^2)^(3/4)*e^(5/2)*arctanh(a^(1/2)*(e*sin(d*x+
c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(7/2)/d+2/15*e*(5*b-3*a*cos(d*x+c))*(e
*sin(d*x+c))^(3/2)/a^2/d+b^2*(a^2-b^2)*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(
a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^4/d/(a-(a^2-b^2)^(1/2))/(e*sin(
d*x+c))^(1/2)+b^2*(a^2-b^2)*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2
*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1
/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^4/d/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/
2)-2/5*(3*a^2-5*b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/
2)/a^3/d/sin(d*x+c)^(1/2)
```

Rubi [A] time = 1.11, antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3872, 2865, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{be^{5/2}\left(a^2-b^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{7/2}d}-\frac{be^{5/2}\left(a^2-b^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{7/2}d}+\frac{2e^2\left(3a^2-5b^2\right)E\left(\frac{1}{2}\left(c+dx-\ldots\right.\right.}{5a^3d\sqrt{\sin(c+\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*(a^2 - b^2)^(3/4)*e^(5/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 -
b^2)^(1/4)*Sqrt[e])])/(a^(7/2)*d) - (b*(a^2 - b^2)^(3/4)*e^(5/2)*ArcTanh[(S
qrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(7/2)*d) - (b
^2*(a^2 - b^2)*e^3*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)
/2, 2]*Sqrt[Sin[c + d*x]])/(a^4*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]
]) - (b^2*(a^2 - b^2)*e^3*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2
 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^4*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c
 + d*x]]) + (2*(3*a^2 - 5*b^2)*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*
Sin[c + d*x]])/(5*a^3*d*Sqrt[Sin[c + d*x]]) + (2*e*(5*b - 3*a*Cos[c + d*x])
*(e*Sin[c + d*x])^(3/2))/(15*a^2*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{2e(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^2d} - \frac{\left(2e^2\right)\int \frac{\left(-ab+\frac{1}{2}\left(3a^2-5b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}\,dx}{5a^2}$$

$$= \frac{2e(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^2d} + \frac{\left(\left(3a^2-5b^2\right)e^2\right)\int \sqrt{e\sin(c+dx)}\,dx}{5a^3} + \frac{(b(}{}$$

$$= \frac{2e(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^2d} + \frac{\left(b^2\left(a^2-b^2\right)e^3\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+\right.}}{2a^4}$$

$$= \frac{2\left(3a^2-5b^2\right)e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^3d\sqrt{\sin(c+dx)}} + \frac{2e(5b-3a\cos(c+dx))(e\sin(c}{15a^2d}$$

$$= -\frac{b^2\left(a^2-b^2\right)e^3\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{a^4\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{b^2\left(a^2-b^2\right)e^3\Pi\left(\frac{2a}{a+\sqrt{a^2}}\right.}{a^4\left(a+\sqrt{a}\right.}$$

$$= \frac{b\left(a^2-b^2\right)^{3/4}e^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{7/2}d} - \frac{b\left(a^2-b^2\right)^{3/4}e^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{7/2}d} - \frac{b}{}$$

Mathematica [C] time = 15.04, size = 853, normalized size = 1.98

$$\frac{(b+a\cos(c+dx))\csc^2(c+dx)\sec(c+dx)(e\sin(c+dx))^{5/2}\left(\frac{2b\sin(c+dx)}{3a^2}-\frac{\sin(2(c+dx))}{5a}\right)}{d(a+b\sec(c+dx))} - \frac{(b+a\cos(c+dx))\sec(c+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/5*((b + a*Cos[c + d*x])*Sec[c + d*x]*(e*Sin[c + d*x])^(5/2)*(((-3*a^2 +
5*b^2)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2
]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*S
qrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqr
```

```
t[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[
Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (a
^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*Sqrt[1 - Sin[c +
 d*x]^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2
)) + (4*a*b*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[
Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[
c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2
- b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] +
 (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]]))
/(Sqrt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2
, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))/(3*(-a^2 + b^2)))*(
b + a*Sqrt[1 - Sin[c + d*x]^2]))/((b + a*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x
]^2])))/(a^2*d*(a + b*Sec[c + d*x])*Sin[c + d*x]^(5/2)) + ((b + a*Cos[c + d
*x])*Csc[c + d*x]^2*Sec[c + d*x]*(e*Sin[c + d*x])^(5/2)*((2*b*Sin[c + d*x])
/(3*a^2) - Sin[2*(c + d*x)]/(5*a)))/(d*(a + b*Sec[c + d*x]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \sin(dx+c))^{\frac{5}{2}}}{b \sec(dx+c)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(5/2)/(b*sec(d*x + c) + a), x)
```

maple [B] time = 10.73, size = 1195, normalized size = 2.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/3/d*b*e/a^2*(e*sin(d*x+c))^(3/2)-1/2/d*b*e^3/a^2/(e^2*(a^2-b^2)/a^2)^(1/4
)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)
-(e^2*(a^2-b^2)/a^2)^(1/4)))+1/2/d*b^3*e^3/a^4/(e^2*(a^2-b^2)/a^2)^(1/4)*ln
(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^
2*(a^2-b^2)/a^2)^(1/4)))+1/d*b*e^3/a^2/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*
sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-1/d*b^3*e^3/a^4/(e^2*(a^2-b^2)
/a^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+2/5/d/a*
e^3*cos(d*x+c)^3/(e*sin(d*x+c))^(1/2)-6/5/d/a*e^3/cos(d*x+c)/(e*sin(d*x+c))
^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ellipt
icE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3/5/d/a*e^3/cos(d*x+c)/(e*sin(d*x+c)
)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Ellip
ticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2/5/d/a*e^3*cos(d*x+c)/(e*sin(d*x+c
))^(1/2)+2/d/a^3*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2*(-sin(d*x+c)+1)^(1
/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2)
,1/2*2^(1/2))-1/d/a^3*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2*(-sin(d*x+c)+
1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^
```

```
(1/2),1/2*2^(1/2))+1/2/d/a^3*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2*(-sin(
d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/
a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/
2/d/a^5*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4*(-sin(d*x+c)+1)^(1/2)*(2*si
n(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d
*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/2/d/a^3*e^3/cos(d*x+c
)/(e*sin(d*x+c))^(1/2)*b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin
(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(
a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/d/a^5*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2
)*b^4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2
-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2
*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^(5/2)/(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)\,\left(e\,\sin(c+d\,x)\right)^{5/2}}{b+a\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(5/2)/(a + b/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(5/2))/(b + a*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(5/2)/(a+b*sec(d*x+c)),x)
[Out] Timed out
```

3.235 $\int \frac{(e\sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=444

$$\frac{2e\sqrt{e\sin(c+dx)}\,(3b-a\cos(c+dx))}{3a^2d}-\frac{be^{3/2}\sqrt[4]{a^2-b^2}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{a^2-b^2}}\right)}{a^{5/2}d}-\frac{be^{3/2}\sqrt[4]{a^2-b^2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+}}{\sqrt{e}\,\sqrt[4]{a^2-b}}\right.}{a^{5/2}d}$$

```
[Out] -b*(a^2-b^2)^(1/4)*e^(3/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1
/4)/e^(1/2))/a^(5/2)/d-b*(a^2-b^2)^(1/4)*e^(3/2)*arctanh(a^(1/2)*(e*sin(d*x
+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(5/2)/d-2/3*(a^2-3*b^2)*e^2*(sin(1/2*
c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/
4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^3/d/(e*sin(d*x+c))^(1/2)-b^2*(a^2
-b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ell
ipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+
c)^(1/2)/a^3/d/(a^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-b^2*(a^2-b^
2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipt
icPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^
(1/2)/a^3/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2/3*e*(3*b-a*c
os(d*x+c))*(e*sin(d*x+c))^(1/2)/a^2/d
```

Rubi [A] time = 1.04, antiderivative size = 444, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3872, 2865, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{be^{3/2}\sqrt[4]{a^2-b^2}\,\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{a^2-b^2}}\right)}{a^{5/2}d}-\frac{be^{3/2}\sqrt[4]{a^2-b^2}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{a^2-b^2}}\right)}{a^{5/2}d}+\frac{2e^2\left(a^2-3b^2\right)\sqrt{\sin(c+dx)}\,F\left(\right.}{3a^3d\sqrt{e\sin(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*(a^2 - b^2)^(1/4)*e^(3/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2
- b^2)^(1/4)*Sqrt[e])])/(a^(5/2)*d)) - (b*(a^2 - b^2)^(1/4)*e^(3/2)*ArcTanh
[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(5/2)*d) +
 (2*(a^2 - 3*b^2)*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/
(3*a^3*d*Sqrt[e*Sin[c + d*x]]) + (b^2*(a^2 - b^2)*e^2*EllipticPi[(2*a)/(a -
 Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^3*(a^2 - b
^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (b^2*(a^2 - b^2)*e^2*Elli
pticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x
]])/(a^3*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*e*(3*
b - a*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*a^2*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{2e(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^2d} - \frac{\left(2e^2\right)\int \frac{-ab+\frac{1}{2}\left(a^2-3b^2\right)\cos(c+dx)}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{3a^2}$$

$$= \frac{2e(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^2d} + \frac{\left(\left(a^2-3b^2\right)e^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3a^3} + \frac{\left(b\left(a^2-b\right.\right.}{}$$

$$= \frac{2e(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^2d} + \frac{\left(b^2\sqrt{a^2-b^2}\,e^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+\right.}}{2a^3}$$

$$= \frac{2\left(a^2-3b^2\right)e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3a^3d\sqrt{e\sin(c+dx)}} + \frac{2e(3b-a\cos(c+dx))\sqrt{e\sin(c}}{3a^2d}$$

$$= \frac{2\left(a^2-3b^2\right)e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3a^3d\sqrt{e\sin(c+dx)}} - \frac{b^2\sqrt{a^2-b^2}\,e^2\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c\right.\right.}{a^3\left(a-\sqrt{a^2-b^2}\right)c}$$

$$= -\frac{b\sqrt[4]{a^2-b^2}\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}d} - \frac{b\sqrt[4]{a^2-b^2}\,e^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}d} + \frac{2\left(\right.}{}$$

Mathematica [C] time = 17.35, size = 1959, normalized size = 4.41

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*Csc[c + d*x]*(e*Sin[c + d*x])^(3/2))/(3*a*d*(a + b
*Sec[c + d*x])) + ((b + a*Cos[c + d*x])*Sec[c + d*x]*(e*Sin[c + d*x])^(3/2)
*((4*a*Cos[c + d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (
Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqr
t[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2]
 - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]
+ Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*
x]] + a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2
- b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*Appe
```

```
lF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] +
 2*(2*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/
(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2
*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x
]^2)))))/((b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) - (2*b*Cos[c + d*x]*(b
 + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 + I
)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sq
rt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I
)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] - Log[Sq
rt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*
Sin[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1,
5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/
(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c +
 d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, 1/2, 2,
 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*Appel
lF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Si
n[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*Sq
rt[1 - Sin[c + d*x]^2]) + (3*b*Cos[c + d*x]*Cos[2*(c + d*x)]*(b + a*Sqrt[1
- Sin[c + d*x]^2])*(((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[1 - ((1 + I)*Sqrt[a]*
Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2 - b^2)^(3/4)) - ((1/2
 - I/2)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2
- b^2)^(1/4)])/(a^(3/2)*(a^2 - b^2)^(3/4)) + ((1/4 - I/4)*(a^2 - 2*b^2)*Log
[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I
*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) - ((1/4 - I/4)*(a^2 - 2*b^2)*
Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]]
+ I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) + (4*Sqrt[Sin[c + d*x]])/a
 + (4*b*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*b*(a^2 - b^2)*AppellF1[
1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Si
n[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1,
 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1
[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2
 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos
[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1 - Sin[c + d*x]^2])))/(6*a*d*(a + b
*Sec[c + d*x])*Sin[c + d*x]^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \sin(dx+c))^{\frac{3}{2}}}{b \sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^(3/2)/(b*sec(d*x + c) + a), x)
```

maple [B] time = 10.48, size = 1120, normalized size = 2.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d*e*b/a^2*(e*sin(d*x+c))^(1/2)+1/d*e^3*b*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*
e^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-1/d*e^3
*b^3/a^2*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c))
^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+1/2/d*e^3*b*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a
^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin
(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-1/2/d*e^3*b^3/a^2*(e^2*(a^2-b^2)
/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)
^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-1/3/d/a*e^2/cos(d
*x+c)/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin
(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2/3/d/a*e^2*cos(
d*x+c)/(e*sin(d*x+c))^(1/2)*sin(d*x+c)+1/d/a^3*e^2/cos(d*x+c)/(e*sin(d*x+c)
)^(1/2)*b^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*E
llipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+1/2/d/a^2*e^2/cos(d*x+c)/(e*sin
(d*x+c))^(1/2)*b^2/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(
1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2
),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/d/a^4*e^2/cos(d*x+c)/(e*sin(d*x+
c))^(1/2)*b^4/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(
1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/d/a^2*e^2/cos(d*x+c)/(e*sin(d*x+c))^(
1/2)*b^2/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d
*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^
2-b^2)^(1/2)/a),1/2*2^(1/2))+1/2/d/a^4*e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*
b^4/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)
^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2
)^(1/2)/a),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^(3/2)/(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)\,\left(e\,\sin(c+d\,x)\right)^{3/2}}{b+a\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(3/2)/(a + b/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(3/2))/(b + a*cos(c + d*x)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.236 $\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=356

$$-\frac{b^2 e\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{a^2 d\left(a-\sqrt{a^2-b^2}\right)\sqrt{e\sin(c+dx)}}-\frac{b^2 e\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{a^2 d\left(\sqrt{a^2-b^2}+a\right)\sqrt{e\sin(c+dx)}}+\frac{b\sqrt{e}\tan^{-1}}{a^{3/2}d}$$

```
[Out] b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*e^(1/2)/a^(3
/2)/(a^2-b^2)^(1/4)/d-b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4
)/e^(1/2))*e^(1/2)/a^(3/2)/(a^2-b^2)^(1/4)/d+b^2*e*(sin(1/2*c+1/4*Pi+1/2*d*
x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),
2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(a-(a^2-b^2)^(1/2))
/(e*sin(d*x+c))^(1/2)+b^2*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2))
,2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2
*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(co
s(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a/d/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.76, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3872, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{3/2}d\sqrt[4]{a^2-b^2}}-\frac{b\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{3/2}d\sqrt[4]{a^2-b^2}}-\frac{b^2 e\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{a^2 d\left(a-\sqrt{a^2-b^2}\right)\sqrt{e\sin(c+dx)}}-\frac{b^2 e}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sin[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*Sqrt[e]*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e]
)])/(a^(3/2)*(a^2 - b^2)^(1/4)*d) - (b*Sqrt[e]*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[
c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(3/2)*(a^2 - b^2)^(1/4)*d) - (b
^2*e*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Si
n[c + d*x]])/(a^2*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (b^2*e*El
lipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d
*x]])/(a^2*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*EllipticE[(c
- Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a*d*Sqrt[Sin[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}\,dx$$
$$= \frac{\int \sqrt{e\sin(c+dx)}\,dx}{a} + \frac{b\int \frac{\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}\,dx}{a}$$
$$= \frac{\left(b^2e\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2a^2} - \frac{\left(b^2e\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{2a^2} + \text{(be}$$
$$= \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}} + \frac{(2be)\,\text{Subst}\left(\int \frac{x^2}{\left(-a^2+b^2\right)e^2+a^2x^4}\,dx, x, \sqrt{e\sin(c+\right.}{d}$$
$$= -\frac{b^2e\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}}; \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{a^2\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{b^2e\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}}; \frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right.}{a^2\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin}}$$
$$= \frac{b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\sqrt[4]{a^2-b^2}\,d} - \frac{b\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\sqrt[4]{a^2-b^2}\,d} - \frac{b^2e\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}}; \frac{1}{2}\left(c-\frac{\pi}{2}\right.\right.}{a^2\left(a-\sqrt{a^2-b^2}\right)}$$

Mathematica [C] time = 20.75, size = 351, normalized size = 0.99

$$\frac{\sqrt{e\sin(c+dx)}\left(a\sqrt{\cos^2(c+dx)}+b\right)\left(3\sqrt{2}b\left(b^2-a^2\right)^{3/4}\left(-\log\left(-\sqrt{2}\sqrt{a}\sqrt[4]{b^2-a^2}\sqrt{\sin(c+dx)}+\sqrt{b^2-a^2}+a\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Sin[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((b + a*Sqrt[Cos[c + d*x]^2])*Sqrt[e*Sin[c + d*x]]*(3*Sqrt[2]*b*(-a^2 + b^2
)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/
4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)]
 - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d
*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2
)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/
2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(
3/2)))/(12*a^(3/2)*(a^2 - b^2)*d*(b + a*Cos[c + d*x])*Sqrt[Sin[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\sin(dx+c)}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

[Out] integrate(sqrt(e*sin(d*x + c))/(b*sec(d*x + c) + a), x)

maple [B] time = 6.69, size = 919, normalized size = 2.58

$$\frac{eb\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}\right)}{d\,a^2\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}-\frac{eb\ln\left(\frac{\sqrt{e\sin(dx+c)}+\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}{\sqrt{e\sin(dx+c)}-\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}\right)}{2d\,a^2\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}-\frac{e\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+c)+2}\,\left(\sqrt{\sin}\,(dx+\right.}{2d\,a^2\left(-a+\sqrt{a^2-b^2}\right)\left(a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x)

[Out] 1/d*e*b/a^2/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-1/2/d*e*b/a^2/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-1/2/d*e*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*b^2/a^2/(-a+(a^2-b^2)^(1/2))/(a+(a^2-b^2)^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-a/(-a+(a^2-b^2)^(1/2)),1/2*2^(1/2))*(a^2-b^2)^(1/2)+1/2/d*e*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*b^2/a^2/(-a+(a^2-b^2)^(1/2))/(a+(a^2-b^2)^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),1/2*2^(1/2))*(a^2-b^2)^(1/2)+2/d*e*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*b^2/a/(-a+(a^2-b^2)^(1/2))/(a+(a^2-b^2)^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/d*e*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*b^2/a/(-a+(a^2-b^2)^(1/2))/(a+(a^2-b^2)^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/2/d*e*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*b^2/a/(-a+(a^2-b^2)^(1/2))/(a+(a^2-b^2)^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-a/(-a+(a^2-b^2)^(1/2)),1/2*2^(1/2))-1/2/d*e*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*b^2/a/(-a+(a^2-b^2)^(1/2))/(a+(a^2-b^2)^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),1/2*2^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\sin(dx+c)}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate(sqrt(e*sin(d*x + c))/(b*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)\,\sqrt{e\,\sin(c+d\,x)}}{b+a\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*sin(c + d*x))^(1/2)/(a + b/cos(c + d*x)),x)

[Out] int((cos(c + d*x)*(e*sin(c + d*x))^(1/2))/(b + a*cos(c + d*x)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(1/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(e*sin(c + d*x))/(a + b*sec(c + d*x)), x)
```

3.237 $\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=370

$$-\frac{b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{\sqrt{a}\,d\sqrt{e}\left(a^2-b^2\right)^{3/4}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{\sqrt{a}\,d\sqrt{e}\left(a^2-b^2\right)^{3/4}}+\frac{b^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{ad\left(-a\sqrt{a^2-b^2}+a^2-b^2\right)\sqrt{e\sin(c+dx)}}+\frac{b^2\sqrt{\sin}}{ad\left(\right.}$$

```
[Out] -b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a^2-b^2)^(
3/4)/d/a^(1/2)/e^(1/2)-b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/
4)/e^(1/2))/(a^2-b^2)^(3/4)/d/a^(1/2)/e^(1/2)-2*(sin(1/2*c+1/4*Pi+1/2*d*x)^
2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1
/2))*sin(d*x+c)^(1/2)/a/d/(e*sin(d*x+c))^(1/2)-b^2*(sin(1/2*c+1/4*Pi+1/2*d*
x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),
2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a/d/(a^2-b^2-a*(a^2-b^2)^
(1/2))/(e*sin(d*x+c))^(1/2)-b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2
*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1
/2)),2^(1/2))*sin(d*x+c)^(1/2)/a/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*sin(d*x+c
))^(1/2)
```

Rubi [A] time = 0.78, antiderivative size = 370, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.440, Rules used = {3872, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{\sqrt{a}\,d\sqrt{e}\left(a^2-b^2\right)^{3/4}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{\sqrt{a}\,d\sqrt{e}\left(a^2-b^2\right)^{3/4}}+\frac{b^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{ad\left(-a\sqrt{a^2-b^2}+a^2-b^2\right)\sqrt{e\sin(c+dx)}}+\frac{b^2\sqrt{\sin}}{ad\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] -((b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(S
qrt[a]*(a^2 - b^2)^(3/4)*d*Sqrt[e])) - (b*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d
*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(Sqrt[a]*(a^2 - b^2)^(3/4)*d*Sqrt[e]) +
 (2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a*d*Sqrt[e*Sin[c
+ d*x]]) + (b^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2,
 2]*Sqrt[Sin[c + d*x]])/(a*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c +
 d*x]]) + (b^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(a*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c +
d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
```

```
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = -\int \frac{\cos(c+dx)}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx$$

$$= \frac{\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{a} + \frac{b\int \frac{1}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{a}$$

$$= \frac{b^2\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2a\sqrt{a^2-b^2}} + \frac{b^2\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(\right.}}{2a\sqrt{a^2-b^2}}$$

$$= \frac{2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{ad\sqrt{e\sin(c+dx)}} + \frac{(2be)\,\mathrm{Subst}\left(\int \frac{1}{(-a^2+b^2)e^2+a^2x}\right.}{d}$$

$$= \frac{2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{ad\sqrt{e\sin(c+dx)}} + \frac{b^2\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+d\right.\right.}{a\left(a^2-b^2-a\sqrt{a^2-b^2}\right)}$$

$$= -\frac{b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} + \frac{2F\left(\frac{1}{2}\left(c-\right.\right.}{ad}$$

Mathematica [C] time = 6.31, size = 546, normalized size = 1.48

$$\frac{2\sqrt{\sin(c+dx)}\left(a\sqrt{\cos^2(c+dx)}+b\right)\left(\frac{b\left(-\log\left(-\sqrt{2}\sqrt{a}\sqrt[4]{b^2-a^2}\sqrt{\sin(c+dx)}+\sqrt{b^2-a^2}+a\sin(c+dx)\right)+\log\left(\sqrt{2}\sqrt{a}\sqrt[4]{b^2-a^2}\sqrt{\sin(c+dx}\right.\right.}{4\sqrt{2}\sqrt{a}\left(b^2-\right.}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (2*(b + a*Sqrt[Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]]*((b*(-2*ArcTan[1 - (Sqrt
[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]
*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - S
qrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Lo
g[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]]
+ a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^
2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 -
b^2)]*Sqrt[Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]])/((-a^2 + b^2 + a^2*Sin[c + d
*x]^2)*(5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[
c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x
]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1,
 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2))))
/(d*(b + a*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)\sqrt{e\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(e*sin(d*x + c))), x)
```

maple [B] time = 7.10, size = 937, normalized size = 2.53

$$\frac{be\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}\ln\left(\frac{\sqrt{e\sin(dx+c)}+\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}{\sqrt{e\sin(dx+c)}-\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}\right)}{2d\left(-a^2e^2+b^2e^2\right)}+\frac{be\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\left(\frac{e^2(a^2-b^2)}{a^2}\right)^{\frac{1}{4}}}\right)}{d\left(-a^2e^2+b^2e^2\right)}-\frac{\sqrt{-\sin(dx+c)+1}\,\sqrt{2\sin(dx+}}{da\left(-a+\sqrt{a^2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*b*e*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(
1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(
1/4)))+1/d*b*e*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d
*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-1/d/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(
d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*(a^2-b^2)/(-a+(a^2-b^2)^(1/2))/(a+(a^2-b^2
)^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/
2*2^(1/2))+1/d*a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1
/2)/(-a+(a^2-b^2)^(1/2))/(a+(a^2-b^2)^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2
)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/2/d/a*(-sin(d*x+c)+1)^(1/2
)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(-a+(a^2-b^2)^(1/2))/(a+(a^2-b^2)
^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-a
/(-a+(a^2-b^2)^(1/2)),1/2*2^(1/2))*b^2-1/2/d*(-sin(d*x+c)+1)^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(a^2-b^2)^(1/2)/(-a+(a^2-b^2)^(1/2))/(a+(a^
2-b^2)^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1
/2),-a/(-a+(a^2-b^2)^(1/2)),1/2*2^(1/2))*b^2-1/2/d/a*(-sin(d*x+c)+1)^(1/2)*
(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(-a+(a^2-b^2)^(1/2))/(a+(a^2-b^2)^(
1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a
+(a^2-b^2)^(1/2)),1/2*2^(1/2))*b^2+1/2/d*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(1/2)/(a^2-b^2)^(1/2)/(-a+(a^2-b^2)^(1/2))/(a+(a^2-b^
2)^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),
a/(a+(a^2-b^2)^(1/2)),1/2*2^(1/2))*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)\sqrt{e\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(e*sin(d*x + c))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)}{\sqrt{e\,\sin(c+d\,x)}\,\left(b+a\,\cos(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(1/2)*(a + b/cos(c + d*x))),x)
```

```
[Out] int(cos(c + d*x)/((e*sin(c + d*x))^(1/2)*(b + a*cos(c + d*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\sin(c+dx)}\,(a+b\sec(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*sin(c + d*x))*(a + b*sec(c + d*x))), x)
```

3.238 $\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=430

$$\frac{\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{a^2-b^2}}\right)}{de^{3/2}\left(a^2-b^2\right)^{5/4}}-\frac{\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{a^2-b^2}}\right)}{de^{3/2}\left(a^2-b^2\right)^{5/4}}-\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\left(a^2-b^2\right)\sqrt{\sin(c+dx)}}+\frac{2(b-a\,\mathrm{co}}{de\left(a^2-b^2\right)}$$

```
[Out] b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*a^(1/2)/(a^2
-b^2)^(5/4)/d/e^(3/2)-b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4
)/e^(1/2))*a^(1/2)/(a^2-b^2)^(5/4)/d/e^(3/2)+2*(b-a*cos(d*x+c))/(a^2-b^2)/d
/e/(e*sin(d*x+c))^(1/2)+b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2))
,2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))
^(1/2)+b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*El
lipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x
+c)^(1/2)/(a^2-b^2)/d/e/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*a*(sin(1
/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c
+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/(a^2-b^2)/d/e^2/sin(d*x+c)^(
1/2)
```

Rubi [A] time = 1.04, antiderivative size = 430, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3872, 2866, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{a^2-b^2}}\right)}{de^{3/2}\left(a^2-b^2\right)^{5/4}}-\frac{\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{e\sin(c+dx)}}{\sqrt{e}\,\sqrt[4]{a^2-b^2}}\right)}{de^{3/2}\left(a^2-b^2\right)^{5/4}}-\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\left(a^2-b^2\right)\sqrt{\sin(c+dx)}}+\frac{2(b-a\,\mathrm{co}}{de\left(a^2-b^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[a]*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e]
)])/((a^2 - b^2)^(5/4)*d*e^(3/2)) - (Sqrt[a]*b*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[
c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(5/4)*d*e^(3/2)) + (2
*(b - a*Cos[c + d*x]))/((a^2 - b^2)*d*e*Sqrt[e*Sin[c + d*x]]) - (b^2*Ellipt
icPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]]
)/((a^2 - b^2)*(a - Sqrt[a^2 - b^2])*d*e*Sqrt[e*Sin[c + d*x]]) - (b^2*Ellip
ticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]
])/((a^2 - b^2)*(a + Sqrt[a^2 - b^2])*d*e*Sqrt[e*Sin[c + d*x]]) - (2*a*Elli
pticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)*d*e^2*Sqrt[
Sin[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = -\int \frac{\cos(c+dx)}{(-b-a\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx$$

$$= \frac{2(b-a\cos(c+dx))}{\left(a^2-b^2\right)de\sqrt{e\sin(c+dx)}} + \frac{2\int \frac{\left(ab+\frac{1}{2}a^2\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}\,dx}{\left(a^2-b^2\right)e^2}$$

$$= \frac{2(b-a\cos(c+dx))}{\left(a^2-b^2\right)de\sqrt{e\sin(c+dx)}} - \frac{a\int \sqrt{e\sin(c+dx)}\,dx}{\left(a^2-b^2\right)e^2} + \frac{(ab)\int \frac{\sqrt{e\sin(c}}{-b-a\cos}}{\left(a^2-b^2\right)}$$

$$= \frac{2(b-a\cos(c+dx))}{\left(a^2-b^2\right)de\sqrt{e\sin(c+dx)}} + \frac{b^2\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2\left(a^2-b^2\right)e} -$$

$$= \frac{2(b-a\cos(c+dx))}{\left(a^2-b^2\right)de\sqrt{e\sin(c+dx)}} - \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{\left(a^2-b^2\right)de^2\sqrt{\sin(c+dx)}}$$

$$= \frac{2(b-a\cos(c+dx))}{\left(a^2-b^2\right)de\sqrt{e\sin(c+dx)}} - \frac{b^2\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin}}{\left(a^2-b^2\right)\left(a-\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c}}$$

$$= \frac{\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{5/4}de^{3/2}} - \frac{\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{5/4}de^{3/2}} + \frac{2(b}{\left(a^2-b\right.}$$

Mathematica [C] time = 14.48, size = 834, normalized size = 1.94

$$-\frac{a(b+a\cos(c+dx))\sec(c+dx)\left(\frac{\left(8F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin^{\frac{3}{2}}(c+dx)a^{5/2}+3\sqrt{2}\,b\left(b^2-a^2\right)^{3/4}\left(2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right.\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] -((a*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[c + d*x]^(3/2)*((Cos[c + d*x]^2*
(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c +
```

```
d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*
x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b
^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqr
t[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]) + 8*a
^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*Sqrt[1 - Sin[c + d*x]^2]))/(12*Sqrt[a]
*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (4*b*Cos[c + d*x]
*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^
2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(
1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c
+ d*x]] + I*a*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 -
b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]]))/(Sqrt[a]*(a^2 - b^2)^(1
/4)) + (b*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(
a^2 - b^2)]*Sin[c + d*x]^(3/2))/(3*(-a^2 + b^2)))*(b + a*Sqrt[1 - Sin[c + d
*x]^2]))/((b + a*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/((a - b)*(a + b)
*d*(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2))) - (2*(b - a*Cos[c + d*x])*
(b + a*Cos[c + d*x])*Tan[c + d*x])/((-a^2 + b^2)*d*(a + b*Sec[c + d*x])*(e*
Sin[c + d*x])^(3/2))
```

fricas [F] time = 175.62, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{e\sin\left(dx+c\right)}}{ae^2\cos\left(dx+c\right)^2-ae^2+\left(be^2\cos\left(dx+c\right)^2-be^2\right)\sec\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*sin(d*x + c))/(a*e^2*cos(d*x + c)^2 - a*e^2 + (b*e^2*cos(d
*x + c)^2 - b*e^2)*sec(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\left(b\sec\left(dx+c\right)+a\right)\left(e\sin\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*sin(d*x + c))^(3/2)), x)
```

maple [B] time = 7.71, size = 1083, normalized size = 2.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/d/e*b/(a-b)/(a+b)/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2)/(
e^2*(a^2-b^2)/a^2)^(1/4))-1/2/d/e*b/(a-b)/(a+b)/(e^2*(a^2-b^2)/a^2)^(1/4)*l
n(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e
^2*(a^2-b^2)/a^2)^(1/4)))+2/d/e*b/(a^2-b^2)/(e*sin(d*x+c))^(1/2)+1/2/d*b^2/
e/(a^2-b^2)^(1/2)/(a+(a^2-b^2)^(1/2))/(-a+(a^2-b^2)^(1/2))/cos(d*x+c)/(e*si
n(d*x+c))^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),1/2*
2^(1/2))*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)-1/2/
d*b^2/e/(a^2-b^2)/(a+(a^2-b^2)^(1/2))/(-a+(a^2-b^2)^(1/2))/cos(d*x+c)/(e*si
n(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/
2)*a*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),1/2*2^(1/2))-2/
d*b^2/e/(a^2-b^2)/(a+(a^2-b^2)^(1/2))/(-a+(a^2-b^2)^(1/2))/cos(d*x+c)/(e*si
```

```
n(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*(-sin(d*x+c)+1
)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*a+1/d*b^2/e/(a^2-b^2)/(a+(a
^2-b^2)^(1/2))/(-a+(a^2-b^2)^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*Ellipti
cF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2
)^(1/2)*sin(d*x+c)^(1/2)*a-1/2/d*b^2/e/(a^2-b^2)^(1/2)/(a+(a^2-b^2)^(1/2))/
(-a+(a^2-b^2)^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*
(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-a
/(-a+(a^2-b^2)^(1/2)),1/2*2^(1/2))-1/2/d*b^2/e/(a^2-b^2)/(a+(a^2-b^2)^(1/2)
)/(-a+(a^2-b^2)^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2
)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*a*EllipticPi((-sin(d*x+c)+1)^(1/2
),-a/(-a+(a^2-b^2)^(1/2)),1/2*2^(1/2))+2/d*b^2/e/(a^2-b^2)/(a+(a^2-b^2)^(1/
2))/(-a+(a^2-b^2)^(1/2))*cos(d*x+c)/(e*sin(d*x+c))^(1/2)*a
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)}{(e\,\sin(c+d\,x))^{3/2}\,(b+a\,\cos(c+d\,x))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(3/2)*(a + b/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/((e*sin(c + d*x))^(3/2)*(b + a*cos(c + d*x))), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\sin(c+dx))^{\frac{3}{2}}\,(a+b\sec(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))**(3/2),x)
[Out] Integral(1/((e*sin(c + d*x))**(3/2)*(a + b*sec(c + d*x))), x)
```

3.239 $\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=452

$$\frac{2a\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3de^2\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}+\frac{ab^2\sqrt{\sin(c+dx)}\,\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{de^2\left(a^2-b^2\right)\left(-a\sqrt{a^2-b^2}+a^2-b^2\right)\sqrt{e\sin(c+dx)}}+\frac{ab^2\sqrt{\sin(c+dx}}{de^2\left(a^2-b^2\right)\left(a\right.}$$

```
[Out] -a^(3/2)*b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a^
2-b^2)^(7/4)/d/e^(5/2)-a^(3/2)*b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-
b^2)^(1/4)/e^(1/2))/(a^2-b^2)^(7/4)/d/e^(5/2)+2/3*(b-a*cos(d*x+c))/(a^2-b^2
)/d/e/(e*sin(d*x+c))^(3/2)-2/3*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/
2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)
^(1/2)/(a^2-b^2)/d/e^2/(e*sin(d*x+c))^(1/2)-a*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x
)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2
*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/(a^2-b^2-a
*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-a*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^
(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a
+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/(a^2-b^2+a*(a^2
-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)
```

Rubi [A] time = 1.05, antiderivative size = 452, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3872, 2866, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{a^{3/2}b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{5/2}\left(a^2-b^2\right)^{7/4}}-\frac{a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{5/2}\left(a^2-b^2\right)^{7/4}}+\frac{2a\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3de^2\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}+\frac{ab^2\sqrt{\sin}}{de^2\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] -((a^(3/2)*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[
e])])/((a^2 - b^2)^(7/4)*d*e^(5/2))) - (a^(3/2)*b*ArcTanh[(Sqrt[a]*Sqrt[e*S
in[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(7/4)*d*e^(5/2)) +
 (2*(b - a*Cos[c + d*x]))/(3*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(3/2)) + (2*a
*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*(a^2 - b^2)*d*e^2*
Sqrt[e*Sin[c + d*x]]) + (a*b^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c -
 Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)*(a^2 - b^2 - a*Sqrt[a^2
 - b^2])*d*e^2*Sqrt[e*Sin[c + d*x]]) + (a*b^2*EllipticPi[(2*a)/(a + Sqrt[a^
2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)*(a^2 - b
^2 + a*Sqrt[a^2 - b^2])*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = -\int \frac{\cos(c+dx)}{(-b-a\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx$$

$$= \frac{2(b-a\cos(c+dx))}{3\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}} + \frac{2\int \frac{ab-\frac{1}{2}a^2\cos(c+dx)}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{3\left(a^2-b^2\right)e^2}$$

$$= \frac{2(b-a\cos(c+dx))}{3\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}} + \frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3\left(a^2-b^2\right)e^2} + \frac{(ab)\int \frac{}{(-b-a}}{(}$$

$$= \frac{2(b-a\cos(c+dx))}{3\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}} + \frac{\left(ab^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c-\right.}}{2\left(a^2-b^2\right)^{3/2}e^2}$$

$$= \frac{2(b-a\cos(c+dx))}{3\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c-}}{3\left(a^2-b^2\right)de^2\sqrt{e\sin(c+dx}}$$

$$= \frac{2(b-a\cos(c+dx))}{3\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c-}}{3\left(a^2-b^2\right)de^2\sqrt{e\sin(c+dx}}$$

$$= -\frac{a^{3/2}b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{7/4}de^{5/2}} - \frac{a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{7/4}de^{5/2}} + \frac{}{3(}$$

Mathematica [C] time = 12.71, size = 1233, normalized size = 2.73

$$-\frac{a(b+a\cos(c+dx))\sec(c+dx)\left(\frac{4b\cos(c+dx)\left(\sqrt{1-\sin^2(c+dx)}\,a+b\right)\left(\frac{5b\left(a^2\right.}{\sqrt{1-\sin^2(c+dx)}\left(2\left(2F_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)a^2+\left(a^2-b^2\right)F_1\right.\right.}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] -1/3*(a*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[c + d*x]^(5/2)*((-2*a*Cos[c +
 d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[a
```

```
]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*S
qrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqr
t[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^
2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c
+ d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^2)*AppellF
1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt
[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*AppellF1[1/4, -1/2
, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*Appe
llF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] +
 (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^
2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b +
 a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (4*b*Cos[c + d*x]*(b + a*Sqrt[1 -
Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt
[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin
[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2
 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] - Log[Sqrt[a^2 - b^2]
+ (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]])
)/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d
*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[
c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*
Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c +
d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*AppellF1[5/4, 3/2,
1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*
(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*Sqrt[1 - Sin[c +
 d*x]^2])))/((a - b)*(a + b)*d*(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2))
 - (2*(b - a*Cos[c + d*x])*(b + a*Cos[c + d*x])*Tan[c + d*x])/(3*(-a^2 + b^
2)*d*(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*sin(d*x + c))^(5/2)), x)
```

maple [A] time = 11.40, size = 681, normalized size = 1.51

$$\frac{2b}{3de\left(a^2-b^2\right)(e\sin(dx+c))^{\frac{3}{2}}}+\frac{b\,a^2\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}\ln\left(\frac{\sqrt{e\sin(dx+c)}+\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}{\sqrt{e\sin(dx+c)}-\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}\right)}{2de(a-b)(a+b)\left(-a^2e^2+b^2e^2\right)}+\frac{b\,a^2\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\left(\frac{e^2\left(a^2-b^2\right)}{a^2}\right)^{\frac{1}{4}}}\right)}{de(a-b)(a+b)\left(-a^2e^2+b^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/3/d*b/e/(a^2-b^2)/(e*sin(d*x+c))^(3/2)+1/2/d*b/e/(a-b)/(a+b)*a^2*(e^2*(a^
2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2
)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+1/d*b/e/(a-
b)/(a+b)*a^2*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x
+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+1/2/d/e^2/cos(d*x+c)/(e*sin(d*x+c))^(
1/2)/(a-b)/(a+b)*b^2/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)
^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1
/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/d/e^2/cos(d*x+c)/(e*sin(d*x+c)
)^(1/2)/(a-b)/(a+b)*b^2/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)
+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)
^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/3/d*a/e^2/cos(d*x+c)/(e*sin(d
*x+c))^(1/2)/(a^2-b^2)/(cos(d*x+c)^2-1)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)
+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2/3
/d*a/e^2*cos(d*x+c)/(e*sin(d*x+c))^(1/2)/(a^2-b^2)/(cos(d*x+c)^2-1)*sin(d*x
+c)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)}{(e\,\sin(c+d\,x))^{5/2}\,\left(b+a\,\cos(c+d\,x)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(5/2)*(a + b/cos(c + d*x))),x)
```

```
[Out] int(cos(c + d*x)/((e*sin(c + d*x))^(5/2)*(b + a*cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.240 $\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=511

$$-\frac{2a\left(3a^2+2b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5de^4\left(a^2-b^2\right)^2\sqrt{\sin(c+dx)}}+\frac{2\left(5a^2b-a\left(3a^2+2b^2\right)\cos(c+dx)\right)}{5de^3\left(a^2-b^2\right)^2\sqrt{e\sin(c+dx)}}-\frac{a^2b^2\sqrt{\sin(c+dx)}\,\Pi\ldots}{de^3\left(a^2-b^2\right)^2\left(a\ldots\right.}$$

```
[Out] a^(5/2)*b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a^2
-b^2)^(9/4)/d/e^(7/2)-a^(5/2)*b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b
^2)^(1/4)/e^(1/2))/(a^2-b^2)^(9/4)/d/e^(7/2)+2/5*(b-a*cos(d*x+c))/(a^2-b^2)
/d/e/(e*sin(d*x+c))^(5/2)+2/5*(5*a^2*b-a*(3*a^2+2*b^2)*cos(d*x+c))/(a^2-b^2
)^2/d/e^3/(e*sin(d*x+c))^(1/2)+a^2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/
sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-
b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^3/(a-(a^2-b^2)^(1/2))
/(e*sin(d*x+c))^(1/2)+a^2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c
+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2
)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^3/(a+(a^2-b^2)^(1/2))/(e*sin(d
*x+c))^(1/2)+2/5*a*(3*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/
2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x
+c))^(1/2)/(a^2-b^2)^2/d/e^4/sin(d*x+c)^(1/2)
```

Rubi [A] time = 1.38, antiderivative size = 511, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3872, 2866, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{a^{5/2}b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{7/2}\left(a^2-b^2\right)^{9/4}}-\frac{a^{5/2}b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{7/2}\left(a^2-b^2\right)^{9/4}}+\frac{2\left(5a^2b-a\left(3a^2+2b^2\right)\cos(c+dx)\right)}{5de^3\left(a^2-b^2\right)^2\sqrt{e\sin(c+dx)}}-\frac{2a\left(3a^2+2b^2\right)\ldots}{5de^4\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(7/2)),x]
```

```
[Out] (a^(5/2)*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e]
)])/((a^2 - b^2)^(9/4)*d*e^(7/2)) - (a^(5/2)*b*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[
c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(9/4)*d*e^(7/2)) + (2
*(b - a*Cos[c + d*x]))/(5*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(5/2)) + (2*(5*a
^2*b - a*(3*a^2 + 2*b^2)*Cos[c + d*x]))/(5*(a^2 - b^2)^2*d*e^3*Sqrt[e*Sin[c
 + d*x]]) - (a^2*b^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*
x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)^2*(a - Sqrt[a^2 - b^2])*d*e^3*Sqr
t[e*Sin[c + d*x]]) - (a^2*b^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c -
Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)^2*(a + Sqrt[a^2 - b^2])*
d*e^3*Sqrt[e*Sin[c + d*x]]) - (2*a*(3*a^2 + 2*b^2)*EllipticE[(c - Pi/2 + d*
x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*(a^2 - b^2)^2*d*e^4*Sqrt[Sin[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx = -\int \frac{\cos(c+dx)}{(-b-a\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} + \frac{2\int \frac{ab-\frac{3}{2}a^2\cos(c+dx)}{(-b-a\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx}{5\left(a^2-b^2\right)e^2}$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} + \frac{2\left(5a^2b-a\left(3a^2+2b^2\right)\cos(c+dx)\right.}{5\left(a^2-b^2\right)^2de^3\sqrt{e\sin(c+dx)}}$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} + \frac{2\left(5a^2b-a\left(3a^2+2b^2\right)\cos(c+dx)\right.}{5\left(a^2-b^2\right)^2de^3\sqrt{e\sin(c+dx)}}$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} + \frac{2\left(5a^2b-a\left(3a^2+2b^2\right)\cos(c+dx)\right.}{5\left(a^2-b^2\right)^2de^3\sqrt{e\sin(c+dx)}}$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} + \frac{2\left(5a^2b-a\left(3a^2+2b^2\right)\cos(c+dx)\right.}{5\left(a^2-b^2\right)^2de^3\sqrt{e\sin(c+dx)}}$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} + \frac{2\left(5a^2b-a\left(3a^2+2b^2\right)\cos(c+dx)\right.}{5\left(a^2-b^2\right)^2de^3\sqrt{e\sin(c+dx)}}$$

$$= \frac{a^{5/2}b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{9/4}de^{7/2}} - \frac{a^{5/2}b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{9/4}de^{7/2}} + \frac{2(}{5\left(a^2-\right.}$$

Mathematica [C] time = 6.89, size = 930, normalized size = 1.82

$$\frac{(b+a\cos(c+dx))\left(-\frac{2(b-a\cos(c+dx))\csc^3(c+dx)}{5\left(b^2-a^2\right)}-\frac{2\left(3\cos(c+dx)a^3-5ba^2+2b^2\cos(c+dx)a\right)\csc(c+dx)}{5\left(b^2-a^2\right)^2}\right)\sin^3(c+dx)\tan(c+dx)}{d(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}} - -$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(7/2)),x]
```

```
[Out] -1/5*(a*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[c + d*x]^(7/2)*(((3*a^3 + 2*a
*b^2)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]
*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sq
rt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt
[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[S
qrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a
*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (a^
2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*Sqrt[1 - Sin[c +
d*x]^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)
) + (2*(8*a^2*b + 2*b^3)*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*
Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt
[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I)*
Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] + Log[Sqrt
[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Si
n[c + d*x]]))/(Sqrt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF1[3/4, 1/2, 1, 7/4, S
in[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))/(3*(-a
^2 + b^2)))*(b + a*Sqrt[1 - Sin[c + d*x]^2]))/((b + a*Cos[c + d*x])*Sqrt[1
- Sin[c + d*x]^2])))/((a - b)^2*(a + b)^2*d*(a + b*Sec[c + d*x])*(e*Sin[c +
 d*x])^(7/2)) + ((b + a*Cos[c + d*x])*((-2*(-5*a^2*b + 3*a^3*Cos[c + d*x] +
 2*a*b^2*Cos[c + d*x])*Csc[c + d*x])/(5*(-a^2 + b^2)^2) - (2*(b - a*Cos[c +
 d*x])*Csc[c + d*x]^3)/(5*(-a^2 + b^2)))*Sin[c + d*x]^3*Tan[c + d*x])/(d*(a
 + b*Sec[c + d*x])*(e*Sin[c + d*x])^(7/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*sin(d*x + c))^(7/2)), x)
```

maple [B] time = 8.49, size = 1672, normalized size = 3.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(7/2),x)
```

```
[Out] 2/5/d*b/e/(a+b)/(a-b)/(e*sin(d*x+c))^(5/2)+2/d*b/e^3/(a-b)^2/(a+b)^2*a^2/(e
*sin(d*x+c))^(1/2)+1/d*b/e^3*a^2/(a-b)^2/(a+b)^2/(e^2*(a^2-b^2)/a^2)^(1/4)*
arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-1/2/d*b/e^3*a^2/(a-b
)^2/(a+b)^2/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^
2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-1/2/d/e^3*
b^2*a^2/(a+(a^2-b^2)^(1/2))/(-a+(a^2-b^2)^(1/2))/(a+b)^2/(a-b)^2*sin(d*x+c)
^(1/2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+
2)^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-a/(-a+(a^2-b^2)^(1/2)),1/2*2^(1/
```

```
2))*(a^2-b^2)^(1/2)+1/2/d/e^3*b^2*a^2/(a+(a^2-b^2)^(1/2))/(-a+(a^2-b^2)^(1/
2))/(a+b)^2/(a-b)^2*sin(d*x+c)^(1/2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(-sin(
d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(
a+(a^2-b^2)^(1/2)),1/2*2^(1/2))*(a^2-b^2)^(1/2)-1/2/d/e^3*b^2*a^3/(a+(a^2-b
^2)^(1/2))/(-a+(a^2-b^2)^(1/2))/(a+b)^2/(a-b)^2*sin(d*x+c)^(1/2)/cos(d*x+c)
/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*Elliptic
Pi((-sin(d*x+c)+1)^(1/2),-a/(-a+(a^2-b^2)^(1/2)),1/2*2^(1/2))-1/2/d/e^3*b^2
*a^3/(a+(a^2-b^2)^(1/2))/(-a+(a^2-b^2)^(1/2))/(a+b)^2/(a-b)^2*sin(d*x+c)^(1
/2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^
(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),1/2*2^(1/2))-6
/5/d/e^3*b^2*a^3/(a+(a^2-b^2)^(1/2))/(-a+(a^2-b^2)^(1/2))/(a+b)^2/(a-b)^2*s
in(d*x+c)^(1/2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*si
n(d*x+c)+2)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-4/5/d/e^3*b^
4*a/(a+(a^2-b^2)^(1/2))/(-a+(a^2-b^2)^(1/2))/(a+b)^2/(a-b)^2*sin(d*x+c)^(1/
2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(
1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3/5/d/e^3*b^2*a^3/(a+(a^2
-b^2)^(1/2))/(-a+(a^2-b^2)^(1/2))/(a+b)^2/(a-b)^2*sin(d*x+c)^(1/2)/cos(d*x+
c)/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*Ellipt
icF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2/5/d/e^3*b^4*a/(a+(a^2-b^2)^(1/2))/
(-a+(a^2-b^2)^(1/2))/(a+b)^2/(a-b)^2*sin(d*x+c)^(1/2)/cos(d*x+c)/(e*sin(d*x
+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*EllipticF((-sin(d*x
+c)+1)^(1/2),1/2*2^(1/2))-6/5/d/e^3*b^2*a^3/(a+(a^2-b^2)^(1/2))/(-a+(a^2-b^
2)^(1/2))/(a+b)^2/(a-b)^2/sin(d*x+c)^2*cos(d*x+c)^3/(e*sin(d*x+c))^(1/2)-4/
5/d/e^3*b^4*a/(a+(a^2-b^2)^(1/2))/(-a+(a^2-b^2)^(1/2))/(a+b)^2/(a-b)^2/sin(
d*x+c)^2*cos(d*x+c)^3/(e*sin(d*x+c))^(1/2)+8/5/d/e^3*b^2*a^3/(a+(a^2-b^2)^(
1/2))/(-a+(a^2-b^2)^(1/2))/(a+b)^2/(a-b)^2/sin(d*x+c)^2*cos(d*x+c)/(e*sin(d
*x+c))^(1/2)+2/5/d/e^3*b^4*a/(a+(a^2-b^2)^(1/2))/(-a+(a^2-b^2)^(1/2))/(a+b)
^2/(a-b)^2/sin(d*x+c)^2*cos(d*x+c)/(e*sin(d*x+c))^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)}{(e\sin(c+dx))^{7/2}\,(b+a\cos(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(7/2)*(a + b/cos(c + d*x))),x)
[Out] int(cos(c + d*x)/((e*sin(c + d*x))^(7/2)*(b + a*cos(c + d*x))), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))**(7/2),x)
[Out] Timed out
```

3.241 $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=1070

$$-\frac{2b^2\left(a^2-b^2\right)^2\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,e^5}{a^7\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}+\frac{7b^4\left(a^2-b^2\right)\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{2a^7\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+}}$$

```
[Out] -7/2*b^3*(a^2-b^2)^(3/4)*e^(9/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b
^2)^(1/4)/e^(1/2))/a^(13/2)/d+2*b*(a^2-b^2)^(7/4)*e^(9/2)*arctan(a^(1/2)*(e
*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(13/2)/d+7/2*b^3*(a^2-b^2)^(3
/4)*e^(9/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a
^(13/2)/d-2*b*(a^2-b^2)^(7/4)*e^(9/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/
(a^2-b^2)^(1/4)/e^(1/2))/a^(13/2)/d-14/45*e^3*cos(d*x+c)*(e*sin(d*x+c))^(3/
2)/a^2/d-7/15*b^2*e^3*(5*b-3*a*cos(d*x+c))*(e*sin(d*x+c))^(3/2)/a^5/d+4/15*
b*e^3*(5*a^2-5*b^2+3*a*b*cos(d*x+c))*(e*sin(d*x+c))^(3/2)/a^5/d+4/7*b*e*(e*
sin(d*x+c))^(7/2)/a^3/d-2/9*e*cos(d*x+c)*(e*sin(d*x+c))^(7/2)/a^2/d+b^2*e*(
e*sin(d*x+c))^(7/2)/a^3/d/(b+a*cos(d*x+c))-7/2*b^4*(a^2-b^2)*e^5*(sin(1/2*c
+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/
4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^7/d/(a-(a
^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*b^2*(a^2-b^2)^2*e^5*(sin(1/2*c+1/4*Pi
+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/
2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^7/d/(a-(a^2-b^2)
^(1/2))/(e*sin(d*x+c))^(1/2)-7/2*b^4*(a^2-b^2)*e^5*(sin(1/2*c+1/4*Pi+1/2*d*
x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),
2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^7/d/(a+(a^2-b^2)^(1/2))
/(e*sin(d*x+c))^(1/2)+2*b^2*(a^2-b^2)^2*e^5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(
1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+
(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^7/d/(a+(a^2-b^2)^(1/2))/(e*sin
(d*x+c))^(1/2)-14/15*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/
2)/a^2/d/sin(d*x+c)^(1/2)+7/5*b^2*(3*a^2-5*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d
*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),
2^(1/2))*(e*sin(d*x+c))^(1/2)/a^6/d/sin(d*x+c)^(1/2)+4/5*b^2*(8*a^2-5*b^2)*
e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE
(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a^6/d/sin(d*x+c)^(
1/2)
```

Rubi [A] time = 2.79, antiderivative size = 1070, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3872, 2912, 2635, 2640, 2639, 2693, 2865, 2867, 2701, 2807, 2805, 329, 298, 205, 208, 2695}

$$-\frac{2b^2\left(a^2-b^2\right)^2\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,e^5}{a^7\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}+\frac{7b^4\left(a^2-b^2\right)\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{2a^7\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(9/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-7*b^3*(a^2 - b^2)^(3/4)*e^(9/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a
^2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(13/2)*d) + (2*b*(a^2 - b^2)^(7/4)*e^(9/2)*
ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(13/
2)*d) + (7*b^3*(a^2 - b^2)^(3/4)*e^(9/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*
x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(13/2)*d) - (2*b*(a^2 - b^2)^(7/4)*
e^(9/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])]
```

```
)/(a^(13/2)*d) + (7*b^4*(a^2 - b^2)*e^5*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^
2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^7*(a - Sqrt[a^2 - b^2]
)*d*Sqrt[e*Sin[c + d*x]]) - (2*b^2*(a^2 - b^2)^2*e^5*EllipticPi[(2*a)/(a -
Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^7*(a - Sqrt
[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (7*b^4*(a^2 - b^2)*e^5*EllipticPi[(2
*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^
7*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (2*b^2*(a^2 - b^2)^2*e^5*
EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
 d*x]])/(a^7*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (14*e^4*Ellipt
icE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(15*a^2*d*Sqrt[Sin[c + d*x
]]) - (7*b^2*(3*a^2 - 5*b^2)*e^4*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Si
n[c + d*x]])/(5*a^6*d*Sqrt[Sin[c + d*x]]) - (4*b^2*(8*a^2 - 5*b^2)*e^4*Elli
pticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a^6*d*Sqrt[Sin[c + d*
x]]) - (14*e^3*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2))/(45*a^2*d) - (7*b^2*e^3
*(5*b - 3*a*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(15*a^5*d) + (4*b*e^3*(5*
(a^2 - b^2) + 3*a*b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(15*a^5*d) + (4*b
*e*(e*Sin[c + d*x])^(7/2))/(7*a^3*d) - (2*e*Cos[c + d*x]*(e*Sin[c + d*x])^(
7/2))/(9*a^2*d) + (b^2*e*(e*Sin[c + d*x])^(7/2))/(a^3*d*(b + a*Cos[c + d*x]
))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
```

```
x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{9/2}}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \int \left(\frac{(e\sin(c+dx))^{9/2}}{a^2} + \frac{b^2(e\sin(c+dx))^{9/2}}{a^2(b+a\cos(c+dx))^2} - \frac{2b(e\sin(c+dx))^{9/2}}{a^2(b+a\cos(c+dx))}\right)dx$$

$$= \frac{\int (e\sin(c+dx))^{9/2}\,dx}{a^2} - \frac{(2b)\int \frac{(e\sin(c+dx))^{9/2}}{b+a\cos(c+dx)}\,dx}{a^2} + \frac{b^2\int \frac{(e\sin(c+dx))^{9/2}}{(b+a\cos(c+dx))^2}\,dx}{a^2}$$

$$= \frac{4be(e\sin(c+dx))^{7/2}}{7a^3d} - \frac{2e\cos(c+dx)(e\sin(c+dx))^{7/2}}{9a^2d} + \frac{b^2e(e\sin(c+dx))^{7/2}}{a^3d(b+a\cos(c+dx))} + \frac{(}{}$$

$$= -\frac{14e^3\cos(c+dx)(e\sin(c+dx))^{3/2}}{45a^2d} - \frac{7b^2e^3(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^5d} +$$

$$= -\frac{14e^3\cos(c+dx)(e\sin(c+dx))^{3/2}}{45a^2d} - \frac{7b^2e^3(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^5d} +$$

$$= \frac{14e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15a^2d\sqrt{\sin(c+dx)}} - \frac{14e^3\cos(c+dx)(e\sin(c+dx))^{3/2}}{45a^2d} - \frac{7b^2e}{}$$

$$= \frac{14e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15a^2d\sqrt{\sin(c+dx)}} - \frac{7b^2\left(3a^2-5b^2\right)e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5a^6d\sqrt{\sin(c+dx)}}$$

$$= \frac{7b^4\left(a^2-b^2\right)e^5\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{2a^7\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{2b^2\left(a^2-b^2\right)^2e^5\Pi\left(\right.}{a^7\left(a-\right.}$$

$$= -\frac{7b^3\left(a^2-b^2\right)^{3/4}e^{9/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{13/2}d} + \frac{2b\left(a^2-b^2\right)^{7/4}e^{9/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right.}{a^{13/2}d}$$

Mathematica [C] time = 15.59, size = 974, normalized size = 0.91

$$(b+a\cos(c+dx))^2\sec^2(c+dx)\left(\frac{(14a^4-159b^2a^2+165b^4)\left(8F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin^{\frac{3}{2}}(c+dx)a^{5/2}+3\sqrt{2}\,b\left(b^2-a^2\right)^{3/4}\left(2\right.\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(9/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(e*Sin[c + d*x])^(9/2)*(((14*a^4 - 1
59*a^2*b^2 + 165*b^4)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*Arc
Tan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan
[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a
^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c
 + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[S
in[c + d*x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin
[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*S
qrt[1 - Sin[c + d*x]^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 -
 Sin[c + d*x]^2)) + (2*(-46*a^3*b + 66*a*b^3)*Cos[c + d*x]*(((1/8 + I/8)*(2
*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*Arc
Tan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[
a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin
[c + d*x]] + Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[S
in[c + d*x]] + I*a*Sin[c + d*x]]))/(Sqrt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF
1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c
 + d*x]^(3/2))/(3*(-a^2 + b^2)))*(b + a*Sqrt[1 - Sin[c + d*x]^2]))/((b + a*
Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(30*a^5*d*(a + b*Sec[c + d*x])^2*
Sin[c + d*x]^(9/2)) + ((b + a*Cos[c + d*x])^2*Csc[c + d*x]^4*Sec[c + d*x]^2
*(e*Sin[c + d*x])^(9/2)*(-1/21*(b*(-37*a^2 + 56*b^2)*Sin[c + d*x])/a^5 + (a
^2*b^2*Sin[c + d*x] - b^4*Sin[c + d*x])/(a^5*(b + a*Cos[c + d*x])) - ((19*a
^2 - 54*b^2)*Sin[2*(c + d*x)])/(90*a^4) - (b*Sin[3*(c + d*x)])/(7*a^3) + Si
n[4*(c + d*x)]/(36*a^2)))/(d*(a + b*Sec[c + d*x])^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 21.44, size = 3808, normalized size = 3.56

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(9/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/d*e^5*sin(d*x+c)^2*cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a^2-b^2)/(-cos(d
*x+c)^2*a^2+b^2)-5/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/a^6*(-sin(d*x+
c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*E
llipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+7/2/d*
e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^6/a^8*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c
)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/d*e^5/cos(d*x+c)/(e*sin(d
*x+c))^(1/2)*b^2/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)
*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2/d*e^5/cos(
d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/a^4/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(
d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2
))+1/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^6/a^6/(a^2-b^2)*(-sin(d*x+c)+1
)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(
1/2),1/2*2^(1/2))-1/2/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^2/(a^2-b^
2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF(
(-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+1/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*
b^4/a^4/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(
1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/2/d*e^5/cos(d*x+c)/(e*s
in(d*x+c))^(1/2)*b^6/a^6/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(
1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3/2/d*e^
5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+
1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-5/d*e^5/cos(d*x+c)/(e*sin(d*x
+c))^(1/2)*b^4/a^6*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^
(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)
^(1/2)/a),1/2*2^(1/2))+7/2/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^6/a^8*(-
sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1
/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2)
)+3/2/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^4*(-sin(d*x+c)+1)^(1/2)*(
2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-s
in(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+34/45/d*e^5*cos(d*x
+c)^3/(e*sin(d*x+c))^(1/2)/a^2-8/15/d*e^5*cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a
^2-8/3/d/a^5*e^3*b^3*(e*sin(d*x+c))^(3/2)+4/3/d/a^3*e^3*b*(e*sin(d*x+c))^(3
/2)-2/9/d*e^5*cos(d*x+c)^5/(e*sin(d*x+c))^(1/2)/a^2+4/7*b*e*(e*sin(d*x+c))^
(7/2)/a^3/d-1/d/a^5*e^5*b^5*(e*sin(d*x+c))^(3/2)/(-a^2*cos(d*x+c)^2*e^2+b^2
*e^2)+2/d/a^3*e^5*b/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2)/(
e^2*(a^2-b^2)/a^2)^(1/4))-1/d/a^3*e^5*b/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*si
n(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-
b^2)/a^2)^(1/4)))-15/2/d/a^5*e^5*b^3/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*si
n(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+15/4/d/a^5*e^5*b^3/(e^2*(a^2-b^2
)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*
x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+11/2/d/a^7*e^5*b^5/(e^2*(a^2-b^2)/a
^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-11/4/d/a^7
*e^5*b^5/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/
a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-6/5/d*e^5*cos
(d*x+c)^3/(e*sin(d*x+c))^(1/2)/a^4*b^2+6/5/d*e^5*cos(d*x+c)/(e*sin(d*x+c))^
(1/2)/a^4*b^2+1/d/a^3*e^5*b^3*(e*sin(d*x+c))^(3/2)/(-a^2*cos(d*x+c)^2*e^2+b
^2*e^2)-1/2/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^2/(a^2-b^2)*(-sin(d
*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a
)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+7/4
/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/a^4/(a^2-b^2)*(-sin(d*x+c)+1)^(1
/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticP
i((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-2/d*e^5/cos(d*
x+c)/(e*sin(d*x+c))^(1/2)*b^6/a^6/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c
)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/4/d*e^5/cos(d*x+c)/(e*sin
```

```
(d*x+c))^(1/2)*b^8/a^8/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/
2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),
1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1
/2)*b^2/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+
c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b
^2)^(1/2)/a),1/2*2^(1/2))+7/4/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/a^4
/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1
+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a
),1/2*2^(1/2))-2/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^6/a^6/(a^2-b^2)*(-
sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1
/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2)
)+3/4/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^8/a^8/(a^2-b^2)*(-sin(d*x+c)+
1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*Elli
pticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+48/5/d*e^
5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2
)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2-1
0/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^6*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))
*b^4-24/5/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^4*b^2*(-sin(d*x+c)+1)^(1/
2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),
1/2*2^(1/2))+5/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^6*b^4*(-sin(d*x+c)+1
)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(
1/2),1/2*2^(1/2))-1/d*e^5*sin(d*x+c)^2*cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^6/
a^4/(a^2-b^2)/(-cos(d*x+c)^2*a^2+b^2)-14/15/d*e^5/cos(d*x+c)/(e*sin(d*x+c))
^(1/2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*El
lipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+7/15/d*e^5/cos(d*x+c)/(e*sin(d*x
+c))^(1/2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2
)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2/d*e^5*sin(d*x+c)^2*cos(d*x
+c)/(e*sin(d*x+c))^(1/2)*b^4/a^2/(a^2-b^2)/(-cos(d*x+c)^2*a^2+b^2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2\,(e\,\sin(c+d\,x))^{9/2}}{(b+a\,\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(9/2)/(a + b/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(9/2))/(b + a*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(9/2)/(a+b*sec(d*x+c))**2,x)
[Out] Timed out
```

3.242 $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=1101

$$-\frac{5b^2\left(a^2-3b^2\right)F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,e^4}{3a^6d\sqrt{e\sin(c+dx)}}-\frac{4b^2\left(4a^2-3b^2\right)F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,e^4}{3a^6d\sqrt{e\sin(c+dx)}}+\frac{10b}{}$$

```
[Out] 5/2*b^3*(a^2-b^2)^(1/4)*e^(7/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^
2)^(1/4)/e^(1/2))/a^(11/2)/d-2*b*(a^2-b^2)^(5/4)*e^(7/2)*arctan(a^(1/2)*(e*
sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(11/2)/d+5/2*b^3*(a^2-b^2)^(1/
4)*e^(7/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^
(11/2)/d-2*b*(a^2-b^2)^(5/4)*e^(7/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(
a^2-b^2)^(1/4)/e^(1/2))/a^(11/2)/d+4/5*b*e*(e*sin(d*x+c))^(5/2)/a^3/d-2/7*e
*cos(d*x+c)*(e*sin(d*x+c))^(5/2)/a^2/d+b^2*e*(e*sin(d*x+c))^(5/2)/a^3/d/(b+
a*cos(d*x+c))-10/21*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*P
i+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^
2/d/(e*sin(d*x+c))^(1/2)+5/3*b^2*(a^2-3*b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)
^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(
1/2))*sin(d*x+c)^(1/2)/a^6/d/(e*sin(d*x+c))^(1/2)+4/3*b^2*(4*a^2-3*b^2)*e^4
*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(co
s(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^6/d/(e*sin(d*x+c))^(1/2
)+5/2*b^4*(a^2-b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*P
i+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(
1/2))*sin(d*x+c)^(1/2)/a^6/d/(a^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/
2)-2*b^2*(a^2-b^2)^2*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^
(1/2))*sin(d*x+c)^(1/2)/a^6/d/(a^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1
/2)+5/2*b^4*(a^2-b^2)*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4
*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2
^(1/2))*sin(d*x+c)^(1/2)/a^6/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(
1/2)-2*b^2*(a^2-b^2)^2*e^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/
4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),
2^(1/2))*sin(d*x+c)^(1/2)/a^6/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^
(1/2)-10/21*e^3*cos(d*x+c)*(e*sin(d*x+c))^(1/2)/a^2/d-5/3*b^2*e^3*(3*b-a*co
s(d*x+c))*(e*sin(d*x+c))^(1/2)/a^5/d+4/3*b*e^3*(3*a^2-3*b^2+a*b*cos(d*x+c))
*(e*sin(d*x+c))^(1/2)/a^5/d
```

Rubi [A] time = 2.93, antiderivative size = 1101, normalized size of antiderivative = 1.00, number of steps used = 35, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3872, 2912, 2635, 2642, 2641, 2693, 2865, 2867, 2702, 2807, 2805, 329, 212, 208, 205, 2695}

$$-\frac{5b^2\left(a^2-3b^2\right)F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,e^4}{3a^6d\sqrt{e\sin(c+dx)}}-\frac{4b^2\left(4a^2-3b^2\right)F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,e^4}{3a^6d\sqrt{e\sin(c+dx)}}+\frac{10b}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(7/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (5*b^3*(a^2 - b^2)^(1/4)*e^(7/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^
2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(11/2)*d) - (2*b*(a^2 - b^2)^(5/4)*e^(7/2)*A
rcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(11/2
)*d) + (5*b^3*(a^2 - b^2)^(1/4)*e^(7/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x
]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(11/2)*d) - (2*b*(a^2 - b^2)^(5/4)*e
^(7/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])
```

```
/(a^(11/2)*d) + (10*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]]
)/(21*a^2*d*Sqrt[e*Sin[c + d*x]]) - (5*b^2*(a^2 - 3*b^2)*e^4*EllipticF[(c -
 Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a^6*d*Sqrt[e*Sin[c + d*x]]) - (4*
b^2*(4*a^2 - 3*b^2)*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]]
)/(3*a^6*d*Sqrt[e*Sin[c + d*x]]) - (5*b^4*(a^2 - b^2)*e^4*EllipticPi[(2*a)/
(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^6*(a
^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*b^2*(a^2 - b^2)^
2*e^4*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(a^6*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]])
- (5*b^4*(a^2 - b^2)*e^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2
+ d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^6*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*S
qrt[e*Sin[c + d*x]]) + (2*b^2*(a^2 - b^2)^2*e^4*EllipticPi[(2*a)/(a + Sqrt[
a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^6*(a^2 - b^2 + a
*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (10*e^3*Cos[c + d*x]*Sqrt[e*Sin
[c + d*x]])/(21*a^2*d) - (5*b^2*e^3*(3*b - a*Cos[c + d*x])*Sqrt[e*Sin[c + d
*x]])/(3*a^5*d) + (4*b*e^3*(3*(a^2 - b^2) + a*b*Cos[c + d*x])*Sqrt[e*Sin[c
+ d*x]])/(3*a^5*d) + (4*b*e*(e*Sin[c + d*x])^(5/2))/(5*a^3*d) - (2*e*Cos[c
+ d*x]*(e*Sin[c + d*x])^(5/2))/(7*a^2*d) + (b^2*e*(e*Sin[c + d*x])^(5/2))/(
a^3*d*(b + a*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
```

```
d}, x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{7/2}}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \int \left(\frac{(e\sin(c+dx))^{7/2}}{a^2} + \frac{b^2(e\sin(c+dx))^{7/2}}{a^2(b+a\cos(c+dx))^2} - \frac{2b(e\sin(c+dx))^{7/2}}{a^2(b+a\cos(c+dx))}\right)dx$$

$$= \frac{\int (e\sin(c+dx))^{7/2}\,dx}{a^2} - \frac{(2b)\int \frac{(e\sin(c+dx))^{7/2}}{b+a\cos(c+dx)}\,dx}{a^2} + \frac{b^2\int \frac{(e\sin(c+dx))^{7/2}}{(b+a\cos(c+dx))^2}\,dx}{a^2}$$

$$= \frac{4be(e\sin(c+dx))^{5/2}}{5a^3d} - \frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7a^2d} + \frac{b^2e(e\sin(c+dx))^{5/2}}{a^3d(b+a\cos(c+dx))} - \cdots$$

$$= -\frac{10e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d} - \frac{5b^2e^3(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^5d} + \cdots$$

$$= -\frac{10e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d} - \frac{5b^2e^3(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^5d} + \cdots$$

$$= \frac{10e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}} - \frac{10e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d} - \cdots$$

$$= \frac{10e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}} - \frac{5b^2\left(a^2-3b^2\right)e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\cdots}{3a^6d\sqrt{e\sin(c+dx)}}$$

$$= \frac{10e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}} - \frac{5b^2\left(a^2-3b^2\right)e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\cdots}{3a^6d\sqrt{e\sin(c+dx)}}$$

$$= \frac{5b^3\sqrt[4]{a^2-b^2}\,e^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{11/2}d} - \frac{2b\left(a^2-b^2\right)^{5/4}e^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{11/2}d}\cdots$$

Mathematica [C] time = 17.62, size = 2095, normalized size = 1.90

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(7/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*(-1/42*((23*a^2 - 84*b^2)*Cos[c + d*x])/a^4 - (b^2*
(-a^2 + b^2))/(a^5*(b + a*Cos[c + d*x])) - (2*b*Cos[2*(c + d*x)])/(5*a^3) +
 Cos[3*(c + d*x)]/(14*a^2))*Csc[c + d*x]^3*Sec[c + d*x]^2*(e*Sin[c + d*x])^
(7/2))/(d*(a + b*Sec[c + d*x])^2) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*
(e*Sin[c + d*x])^(7/2)*((2*(50*a^4 - 273*a^2*b^2 + 105*b^4)*Cos[c + d*x]^2*
(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[S
in[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[
c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a
^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2]
 + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]])
)/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^2)*AppellF1[1/4, -
1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c +
 d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4
, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4
, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (-a^2 +
 b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2
- b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c
 + d*x])*(1 - Sin[c + d*x]^2)) + (2*(-139*a^3*b + 210*a*b^3)*Cos[c + d*x]*(
b + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 +
I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*S
qrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 +
I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] - Log[S
qrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a
*Sin[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1,
 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])
/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c
+ d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, 1/2, 2
, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*Appe
llF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*S
in[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*S
qrt[1 - Sin[c + d*x]^2]) + ((231*a^3*b - 420*a*b^3)*Cos[c + d*x]*Cos[2*(c +
 d*x)]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[
1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2
- b^2)^(3/4)) - ((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt
[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2 - b^2)^(3/4)) + ((1/4 - I
/4)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*S
qrt[Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) - ((1/4
- I/4)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4
)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) + (4*
Sqrt[Sin[c + d*x]])/a + (4*b*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^
2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*b*
(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)
/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*
AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)
] + 2*(2*a^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2
)/(a^2 - b^2)] + (a^2 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^
2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*
x]^2)))))/((b + a*Cos[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1 - Sin[c + d*x
]^2])))/(210*a^5*d*(a + b*Sec[c + d*x])^2*Sin[c + d*x]^(7/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [B] time = 21.39, size = 3412, normalized size = 3.10

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/2/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^6/a^6/(a^2-b^2)*(-sin(d*x+c)+1
)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(
1/2),1/2*2^(1/2))+1/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/a^4/(a^2-b^2)
*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-
sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/2/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*
b^2/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(
1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/d*e^4*sin(d*x+c)*cos(d*
x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a^2-b^2)/(-cos(d*x+c)^2*a^2+b^2)+4/5*b*e*(e*
sin(d*x+c))^(5/2)/a^3/d+1/d/a^3*e^5*b^3*(e*sin(d*x+c))^(1/2)/(-a^2*cos(d*x+
c)^2*e^2+b^2*e^2)-1/d/a^5*e^5*b^5*(e*sin(d*x+c))^(1/2)/(-a^2*cos(d*x+c)^2*e
^2+b^2*e^2)+2/7/d*e^4*cos(d*x+c)^3/(e*sin(d*x+c))^(1/2)/a^2*sin(d*x+c)-16/2
1/d*e^4*cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^2*sin(d*x+c)+2/d*e^4*cos(d*x+c)/(
e*sin(d*x+c))^(1/2)/a^4*b^2*sin(d*x+c)+1/d/a*e^5*b*(e^2*(a^2-b^2)/a^2)^(1/4
)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((
e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+9/4/d/a^5*e^5*b^5*(e^2*(a^2
-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)
/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+2/d/a*e^5*b*
(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e
^2*(a^2-b^2)/a^2)^(1/4))-13/2/d/a^3*e^5*b^3*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2
*e^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+9/2/d/
a^5*e^5*b^5*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+
c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-13/4/d/a^3*e^5*b^3*(e^2*(a^2-b^2)/a^2)
^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4
))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+3/d*e^4/cos(d*x+c)/(e*
sin(d*x+c))^(1/2)*b^6/a^5/(a^2-b^2)^(3/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+
1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-5/4/d*e^4/cos(d*x+c)/(e*sin(d
*x+c))^(1/2)*b^8/a^7/(a^2-b^2)^(3/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)
^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1
/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-3/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^
(1/2)*b^6/a^5/(a^2-b^2)^(3/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*
sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(
1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-9/4/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)
*b^4/a^3/(a^2-b^2)^(3/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d
*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^
2-b^2)^(1/2)/a),1/2*2^(1/2))+5/4/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^8/
a^7/(a^2-b^2)^(3/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)
^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2
```

```
)^(1/2)/a),1/2*2^(1/2))+1/2/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a/(a^
2-b^2)^(3/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/
(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)
/a),1/2*2^(1/2))+4/d/a^3*e^3*b*(e*sin(d*x+c))^(1/2)-8/d/a^5*e^3*b^3*(e*sin(
d*x+c))^(1/2)+9/4/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/a^3/(a^2-b^2)^(
3/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-
b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*
2^(1/2))-5/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/a^5/(a^2-b^2)^(1/2)*(-
sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1
/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2)
)+4/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^4*b^2*(-sin(d*x+c)+1)^(1/2)*(2*
sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^
(1/2))-5/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^6*b^4*(-sin(d*x+c)+1)^(1/2
)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1
/2*2^(1/2))+2/d*e^4*sin(d*x+c)*cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/a^2/(a^2
-b^2)/(-cos(d*x+c)^2*a^2+b^2)-1/d*e^4*sin(d*x+c)*cos(d*x+c)/(e*sin(d*x+c))^
(1/2)*b^6/a^4/(a^2-b^2)/(-cos(d*x+c)^2*a^2+b^2)-5/21/d*e^4/cos(d*x+c)/(e*si
n(d*x+c))^(1/2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)
^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+7/2/d*e^4/cos(d*x+c)/(e
*sin(d*x+c))^(1/2)*b^6/a^7/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)
+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-3/2/d*e^4/cos(d*x+c)/(e*sin(
d*x+c))^(1/2)*b^2/a^3/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2
)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(
1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+5/d*e^4/cos(d*x+c)/(e*sin(d*x+c))
^(1/2)*b^4/a^5/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)
*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/
(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-7/2/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2
)*b^6/a^7/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(
d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a
^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/2/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2
/a^3/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^
2)^(1/2)/a),1/2*2^(1/2))-1/2/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a/(a
^2-b^2)^(3/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)
/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2
)/a),1/2*2^(1/2))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)^2\,(e\,\sin(c+dx))^{7/2}}{(b+a\,\cos(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(7/2)/(a + b/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(7/2))/(b + a*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(7/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.243 $\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=850

$$\frac{3e^3\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^5\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}+\frac{3e^3\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^5\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}-\frac{3e^{5/2}\,\text{ta}}{}$$

```
[Out] -3/2*b^3*e^(5/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2
))/a^(9/2)/(a^2-b^2)^(1/4)/d+2*b*(a^2-b^2)^(3/4)*e^(5/2)*arctan(a^(1/2)*(e*
sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(9/2)/d+3/2*b^3*e^(5/2)*arctan
h(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(9/2)/(a^2-b^2)^(
1/4)/d-2*b*(a^2-b^2)^(3/4)*e^(5/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^
2-b^2)^(1/4)/e^(1/2))/a^(9/2)/d+4/3*b*e*(e*sin(d*x+c))^(3/2)/a^3/d-2/5*e*co
s(d*x+c)*(e*sin(d*x+c))^(3/2)/a^2/d+b^2*e*(e*sin(d*x+c))^(3/2)/a^3/d/(b+a*c
os(d*x+c))-3/2*b^4*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi
+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1
/2))*sin(d*x+c)^(1/2)/a^5/d/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*b^2*
(a^2-b^2)*e^3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)
*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(
d*x+c)^(1/2)/a^5/d/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-3/2*b^4*e^3*(si
n(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1
/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^5/
d/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*b^2*(a^2-b^2)*e^3*(sin(1/2*c+1
/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*
Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^5/d/(a+(a^2
-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-6/5*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/
2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(
e*sin(d*x+c))^(1/2)/a^2/d/sin(d*x+c)^(1/2)+7*b^2*e^2*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x)
,2^(1/2))*(e*sin(d*x+c))^(1/2)/a^4/d/sin(d*x+c)^(1/2)
```

Rubi [A] time = 2.13, antiderivative size = 850, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3872, 2912, 2635, 2640, 2639, 2693, 2867, 2701, 2807, 2805, 329, 298, 205, 208, 2695}

$$\frac{3e^3\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^5\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}+\frac{3e^3\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^5\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}-\frac{3e^{5/2}\,\text{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(5/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-3*b^3*e^(5/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sq
rt[e])])/(2*a^(9/2)*(a^2 - b^2)^(1/4)*d) + (2*b*(a^2 - b^2)^(3/4)*e^(5/2)*A
rcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(9/2)
*d) + (3*b^3*e^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1
/4)*Sqrt[e])])/(2*a^(9/2)*(a^2 - b^2)^(1/4)*d) - (2*b*(a^2 - b^2)^(3/4)*e^(
5/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(
a^(9/2)*d) + (3*b^4*e^3*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 +
 d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^5*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c
 + d*x]]) - (2*b^2*(a^2 - b^2)*e^3*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]),
(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^5*(a - Sqrt[a^2 - b^2])*d*Sqr
t[e*Sin[c + d*x]]) + (3*b^4*e^3*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c
- Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^5*(a + Sqrt[a^2 - b^2])*d*Sqrt
```

```
[e*Sin[c + d*x]]) - (2*b^2*(a^2 - b^2)*e^3*EllipticPi[(2*a)/(a + Sqrt[a^2 -
 b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^5*(a + Sqrt[a^2 - b^2
])*d*Sqrt[e*Sin[c + d*x]]) + (6*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e
*Sin[c + d*x]])/(5*a^2*d*Sqrt[Sin[c + d*x]]) - (7*b^2*e^2*EllipticE[(c - Pi
/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a^4*d*Sqrt[Sin[c + d*x]]) + (4*b*e*(
e*Sin[c + d*x])^(3/2))/(3*a^3*d) - (2*e*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2)
)/(5*a^2*d) + (b^2*e*(e*Sin[c + d*x])^(3/2))/(a^3*d*(b + a*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{5/2}}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \int \left(\frac{(e\sin(c+dx))^{5/2}}{a^2} + \frac{b^2(e\sin(c+dx))^{5/2}}{a^2(b+a\cos(c+dx))^2} - \frac{2b(e\sin(c+dx))^{5/2}}{a^2(b+a\cos(c+dx))}\right) dx$$

$$= \frac{\int (e\sin(c+dx))^{5/2}\,dx}{a^2} - \frac{(2b)\int \frac{(e\sin(c+dx))^{5/2}}{b+a\cos(c+dx)}\,dx}{a^2} + \frac{b^2\int \frac{(e\sin(c+dx))^{5/2}}{(b+a\cos(c+dx))^2}\,dx}{a^2}$$

$$= \frac{4be(e\sin(c+dx))^{3/2}}{3a^3d} - \frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d} + \frac{b^2e(e\sin(c+dx))^{3/2}}{a^3d(b+a\cos(c+dx))}$$

$$= \frac{4be(e\sin(c+dx))^{3/2}}{3a^3d} - \frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d} + \frac{b^2e(e\sin(c+dx))^{3/2}}{a^3d(b+a\cos(c+dx))}$$

$$= \frac{6e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}} + \frac{4be(e\sin(c+dx))^{3/2}}{3a^3d} - \frac{2e\cos(c+dx)(}{5}$$

$$= \frac{6e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}} - \frac{7b^2e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+}}{a^4d\sqrt{\sin(c+dx)}}$$

$$= \frac{3b^4e^3\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{2a^5\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{2b^2\left(a^2-b^2\right)e^3\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}}\right.}{a^5\left(a-\sqrt{a^2-\right.}}$$

$$= -\frac{3b^3e^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{9/2}\sqrt[4]{a^2-b^2}\,d} + \frac{2b\left(a^2-b^2\right)^{3/4}e^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{9/2}d} + \frac{3b^3}{}$$

Mathematica [C] time = 15.15, size = 886, normalized size = 1.04

$$\frac{(b+a\cos(c+dx))^2\csc^2(c+dx)\sec^2(c+dx)(e\sin(c+dx))^{5/2}\left(\frac{\sin(c+dx)b^2}{a^3(b+a\cos(c+dx))} + \frac{4\sin(c+dx)b}{3a^3} - \frac{\sin(2(c+dx))}{5a^2}\right)}{d(a+b\sec(c+dx))^2} - \frac{(b+}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(5/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/10*((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(e*Sin[c + d*x])^(5/2)*(((-6*a
^2 + 35*b^2)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTan[1 - (
Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (Sqr
t[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2]
 - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]
+ Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*
x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]
^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*Sqrt[1 - S
in[c + d*x]^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 - Sin[c +
d*x]^2)) + (28*a*b*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[a
]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sq
rt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a
```

```
]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] + Log[Sqrt[a^2 -
 b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c +
d*x]]))/(Sqrt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF1[3/4, 1/2, 1, 7/4, Sin[c +
 d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))/(3*(-a^2 + b
^2)))*(b + a*Sqrt[1 - Sin[c + d*x]^2]))/((b + a*Cos[c + d*x])*Sqrt[1 - Sin[
c + d*x]^2])))/(a^3*d*(a + b*Sec[c + d*x])^2*Sin[c + d*x]^(5/2)) + ((b + a*
Cos[c + d*x])^2*Csc[c + d*x]^2*Sec[c + d*x]^2*(e*Sin[c + d*x])^(5/2)*((4*b*
Sin[c + d*x])/(3*a^3) + (b^2*Sin[c + d*x])/(a^3*(b + a*Cos[c + d*x])) - Sin
[2*(c + d*x)]/(5*a^2)))/(d*(a + b*Sec[c + d*x])^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [B] time = 16.08, size = 2540, normalized size = 2.99

Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/d*e^3*sin(d*x+c)^2*cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a^2-b^2)/(-cos(d
*x+c)^2*a^2+b^2)+6/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^4*(-sin(d*x+
c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+
1)^(1/2),1/2*2^(1/2))-3/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^4*(-sin
(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*
x+c)+1)^(1/2),1/2*2^(1/2))-2/5*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^(1/2)+2/
5*e^3*cos(d*x+c)^3/a^2/d/(e*sin(d*x+c))^(1/2)+4/3*b*e*(e*sin(d*x+c))^(3/2)/
a^3/d+1/d/a^3*e^3*b^3*(e*sin(d*x+c))^(3/2)/(-a^2*cos(d*x+c)^2*e^2+b^2*e^2)+
2/d/a^3*e^3*b/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a
^2-b^2)/a^2)^(1/4))-1/d/a^3*e^3*b/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+
c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a
^2)^(1/4)))-7/2/d/a^5*e^3*b^3/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c
))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+7/4/d/a^5*e^3*b^3/(e^2*(a^2-b^2)/a^2)^(
1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1
/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-1/2/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a
^2*b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1
/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+1/2/d*e^3/cos(d*x+c)/(e*si
n(d*x+c))^(1/2)/a^4*b^4/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1
/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3/2/d*e^3
/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c
)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1
)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-5/2/d*e^3/cos(d*x+c)/(e*sin(d*
x+c))^(1/2)*b^4/a^6*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)
^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2
```

```
)^(1/2)/a),1/2*2^(1/2))+3/2/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^4*(
-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(
1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2
))-1/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^4*b^4/(a^2-b^2)*(-sin(d*x+c)+1
)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(
1/2),1/2*2^(1/2))+1/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^2*b^2/(a^2-b^2)
*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-
sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-5/2/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*
b^4/a^6*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a
^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1
/2*2^(1/2))-1/2/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/(a^2-b^2)/a^2*(-sin(d
*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a
)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))*b^2
+5/4/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^4*b^4/(a^2-b^2)*(-sin(d*x+c)+1
)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*Ellip
ticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-3/4/d*e^3/
cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^6*b^6/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*
sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin
(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/d*e^3/cos(d*x+c)/
(e*sin(d*x+c))^(1/2)/(a^2-b^2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(
1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2
),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))*b^2+5/4/d*e^3/cos(d*x+c)/(e*sin(d*x+
c))^(1/2)/a^4*b^4/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*si
n(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+
(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-3/4/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a
^6*b^6/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1
/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(
1/2)/a),1/2*2^(1/2))+1/d*e^3*sin(d*x+c)^2*cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a
^2*b^4/(a^2-b^2)/(-cos(d*x+c)^2*a^2+b^2)-6/5/d*e^3/cos(d*x+c)/(e*sin(d*x+c)
)^(1/2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*E
llipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3/5/d*e^3/cos(d*x+c)/(e*sin(d*x
+c))^(1/2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2
)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2\,(e\,\sin(c+d\,x))^{5/2}}{(b+a\,\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(5/2)/(a + b/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(5/2))/(b + a*cos(c + d*x))^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(5/2)/(a+b*sec(d*x+c))**2,x)
[Out] Timed out
```

3.244 $\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=882

$$-\frac{e^2\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^4\left(a^2-\sqrt{a^2-b^2}\,a-b^2\right)d\sqrt{e\sin(c+dx)}}-\frac{e^2\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^4\left(a^2+\sqrt{a^2-b^2}\,a-b^2\right)d\sqrt{e\sin(c+dx)}}+\frac{e^{3/2}\tan^{-}\cdots}{2a^{7}\cdots}$$

```
[Out] 1/2*b^3*e^(3/2)*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2)
)/a^(7/2)/(a^2-b^2)^(3/4)/d-2*b*(a^2-b^2)^(1/4)*e^(3/2)*arctan(a^(1/2)*(e*s
in(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(7/2)/d+1/2*b^3*e^(3/2)*arctanh
(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(7/2)/(a^2-b^2)^(3
/4)/d-2*b*(a^2-b^2)^(1/4)*e^(3/2)*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2
-b^2)^(1/4)/e^(1/2))/a^(7/2)/d-2/3*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/
sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(
d*x+c)^(1/2)/a^2/d/(e*sin(d*x+c))^(1/2)+5*b^2*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x
)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^
(1/2))*sin(d*x+c)^(1/2)/a^4/d/(e*sin(d*x+c))^(1/2)+1/2*b^4*e^2*(sin(1/2*c+1
/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*
Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^4/d/(a^2-b^
2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*b^2*(a^2-b^2)*e^2*(sin(1/2*c+1/
4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*P
i+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^4/d/(a^2-b^2
-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+1/2*b^4*e^2*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x
),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^4/d/(a^2-b^2+a*(a^2-b
^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*b^2*(a^2-b^2)*e^2*(sin(1/2*c+1/4*Pi+1/2*d
*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x)
,2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^4/d/(a^2-b^2+a*(a^2-b^
2)^(1/2))/(e*sin(d*x+c))^(1/2)+4*b*e*(e*sin(d*x+c))^(1/2)/a^3/d-2/3*e*cos(d
*x+c)*(e*sin(d*x+c))^(1/2)/a^2/d+b^2*e*(e*sin(d*x+c))^(1/2)/a^3/d/(b+a*cos(
d*x+c))
```

Rubi [A] time = 2.17, antiderivative size = 882, normalized size of antiderivative = 1.00, number of steps used = 32, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3872, 2912, 2635, 2642, 2641, 2693, 2867, 2702, 2807, 2805, 329, 212, 208, 205, 2695}

$$-\frac{e^2\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^4\left(a^2-\sqrt{a^2-b^2}\,a-b^2\right)d\sqrt{e\sin(c+dx)}}-\frac{e^2\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^4\left(a^2+\sqrt{a^2-b^2}\,a-b^2\right)d\sqrt{e\sin(c+dx)}}+\frac{e^{3/2}\tan^{-}\cdots}{2a^{7}\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (b^3*e^(3/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[
e])])/(2*a^(7/2)*(a^2 - b^2)^(3/4)*d) - (2*b*(a^2 - b^2)^(1/4)*e^(3/2)*ArcT
an[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(7/2)*d)
 + (b^3*e^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*S
qrt[e])])/(2*a^(7/2)*(a^2 - b^2)^(3/4)*d) - (2*b*(a^2 - b^2)^(1/4)*e^(3/2)*
ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(7/
2)*d) + (2*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a^2*
d*Sqrt[e*Sin[c + d*x]]) - (5*b^2*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[
Sin[c + d*x]])/(a^4*d*Sqrt[e*Sin[c + d*x]]) - (b^4*e^2*EllipticPi[(2*a)/(a
 - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^4*(a^2
 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*b^2*(a^2 - b^2)*e^2
```

```
*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
+ d*x]])/(a^4*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (b^
4*e^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(2*a^4*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]
) + (2*b^2*(a^2 - b^2)*e^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/
2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^4*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*S
qrt[e*Sin[c + d*x]]) + (4*b*e*Sqrt[e*Sin[c + d*x]])/(a^3*d) - (2*e*Cos[c +
d*x]*Sqrt[e*Sin[c + d*x]])/(3*a^2*d) + (b^2*e*Sqrt[e*Sin[c + d*x]])/(a^3*d*
(b + a*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
```

```
tegersQ[2*m, 2*p]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{3/2}}{(-b-a\cos(c+dx))^2}\,dx$$
$$= \int \left(\frac{(e\sin(c+dx))^{3/2}}{a^2} + \frac{b^2(e\sin(c+dx))^{3/2}}{a^2(b+a\cos(c+dx))^2} - \frac{2b(e\sin(c+dx))^{3/2}}{a^2(b+a\cos(c+dx))}\right)dx$$
$$= \frac{\int (e\sin(c+dx))^{3/2}\,dx}{a^2} - \frac{(2b)\int \frac{(e\sin(c+dx))^{3/2}}{b+a\cos(c+dx)}\,dx}{a^2} + \frac{b^2\int \frac{(e\sin(c+dx))^{3/2}}{(b+a\cos(c+dx))^2}\,dx}{a^2}$$
$$= \frac{4be\sqrt{e\sin(c+dx)}}{a^3d} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d} + \frac{b^2e\sqrt{e\sin(c+dx)}}{a^3d(b+a\cos(c+dx))} + \frac{e^2}{}$$
$$= \frac{4be\sqrt{e\sin(c+dx)}}{a^3d} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d} + \frac{b^2e\sqrt{e\sin(c+dx)}}{a^3d(b+a\cos(c+dx))} - \frac{(b}{}$$
$$= \frac{2e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}} + \frac{4be\sqrt{e\sin(c+dx)}}{a^3d} - \frac{2e\cos(c+dx)\sqrt{e\,s}}{3a^2d}$$
$$= \frac{2e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}} - \frac{5b^2e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{a^4d\sqrt{e\sin(c+dx)}}$$
$$= \frac{2e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}} - \frac{5b^2e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{a^4d\sqrt{e\sin(c+dx)}}$$
$$= \frac{b^3e^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{7/2}\left(a^2-b^2\right)^{3/4}d} - \frac{2b\sqrt[4]{a^2-b^2}\,e^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{7/2}d} + \frac{b^3e^{3/2}\tan}{2a^{7/}}$$

Mathematica [C] time = 16.97, size = 2012, normalized size = 2.28

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*((-2*Cos[c + d*x])/(3*a^2) + b^2/(a^3*(b + a*Cos[c
+ d*x])))*Csc[c + d*x]*Sec[c + d*x]^2*(e*Sin[c + d*x])^(3/2))/(d*(a + b*Sec
[c + d*x])^2) - ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(e*Sin[c + d*x])^(3/
2)*((2*(-2*a^2 + 3*b^2)*Cos[c + d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b
*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] +
2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log
[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
 a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4
)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3
/4)) - (5*a*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Si
n[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5
*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^
2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2
*Sin[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin
[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*
(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (8*
a*b*Cos[c + d*x]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2
*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*Arc
Tan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[
```

```
a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin
[c + d*x]] - Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[S
in[c + d*x]] + I*a*Sin[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*App
ellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*S
qrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1
/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*Ap
pellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]
+ (a^2 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^
2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b +
 a*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2]) - (6*a*b*Cos[c + d*x]*Cos[2*(c +
 d*x)]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[
1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2
- b^2)^(3/4)) - ((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt
[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2 - b^2)^(3/4)) + ((1/4 - I
/4)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*S
qrt[Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) - ((1/4
- I/4)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4
)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) + (4*
Sqrt[Sin[c + d*x]])/a + (4*b*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^
2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*b*
(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)
/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*
AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)
] + 2*(2*a^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2
)/(a^2 - b^2)] + (a^2 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^
2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*
x]^2)))))/((b + a*Cos[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1 - Sin[c + d*x
]^2])))/(6*a^3*d*(a + b*Sec[c + d*x])^2*Sin[c + d*x]^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \sin(dx+c))^{\frac{3}{2}}}{(b \sec(dx+c)+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

maple [B] time = 16.85, size = 2282, normalized size = 2.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 4*b*e*(e*sin(d*x+c))^(1/2)/a^3/d+1/d/a^3*e^3*b^3*(e*sin(d*x+c))^(1/2)/(-a^2
*cos(d*x+c)^2*e^2+b^2*e^2)+1/d/a*e^3*b*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+
b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c)
)^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-5/4/d/a^3*e^3*b^3*(e^2*(a^2-b^2)/a^2)^(
```

```
1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))
/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+2/d/a*e^3*b*(e^2*(a^2-b^
2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)
/a^2)^(1/4))-5/2/d/a^3*e^3*b^3*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)
*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-1/3/d*e^2/cos(d*x+c
)/(e*sin(d*x+c))^(1/2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin
(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2/3/d*e^2*cos(d*
x+c)/(e*sin(d*x+c))^(1/2)/a^2*sin(d*x+c)+3/d*e^2/cos(d*x+c)/(e*sin(d*x+c))^
(1/2)*b^2/a^4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)
*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/d*e^2*sin(d*x+c)*cos(d*x+c)
/(e*sin(d*x+c))^(1/2)*b^2/(a^2-b^2)/(-cos(d*x+c)^2*a^2+b^2)+1/d*e^2*sin(d*x
+c)*cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^2*b^4/(a^2-b^2)/(-cos(d*x+c)^2*a^2+b^
2)-1/2/d*e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^2*b^2/(a^2-b^2)*(-sin(d*x+c)
+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)
^(1/2),1/2*2^(1/2))+1/2/d*e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^4*b^4/(a^2-
b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Elliptic
F((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/2/d*e^2/cos(d*x+c)/(e*sin(d*x+c))^(1
/2)/(a^2-b^2)^(3/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+
c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b
^2)^(1/2)/a),1/2*2^(1/2))*b^2+7/4/d*e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^3
*b^4/(a^2-b^2)^(3/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^
2)^(1/2)/a),1/2*2^(1/2))-5/4/d*e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^5*b^6/
(a^2-b^2)^(3/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/
2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1
/2)/a),1/2*2^(1/2))+1/2/d*e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(3/
2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-
b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*
2^(1/2))*b^2-7/4/d*e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^3*b^4/(a^2-b^2)^(3
/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b
^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2
^(1/2))+5/4/d*e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^5*b^6/(a^2-b^2)^(3/2)*(
-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(
1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2
))+3/2/d*e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^3/(a^2-b^2)^(1/2)*(-sin(
d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/
a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-5/
2/d*e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/a^5/(a^2-b^2)^(1/2)*(-sin(d*x+c
)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*El
lipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-3/2/d*e
^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^3/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^
(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*Ellipti
cPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+5/2/d*e^2/co
s(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/a^5/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)
*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((
-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+dx)^2 (e\sin(c+dx))^{3/2}}{(b+a\cos(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(3/2)/(a + b/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(3/2))/(b + a*cos(c + d*x))^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(3/2)/(a+b*sec(d*x+c))**2,x)
[Out] Timed out
```

3.245 $\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=809

$$-\frac{e\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^3\left(a^2-b^2\right)\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}-\frac{e\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^3\left(a^2-b^2\right)\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}+\frac{\sqrt{e}\tan}{2a}$$

```
[Out] b^2*(e*sin(d*x+c))^(3/2)/a/(a^2-b^2)/d/e/(b+a*cos(d*x+c))+1/2*b^3*arctan(a^
(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*e^(1/2)/a^(5/2)/(a^2-b^
2)^(5/4)/d+2*b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))
*e^(1/2)/a^(5/2)/(a^2-b^2)^(1/4)/d-1/2*b^3*arctanh(a^(1/2)*(e*sin(d*x+c))^(
1/2)/(a^2-b^2)^(1/4)/e^(1/2))*e^(1/2)/a^(5/2)/(a^2-b^2)^(5/4)/d-2*b*arctanh
(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*e^(1/2)/a^(5/2)/(a^2
-b^2)^(1/4)/d+2*b^2*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+
1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/
2))*sin(d*x+c)^(1/2)/a^3/d/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+1/2*b^4
*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi
(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2
)/a^3/(a^2-b^2)/d/(a-(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*b^2*e*(sin(1/2
*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+
1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^3/d/(a+
(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+1/2*b^4*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^
2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a
/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^3/(a^2-b^2)/d/(a+(a^2-b^2)
^(1/2))/(e*sin(d*x+c))^(1/2)-2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*
c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c
))^(1/2)/a^2/d/sin(d*x+c)^(1/2)+b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin
(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(
d*x+c))^(1/2)/a^2/(a^2-b^2)/d/sin(d*x+c)^(1/2)
```

Rubi [A] time = 1.83, antiderivative size = 809, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3872, 2912, 2640, 2639, 2694, 2867, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{e\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^3\left(a^2-b^2\right)\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}-\frac{e\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^3\left(a^2-b^2\right)\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}+\frac{\sqrt{e}\tan}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sin[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (b^3*Sqrt[e]*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[
e])])/(2*a^(5/2)*(a^2 - b^2)^(5/4)*d) + (2*b*Sqrt[e]*ArcTan[(Sqrt[a]*Sqrt[e
*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(5/2)*(a^2 - b^2)^(1/4)*d)
 - (b^3*Sqrt[e]*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*S
qrt[e])])/(2*a^(5/2)*(a^2 - b^2)^(5/4)*d) - (2*b*Sqrt[e]*ArcTanh[(Sqrt[a]*S
qrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(5/2)*(a^2 - b^2)^(1/
4)*d) - (2*b^2*e*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2
, 2]*Sqrt[Sin[c + d*x]])/(a^3*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]])
 - (b^4*e*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sq
rt[Sin[c + d*x]])/(2*a^3*(a^2 - b^2)*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c +
 d*x]]) - (2*b^2*e*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)
/2, 2]*Sqrt[Sin[c + d*x]])/(a^3*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]
]) - (b^4*e*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*
```

```
Sqrt[Sin[c + d*x]])/(2*a^3*(a^2 - b^2)*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c
 + d*x]]) + (2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a^2*
d*Sqrt[Sin[c + d*x]]) - (b^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c
+ d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[Sin[c + d*x]]) + (b^2*(e*Sin[c + d*x])^(3/
2))/(a*(a^2 - b^2)*d*e*(b + a*Cos[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sqrt{e\sin(c+dx)}}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \int \left(\frac{\sqrt{e\sin(c+dx)}}{a^2} + \frac{b^2\sqrt{e\sin(c+dx)}}{a^2(b+a\cos(c+dx))^2} - \frac{2b\sqrt{e\sin(c+dx)}}{a^2(b+a\cos(c+dx))}\right) dx$$

$$= \frac{\int \sqrt{e\sin(c+dx)}\,dx}{a^2} - \frac{(2b)\int \frac{\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}\,dx}{a^2} + \frac{b^2\int \frac{\sqrt{e\sin(c+dx)}}{(b+a\cos(c+dx))^2}\,dx}{a^2}$$

$$= \frac{b^2(e\sin(c+dx))^{3/2}}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))} + \frac{b^2\int \frac{\left(-b-\frac{1}{2}a\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}\,dx}{a^2\left(a^2-b^2\right)} + \frac{\left(b^2e\right)\int \cdots}{\cdots}$$

$$= \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2d\sqrt{\sin(c+dx)}} + \frac{b^2(e\sin(c+dx))^{3/2}}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))} - \frac{b^2\int \sqrt{e \cdots}}{2a^2 \cdots}$$

$$= -\frac{2b^2e\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{a^3\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{2b^2e\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+\cdots\right.\right.}{a^3\left(a+\sqrt{a^2-b^2}\right)d\sqrt{\cdots}}$$

$$= \frac{2b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}\,d} - \frac{2b\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}\,d} - \frac{2b^2e\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(\cdots\right.\right.}{a^3\left(a-\sqrt{a^2-\cdots}\right.}$$

$$= \frac{2b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}\,d} - \frac{2b\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}\,d} - \frac{2b^2e\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(\cdots\right.\right.}{a^3\left(a-\sqrt{a^2-\cdots}\right.}$$

$$= \frac{b^3\sqrt{e}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{5/2}\left(a^2-b^2\right)^{5/4}d} + \frac{2b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}\,d} - \frac{b^3\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin\cdots}}{\sqrt[4]{a^2-b\cdots}}\right)}{2a^{5/2}\left(a^2-b^2\right)^{5/4}}$$

Mathematica [C] time = 15.27, size = 854, normalized size = 1.06

$$\frac{(b+a\cos(c+dx))\sec(c+dx)\sqrt{e\sin(c+dx)}\,\tan(c+dx)b^2}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\sqrt{e\sin(c+dx)}\left(\frac{(3b\cdots}{\cdots}\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Sin[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Sqrt[e*Sin[c + d*x]]*(((-2*a^2 + 3*b
^2)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*S
qrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt
[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2
]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqr
t[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*S
in[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*
Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*Sqrt[1 - Sin[c + d*
x]^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2))
```

```
+ (4*a*b*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin
[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c +
 d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b
^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + (1
 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]]))/(S
qrt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (
a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))/(3*(-a^2 + b^2)))*(b +
 a*Sqrt[1 - Sin[c + d*x]^2]))/((b + a*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2
])))/(2*a*(-a + b)*(a + b)*d*(a + b*Sec[c + d*x])^2*Sqrt[Sin[c + d*x]]) + (
b^2*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sqrt[e*Sin[c + d*x]]*Tan[c + d*x])/(a
*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e \sin(dx + c)}}{(b \sec(dx + c) + a)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
[Out] integrate(sqrt(e*sin(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

maple [A] time = 13.40, size = 1563, normalized size = 1.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c))^2,x)
[Out] 1/d/a*e*b^3/(a^2-b^2)*(e*sin(d*x+c))^(3/2)/(-a^2*cos(d*x+c)^2*e^2+b^2*e^2)+
2/d/a*e*b/(a^2-b^2)/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2)/(
e^2*(a^2-b^2)/a^2)^(1/4))-3/2/d/a^3*e*b^3/(a^2-b^2)/(e^2*(a^2-b^2)/a^2)^(1/
4)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-1/d/a*e*b/(a^2-b^
2)/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(
1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+3/4/d/a^3*e*b^3/(a^
2-b^2)/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^
2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-2/d*e/cos(d*x+c
)/(e*sin(d*x+c))^(1/2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin
(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+1/d*e/cos(d*x+c)
/(e*sin(d*x+c))^(1/2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(
d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/d*e*sin(d*x+c)^
2*cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a^2-b^2)/(-cos(d*x+c)^2*a^2+b^2)+1/d
*e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*
(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2
*2^(1/2))-1/2/d*e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^2/(a^2-b^2)*(-sin(d
*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+
c)+1)^(1/2),1/2*2^(1/2))-1/2/d*e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^2/(a
^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a
^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1
/2*2^(1/2))+3/4/d*e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/a^4/(a^2-b^2)*(-sin
```

```
(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)
/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1
/2/d*e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1
/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticP
i((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/4/d*e/cos(d*
x+c)/(e*sin(d*x+c))^(1/2)*b^4/a^4/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c
)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/2/d*e/cos(d*x+c)/(e*sin(d
*x+c))^(1/2)*b^2/a^4*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^
2)^(1/2)/a),1/2*2^(1/2))+3/2/d*e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^4*(-
sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1
/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2)
)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\sin(dx+c)}}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
[Out] integrate(sqrt(e*sin(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2\,\sqrt{e\,\sin(c+d\,x)}}{(b+a\,\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^(1/2)/(a + b/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^(1/2))/(b + a*cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(1/2)/(a+b*sec(d*x+c))**2,x)
[Out] Integral(sqrt(e*sin(c + d*x))/(a + b*sec(c + d*x))**2, x)
```

3.246 $\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=838

$$\frac{3\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^2\left(a^2-b^2\right)\left(a^2-\sqrt{a^2-b^2}\,a-b^2\right)d\sqrt{e\sin(c+dx)}}+\frac{3\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^2\left(a^2-b^2\right)\left(a^2+\sqrt{a^2-b^2}\,a-b^2\right)d\sqrt{e\sin(c+dx)}}\ 3$$

```
[Out] -3/2*b^3*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(3/
2)/(a^2-b^2)^(7/4)/d/e^(1/2)-2*b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b
^2)^(1/4)/e^(1/2))/a^(3/2)/(a^2-b^2)^(3/4)/d/e^(1/2)-3/2*b^3*arctanh(a^(1/2
)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^(3/2)/(a^2-b^2)^(7/4)/d/e
^(1/2)-2*b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/a^
(3/2)/(a^2-b^2)^(3/4)/d/e^(1/2)-2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c
)^(1/2)/a^2/d/(e*sin(d*x+c))^(1/2)-b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/
sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(
d*x+c)^(1/2)/a^2/(a^2-b^2)/d/(e*sin(d*x+c))^(1/2)-2*b^2*(sin(1/2*c+1/4*Pi+1
/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*
d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(a^2-b^2-a*(a^
2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-3/2*b^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1
/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(
a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^2/(a^2-b^2)/d/(a^2-b^2-a*(a^2-b
^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/s
in(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b
^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*s
in(d*x+c))^(1/2)-3/2*b^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*
Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^
(1/2))*sin(d*x+c)^(1/2)/a^2/(a^2-b^2)/d/(a^2-b^2+a*(a^2-b^2)^(1/2))/(e*sin(
d*x+c))^(1/2)+b^2*(e*sin(d*x+c))^(1/2)/a/(a^2-b^2)/d/e/(b+a*cos(d*x+c))
```

Rubi [A] time = 1.92, antiderivative size = 838, normalized size of antiderivative = 1.00, number of steps used = 27, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3872, 2912, 2642, 2641, 2694, 2867, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{3\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^2\left(a^2-b^2\right)\left(a^2-\sqrt{a^2-b^2}\,a-b^2\right)d\sqrt{e\sin(c+dx)}}+\frac{3\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^2\left(a^2-b^2\right)\left(a^2+\sqrt{a^2-b^2}\,a-b^2\right)d\sqrt{e\sin(c+dx)}}\ 3$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (-3*b^3*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])
/(2*a^(3/2)*(a^2 - b^2)^(7/4)*d*Sqrt[e]) - (2*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[
c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(3/2)*(a^2 - b^2)^(3/4)*d*Sqrt[
e]) - (3*b^3*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt
[e])])/(2*a^(3/2)*(a^2 - b^2)^(7/4)*d*Sqrt[e]) - (2*b*ArcTanh[(Sqrt[a]*Sqrt
[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(3/2)*(a^2 - b^2)^(3/4)*
d*Sqrt[e]) + (2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^2*d
*Sqrt[e*Sin[c + d*x]]) + (b^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
 d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[e*Sin[c + d*x]]) + (2*b^2*EllipticPi[(2*a)/
(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^2*(a^2
 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (3*b^4*EllipticPi[(2*
a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^2
*(a^2 - b^2)*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*b
```

```
^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[
c + d*x]])/(a^2*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (
3*b^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(2*a^2*(a^2 - b^2)*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*S
in[c + d*x]]) + (b^2*Sqrt[e*Sin[c + d*x]])/(a*(a^2 - b^2)*d*e*(b + a*Cos[c
+ d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{\cos^2(c+dx)}{(-b-a\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$$

$$= \int \left(\frac{1}{a^2\sqrt{e\sin(c+dx)}} + \frac{b^2}{a^2(-b-a\cos(c+dx))^2\sqrt{e\sin(c+dx)}} + \frac{}{a^2(}\right.$$

$$= \frac{\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{a^2} + \frac{(2b)\int \frac{1}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{a^2} + \frac{b^2\int \frac{}{(-b-a\cos(}}{}$$

$$= \frac{b^2\sqrt{e\sin(c+dx)}}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))} + \frac{b^2\int \frac{b-\frac{1}{2}a\cos(c+dx)}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{a^2\left(a^2-b^2\right)} +$$

$$= \frac{2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{a^2d\sqrt{e\sin(c+dx)}} + \frac{b^2\sqrt{e\sin(c+dx)}}{a\left(a^2-b^2\right)de(b+a\cos(c+d}$$

$$= \frac{2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{a^2d\sqrt{e\sin(c+dx)}} + \frac{2b^2\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+d\right.\right.}{a^2\left(a^2-b^2-a\sqrt{a^2-b^2}\right)}$$

$$= -\frac{2b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} - \frac{2b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} + \frac{2F\left(\frac{1}{2}\left(c-\right.\right.}{a^2}$$

$$= -\frac{2b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} - \frac{2b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} + \frac{2F\left(\frac{1}{2}\left(c-\right.\right.}{a^2}$$

$$= -\frac{3b^3\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{3/2}\left(a^2-b^2\right)^{7/4}d\sqrt{e}} - \frac{2b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} - \frac{3b^3\tanh^{-1}}{2a^{3/2}\left(a^2\right.}$$

Mathematica [C] time = 13.31, size = 1246, normalized size = 1.49

$$\frac{(b+a\cos(c+dx))\sec(c+dx)\tan(c+dx)b^2}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}} + \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\sqrt{\sin(c+dx)}\left(\frac{2\left(b^2-2a^2\right)\left(\sqrt{1-\sin^2(}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]]*((2*(-2*a^2 + b^2
)*Cos[c + d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (Sqrt[
2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*
Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sq
rt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log
[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
 a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^2
)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b
^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*AppellF1[
```

```
1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2
*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2
- b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[
c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2))
)))/((b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (4*a*b*Cos[c + d*x]*(b +
a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 + I)*S
qrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[
a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I)*S
qrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] - Log[Sqrt[
a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin
[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4
, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/(Sq
rt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*
x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, 1/2, 2, 9/
4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*AppellF1
[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c
 + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*Sqrt[
1 - Sin[c + d*x]^2])))/(2*a*(-a + b)*(a + b)*d*(a + b*Sec[c + d*x])^2*Sqrt[
e*Sin[c + d*x]]) + (b^2*(b + a*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(a*
(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^2\sqrt{e\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/((b*sec(d*x + c) + a)^2*sqrt(e*sin(d*x + c))), x)
```

maple [A] time = 15.31, size = 1475, normalized size = 1.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x)
[Out] 1/d/a*e*b^3/(a^2-b^2)*(e*sin(d*x+c))^(1/2)/(-a^2*cos(d*x+c)^2*e^2+b^2*e^2)+
2/d*a*e*b/(a^2-b^2)*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*
sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-1/2/d/a*e*b^3/(a^2-b^2)*(e^2*(
a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^
2-b^2)/a^2)^(1/4))+1/d*a*e*b/(a^2-b^2)*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+
b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c)
)^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-1/4/d/a*e*b^3/(a^2-b^2)*(e^2*(a^2-b^2)/
a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^
(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-1/d/cos(d*x+c)/(e*
sin(d*x+c))^(1/2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+
c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/d*sin(d*x+c)*cos(d*
x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a^2-b^2)/(-cos(d*x+c)^2*a^2+b^2)-1/2/d/cos(d
```

```
*x+c)/(e*sin(d*x+c))^(1/2)/a^2*b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d
*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2)
)-1/2/d/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a*b^2/(a^2-b^2)^(3/2)*(-sin(d*x+c)+
1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*Elli
pticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+5/4/d/cos
(d*x+c)/(e*sin(d*x+c))^(1/2)/a^3*b^4/(a^2-b^2)^(3/2)*(-sin(d*x+c)+1)^(1/2)*
(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-
sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/2/d/cos(d*x+c)/(
e*sin(d*x+c))^(1/2)/a*b^2/(a^2-b^2)^(3/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+
1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-5/4/d/cos(d*x+c)/(e*sin(d*x+c
))^(1/2)/a^3*b^4/(a^2-b^2)^(3/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/
2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),
1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/2/d/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*
b^2/a^3/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*
x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2
-b^2)^(1/2)/a),1/2*2^(1/2))-3/2/d/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^3/(
a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2
)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/
2)/a),1/2*2^(1/2))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos (c+d\,x)^2}{\sqrt{e\,\sin (c+d\,x)}\,\left(b+a\,\cos (c+d\,x)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(1/2)*(a + b/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/((e*sin(c + d*x))^(1/2)*(b + a*cos(c + d*x))^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\sin (c+dx)}\,(a+b\sec (c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(e*sin(c + d*x))*(a + b*sec(c + d*x))**2), x)
```

3.247 $\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=1054

$$-\frac{5\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a\left(a^2-b^2\right)^2\left(a-\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}-\frac{5\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a\left(a^2-b^2\right)^2\left(a+\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}+\frac{5\tan^{-}}{2\sqrt{a}}$$

```
[Out] 5/2*b^3*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a^2-b
^2)^(9/4)/d/e^(3/2)/a^(1/2)+2*b*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^
2)^(1/4)/e^(1/2))/(a^2-b^2)^(5/4)/d/e^(3/2)/a^(1/2)-5/2*b^3*arctanh(a^(1/2)
*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a^2-b^2)^(9/4)/d/e^(3/2)/a^
(1/2)-2*b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))/(a^
2-b^2)^(5/4)/d/e^(3/2)/a^(1/2)-2*cos(d*x+c)/a^2/d/e/(e*sin(d*x+c))^(1/2)+b^
2/a/(a^2-b^2)/d/e/(b+a*cos(d*x+c))/(e*sin(d*x+c))^(1/2)+4*b*(a-b*cos(d*x+c)
)/a^2/(a^2-b^2)/d/e/(e*sin(d*x+c))^(1/2)+b^2*(5*a*b-(3*a^2+2*b^2)*cos(d*x+c
))/a^2/(a^2-b^2)^2/d/e/(e*sin(d*x+c))^(1/2)+5/2*b^4*(sin(1/2*c+1/4*Pi+1/2*d
*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x)
,2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/e/(a-(a^
2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2
)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a-(a^
2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/a/(a^2-b^2)/d/e/(a-(a^2-b^2)^(1/2))
/(e*sin(d*x+c))^(1/2)+5/2*b^4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c
+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2
)),2^(1/2))*sin(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/e/(a+(a^2-b^2)^(1/2))/(e*sin(d
*x+c))^(1/2)+2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2
*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))
*sin(d*x+c)^(1/2)/a/(a^2-b^2)/d/e/(a+(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)+
2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(c
os(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a^2/d/e^2/sin(d*x+c)
^(1/2)+4*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*
EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*sin(d*x+c))^(1/2)/a^2/(a^2-
b^2)/d/e^2/sin(d*x+c)^(1/2)+b^2*(3*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2
))*(e*sin(d*x+c))^(1/2)/a^2/(a^2-b^2)^2/d/e^2/sin(d*x+c)^(1/2)
```

Rubi [A] time = 2.69, antiderivative size = 1054, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3872, 2912, 2636, 2640, 2639, 2694, 2866, 2867, 2701, 2807, 2805, 329, 298, 205, 208, 2696}

$$-\frac{5\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a\left(a^2-b^2\right)^2\left(a-\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}-\frac{5\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a\left(a^2-b^2\right)^2\left(a+\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}+\frac{5\tan^{-}}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (5*b^3*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/
(2*Sqrt[a]*(a^2 - b^2)^(9/4)*d*e^(3/2)) + (2*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c
 + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(Sqrt[a]*(a^2 - b^2)^(5/4)*d*e^(3/2
)) - (5*b^3*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[
e])])/(2*Sqrt[a]*(a^2 - b^2)^(9/4)*d*e^(3/2)) - (2*b*ArcTanh[(Sqrt[a]*Sqrt[
e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(Sqrt[a]*(a^2 - b^2)^(5/4)*d
*e^(3/2)) - (2*Cos[c + d*x])/(a^2*d*e*Sqrt[e*Sin[c + d*x]]) + b^2/(a*(a^2 -
 b^2)*d*e*(b + a*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]]) + (4*b*(a - b*Cos[c +
```

```
d*x]))/(a^2*(a^2 - b^2)*d*e*Sqrt[e*Sin[c + d*x]]) + (b^2*(5*a*b - (3*a^2 +
2*b^2)*Cos[c + d*x]))/(a^2*(a^2 - b^2)^2*d*e*Sqrt[e*Sin[c + d*x]]) - (5*b^4
*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
+ d*x]])/(2*a*(a^2 - b^2)^2*(a - Sqrt[a^2 - b^2])*d*e*Sqrt[e*Sin[c + d*x]])
 - (2*b^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sq
rt[Sin[c + d*x]])/(a*(a^2 - b^2)*(a - Sqrt[a^2 - b^2])*d*e*Sqrt[e*Sin[c + d
*x]]) - (5*b^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(2*a*(a^2 - b^2)^2*(a + Sqrt[a^2 - b^2])*d*e*Sqrt[e*
Sin[c + d*x]]) - (2*b^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 +
 d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a*(a^2 - b^2)*(a + Sqrt[a^2 - b^2])*d*e*Sq
rt[e*Sin[c + d*x]]) - (2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*
x]])/(a^2*d*e^2*Sqrt[Sin[c + d*x]]) - (4*b^2*EllipticE[(c - Pi/2 + d*x)/2,
2]*Sqrt[e*Sin[c + d*x]])/(a^2*(a^2 - b^2)*d*e^2*Sqrt[Sin[c + d*x]]) - (b^2*
(3*a^2 + 2*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a^2
*(a^2 - b^2)^2*d*e^2*Sqrt[Sin[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
```

```
^2, 0]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \int \frac{\cos^2(c+dx)}{(-b-a\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$$

$$= \int \left(\frac{1}{a^2(e\sin(c+dx))^{3/2}} + \frac{b^2}{a^2(-b-a\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\right.$$

$$= \frac{\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx}{a^2} + \frac{(2b)\int \frac{1}{(-b-a\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx}{a^2} + \frac{b^2\int \frac{}{(-b-}}{}$$

$$= -\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))\sqrt{e\sin(c+}}$$

$$= -\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))\sqrt{e\sin(c+}}$$

$$= -\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))\sqrt{e\sin(c+}}$$

$$= -\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))\sqrt{e\sin(c+}}$$

$$= -\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))\sqrt{e\sin(c+}}$$

$$= \frac{2b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\left(a^2-b^2\right)^{5/4}de^{3/2}} - \frac{2b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\left(a^2-b^2\right)^{5/4}de^{3/2}} - \frac{2\cos(c}{a^2de\sqrt{e\sin}}$$

$$= \frac{5b^3\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2\sqrt{a}\left(a^2-b^2\right)^{9/4}de^{3/2}} + \frac{2b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\left(a^2-b^2\right)^{5/4}de^{3/2}} - \frac{5b^3\tanh^{-1}}{2\sqrt{a}\left(a^2\right.}$$

Mathematica [C] time = 6.93, size = 922, normalized size = 0.87

$$\frac{(b + a\cos(c + dx))^2\left(\frac{ab^2\sin(c+dx)}{(b^2-a^2)^2(b+a\cos(c+dx))} - \frac{2(\cos(c+dx)a^2-2ba+b^2\cos(c+dx))\csc(c+dx)}{(b^2-a^2)^2}\right)\tan^2(c + dx)}{d(a + b\sec(c + dx))^2(e\sin(c + dx))^{3/2}} - \frac{(b + a\cos(c + dx}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] -1/2*((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Sin[c + d*x]^(3/2)*(((2*a^3 + 3
*a*b^2)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[
2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*
Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sq
rt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log
[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
 a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (
a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*Sqrt[1 - Sin[c
+ d*x]^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^
2)) + (2*(6*a^2*b + 4*b^3)*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I
)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sq
rt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I
)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] + Log[Sq
rt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*
Sin[c + d*x]]))/(Sqrt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF1[3/4, 1/2, 1, 7/4,
 Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))/(3*(
-a^2 + b^2)))*(b + a*Sqrt[1 - Sin[c + d*x]^2]))/((b + a*Cos[c + d*x])*Sqrt[
1 - Sin[c + d*x]^2])))/((a - b)^2*(a + b)^2*d*(a + b*Sec[c + d*x])^2*(e*Sin
[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])^2*((-2*(-2*a*b + a^2*Cos[c + d*x]
 + b^2*Cos[c + d*x])*Csc[c + d*x])/(-a^2 + b^2)^2 + (a*b^2*Sin[c + d*x])/((
-a^2 + b^2)^2*(b + a*Cos[c + d*x])))*Tan[c + d*x]^2)/(d*(a + b*Sec[c + d*x]
)^2*(e*Sin[c + d*x])^(3/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx + c) + a)^2 (e\sin(dx + c))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2)), x)
```

maple [A] time = 17.21, size = 2263, normalized size = 2.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x)
```

```
[Out] 4/d/e*a*b/(a^2-b^2)^2/(e*sin(d*x+c))^(1/2)+1/d*a/e*b^3/(a-b)^2/(a+b)^2*(e*s
in(d*x+c))^(3/2)/(-a^2*cos(d*x+c)^2*e^2+b^2*e^2)+2/d*a/e*b/(a-b)^2/(a+b)^2/
(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(
1/4))-1/d*a/e*b/(a-b)^2/(a+b)^2/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c)
)^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2
)^(1/4)))+1/2/d/a/e*b^3/(a-b)^2/(a+b)^2/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e
*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-1/4/d/a/e*b^3/(a-b)^2/(a+b)^2
/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/
4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-1/d/e*sin(d*x+c)^2*co
s(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a-b)/(a+b)*a^2/(a^2-b^2)/(-cos(d*x+c)^2*
a^2+b^2)+1/d/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a-b)/(a+b)/(a^2-b^2)*(-
sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin
(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/2/d/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(
a-b)/(a+b)/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c
)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/2/d/e/cos(d*x+c)/(e*
sin(d*x+c))^(1/2)*b^2/(a-b)/(a+b)/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*
x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c
)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/4/d/e/cos(d*x+c)/(e*sin(d
*x+c))^(1/2)*b^4/(a-b)/(a+b)/(a^2-b^2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x
+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)
+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/d/e/cos(d*x+c)/(e*sin(d*
x+c))^(1/2)*b^2/(a-b)/(a+b)/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2
)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(
1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/4/d/e/cos(d*x+c)/(e*sin(d*x+c))
^(1/2)*b^4/(a-b)/(a+b)/(a^2-b^2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)
^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1
/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/2/d/e/cos(d*x+c)/(e*sin(d*x+c))^
(1/2)*b^2/(a-b)^2/(a+b)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(
d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a
^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/d/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/(
a-b)^2/(a+b)^2/a^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^
(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)
^(1/2)/a),1/2*2^(1/2))+3/2/d/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a-b)^2/
(a+b)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a
^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1
/2*2^(1/2))-1/2/d/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/(a-b)^2/(a+b)^2/a^2
*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)
^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1
/2))+2/d/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2
)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1
/2*2^(1/2))*a^2+2/d/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2*(-sin(d*x
+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)
+1)^(1/2),1/2*2^(1/2))*b^2-1/d/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/(a^2-b^2)^
2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((
-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2-1/d/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)
/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*
EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2-2/d/e*cos(d*x+c)/(e*sin(d*
x+c))^(1/2)/(a^2-b^2)^2*a^2-2/d/e*cos(d*x+c)/(e*sin(d*x+c))^(1/2)/(a^2-b^2)
^2*b^2
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2}{(e\,\sin(c+d\,x))^{3/2}\,(b+a\,\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(3/2)*(a + b/cos(c + d*x))^2),x)
[Out] int(cos(c + d*x)^2/((e*sin(c + d*x))^(3/2)*(b + a*cos(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
[Out] Timed out
```

3.248 $\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=1089

$$\frac{7\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2\left(a^2-b^2\right)^2\left(a^2-\sqrt{a^2-b^2}\,a-b^2\right)de^2\sqrt{e\sin(c+dx)}}+\frac{7\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2\left(a^2-b^2\right)^2\left(a^2+\sqrt{a^2-b^2}\,a-b^2\right)de^2\sqrt{e\sin(c+dx)}}-\cdots$$

```
[Out] -2/3*cos(d*x+c)/a^2/d/e/(e*sin(d*x+c))^(3/2)+b^2/a/(a^2-b^2)/d/e/(b+a*cos(d
*x+c))/(e*sin(d*x+c))^(3/2)+4/3*b*(a-b*cos(d*x+c))/a^2/(a^2-b^2)/d/e/(e*sin
(d*x+c))^(3/2)+1/3*b^2*(7*a*b-(5*a^2+2*b^2)*cos(d*x+c))/a^2/(a^2-b^2)^2/d/e
/(e*sin(d*x+c))^(3/2)-7/2*b^3*arctan(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)
^(1/4)/e^(1/2))*a^(1/2)/(a^2-b^2)^(11/4)/d/e^(5/2)-2*b*arctan(a^(1/2)*(e*si
n(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*a^(1/2)/(a^2-b^2)^(7/4)/d/e^(5/2)-
7/2*b^3*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(1/4)/e^(1/2))*a^(1/
2)/(a^2-b^2)^(11/4)/d/e^(5/2)-2*b*arctanh(a^(1/2)*(e*sin(d*x+c))^(1/2)/(a^2
-b^2)^(1/4)/e^(1/2))*a^(1/2)/(a^2-b^2)^(7/4)/d/e^(5/2)-2/3*(sin(1/2*c+1/4*P
i+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/
2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^2/d/e^2/(e*sin(d*x+c))^(1/2)-4/3*b^2*(si
n(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/
2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c)^(1/2)/a^2/(a^2-b^2)/d/e^2/(e*sin(d*
x+c))^(1/2)-1/3*b^2*(5*a^2+2*b^2)*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*sin(d*x+c
)^(1/2)/a^2/(a^2-b^2)^2/d/e^2/(e*sin(d*x+c))^(1/2)-7/2*b^4*(sin(1/2*c+1/4*P
i+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1
/2*d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^2
/(a^2-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*b^2*(sin(1/2*c+1/4*Pi+1
/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*
d*x),2*a/(a-(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/(a^2
-b^2-a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-7/2*b^4*(sin(1/2*c+1/4*Pi+1/2*
d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x
),2*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)^2/d/e^2/(a^2-
b^2+a*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)-2*b^2*(sin(1/2*c+1/4*Pi+1/2*d*x
)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticPi(cos(1/2*c+1/4*Pi+1/2*d*x),2
*a/(a+(a^2-b^2)^(1/2)),2^(1/2))*sin(d*x+c)^(1/2)/(a^2-b^2)/d/e^2/(a^2-b^2+a
*(a^2-b^2)^(1/2))/(e*sin(d*x+c))^(1/2)
```

Rubi [A] time = 2.78, antiderivative size = 1089, normalized size of antiderivative = 1.00, number of steps used = 33, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.640, Rules used = {3872, 2912, 2636, 2642, 2641, 2694, 2866, 2867, 2702, 2807, 2805, 329, 212, 208, 205, 2696}

$$\frac{7\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2\left(a^2-b^2\right)^2\left(a^2-\sqrt{a^2-b^2}\,a-b^2\right)de^2\sqrt{e\sin(c+dx)}}+\frac{7\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2\left(a^2-b^2\right)^2\left(a^2+\sqrt{a^2-b^2}\,a-b^2\right)de^2\sqrt{e\sin(c+dx)}}-\cdots$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-7*Sqrt[a]*b^3*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sq
rt[e])])/(2*(a^2 - b^2)^(11/4)*d*e^(5/2)) - (2*Sqrt[a]*b*ArcTan[(Sqrt[a]*Sq
rt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(7/4)*d*e^(5
/2)) - (7*Sqrt[a]*b^3*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(
1/4)*Sqrt[e])])/(2*(a^2 - b^2)^(11/4)*d*e^(5/2)) - (2*Sqrt[a]*b*ArcTanh[(Sq
rt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(7/4
)*d*e^(5/2)) - (2*Cos[c + d*x])/(3*a^2*d*e*(e*Sin[c + d*x])^(3/2)) + b^2/(a
```

```
*(a^2 - b^2)*d*e*(b + a*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2)) + (4*b*(a - b
*Cos[c + d*x]))/(3*a^2*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(3/2)) + (b^2*(7*a*
b - (5*a^2 + 2*b^2)*Cos[c + d*x]))/(3*a^2*(a^2 - b^2)^2*d*e*(e*Sin[c + d*x]
)^(3/2)) + (2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a^2*d
*e^2*Sqrt[e*Sin[c + d*x]]) + (4*b^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(3*a^2*(a^2 - b^2)*d*e^2*Sqrt[e*Sin[c + d*x]]) + (b^2*(5*a^2
+ 2*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a^2*(a^2 -
 b^2)^2*d*e^2*Sqrt[e*Sin[c + d*x]]) + (7*b^4*EllipticPi[(2*a)/(a - Sqrt[a^2
 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*(a^2 - b^2)^2*(a^2
- b^2 - a*Sqrt[a^2 - b^2])*d*e^2*Sqrt[e*Sin[c + d*x]]) + (2*b^2*EllipticPi[
(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a
^2 - b^2)*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*e^2*Sqrt[e*Sin[c + d*x]]) + (7*
b^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin
[c + d*x]])/(2*(a^2 - b^2)^2*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*e^2*Sqrt[e*S
in[c + d*x]]) + (2*b^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 +
d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)*(a^2 - b^2 + a*Sqrt[a^2 - b^2])
*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
```

```
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \int \frac{\cos^2(c+dx)}{(-b-a\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$$

$$= \int \left(\frac{1}{a^2(e\sin(c+dx))^{5/2}} + \frac{b^2}{a^2(-b-a\cos(c+dx))^2(e\sin(c+dx))^{5/2}} \cdots\right.$$

$$= \frac{\int \frac{1}{(e\sin(c+dx))^{5/2}}\,dx}{a^2} + \frac{(2b)\int \frac{1}{(-b-a\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx}{a^2} + \frac{b^2\int \cdots}{\cdots}$$

$$= -\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))(e\sin \cdots}$$

$$= -\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))(e\sin \cdots}$$

$$= -\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))(e\sin \cdots}$$

$$= -\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))(e\sin \cdots}$$

$$= -\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))(e\sin \cdots}$$

$$= -\frac{2\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{7/4}de^{5/2}} - \frac{2\sqrt{a}\,b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{7/4}de^{5/2}} - \cdots$$

$$= -\frac{7\sqrt{a}\,b^3\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2\left(a^2-b^2\right)^{11/4}de^{5/2}} - \frac{2\sqrt{a}\,b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{7/4}de^{5/2}} - \cdots$$

Mathematica [C] time = 15.77, size = 1320, normalized size = 1.21

$$\frac{(b+a\cos(c+dx))^2\left(\frac{ab^2}{\left(b^2-a^2\right)^2(b+a\cos(c+dx))}-\frac{2\left(\cos(c+dx)a^2-2ba+b^2\cos(c+dx)\right)\csc^2(c+dx)}{3\left(b^2-a^2\right)^2}\right)\sin(c+dx)\tan^2(c+dx)}{d(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}-\ldots(b+a\,c$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] -1/6*((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Sin[c + d*x]^(5/2)*((2*(-2*a^3
 - 5*a*b^2)*Cos[c + d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1
 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 +
(Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 +
b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*
x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c
+ d*x]] + a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(
a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)
/(a^2 - b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*
AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2
)] + 2*(2*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]
^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2,
(a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c +
 d*x]^2)))))/((b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(10*a^2*b + 4
*b^3)*Cos[c + d*x]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*
(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*A
rcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqr
t[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*S
in[c + d*x]] - Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt
[Sin[c + d*x]] + I*a*Sin[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*A
ppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]
*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4,
 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*
AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)
] + (a^2 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x
]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b
 + a*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/((a - b)^2*(a + b)^2*d*(a +
b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)) + ((b + a*Cos[c + d*x])^2*((a*b^2
)/((-a^2 + b^2)^2*(b + a*Cos[c + d*x])) - (2*(-2*a*b + a^2*Cos[c + d*x] + b
^2*Cos[c + d*x])*Csc[c + d*x]^2)/(3*(-a^2 + b^2)^2))*Sin[c + d*x]*Tan[c + d
*x]^2)/(d*(a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(b\sec(dx+c)+a)^2(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2)), x)
```

maple [A] time = 18.90, size = 2159, normalized size = 1.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x)
```

```
[Out] 4/3/d/e*a*b/(a^2-b^2)^2/(e*sin(d*x+c))^(3/2)+1/d*a/e*b^3/(a-b)^2/(a+b)^2*(e
*sin(d*x+c))^(1/2)/(-a^2*cos(d*x+c)^2*e^2+b^2*e^2)+1/d*a^3/e*b/(a-b)^2/(a+b
)^2*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(
e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))
+3/4/d*a/e*b^3/(a-b)^2/(a+b)^2*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)
*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-
(e^2*(a^2-b^2)/a^2)^(1/4)))+2/d*a^3/e*b/(a-b)^2/(a+b)^2*(e^2*(a^2-b^2)/a^2)
^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(
1/4))+3/2/d*a/e*b^3/(a-b)^2/(a+b)^2*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2
*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-1/d/e^2*sin(d*
x+c)*cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a-b)/(a+b)*a^2/(a^2-b^2)/(-cos(d*
x+c)^2*a^2+b^2)-1/2/d/e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a-b)/(a+b)/(
a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)*Elli
pticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/2/d/e^2/cos(d*x+c)/(e*sin(d*x+c)
)^(1/2)*b^2/(a-b)/(a+b)/(a^2-b^2)^(3/2)*a*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+
c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+
1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+5/4/d/e^2/cos(d*x+c)/(e*sin(d
*x+c))^(1/2)*b^4/(a-b)/(a+b)/(a^2-b^2)^(3/2)/a*(-sin(d*x+c)+1)^(1/2)*(2*sin
(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*
x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/2/d/e^2/cos(d*x+c)/(e*
sin(d*x+c))^(1/2)*b^2/(a-b)/(a+b)/(a^2-b^2)^(3/2)*a*(-sin(d*x+c)+1)^(1/2)*(
2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-s
in(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-5/4/d/e^2/cos(d*x+c
)/(e*sin(d*x+c))^(1/2)*b^4/(a-b)/(a+b)/(a^2-b^2)^(3/2)/a*(-sin(d*x+c)+1)^(1
/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticP
i((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/2/d/e^2/cos(
d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a-b)^2/(a+b)^2*a/(a^2-b^2)^(1/2)*(-sin(d*x
+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*
EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/d
/e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/(a-b)^2/(a+b)^2/a/(a^2-b^2)^(1/2)*
(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^
(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/
2))-3/2/d/e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a-b)^2/(a+b)^2*a/(a^2-b^
2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(1/2)/(1+(
a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),
1/2*2^(1/2))+1/2/d/e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/(a-b)^2/(a+b)^2/
a/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(
1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^
(1/2)/a),1/2*2^(1/2))+1/3/d/e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2
/(cos(d*x+c)^2-1)*(-sin(d*x+c)+1)^(1/2)*(2*sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(
5/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2+1/3/d/e^2/cos(d*x+c)/
(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2/(cos(d*x+c)^2-1)*(-sin(d*x+c)+1)^(1/2)*(2*
sin(d*x+c)+2)^(1/2)*sin(d*x+c)^(5/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^
(1/2))*b^2+2/3/d/e^2*cos(d*x+c)/(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2/(cos(d*x+c
)^2-1)*sin(d*x+c)*a^2+2/3/d/e^2*cos(d*x+c)/(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2
/(cos(d*x+c)^2-1)*sin(d*x+c)*b^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2}{{\left(e\,\sin\left(c+d\,x\right)\right)}^{5/2}\,{\left(b+a\,\cos\left(c+d\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((e*sin(c + d*x))^(5/2)*(a + b/cos(c + d*x))^2),x)
```

```
[Out] int(cos(c + d*x)^2/((e*sin(c + d*x))^(5/2)*(b + a*cos(c + d*x))^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.249 $\int \sqrt{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=125

$$-\frac{2\cot(e+fx)\sqrt{-\frac{b(1-\sec(e+fx))}{a+b\sec(e+fx)}}\sqrt{\frac{b(\sec(e+fx)+1)}{a+b\sec(e+fx)}}(a+b\sec(e+fx))\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{a-b}{a+b}\right)}{f\sqrt{a+b}}$$

```
[Out] -2*cot(f*x+e)*EllipticPi((a+b)^(1/2)/(a+b*sec(f*x+e))^(1/2),a/(a+b),((a-b)/
(a+b))^(1/2))*(a+b*sec(f*x+e))*(-b*(1-sec(f*x+e))/(a+b*sec(f*x+e)))^(1/2)*(
b*(1+sec(f*x+e))/(a+b*sec(f*x+e)))^(1/2)/f/(a+b)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3780}

$$-\frac{2\cot(e+fx)\sqrt{-\frac{b(1-\sec(e+fx))}{a+b\sec(e+fx)}}\sqrt{\frac{b(\sec(e+fx)+1)}{a+b\sec(e+fx)}}(a+b\sec(e+fx))\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{a-b}{a+b}\right)}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-2*Cot[e + f*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[e
+ f*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[e + f*x]))/(a + b*Sec[e + f*x
]))]*Sqrt[(b*(1 + Sec[e + f*x]))/(a + b*Sec[e + f*x])]*(a + b*Sec[e + f*x])
)/(Sqrt[a + b]*f)
```

Rule 3780

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*(a + b
*Csc[c + d*x])*Sqrt[(b*(1 + Csc[c + d*x]))/(a + b*Csc[c + d*x])]*Sqrt[-((b*
(1 - Csc[c + d*x]))/(a + b*Csc[c + d*x]))]*EllipticPi[a/(a + b), ArcSin[Rt[
a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)/(a + b)])/(d*Rt[a + b, 2]*Cot[
c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(e+fx)}\,dx = -\frac{2\cot(e+fx)\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(e+fx))}{a+b\sec(e+fx)}}\sqrt{\frac{b(1+\sec(e+fx))}{a+b\sec(e+fx)}}(}{\sqrt{a+b}f}$$

Mathematica [A] time = 0.27, size = 151, normalized size = 1.21

$$\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\sqrt{a+b\sec(e+fx)}\left((b-a)F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)\right.}{f(a\cos(e+fx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Cos
[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((-a + b)*EllipticF[ArcSin[Tan[(e
+ f*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]],
 (a - b)/(a + b)])*Sqrt[a + b*Sec[e + f*x]])/(f*(b + a*Cos[e + f*x]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(f*x + e) + a), x)
```

maple [A] time = 1.34, size = 215, normalized size = 1.72

$$-\frac{2\sqrt{\frac{b+a\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{\frac{b+a\cos\left(fx+e\right)}{\left(1+\cos\left(fx+e\right)\right)(a+b)}}\left(1+\cos\left(fx+e\right)\right)^{2}\left(\mathrm{EllipticF}\left(\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)},\sqrt{\frac{a-b}{a+b}}\right)a-\mathrm{Ellipt}\right.}{f\left(b+a\cos\left(fx+e\right)\right)\sin\left(fx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(1/2),x)
[Out] -2/f*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*(1+cos(f*x+e))^2*(EllipticF((
-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a-EllipticF((-1+cos(f*x+e))/
sin(f*x+e),((a-b)/(a+b))^(1/2))*b-2*a*EllipticPi((-1+cos(f*x+e))/sin(f*x+e)
,-1,((a-b)/(a+b))^(1/2)))*(-1+cos(f*x+e))/(b+a*cos(f*x+e))/sin(f*x+e)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec\left(fx+e\right)+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sec(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/2),x)
[Out] int((a + b/cos(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec\left(e+fx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)), x)
```

3.250 $\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=121

$$\frac{\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\sec(e+fx)}}{f}$$

```
[Out] cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2)
)*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2
)/f-cot(f*x+e)*(a+b*sec(f*x+e))^(1/2)/f
```

Rubi [A] time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3875, 3832}

$$\frac{\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[e + f*x]))/(a - b))])/f - (Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/f
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rubi steps

$$\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx = -\frac{\cot(e+fx)\sqrt{a+b\sec(e+fx)}}{f}+\frac{1}{2}b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$$

$$=\frac{\sqrt{a+b}\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+s}{}}}{f}$$

Mathematica [A] time = 1.27, size = 120, normalized size = 0.99

$$\frac{b\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(\sec(e+fx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)-\csc(e+fx)\sqrt{\frac{1}{\sec(e+fx)+1}}(a\cos(e+fx)+b)}{f\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-((b + a*Cos[e + f*x])*Csc[e + f*x]*Sqrt[(1 + Sec[e + f*x])^(-1)]) + b*Ell
ipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[(a + b*Sec[e + f*x])
/((a + b)*(1 + Sec[e + f*x]))])/(f*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[a + b
*Sec[e + f*x]])
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{b\sec\left(fx+e\right)+a}\,\csc\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*csc(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sec\left(fx+e\right)+a}\,\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*csc(f*x + e)^2, x)
```

maple [B] time = 1.34, size = 264, normalized size = 2.18

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}\left(\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\,\sqrt{\frac{b+a\cos\left(fx+e\right)}{\left(1+\cos\left(fx+e\right)\right)\left(a+b\right)}}\;\mathrm{EllipticF}\left(\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)},\sqrt{\frac{a-b}{a+b}}\right)b\sin\left(fx+e\right)\cos\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(-1+cos(f*x+e))^2*((cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))
/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a
+b))^(1/2))*b*sin(f*x+e)*cos(f*x+e)+EllipticF((-1+cos(f*x+e))/sin(f*x+e),((
a-b)/(a+b))^(1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+c
os(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*b+a*cos(f*x+e)^2+b*cos(f*x+e))*(1+cos(f*
x+e))^2*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)/(b+a*cos(f*x+e))/sin(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sec\left(fx+e\right)+a}\,\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*csc(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}}{{\sin\left(e+f\,x\right)}^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(1/2)/sin(e + f*x)^2,x)
```

```
[Out] int((a + b/cos(e + f*x))^(1/2)/sin(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sec\left(e + fx\right)}\csc^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))*csc(e + f*x)**2, x)
```

3.251 $\int (a+b\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=309

$$\frac{2(2a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f}-\frac{2(a-b)\sqrt{a+b}\cot}{}$$

```
[Out] -2*(a-b)*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-
b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a
-b))^(1/2)/f+2*(2*a-b)*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1
/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+
sec(f*x+e))/(a-b))^(1/2)/f-2*a*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)
/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+
b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f
```

Rubi [A] time = 0.23, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3781, 3921, 3784, 3832, 4004}

$$\frac{2(2a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f}-\frac{2(a-b)\sqrt{a+b}\cot}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[e + f*x]))/(a - b))])/f + (2*(2*a - b)*Sqrt[a + b]*Cot[e +
f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a -
b))])/f - (2*a*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a
 + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/f
```

Rule 3781

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(3/2), x_Symbol] :> Int[(a^2 + b
*(2*a - b)*Csc[c + d*x])/Sqrt[a + b*Csc[c + d*x]], x] + Dist[b^2, Int[(Csc[
c + d*x]*(1 + Csc[c + d*x]))/Sqrt[a + b*Csc[c + d*x]], x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int (a+b\sec(e+fx))^{3/2}\,dx = b^2\int \frac{\sec(e+fx)(1+\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}\,dx + \int \frac{a^2+(2a-b)b\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$$

$$= -\frac{2(a-b)\sqrt{a+b}\,\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a-}}}{f}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec}{a-}}}{f}$$

Mathematica [C] time = 6.14, size = 882, normalized size = 2.85

$$\frac{2b\cos(e+fx)\sin(e+fx)(a+b\sec(e+fx))^{3/2}}{f(b+a\cos(e+fx))} + \frac{2\left(-b^2\sqrt{\frac{b-a}{a+b}}\tan^5\left(\frac{1}{2}(e+fx)\right)+ab\sqrt{\frac{b-a}{a+b}}\tan^5\left(\frac{1}{2}(e+fx)\right)-2ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*b*Cos[e + f*x]*(a + b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(f*(b + a*Cos[e
+ f*x])) + (2*(a + b*Sec[e + f*x])^(3/2)*(a*b*Sqrt[(-a + b)/(a + b)]*Tan[(e
 + f*x)/2] + b^2*Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2] - 2*a*b*Sqrt[(-a +
 b)/(a + b)]*Tan[(e + f*x)/2]^3 + a*b*Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/
2]^5 - b^2*Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]^5 + (2*I)*a^2*EllipticPi
[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a
 + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2
]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + (2*I)*a^2*EllipticPi[-((a + b)/(a -
b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*T
an[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x
)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - I*(a - b)*b*EllipticE[I*ArcSinh[S
qrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e +
 f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b
*Tan[(e + f*x)/2]^2)/(a + b)] - I*(a - b)^2*EllipticF[I*ArcSinh[Sqrt[(-a +
b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2
]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e +
f*x)/2]^2)/(a + b)]))/(Sqrt[(-a + b)/(a + b)]*f*(b + a*Cos[e + f*x])^(3/2)*
```

```
Sec[e + f*x]^(3/2)*Sqrt[(1 - Tan[(e + f*x)/2]^2)^(-1)]*(-1 + Tan[(e + f*x)/
2]^2)*(1 + Tan[(e + f*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b
*Tan[(e + f*x)/2]^2)/(1 + Tan[(e + f*x)/2]^2)])
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec \left(fx+e\right)+a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2), x)
```

maple [B] time = 1.37, size = 1199, normalized size = 3.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(3/2),x)
```

```
[Out] 2/f*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*(1+cos(f*x+e))^2*(-1+cos(f*x+e))^2*
(cos(f*x+e)*a^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(
f*x+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)
/(a+b))^(1/2))-2*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x
+e))/(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f
*x+e),((a-b)/(a+b))^(1/2))*a*b-cos(f*x+e)*b^2*(cos(f*x+e)/(1+cos(f*x+e)))^(
1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticF((-1
+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))+cos(f*x+e)*(cos(f*x+e)/(1+cos(
f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*Ell
ipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b+cos(f*x+e)*b^2*(
cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1
/2)*sin(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))-2*
cos(f*x+e)*a^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f
*x+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a
-b)/(a+b))^(1/2))+(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+co
s(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(
1/2))*a^2*sin(f*x+e)-2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/
(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e)
,((a-b)/(a+b))^(1/2))*a*b-b^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f
*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticF((-1+cos(f*x+e))/sin
(f*x+e),((a-b)/(a+b))^(1/2))+(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*
x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(
f*x+e),((a-b)/(a+b))^(1/2))*a*b+b^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a
*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticE((-1+cos(f*x+e
))/sin(f*x+e),((a-b)/(a+b))^(1/2))-2*a^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticPi((-1+cos
(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))-cos(f*x+e)^2*a*b+a*b*cos(f*x+e)
-cos(f*x+e)*b^2+b^2)/sin(f*x+e)^5/(b+a*cos(f*x+e))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(3/2),x)
```

```
[Out] int((a + b/cos(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(e+fx\right)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x))**(3/2), x)
```

3.252 $\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=228

$$-\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f}+\frac{3(a-b)\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{s}}}{\sqrt{}}\right.\right.}{f}$$

```
[Out] -cot(f*x+e)*(a+b*sec(f*x+e))^(3/2)/f-3*(a-b)*cot(f*x+e)*EllipticE((a+b*sec(
f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e)
)/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f+3*(a-b)*cot(f*x+e)*Ellipti
cF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(
1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f
```

Rubi [A] time = 0.24, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3875, 3829, 3832, 4004}

$$-\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f}+\frac{3(a-b)\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\frac{\sqrt{a+b\text{s}}}{\sqrt{}}\right.\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-3*(a - b)*Sqrt[a + b]*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[e + f*x]))/(a - b))])/f + (3*(a - b)*Sqrt[a + b]*Cot[e + f*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b)
)])/f - (Cot[e + f*x]*(a + b*Sec[e + f*x])^(3/2))/f
```

Rule 3829

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[a - b, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[b, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x],
x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
```

```
2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx = -\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f} + \frac{1}{2}(3b)\int \sec(e+fx)\sqrt{a+b\sec(e$$
$$= -\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f} + \frac{1}{2}(3(a-b)b)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+f}}$$
$$= -\frac{3(a-b)\sqrt{a+b}\,\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+f}{a+b}}}{f}$$

Mathematica [A] time = 11.25, size = 276, normalized size = 1.21

$$\frac{\cos(e+fx)(a+b\sec(e+fx))^{3/2}(\csc(e+fx)(-a\cos(e+fx)-b)+3b\sin(e+fx))}{f(a\cos(e+fx)+b)} + \frac{3b(a+b\sec(e+fx))^{3/2}\left(-t\right.}{f\sqrt{s}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Cos[e + f*x]*(a + b*Sec[e + f*x])^(3/2)*((-b - a*Cos[e + f*x])*Csc[e + f*x
] + 3*b*Sin[e + f*x]))/(f*(b + a*Cos[e + f*x])) + (3*b*(a + b*Sec[e + f*x])
^(3/2)*(-(((a + b)*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*
(EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - EllipticF[ArcSin[Ta
n[(e + f*x)/2]], (a - b)/(a + b)]))/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])])
- (b + a*Cos[e + f*x])*Tan[(e + f*x)/2]))/(f*(b + a*Cos[e + f*x])^2*Sqrt[Se
c[(e + f*x)/2]^2]*Sec[e + f*x]^(3/2)*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]])
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\csc\left(fx+e\right)^2\sec\left(fx+e\right)+a\csc\left(fx+e\right)^2\right)\sqrt{b\sec\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e)^2*sec(f*x + e) + a*csc(f*x + e)^2)*sqrt(b*sec(f*x
+ e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)*csc(f*x + e)^2, x)
```

maple [B] time = 1.39, size = 849, normalized size = 3.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/f*(-1+cos(f*x+e))^2*(3*cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a
*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticE((-1+cos(f*x+e
))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b+3*cos(f*x+e)*b^2*(cos(f*x+e)/(1+cos(
f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*Ell
ipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))-3*cos(f*x+e)*(cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*si
n(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b-3*co
s(f*x+e)*b^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x
+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a
+b))^(1/2))+3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*
x+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(
a+b))^(1/2))*a*b+3*b^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/
(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e)
,((a-b)/(a+b))^(1/2))-3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))
/(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e
),((a-b)/(a+b))^(1/2))*a*b-3*b^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*co
s(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticF((-1+cos(f*x+e))/
sin(f*x+e),((a-b)/(a+b))^(1/2))-a^2*cos(f*x+e)^2-3*cos(f*x+e)^2*a*b+a*b*cos
(f*x+e)-3*cos(f*x+e)*b^2+2*b^2)*(1+cos(f*x+e))^2*((b+a*cos(f*x+e))/cos(f*x+
e))^(1/2)/(b+a*cos(f*x+e))/sin(f*x+e)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sec\left(fx + e\right) + a\right)^{\frac{3}{2}} \csc\left(fx + e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)*csc(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)^{3/2}}{{\sin\left(e+f\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(e + f*x))^(3/2)/sin(e + f*x)^2,x)
```

```
[Out] int((a + b/cos(e + f*x))^(3/2)/sin(e + f*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

3.253 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal. Leaf size=106

$$-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{af}$$

```
[Out] -2*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))
/(a-b))^(1/2)/a/f
```

Rubi [A] time = 0.02, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {3784}

$$-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e
+ f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a*f)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx = -\frac{2\sqrt{a+b}\cot(e+fx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+}{a-b}}}{af}$$

Mathematica [A] time = 0.22, size = 138, normalized size = 1.30

$$-\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}\sec(e+fx)\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\left(F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)-2\Pi\left(-1;\sin^{-}\right.\right.}{f\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Co
s[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*(EllipticF[ArcSin[Tan[(e + f*x)/2
]], (a - b)/(a + b)] - 2*EllipticPi[-1, ArcSin[Tan[(e + f*x)/2]], (a - b)/(
a + b)])*Sec[e + f*x])/(f*Sqrt[a + b*Sec[e + f*x]])
```

fricas [F] time = 31.82, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{b\sec\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sec(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sec(f*x + e) + a), x)
```

maple [A] time = 1.36, size = 178, normalized size = 1.68

$$-\frac{2\sqrt{\frac{b+a\cos\left(fx+e\right)}{\cos\left(fx+e\right)}}\,\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\,\sqrt{\frac{b+a\cos\left(fx+e\right)}{\left(1+\cos\left(fx+e\right)\right)\left(a+b\right)}}\,\left(1+\cos\left(fx+e\right)\right)^{2}\left(\text{EllipticF}\left(\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)},\sqrt{\frac{a-b}{a+b}}\right)-2\,\text{Ell}\right.}{f\left(b+a\cos\left(fx+e\right)\right)\sin\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/f*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*(1+cos(f*x+e))^2*(EllipticF((
-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))-2*EllipticPi((-1+cos(f*x+e))
/sin(f*x+e),-1,((a-b)/(a+b))^(1/2)))*(-1+cos(f*x+e))/(b+a*cos(f*x+e))/sin(f
*x+e)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sec(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\frac{b}{\cos\left(e+fx\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x))^(1/2),x)
```

```
[Out] int(1/(a + b/cos(e + f*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\sec\left(e + fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sec(e + f*x)), x)
```

3.254 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal. Leaf size=255

$$\frac{b^2\tan(e+fx)}{f\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}-\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}}-\frac{\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\sqrt{a}\right.\right.}{f\sqrt{a+b}}$$

```
[Out] cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2)
)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f/(a+b)^(1
/2)-cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(
1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/f/(a+b
)^(1/2)-cot(f*x+e)/f/(a+b*sec(f*x+e))^(1/2)+b^2*tan(f*x+e)/(a^2-b^2)/f/(a+b
*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3875, 3833, 21, 3829, 3832, 4004}

$$\frac{b^2\tan(e+fx)}{f\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}-\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}}-\frac{\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,F\left(\sin^{-1}\left(\sqrt{a}\right.\right.}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x
]))/(a - b))])/(Sqrt[a + b]*f) - (Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*
Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(Sqrt[a + b]*f) - Cot[e + f
*x]/(f*Sqrt[a + b*Sec[e + f*x]]) + (b^2*Tan[e + f*x])/((a^2 - b^2)*f*Sqrt[a
 + b*Sec[e + f*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3829

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[a - b, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[b, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x],
x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3833

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx = -\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}} - \frac{1}{2}b\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx$$

$$= -\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}} + \frac{b^2\tan(e+fx)}{\left(a^2-b^2\right)f\sqrt{a+b\sec(e+fx)}} + \frac{b\int \frac{\sec(e+fx)\left(-\frac{a}{2}-\frac{1}{2}b\sec(e+fx\right.}{\sqrt{a+b\sec(e+fx)}}}{a^2-b^2}$$

$$= -\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}} + \frac{b^2\tan(e+fx)}{\left(a^2-b^2\right)f\sqrt{a+b\sec(e+fx)}} - \frac{b\int \sec(e+fx)\sqrt{a+b\sec}}{2\left(a^2-b^2\right)}$$

$$= -\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}} + \frac{b^2\tan(e+fx)}{\left(a^2-b^2\right)f\sqrt{a+b\sec(e+fx)}} - \frac{b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{2(a+b)} - \frac{b^2}{}$$

$$= \frac{\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{\sqrt{a+b}\,f} - \frac{\cot(e+}{}$$

Mathematica [A] time = 7.75, size = 259, normalized size = 1.02

$$\frac{\sqrt{\sec(e+fx)}\left(\frac{\csc(e+fx)(a\cos(e+fx)+b)(b\cos(e+fx)-a)}{\left(a^2-b^2\right)\sqrt{\sec(e+fx)}} + \frac{b\left(-\tan\left(\frac{1}{2}(e+fx)\right)(a\cos(e+fx)+b) - \frac{(a+b)\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\left(E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\right.\right.}{\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}}}{\left(b^2-a^2\right)\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+f}}\right.}{f\sqrt{a+b\sec(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[Sec[e + f*x]]*(((b + a*Cos[e + f*x])*(-a + b*Cos[e + f*x])*Csc[e + f*
x])/((a^2 - b^2)*Sqrt[Sec[e + f*x]]) + (b*(-(((a + b)*Sqrt[(b + a*Cos[e + f
```

```
*x])/((a + b)*(1 + Cos[e + f*x]))]*(EllipticE[ArcSin[Tan[(e + f*x)/2]], (a
- b)/(a + b)] - EllipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]))/Sqrt
[Cos[e + f*x]/(1 + Cos[e + f*x])]) - (b + a*Cos[e + f*x])*Tan[(e + f*x)/2])
)/((-a^2 + b^2)*Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*
x]])))/(f*Sqrt[a + b*Sec[e + f*x]])
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc\left(fx+e\right)^{2}}{\sqrt{b\sec\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(csc(f*x + e)^2/sqrt(b*sec(f*x + e) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{2}}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*sec(f*x + e) + a), x)
```

maple [B] time = 1.49, size = 852, normalized size = 3.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(-1+cos(f*x+e))^2*(cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*
cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticE((-1+cos(f*x+e)
)/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b+cos(f*x+e)*b^2*(cos(f*x+e)/(1+cos(f*x
+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*Ellipt
icE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))-cos(f*x+e)*(cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*x
+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b-cos(f*x+e
)*b^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a
+b))^(1/2)*sin(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1
/2))+(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+
b))^(1/2)*sin(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/
2))*a*b+b^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+
e))/(a+b))^(1/2)*sin(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+
b))^(1/2))-(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e
))/(a+b))^(1/2)*sin(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b
))^(1/2))*a*b-b^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+co
s(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-
b)/(a+b))^(1/2))+a^2*cos(f*x+e)^2-cos(f*x+e)^2*a*b+a*b*cos(f*x+e)-cos(f*x+e
)*b^2)*(1+cos(f*x+e))^2*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)/(b+a*cos(f*x+e)
)/sin(f*x+e)^5/(a-b)/(a+b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{2}}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*sec(f*x + e) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin\left(e+f\,x\right)^2\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x))^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{2}\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/sqrt(a + b*sec(e + f*x)), x)
```

3.255 $\int \frac{1}{(a+b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=347

$$\frac{2b^2\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}-\frac{2\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\,\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+}}{\sqrt{a+b}}\right.\right.}{a^2f}$$

```
[Out] 2*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/a/f/(a+b
)^(1/2)-2*cot(f*x+e)*EllipticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/
a/f/(a+b)^(1/2)-2*cot(f*x+e)*EllipticPi((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),
(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b
*(1+sec(f*x+e))/(a-b))^(1/2)/a^2/f+2*b^2*tan(f*x+e)/a/(a^2-b^2)/f/(a+b*sec(
f*x+e))^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3785, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b^2\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}-\frac{2\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\,\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+}}{\sqrt{a+b}}\right.\right.}{a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(-3/2),x]
```

```
[Out] (2*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f
*x]))/(a - b))])/(a*Sqrt[a + b]*f) - (2*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[
a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a*Sqrt[a + b]*f) - (
2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e +
f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a^2*f) + (2*b^2*Tan[e + f*x])/(a*(
a^2 - b^2)*f*Sqrt[a + b*Sec[e + f*x]])
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
```

```
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(e+fx))^{3/2}}\,dx = \frac{2b^2\tan(e+fx)}{a\left(a^2-b^2\right)f\sqrt{a+b\sec(e+fx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\frac{1}{2}ab\sec(e+fx)+\frac{1}{2}b^2\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{a\left(a^2-b^2\right)}$$

$$= \frac{2b^2\tan(e+fx)}{a\left(a^2-b^2\right)f\sqrt{a+b\sec(e+fx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\left(\frac{ab}{2}-\frac{b^2}{2}\right)\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{a\left(a^2-b^2\right)} - \frac{b^2\int \frac{\sec(e+fx)(}{\sqrt{a+b\,}}}{a\left(a^2\right.}$$

$$= \frac{2\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{a\sqrt{a+b}\,f} + \frac{}{a\left(a^2\right.}$$

$$= \frac{2\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{a\sqrt{a+b}\,f} - \frac{2\cot}{}$$

Mathematica [C] time = 6.15, size = 1249, normalized size = 3.60

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x])^(-3/2),x]
```

```
[Out] ((b + a*Cos[e + f*x])^2*Sec[e + f*x]^2*((2*b*Sin[e + f*x])/(a*(-a^2 + b^2))
 + (2*b^2*Sin[e + f*x])/(a*(a^2 - b^2)*(b + a*Cos[e + f*x]))))/(f*(a + b*Se
c[e + f*x])^(3/2)) + (2*(b + a*Cos[e + f*x])^(3/2)*Sec[e + f*x]^(3/2)*Sqrt[
```

```
(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(1 + Tan[(e + f*x)/2]
^2)]*(a*b*Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2] + b^2*Sqrt[(-a + b)/(a +
b)]*Tan[(e + f*x)/2] - 2*a*b*Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]^3 + a*
b*Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]^5 - b^2*Sqrt[(-a + b)/(a + b)]*Ta
n[(e + f*x)/2]^5 - (2*I)*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[
(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x
)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)]
 + (2*I)*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)
]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a
+ b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - (2*I)*a^2*Ell
ipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/
2]], (a + b)/(a - b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[
(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + (2*I)*b^2*
EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*
x)/2]], (a + b)/(a - b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sq
rt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - I*(a -
b)*b*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a + b
- a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + I*(a^2 + a*b - 2*
b^2)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a + b
- a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)]))/(a*Sqrt[(-a + b)/
(a + b)]*(a^2 - b^2)*f*(a + b*Sec[e + f*x])^(3/2)*(-1 + Tan[(e + f*x)/2]^2)
*Sqrt[(1 + Tan[(e + f*x)/2]^2)/(1 - Tan[(e + f*x)/2]^2)]*(a*(-1 + Tan[(e +
f*x)/2]^2) - b*(1 + Tan[(e + f*x)/2]^2)))
```

fricas [F] time = 26.22, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)+a}}{b^2\sec\left(fx+e\right)^2+2\,ab\sec\left(fx+e\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)/(b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e)
+ a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(-3/2), x)
```

maple [B] time = 1.45, size = 1209, normalized size = 3.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/f*4^(1/2)*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*(cos(f*x+e)*a^2*(cos(f*x+e)
/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*
x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))+cos(f*x+e)*(
```

```
cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1
/2)*sin(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*
b-cos(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x
+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a
+b))^(1/2))*a*b-cos(f*x+e)*b^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(
f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticE((-1+cos(f*x+e))/si
n(f*x+e),((a-b)/(a+b))^(1/2))-2*cos(f*x+e)*a^2*(cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticPi((
-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))+2*sin(f*x+e)*cos(f*x+e)*(
cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1
/2)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*b^2+(cos(
f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*
EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)+(c
os(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/
2)*sin(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b
-(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^
(1/2)*sin(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*
a*b-b^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/
(a+b))^(1/2)*sin(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^
(1/2))-2*a^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x
+e))/(a+b))^(1/2)*sin(f*x+e)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b
)/(a+b))^(1/2))+2*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*
x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,
((a-b)/(a+b))^(1/2))*b^2+cos(f*x+e)^2*a*b-b^2*cos(f*x+e)^2-a*b*cos(f*x+e)+c
os(f*x+e)*b^2)/(b+a*cos(f*x+e))/sin(f*x+e)/a/(a+b)/(a-b)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{{\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b/cos(e + f*x))^(3/2),x)
```

```
[Out] int(1/(a + b/cos(e + f*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b \sec{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x))**(-3/2), x)
```

3.256 $\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=318

$$\frac{4ab^2\tan(e+fx)}{f\left(a^2-b^2\right)^2\sqrt{a+b\sec(e+fx)}}+\frac{b^2\tan(e+fx)}{f\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}-\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}}-\frac{(3a-b)\cot(e+f}{}$$

```
[Out] -cot(f*x+e)/f/(a+b*sec(f*x+e))^(3/2)+4*a*cot(f*x+e)*EllipticE((a+b*sec(f*x+
e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(
-b*(1+sec(f*x+e))/(a-b))^(1/2)/(a-b)/(a+b)^(3/2)/f-(3*a-b)*cot(f*x+e)*Ellip
ticF((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(f*x+
e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/(a-b)/(a+b)^(3/2)/f+b^2*ta
n(f*x+e)/(a^2-b^2)/f/(a+b*sec(f*x+e))^(3/2)+4*a*b^2*tan(f*x+e)/(a^2-b^2)^2/
f/(a+b*sec(f*x+e))^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 318, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3875, 3833, 4003, 4005, 3832, 4004}

$$\frac{4ab^2\tan(e+fx)}{f\left(a^2-b^2\right)^2\sqrt{a+b\sec(e+fx)}}+\frac{b^2\tan(e+fx)}{f\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}-\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}}-\frac{(3a-b)\cot(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (4*a*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e +
 f*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*f) - ((3*a - b)*Cot[e + f*x]*Elli
pticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/((a
 - b)*(a + b)^(3/2)*f) - Cot[e + f*x]/(f*(a + b*Sec[e + f*x])^(3/2)) + (b^2
*Tan[e + f*x])/((a^2 - b^2)*f*(a + b*Sec[e + f*x])^(3/2)) + (4*a*b^2*Tan[e
+ f*x])/((a^2 - b^2)^2*f*Sqrt[a + b*Sec[e + f*x]])
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3833

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
```

```
m}, x]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = -\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}} - \frac{1}{2}(3b)\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx$$

$$= -\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}} + \frac{b^2\tan(e+fx)}{(a^2-b^2)f(a+b\sec(e+fx))^{3/2}} + \frac{b\int \frac{\sec(e+fx)\left(-\frac{3a}{2}+\frac{1}{2}b\,\mathrm{s}\right.}{(a+b\sec(e+fx}}{a^2-b^2}$$

$$= -\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}} + \frac{b^2\tan(e+fx)}{(a^2-b^2)f(a+b\sec(e+fx))^{3/2}} + \frac{4ab^2\tan(e+}{(a^2-b^2)^2 f\sqrt{a+b\,\mathrm{s}}}$$

$$= -\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}} + \frac{b^2\tan(e+fx)}{(a^2-b^2)f(a+b\sec(e+fx))^{3/2}} + \frac{4ab^2\tan(e+}{(a^2-b^2)^2 f\sqrt{a+b\,\mathrm{s}}}$$

$$= \frac{4a\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{(a-b)(a+b)^{3/2}f} - \frac{(3a}{}$$

Mathematica [A] time = 8.49, size = 259, normalized size = 0.81

$$\frac{-2b\left(3a^2+4ab+b^2\right)\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}F\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-((a - b)*((3*a - b)*b + a*(a - 3*b)*Cos[e + f*x])*Csc[e + f*x]) + 8*a*b*(
a + b)*Cos[(e + f*x)/2]^2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f
*x]))]*EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sec[e + f*x]*Sq
rt[(1 + Sec[e + f*x])^(-1)] - 2*b*(3*a^2 + 4*a*b + b^2)*Cos[(e + f*x)/2]^2*
Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*EllipticF[ArcSin[Ta
n[(e + f*x)/2]], (a - b)/(a + b)]*Sec[e + f*x]*Sqrt[(1 + Sec[e + f*x])^(-1)
])/((a^2 - b^2)^2*f*Sqrt[a + b*Sec[e + f*x]])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)+a}\,\csc\left(fx+e\right)^{2}}{b^{2}\sec\left(fx+e\right)^{2}+2\,ab\sec\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*csc(f*x + e)^2/(b^2*sec(f*x + e)^2 + 2*a*
b*sec(f*x + e) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(fx+e\right)^{2}}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e) + a)^(3/2), x)
```

maple [B] time = 1.32, size = 1065, normalized size = 3.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/2/f*(3*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+
cos(f*x+e))/(a+b))^(1/2)*cos(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((
a-b)/(a+b))^(1/2))*a^2*b+4*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b
+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*cos(f*x+e)*EllipticF((-1+cos(f*x
+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b^2+sin(f*x+e)*(cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*cos(f*x+e)*Ellip
ticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^3-4*sin(f*x+e)*(cos(
f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*
cos(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2*b-
4*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x
+e))/(a+b))^(1/2)*cos(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a
+b))^(1/2))*a*b^2+3*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b+a*cos(
f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((
a-b)/(a+b))^(1/2))*a^2*b+4*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*((b
+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*
x+e),((a-b)/(a+b))^(1/2))*a*b^2+sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/s
in(f*x+e),((a-b)/(a+b))^(1/2))*b^3-4*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))
^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+cos(f*x+
```

```
e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2*b-4*sin(f*x+e)*(cos(f*x+e)/(1+cos(f
*x+e)))^(1/2)*((b+a*cos(f*x+e))/(1+cos(f*x+e))/(a+b))^(1/2)*EllipticE((-1+c
os(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b^2-cos(f*x+e)^2*a^3+4*a^2*cos
(f*x+e)^2*b-3*cos(f*x+e)^2*a*b^2-3*cos(f*x+e)*a^2*b+4*cos(f*x+e)*a*b^2-cos(
f*x+e)*b^3)*((b+a*cos(f*x+e))/cos(f*x+e))^(1/2)*4^(1/2)/(b+a*cos(f*x+e))/si
n(f*x+e)/(a-b)^2/(a+b)^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{2}}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e) + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin\left(e+f\,x\right)^{2}\,{\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x))^(3/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^2*(a + b/cos(e + f*x))^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^{2}\left(e+fx\right)}{\left(a+b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*sec(e + f*x))**(3/2), x)
```

3.257 $\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=249

$$\frac{a^3\cos(c+dx)(e\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)\sqrt{\cos^2(c+dx)}}+\frac{3a^2b(e\sin(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2\right.}{de(m+1)}$$

```
[Out] 3*a^2*b*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(
1+m)/d/e/(1+m)+b^3*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*si
n(d*x+c))^(1+m)/d/e/(1+m)+a^3*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/
2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+m)/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)+3*a*
b^2*hypergeom([3/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*sec(d*x+c)*(e*sin(
d*x+c))^(1+m)*(cos(d*x+c)^2)^(1/2)/d/e/(1+m)
```

Rubi [A] time = 0.39, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3872, 2912, 2643, 2564, 364, 2577}

$$\frac{3a^2b(e\sin(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{a^3\cos(c+dx)(e\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2\right.}{de(m+1)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a^3*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (3*a^2*b
*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x]
)^(1 + m))/(d*e*(1 + m)) + (b^3*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2,
Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)) + (3*a*b^2*Sqrt[Cos
[c + d*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*S
ec[c + d*x]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
```

```
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx = -\int (-b-a\cos(c+dx))^3\sec^3(c+dx)(e\sin(c+dx))^m\,dx$$
$$= -\int \left(-a^3(e\sin(c+dx))^m - 3a^2b\sec(c+dx)(e\sin(c+dx))^m - 3ab^2\,\mathrm{se}\right.$$
$$= a^3\int (e\sin(c+dx))^m\,dx + \left(3a^2b\right)\int \sec(c+dx)(e\sin(c+dx))^m\,dx +$$

$$= \frac{a^3\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \frac{3a}{}$$
$$= \frac{a^3\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \frac{3a}{}$$

Mathematica [A] time = 0.34, size = 182, normalized size = 0.73

$$\frac{\tan(c+dx)(e\sin(c+dx))^m\left(a^3\sqrt{\cos^2(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)+b\left(3a^2\cos(c+dx)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+}{2}\right.\right.\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*(e*Sin[c + d*x])^m,x]
```

```
[Out] ((a^3*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin
[c + d*x]^2] + b*(3*a^2*Cos[c + d*x]*Hypergeometric2F1[1, (1 + m)/2, (3 + m
)/2, Sin[c + d*x]^2] + b*(3*a*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (
1 + m)/2, (3 + m)/2, Sin[c + d*x]^2] + b*Cos[c + d*x]*Hypergeometric2F1[2,
(1 + m)/2, (3 + m)/2, Sin[c + d*x]^2])))*(e*Sin[c + d*x])^m*Tan[c + d*x])/(
d*(1 + m))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3\right)(e\sin(dx+c))^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)*(e*sin(d*x + c))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^3 (e\sin(dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*(e*sin(d*x + c))^m, x)
```

maple [F] time = 4.12, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^3 (e\sin(dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^3 (e\sin(dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*(e*sin(d*x + c))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int (e\,\sin(c+d\,x))^m \left(a+\frac{b}{\cos(c+d\,x)}\right)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^3,x)
```

```
[Out] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\sin(c+dx))^m (a+b\sec(c+dx))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(e*sin(d*x+c))**m,x)
```

```
[Out] Integral((e*sin(c + d*x))**m*(a + b*sec(c + d*x))**3, x)
```

3.258 $\int (a+b\sec(c+dx))^2(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=190

$$\frac{a^2\sin(c+dx)\cos(c+dx)(e\sin(c+dx))^m\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}+\frac{2ab(e\sin(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2}\right.}{de(m+1)}$$

```
[Out] 2*a*b*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+
m)/d/e/(1+m)+a^2*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+
c)^2)*sin(d*x+c)*(e*sin(d*x+c))^m/d/(1+m)/(cos(d*x+c)^2)^(1/2)+b^2*hypergeo
m([3/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^m*(cos(d*x+c)^2
)^(1/2)*tan(d*x+c)/d/(1+m)
```

Rubi [A] time = 0.84, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3872, 2911, 2564, 364, 4398, 4401, 2643, 2577}

$$\frac{a^2\sin(c+dx)\cos(c+dx)(e\sin(c+dx))^m\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}+\frac{2ab(e\sin(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2}\right.}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*Sin[c + d*x]*(e*Sin[c + d*x])^m)/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (2*
a*b*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d
*x])^(1 + m))/(d*e*(1 + m)) + (b^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3
/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^m*Tan[c + d*x])/
(d*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
```

```
&& !IntegerQ[2*n]
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 4398

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[(a^IntPart[p]*(a*vv)^FracPart[p])/vv^FracPart[p], Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] && !IntegerQ[p] && !InertTrigFreeQ[
v]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2(e\sin(c+dx))^m\,dx = \int (-b-a\cos(c+dx))^2\sec^2(c+dx)(e\sin(c+dx))^m\,dx$$
$$= (2ab)\int \sec(c+dx)(e\sin(c+dx))^m\,dx + \int \left(b^2+a^2\cos^2(c+dx)\right)$$
$$= \frac{(2ab)\,\mathrm{Subst}\left(\int \frac{x^m}{1-\frac{x^2}{e^2}}\,dx, x, e\sin(c+dx)\right)}{de} + (\sin^{-m}(c+dx)(e\sin(c$$
$$= \frac{2ab\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)} + (\sin^{-m}(c+$$
$$= \frac{2ab\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)} + \left(a^2\sin^{-m}(\right.$$
$$= \frac{a^2\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin(c+dx)(e\sin(c+}{d(1+m)\sqrt{\cos^2(c+dx)}}$$

Mathematica [A] time = 0.30, size = 134, normalized size = 0.71

$$\frac{(e\sin(c+dx))^m\left(\sqrt{\cos^2(c+dx)}\,\tan(c+dx)\left(a^2\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)+b^2\,{}_2F_1\left(\frac{3}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+\right.\right.}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^m,x]
```

```
[Out] ((e*Sin[c + d*x])^m*(2*a*b*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c
 + d*x]^2]*Sin[c + d*x] + Sqrt[Cos[c + d*x]^2]*(a^2*Hypergeometric2F1[1/2,
(1 + m)/2, (3 + m)/2, Sin[c + d*x]^2] + b^2*Hypergeometric2F1[3/2, (1 + m)/
2, (3 + m)/2, Sin[c + d*x]^2])*Tan[c + d*x]))/(d*(1 + m))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2 \sec\left(dx+c\right)^2 + 2\,ab\sec\left(dx+c\right) + a^2\right)\left(e\sin\left(dx+c\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*(e*sin(d*x + c))^m
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(dx+c\right)+a\right)^2 \left(e\sin\left(dx+c\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*(e*sin(d*x + c))^m, x)
```

maple [F] time = 4.09, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(dx+c\right)\right)^2 \left(e\sin\left(dx+c\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(dx+c\right)+a\right)^2 \left(e\sin\left(dx+c\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*(e*sin(d*x + c))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(e\,\sin\left(c+d\,x\right)\right)}^m\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^2,x)
```

```
[Out] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(e\sin\left(c+dx\right)\right)^m \left(a+b\sec\left(c+dx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(e*sin(d*x+c))**m,x)
```

```
[Out] Integral((e*sin(c + d*x))**m*(a + b*sec(c + d*x))**2, x)
```

3.259 $\int (a+b\sec(c+dx))(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=119

$$\frac{a\cos(c+dx)(e\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)\sqrt{\cos^2(c+dx)}}+\frac{b(e\sin(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+\right.}{de(m+1)}$$

```
[Out] b*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+m)/d
/e/(1+m)+a*cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*
(e*sin(d*x+c))^(1+m)/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2838, 2564, 364, 2643}

$$\frac{a\cos(c+dx)(e\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)\sqrt{\cos^2(c+dx)}}+\frac{b(e\sin(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+\right.}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2
]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (b*Hyperge
ometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1 + m
))/(d*e*(1 + m))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
```

```
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int (a+b\sec(c+dx))(e\sin(c+dx))^m\,dx = -\int (-b-a\cos(c+dx))\sec(c+dx)(e\sin(c+dx))^m\,dx$$
$$= a\int (e\sin(c+dx))^m\,dx + b\int \sec(c+dx)(e\sin(c+dx))^m\,dx$$
$$= \frac{a\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \frac{b\,\mathrm{Su}}{}$$
$$= \frac{a\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \frac{b\,{}_2F}{}$$

Mathematica [A] time = 0.11, size = 98, normalized size = 0.82

$$\frac{\tan(c+dx)(e\sin(c+dx))^m\left(a\sqrt{\cos^2(c+dx)}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)+b\cos(c+dx)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin\right.\right.}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^m,x]
[Out] ((a*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c
 + d*x]^2] + b*Cos[c + d*x]*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[
c + d*x]^2])*(e*Sin[c + d*x])^m*Tan[c + d*x])/(d*(1 + m))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)(e\sin(dx+c))^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

maple [F] time = 2.64, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)\,(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int (e\,\sin(c+d\,x))^m\,\left(a+\frac{b}{\cos(c+d\,x)}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x)),x)
[Out] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\sin(c+dx))^m\,(a+b\sec(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(e*sin(d*x+c))**m,x)
[Out] Integral((e*sin(c + d*x))**m*(a + b*sec(c + d*x)), x)
```

3.260 $\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=232

$$\frac{\cos(c+dx)(e\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{ade(m+1)\sqrt{\cos^2(c+dx)}}-\frac{be(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)}{a\cos(c+dx}\right.}{}$$

```
[Out] -b*e*AppellF1(1-m,1/2-1/2*m,1/2-1/2*m,2-m,(-a+b)/(b+a*cos(d*x+c)),(a+b)/(b+
a*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(a*(1+cos(d
*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(e*sin(d*x+c))^(-1+m)/a^2/d/(1-m)+cos(
d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^
(1+m)/a/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3872, 2867, 2643, 2703}

$$\frac{\cos(c+dx)(e\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{ade(m+1)\sqrt{\cos^2(c+dx)}}-\frac{be(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)}{a\cos(c+dx}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*e*AppellF1[1 - m, (1 - m)/2, (1 - m)/2, 2 - m, -((a - b)/(b + a*Cos[c
+ d*x])), (a + b)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Co
s[c + d*x])))^((1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1
 - m)/2)*(e*Sin[c + d*x])^(-1 + m))/(a^2*d*(1 - m))) + (Cos[c + d*x]*Hyperg
eometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1
+ m))/(a*d*e*(1 + m)*Sqrt[Cos[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2703

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1)*AppellF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b
*Sin[e + f*x]), (a - b)/(a + b*Sin[e + f*x])])/(b*f*(m + p)*(-((b*(1 - Sin[
e + f*x]))/(a + b*Sin[e + f*x])))^((p - 1)/2)*((b*(1 + Sin[e + f*x]))/(a +
b*Sin[e + f*x]))^((p - 1)/2)), x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)(e\sin(c+dx))^m}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{\int (e\sin(c+dx))^m\,dx}{a} + \frac{b\int \frac{(e\sin(c+dx))^m}{-b-a\cos(c+dx)}\,dx}{a}$$

$$= -\frac{beF_1\left(1-m;\frac{1-m}{2},\frac{1-m}{2};2-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+c\cdots}{b+a\,c\cdots}\right.}{a^2d(1-m)}$$

Mathematica [B] time = 5.97, size = 687, normalized size = 2.96

$$\frac{\cdots}{d(a+b\sec(c+dx))\left(2m\tan^2\left(\frac{1}{2}(c+dx)\right)\left((a+b)\,{}_2F_1\left(\frac{m+1}{2},m+1;\frac{m+3}{2};-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-bF_1\left(\frac{m+1}{2};m,1;\cdots\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(-(b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*
Tan[(c + d*x)/2]^2)/(a + b)]) + (a + b)*Hypergeometric2F1[(1 + m)/2, 1 + m,
 (3 + m)/2, -Tan[(c + d*x)/2]^2])*(e*Sin[c + d*x])^m*Tan[(c + d*x)/2])/(d*(
a + b*Sec[c + d*x])*((-(b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*
x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (a + b)*Hypergeometric2F1
[(1 + m)/2, 1 + m, (3 + m)/2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2 + 2*
m*Cot[c + d*x]*(-(b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^
2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (a + b)*Hypergeometric2F1[(1 +
m)/2, 1 + m, (3 + m)/2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2] + 2*m*(-(b*A
ppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c +
d*x)/2]^2)/(a + b)]) + (a + b)*Hypergeometric2F1[(1 + m)/2, 1 + m, (3 + m)/
2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2 + ((1 + m)*Sec[(c + d*x)/2]^2*(
-((a + b)^2*(Hypergeometric2F1[(1 + m)/2, 1 + m, (3 + m)/2, -Tan[(c + d*x)/
2]^2] - (Sec[(c + d*x)/2]^2)^(-1 - m))) + (2*b*((-a + b)*AppellF1[(3 + m)/2
, m, 2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b
)] + (a + b)*m*AppellF1[(3 + m)/2, 1 + m, 1, (5 + m)/2, -Tan[(c + d*x)/2]^2
, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2)/(3 + m)))/(a +
 b)))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(e\sin(dx+c))^m}{b\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x, algorithm="fricas")
[Out] integral((e*sin(d*x + c))^m/(b*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a), x)
```

maple [F] time = 1.98, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{a+b\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x)
[Out] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)\,\left(e\,\sin(c+d\,x)\right)^m}{b+a\,\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m/(a + b/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e*sin(c + d*x))^m)/(b + a*cos(c + d*x)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c)),x)
[Out] Integral((e*sin(c + d*x))**m/(a + b*sec(c + d*x)), x)
```

3.261 $\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=405

$$\frac{b^2e(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}F_1\left(2-m;\frac{1-m}{2},\frac{1-m}{2};3-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+d}\right.}{a^3d(2-m)(a\cos(c+dx)+b)}$$

```
[Out] -2*b*e*AppellF1(1-m,1/2-1/2*m,1/2-1/2*m,2-m,(-a+b)/(b+a*cos(d*x+c)),(a+b)/(
b+a*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(a*(1+cos
(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(e*sin(d*x+c))^(-1+m)/a^3/d/(1-m)+b^
2*e*AppellF1(2-m,1/2-1/2*m,1/2-1/2*m,3-m,(-a+b)/(b+a*cos(d*x+c)),(a+b)/(b+a
*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(a*(1+cos(d*
x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(e*sin(d*x+c))^(-1+m)/a^3/d/(2-m)/(b+a*
cos(d*x+c))+cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/2*m],sin(d*x+c)^2)
*(e*sin(d*x+c))^(1+m)/a^2/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 405, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3872, 2912, 2643, 2703}

$$\frac{b^2e(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}F_1\left(2-m;\frac{1-m}{2},\frac{1-m}{2};3-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+d}\right.}{a^3d(2-m)(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*b*e*AppellF1[1 - m, (1 - m)/2, (1 - m)/2, 2 - m, -((a - b)/(b + a*Cos[c
 + d*x])), (a + b)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*C
os[c + d*x])))^((1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((
1 - m)/2)*(e*Sin[c + d*x])^(-1 + m))/(a^3*d*(1 - m)) + (b^2*e*AppellF1[2 -
m, (1 - m)/2, (1 - m)/2, 3 - m, -((a - b)/(b + a*Cos[c + d*x])), (a + b)/(b
 + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Cos[c + d*x])))^((1 -
m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1 - m)/2)*(e*Sin[c +
d*x])^(-1 + m))/(a^3*d*(2 - m)*(b + a*Cos[c + d*x])) + (Cos[c + d*x]*Hyperg
eometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1
+ m))/(a^2*d*e*(1 + m)*Sqrt[Cos[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2703

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1)*AppellF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b
*Sin[e + f*x]), (a - b)/(a + b*Sin[e + f*x])])/(b*f*(m + p)*(-((b*(1 - Sin[
e + f*x]))/(a + b*Sin[e + f*x])))^((p - 1)/2)*((b*(1 + Sin[e + f*x]))/(a +
b*Sin[e + f*x]))^((p - 1)/2)), x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)(e\sin(c+dx))^m}{(-b-a\cos(c+dx))^2}\,dx\\ &= \int \left(\frac{(e\sin(c+dx))^m}{a^2}+\frac{b^2(e\sin(c+dx))^m}{a^2(b+a\cos(c+dx))^2}-\frac{2b(e\sin(c+dx))^m}{a^2(b+a\cos(c+dx))}\right)dx\\ &= \frac{\int (e\sin(c+dx))^m\,dx}{a^2}-\frac{(2b)\int \frac{(e\sin(c+dx))^m}{b+a\cos(c+dx)}\,dx}{a^2}+\frac{b^2\int \frac{(e\sin(c+dx))^m}{(b+a\cos(c+dx))^2}\,dx}{a^2}\\ &= -\frac{2beF_1\left(1-m;\frac{1-m}{2},\frac{1-m}{2};2-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cdots}{b+a\cdots}\right.}{a^3d(1-m)}\end{aligned}$$

Mathematica [B] time = 11.75, size = 1433, normalized size = 3.54

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-4*b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Ta
n[(c + d*x)/2]^2)/(a + b)]*(b + a*Cos[c + d*x])*Sec[c + d*x]^2*(e*Sin[c + d
*x])^m*Tan[(c + d*x)/2])/(a^2*d*(a + b*Sec[c + d*x])^2*(AppellF1[(1 + m)/2,
 m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)
]*Sec[(c + d*x)/2]^2 + 2*m*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d
*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Cot[c + d*x]*Tan[(c + d*x)/
2] + 2*m*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)
*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2 - (2*(1 + m)*((-a + b)*App
ellF1[(3 + m)/2, m, 2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*
x)/2]^2)/(a + b)] + (a + b)*m*AppellF1[(3 + m)/2, 1 + m, 1, (5 + m)/2, -Tan
[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^2*
Tan[(c + d*x)/2]^2)/((a + b)*(3 + m)))) + (2*b^2*((a + b)*AppellF1[(1 + m)/
2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a +
b)] - 2*a*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b
)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[c + d*x]^2*(e*Sin[c + d*x])^m*Tan[(c +
d*x)/2])/(a^2*d*(a + b*Sec[c + d*x])^2*(((a + b)*AppellF1[(1 + m)/2, m, 1,
(3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a
*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c
+ d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^2 + 2*m*((a + b)*AppellF1[(1 + m)/2
, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b
)] - 2*a*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)
*Tan[(c + d*x)/2]^2)/(a + b)])*Cot[c + d*x]*Tan[(c + d*x)/2] + 2*m*((a + b)
*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c
```

```
+ d*x)/2]^2)/(a + b)] - 2*a*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c +
d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2 - (2*(
1 + m)*((-a^2 + b^2)*AppellF1[(3 + m)/2, m, 2, (5 + m)/2, -Tan[(c + d*x)/2]
^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 4*a*(a - b)*AppellF1[(3 + m)/2,
 m, 3, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)
] + (a + b)*m*((a + b)*AppellF1[(3 + m)/2, 1 + m, 1, (5 + m)/2, -Tan[(c + d
*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*AppellF1[(3 + m)/2, 1
 + m, 2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a +
b)]))*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2)/((a + b)*(3 + m)))) - ((b + a*
Cos[c + d*x])^2*Hypergeometric2F1[1/2, (1 - m)/2, 3/2, Cos[c + d*x]^2]*(e*S
in[c + d*x])^m*(Sin[c + d*x]^2)^((-1 - m)/2)*Tan[c + d*x])/(a^2*d*(a + b*Se
c[c + d*x])^2)
```

fricas [F] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(e\sin(dx+c))^m}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((e*sin(d*x + c))^m/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(e\sin(dx+c))^m}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^2, x)
```

maple [F] time = 1.14, size = 0, normalized size = 0.00

$$\int\frac{(e\sin(dx+c))^m}{(a+b\sec(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x)
```

```
[Out] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(e\sin(dx+c))^m}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\cos(c+d\,x)^2\,(e\,\sin(c+d\,x))^m}{(b+a\,\cos(c+d\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^2*(e*sin(c + d*x))^m)/(b + a*cos(c + d*x))^2, x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((e*sin(c + d*x))**m/(a + b*sec(c + d*x))**2, x)
```

3.262 $\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=580

$$-\frac{b^3 e (e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}} F_1\left(3-m;\frac{1-m}{2},\frac{1-m}{2};4-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c}\right.}{a^4 d(3-m)(a\cos(c+dx)+b)^2}$$

```
[Out] -3*b*e*AppellF1(1-m,1/2-1/2*m,1/2-1/2*m,2-m,(-a+b)/(b+a*cos(d*x+c)),(a+b)/(
b+a*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(a*(1+cos
(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(e*sin(d*x+c))^(-1+m)/a^4/d/(1-m)-b^
3*e*AppellF1(3-m,1/2-1/2*m,1/2-1/2*m,4-m,(-a+b)/(b+a*cos(d*x+c)),(a+b)/(b+a
*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(a*(1+cos(d*
x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(e*sin(d*x+c))^(-1+m)/a^4/d/(3-m)/(b+a*
cos(d*x+c))^2+3*b^2*e*AppellF1(2-m,1/2-1/2*m,1/2-1/2*m,3-m,(-a+b)/(b+a*cos(
d*x+c)),(a+b)/(b+a*cos(d*x+c)))*(-a*(1-cos(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1
/2*m)*(a*(1+cos(d*x+c))/(b+a*cos(d*x+c)))^(1/2-1/2*m)*(e*sin(d*x+c))^(-1+m)
/a^4/d/(2-m)/(b+a*cos(d*x+c))+cos(d*x+c)*hypergeom([1/2, 1/2+1/2*m],[3/2+1/
2*m],sin(d*x+c)^2)*(e*sin(d*x+c))^(1+m)/a^3/d/e/(1+m)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.59, antiderivative size = 580, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3872, 2912, 2643, 2703}

$$\frac{3b^2 e (e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}} F_1\left(2-m;\frac{1-m}{2},\frac{1-m}{2};3-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c}\right.}{a^4 d(2-m)(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-3*b*e*AppellF1[1 - m, (1 - m)/2, (1 - m)/2, 2 - m, -((a - b)/(b + a*Cos[c
 + d*x])), (a + b)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*C
os[c + d*x])))^((1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((
1 - m)/2)*(e*Sin[c + d*x])^(-1 + m))/(a^4*d*(1 - m)) - (b^3*e*AppellF1[3 -
m, (1 - m)/2, (1 - m)/2, 4 - m, -((a - b)/(b + a*Cos[c + d*x])), (a + b)/(b
 + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Cos[c + d*x])))^((1 -
m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1 - m)/2)*(e*Sin[c +
d*x])^(-1 + m))/(a^4*d*(3 - m)*(b + a*Cos[c + d*x])^2) + (3*b^2*e*AppellF1[
2 - m, (1 - m)/2, (1 - m)/2, 3 - m, -((a - b)/(b + a*Cos[c + d*x])), (a + b
)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Cos[c + d*x])))^((
1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1 - m)/2)*(e*Sin[
c + d*x])^(-1 + m))/(a^4*d*(2 - m)*(b + a*Cos[c + d*x])) + (Cos[c + d*x]*Hy
pergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])
^(1 + m))/(a^3*d*e*(1 + m)*Sqrt[Cos[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2703

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1)*AppellF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b
```

```
*Sin[e + f*x]), (a - b)/(a + b*Sin[e + f*x])])/(b*f*(m + p)*(-((b*(1 - Sin[
e + f*x]))/(a + b*Sin[e + f*x])))^((p - 1)/2)*((b*(1 + Sin[e + f*x]))/(a +
b*Sin[e + f*x]))^((p - 1)/2)), x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)(e\sin(c+dx))^m}{(-b-a\cos(c+dx))^3}\,dx$$

$$= -\int \left(-\frac{(e\sin(c+dx))^m}{a^3} + \frac{b^3(e\sin(c+dx))^m}{a^3(b+a\cos(c+dx))^3} - \frac{3b^2(e\sin(c+dx))^m}{a^3(b+a\cos(c+dx))^2} + \frac{3b(e\sin(c+dx))^m}{a^3(b+\cdots}\right.$$

$$= \frac{\int (e\sin(c+dx))^m\,dx}{a^3} - \frac{(3b)\int \frac{(e\sin(c+dx))^m}{b+a\cos(c+dx)}\,dx}{a^3} + \frac{\left(3b^2\right)\int \frac{(e\sin(c+dx))^m}{(b+a\cos(c+dx))^2}\,dx}{a^3} - \frac{b^3\int \frac{(e\sin\cdots}{(b+a\cdots}}{\cdots}$$

$$= -\frac{3beF_1\left(1-m;\frac{1-m}{2},\frac{1-m}{2};2-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cdots}{b+a\cdots}\right.}{a^4d(1-m)}$$

Mathematica [B] time = 17.97, size = 2700, normalized size = 4.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-6*b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Ta
n[(c + d*x)/2]^2)/(a + b)]*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^3*(e*Sin[c +
 d*x])^m*Tan[(c + d*x)/2])/(a^3*d*(a + b*Sec[c + d*x])^3*(AppellF1[(1 + m)/
2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a +
b)]*Sec[(c + d*x)/2]^2 + 2*m*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c +
 d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Cot[c + d*x]*Tan[(c + d*x
)/2] + 2*m*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a -
b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2 - (2*(1 + m)*((-a + b)*A
ppellF1[(3 + m)/2, m, 2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c +
d*x)/2]^2)/(a + b)] + (a + b)*m*AppellF1[(3 + m)/2, 1 + m, 1, (5 + m)/2, -T
an[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2]^2)/((a + b)*(3 + m)))) + (6*b^2*((a + b)*AppellF1[(1 + m
)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a
+ b)] - 2*a*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a -
 b)*Tan[(c + d*x)/2]^2)/(a + b)])*(b + a*Cos[c + d*x])*Sec[c + d*x]^3*(e*Si
n[c + d*x])^m*Tan[(c + d*x)/2])/(a^3*d*(a + b*Sec[c + d*x])^3*(((a + b)*App
```

```
ellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*
x)/2]^2)/(a + b)] - 2*a*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)
/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^2 + 2*m*((a
+ b)*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan
[(c + d*x)/2]^2)/(a + b)] - 2*a*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(
c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Cot[c + d*x]*Tan[(c +
 d*x)/2] + 2*m*((a + b)*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)
/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*AppellF1[(1 + m)/2, m, 2
, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Ta
n[(c + d*x)/2]^2 - (2*(1 + m)*((-a^2 + b^2)*AppellF1[(3 + m)/2, m, 2, (5 +
m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 4*a*(a -
 b)*AppellF1[(3 + m)/2, m, 3, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[
(c + d*x)/2]^2)/(a + b)] + (a + b)*m*((a + b)*AppellF1[(3 + m)/2, 1 + m, 1,
 (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - 2*
a*AppellF1[(3 + m)/2, 1 + m, 2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Ta
n[(c + d*x)/2]^2)/(a + b)]))*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2)/((a + b
)*(3 + m)))) - (2*b^3*((a + b)^2*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[
(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - 4*a*(a + b)*AppellF
1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2
]^2)/(a + b)] + 4*a^2*AppellF1[(1 + m)/2, m, 3, (3 + m)/2, -Tan[(c + d*x)/2
]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Sec[c + d*x]^3*(e*Sin[c + d*x])
^m*Tan[(c + d*x)/2])/(a^3*d*(a + b*Sec[c + d*x])^3*(((a + b)^2*AppellF1[(1
+ m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/
(a + b)] - 4*a*(a + b)*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/
2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 4*a^2*AppellF1[(1 + m)/2, m,
3, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*S
ec[(c + d*x)/2]^2 + 2*m*((a + b)^2*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Ta
n[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - 4*a*(a + b)*Appel
lF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)
/2]^2)/(a + b)] + 4*a^2*AppellF1[(1 + m)/2, m, 3, (3 + m)/2, -Tan[(c + d*x)
/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Cot[c + d*x]*Tan[(c + d*x)/2]
 + 2*m*((a + b)^2*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2,
 ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - 4*a*(a + b)*AppellF1[(1 + m)/2, m,
 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] +
 4*a^2*AppellF1[(1 + m)/2, m, 3, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*T
an[(c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2 + (2*(1 + m)*((a + b)^2*((a
 - b)*AppellF1[(3 + m)/2, m, 2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Ta
n[(c + d*x)/2]^2)/(a + b)] - (a + b)*m*AppellF1[(3 + m)/2, 1 + m, 1, (5 + m
)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) - 4*a*(a +
 b)*(2*(a - b)*AppellF1[(3 + m)/2, m, 3, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((
a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*m*AppellF1[(3 + m)/2, 1 + m,
2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) +
 4*a^2*(3*(a - b)*AppellF1[(3 + m)/2, m, 4, (5 + m)/2, -Tan[(c + d*x)/2]^2,
 ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] - (a + b)*m*AppellF1[(3 + m)/2, 1 +
m, 3, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]
))*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2)/((a + b)*(3 + m)))) - ((b + a*Cos
[c + d*x])^3*Hypergeometric2F1[1/2, (1 - m)/2, 3/2, Cos[c + d*x]^2]*Sec[c +
 d*x]*(e*Sin[c + d*x])^m*(Sin[c + d*x]^2)^((-1 - m)/2)*Tan[c + d*x])/(a^3*d
*(a + b*Sec[c + d*x])^3)
```

fricas [F] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{(e\sin(dx+c))^m}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((e*sin(d*x + c))^m/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 +
3*a^2*b*sec(d*x + c) + a^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{(b\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^3, x)
```

maple [F] time = 1.22, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{(a+b\sec(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x)
[Out] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{(b\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^3\,(e\,\sin(c+d\,x))^m}{(b+a\,\cos(c+d\,x))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^3,x)
[Out] int((cos(c + d*x)^3*(e*sin(c + d*x))^m)/(b + a*cos(c + d*x))^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c))**3,x)
[Out] Integral((e*sin(c + d*x))**m/(a + b*sec(c + d*x))**3, x)
```

3.263 $\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=28

$$\text{Int}\left((a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m,x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m,x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx=\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$$

Mathematica [A] time = 7.69, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m,x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m, x]
```

fricas [A] time = 0.84, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^{\frac{3}{2}}\,(e\sin(dx+c))^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}\,(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

maple [A] time = 1.13, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^{\frac{3}{2}}\,(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec (dx+c)+a)^{\frac{3}{2}} (e\sin (dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int {\left(e\,\sin \left(c+d\,x\right)\right)}^m\,{\left(a+\frac{b}{\cos \left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(e*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

3.264 $\int \sqrt{a+b\sec(c+dx)}\,(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\sqrt{a+b\sec(c+dx)}\,(e\sin(c+dx))^m,x\right)$$

```
[Out] Unintegrable((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{a+b\sec(c+dx)}\,(e\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*(e*Sin[c + d*x])^m,x]
[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]*(e*Sin[c + d*x])^m, x]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\,(e\sin(c+dx))^m\,dx=\int \sqrt{a+b\sec(c+dx)}\,(e\sin(c+dx))^m\,dx$$

Mathematica [A] time = 0.52, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sec(c+dx)}\,(e\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*(e*Sin[c + d*x])^m,x]
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]*(e*Sin[c + d*x])^m, x]
```

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\sec\left(dx+c\right)+a}\,\left(e\sin\left(dx+c\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sec\left(dx+c\right)+a}\,\left(e\sin\left(dx+c\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

maple [A] time = 1.08, size = 0, normalized size = 0.00

$$\int \left(e\sin\left(dx+c\right)\right)^m\sqrt{a+b\sec\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sec (dx + c) + a}\, (e \sin (dx + c))^{m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (e\, \sin (c + d\, x))^{m}\, \sqrt{a + \frac{b}{\cos (c + d\, x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (e \sin (c + dx))^{m}\, \sqrt{a + b \sec (c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*sin(c + d*x))**m*sqrt(a + b*sec(c + d*x)), x)
```

3.265 $\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=28

$$\mathrm{Int}\left(\frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*Sin[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Defer[Int][(e*Sin[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 2.63, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Sin[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]],x]
[Out] Integrate[(e*Sin[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]], x]
```

fricas [A] time = 0.68, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(e\sin(dx+c))^m}{\sqrt{b\sec(dx+c)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral((e*sin(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{\sqrt{b\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((e*sin(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

maple [A] time = 1.02, size = 0, normalized size = 0.00

$$\int \frac{(e \sin (dx+c))^m}{\sqrt{a+b \sec (dx+c)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
[Out] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \sin (dx+c))^m}{\sqrt{b \sec (dx+c)+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(e\,\sin (c+d\,x))^m}{\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^(1/2),x)
[Out] int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \sin (c+dx))^m}{\sqrt{a+b \sec (c+dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral((e*sin(c + d*x))**m/sqrt(a + b*sec(c + d*x)), x)
```

3.266 $\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=28

$$\text{Int}\left(\frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
```

Rubi [A] time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Defer[Int][(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 2.80, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2), x]
```

fricas [A] time = 0.93, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{b\sec(dx+c)+a}\,(e\sin(dx+c))^m}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*(e*sin(d*x + c))^m/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\sin(dx+c))^m}{(b\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^(3/2), x)
```

maple [A] time = 0.99, size = 0, normalized size = 0.00

$$\int \frac{(e \sin (dx + c))^{m}}{(a + b \sec (dx + c))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
[Out] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \sin (dx + c))^{m}}{(b \sec (dx + c) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(e\ \sin (c + d\, x))^{m}}{\left(a + \frac{b}{\cos(c+d\, x)}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^(3/2),x)
[Out] int((e*sin(c + d*x))^m/(a + b/cos(c + d*x))^(3/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e \sin (c + dx))^{m}}{(a + b \sec (c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral((e*sin(c + d*x))**m/(a + b*sec(c + d*x))**(3/2), x)
```

3.267 $\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=26

$$\text{Int}\left((e\sin(c+dx))^m(a+b\sec(c+dx))^n,x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*(e*Sin[c + d*x])^m,x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*(e*Sin[c + d*x])^m, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx=\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx$$

Mathematica [A] time = 3.38, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*(e*Sin[c + d*x])^m,x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*(e*Sin[c + d*x])^m, x]
```

fricas [A] time = 0.80, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^n\,(e\sin(dx+c))^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n\,(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

maple [A] time = 2.44, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^n\,(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
[Out] int((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n (e\sin(dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int (e\,\sin(c+d\,x))^m \left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^n,x)
[Out] int((e*sin(c + d*x))^m*(a + b/cos(c + d*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*(e*sin(d*x+c))**m,x)
[Out] Timed out
```

3.268 $\int (a + b\sec(c + dx))^n \sin^5(c + dx)\, dx$

Optimal. Leaf size=150

$$\frac{b^5(a + b\sec(c + dx))^{n+1}\,{}_2F_1\left(6, n + 1; n + 2; \frac{b\sec(c+dx)}{a} + 1\right)}{a^6 d(n + 1)} - \frac{2b^3(a + b\sec(c + dx))^{n+1}\,{}_2F_1\left(4, n + 1; n + 2; \frac{b\sec}{}\right.}{a^4 d(n + 1)}$$

```
[Out] b*hypergeom([2, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c))^(1+n)/a^2/d/(
1+n)-2*b^3*hypergeom([4, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c))^(1+n
)/a^4/d/(1+n)+b^5*hypergeom([6, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c
))^(1+n)/a^6/d/(1+n)
```

Rubi [A] time = 0.13, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3874, 180, 65}

$$-\frac{2b^3(a + b\sec(c + dx))^{n+1}\,{}_2F_1\left(4, n + 1; n + 2; \frac{b\sec(c+dx)}{a} + 1\right)}{a^4 d(n + 1)} + \frac{b^5(a + b\sec(c + dx))^{n+1}\,{}_2F_1\left(6, n + 1; n + 2; \frac{b\,s}{}\right.}{a^6 d(n + 1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^5,x]
```

```
[Out] (b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
+ d*x])^(1 + n))/(a^2*d*(1 + n)) - (2*b^3*Hypergeometric2F1[4, 1 + n, 2 + n
, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(1 + n))/(a^4*d*(1 + n)) + (
b^5*Hypergeometric2F1[6, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
 + d*x])^(1 + n))/(a^6*d*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 3874

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[f^(-1), Subst[Int[((-1 + x)^((p - 1)/2)*(1 + x)^((p
- 1)/2)*(a + b*x)^m)/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e,
 f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n \sin^5(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(-1+x)^2(1+x)^2(a-bx)^n}{x^6}dx, x, -\sec(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{(a-bx)^n}{x^6} - \frac{2(a-bx)^n}{x^4} + \frac{(a-bx)^n}{x^2}\right)dx, x, -\sec(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{(a-bx)^n}{x^6}dx, x, -\sec(c+dx)\right)}{d} - \frac{\text{Subst}\left(\int \frac{(a-bx)^n}{x^2}dx, x, -\sec(c+\right.}{d}$$
$$= \frac{b\,{}_2F_1\left(2,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{a^2d(1+n)} - \frac{2b^3\,{}_2F_1\left(4,\right.}{}$$

Mathematica [B] time = 8.93, size = 562, normalized size = 3.75

$$-\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\cos(c+dx)(a+b\sec(c+dx))^n\left(192a^3(n-1)(a\cos(c+dx)+b)^2 - 240a^3(n-1)\sec^2\left(\frac{1}{2}(c+d\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^5,x]
```

```
[Out] -1/120*(Cos[(c + d*x)/2]^6*Cos[c + d*x]*(192*a^3*(-1 + n)*(b + a*Cos[c + d*
x])^2 - 240*a^3*(-1 + n)*(b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^2 - 24*a^2
*(2*a - b*(-4 + n))*(-1 + n)*(b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^2 + 40
*a^2*(2*a - b*(-3 + n))*(-1 + n)*(b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^4
+ a*(1 - n)*(96*a^2 + 4*a*b*(6 - 4*n) - 4*b^2*(12 - 7*n + n^2))*(b + a*Cos[
c + d*x])^2*Sec[(c + d*x)/2]^4 - 10*a*((-1 + n)*(-14*a^2 + 2*a*b*(-1 + n) +
 b^2*(6 - 5*n + n^2))*(b + a*Cos[c + d*x])^2 + b*(24*a^3 + 12*a^2*b*(-1 + n
) - 4*a*b^2*(2 - 3*n + n^2) - b^3*(-6 + 11*n - 6*n^2 + n^3))*Hypergeometric
2F1[2, 1 - n, 2 - n, (a*Cos[c + d*x])/(b + a*Cos[c + d*x])])*Sec[(c + d*x)/
2]^6 + ((-1 + n)*(-84*a^3 + 2*a^2*b*(18 - 7*n) + 4*a*b^2*(9 - 9*n + 2*n^2)
+ b^3*(-24 + 26*n - 9*n^2 + n^3))*(b + a*Cos[c + d*x])^2 + b*(120*a^4 + 120
*a^3*b*(-1 + n) - 10*a*b^3*(-6 + 11*n - 6*n^2 + n^3) - b^4*(24 - 50*n + 35*
n^2 - 10*n^3 + n^4))*Hypergeometric2F1[2, 1 - n, 2 - n, (a*Cos[c + d*x])/(b
 + a*Cos[c + d*x])])*Sec[(c + d*x)/2]^6)*(a + b*Sec[c + d*x])^n)/(a^4*d*(-1
 + n)*(b + a*Cos[c + d*x]))
```

fricas [F] time = 0.69, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1\right)(b\sec(dx+c)+a)^n \sin(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(b*sec(d*x + c) + a)^n*sin
(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n \sin(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^5, x)
```

maple [F] time = 2.72, size = 0, normalized size = 0.00

$$\int (a + b \sec (dx + c))^{n} \left(\sin^{5} (dx + c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x)
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec (dx + c) + a)^{n} \sin (dx + c)^{5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin (c + d\,x)^{5} \left(a + \frac{b}{\cos (c + d\,x)}\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5*(a + b/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^5*(a + b/cos(c + d*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**5,x)
[Out] Timed out
```

3.269 $\int (a+b\sec(c+dx))^n \sin^3(c+dx)\,dx$

Optimal. Leaf size=121

$$\frac{\cos^3(c+dx)(2a-b(2-n)\sec(c+dx))(a+b\sec(c+dx))^{n+1}}{6a^2d}+\frac{b\left(6a^2-b^2\left(n^2-3n+2\right)\right)(a+b\sec(c+dx))^{n+1}\,{}_2}{6a^4d(n+1)}$$

[Out] 1/6*b*(6*a^2-b^2*(n^2-3*n+2))*hypergeom([2, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c))^(1+n)/a^4/d/(1+n)+1/6*cos(d*x+c)^3*(a+b*sec(d*x+c))^(1+n)*(2*a-b*(2-n)*sec(d*x+c))/a^2/d

Rubi [A] time = 0.10, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3874, 145, 65}

$$\frac{b\left(6a^2-b^2\left(n^2-3n+2\right)\right)(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{b\sec(c+dx)}{a}+1\right)}{6a^4d(n+1)}+\frac{\cos^3(c+dx)(2a-b(2-n)\,\text{s}}{}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^3,x]

[Out] (b*(6*a^2 - b^2*(2 - 3*n + n^2))*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(1 + n))/(6*a^4*d*(1 + n)) + (Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(1 + n)*(2*a - b*(2 - n)*Sec[c + d*x]))/(6*a^2*d)

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 145

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)*(
a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x]
 + Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] && !LtQ[n, -2]))
```

Rule 3874

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[f^(-1), Subst[Int[((-1 + x)^((p - 1)/2)*(1 + x)^((p
- 1)/2)*(a + b*x)^m)/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e,
 f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n \sin^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(-1+x)(1+x)(a-bx)^n}{x^4}\,dx, x, -\sec(c+dx)\right)}{d}$$

$$= \frac{\cos^3(c+dx)(a+b\sec(c+dx))^{1+n}(2a-b(2-n)\sec(c+dx))}{6a^2d} - \frac{\left(6-\right.}{}$$

$$= \frac{b\left(6a^2-b^2\left(2-3n+n^2\right)\right)\,{}_2F_1\left(2,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\,\mathrm{se}}{6a^4d(1+n)}$$

Mathematica [A] time = 1.86, size = 155, normalized size = 1.28

$$\frac{\cos(c+dx)(a+b\sec(c+dx))^n\left(-\frac{2b\left(b^2\left(n^2-3n+2\right)-6a^2\right){}_2F_1\left(2,1-n;2-n;\frac{a\cos(c+dx)}{b+a\cos(c+dx)}\right)}{a(n-1)}-\frac{2(2a-b(n-2))(a\cos(c+dx)+b)^2}{a}+8\cos^2\right.}{12ad(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^3,x]
```

```
[Out] (Cos[c + d*x]*((-2*(2*a - b*(-2 + n))*(b + a*Cos[c + d*x])^2)/a + 8*Cos[(c
+ d*x)/2]^2*(b + a*Cos[c + d*x])^2 - (2*b*(-6*a^2 + b^2*(2 - 3*n + n^2))*Hy
pergeometric2F1[2, 1 - n, 2 - n, (a*Cos[c + d*x])/(b + a*Cos[c + d*x])])/(a
*(-1 + n)))*(a + b*Sec[c + d*x])^n)/(12*a*d*(b + a*Cos[c + d*x]))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos(dx+c)^2-1\right)(b\sec(dx+c)+a)^n\sin(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(b*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError >> An error occurred running a Giac command:
INPUT:sage2OUTPUT:Evaluation time: 1.34sym2poly/r2sym(const gen & e,const i
ndex_m & i,const vecteur & l) Error: Bad Argument Value
```

maple [F] time = 2.71, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^n\left(\sin^3(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n\sin(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(c+d\,x\right)^3\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + b/cos(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)^3*(a + b/cos(c + d*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.270 $\int (a+b\sec(c+dx))^n \sin(c+dx)\,dx$

Optimal. Leaf size=48

$$\frac{b(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{b\sec(c+dx)}{a}+1\right)}{a^2d(n+1)}$$

```
[Out] b*hypergeom([2, 1+n],[2+n],1+b*sec(d*x+c)/a)*(a+b*sec(d*x+c))^(1+n)/a^2/d/(
1+n)
```

Rubi [A] time = 0.04, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3874, 65}

$$\frac{b(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{b\sec(c+dx)}{a}+1\right)}{a^2d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x],x]
```

```
[Out] (b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
+ d*x])^(1 + n))/(a^2*d*(1 + n))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 3874

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[f^(-1), Subst[Int[((-1 + x)^((p - 1)/2)*(1 + x)^((p
- 1)/2)*(a + b*x)^m)/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e,
 f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^n \sin(c+dx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(a-bx)^n}{x^2}\,dx,x,-\sec(c+dx)\right)}{d}\\ &= \frac{b\,{}_2F_1\left(2,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{a^2d(1+n)}\end{aligned}$$

Mathematica [A] time = 0.54, size = 72, normalized size = 1.50

$$\frac{b\cos(c+dx)(a+b\sec(c+dx))^n\,{}_2F_1\left(2,1-n;2-n;\frac{a\cos(c+dx)}{b+a\cos(c+dx)}\right)}{d(n-1)(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x],x]
```

```
[Out] (b*Cos[c + d*x]*Hypergeometric2F1[2, 1 - n, 2 - n, (a*Cos[c + d*x])/(b + a*
Cos[c + d*x])]*(a + b*Sec[c + d*x])^n)/(d*(-1 + n)*(b + a*Cos[c + d*x]))
```

fricas [F] time = 0.55, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^n\sin(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

maple [F] time = 0.99, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^n\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c),x)
```

```
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n\sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

mupad [B] time = 1.39, size = 73, normalized size = 1.52

$$\frac{\cos(c+d\,x)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^n\,{}_2\mathrm{F}_1\left(1-n,-n;\;2-n;\;-\frac{a\,\cos(c+d\,x)}{b}\right)}{d\,\left(\frac{a\,\cos(c+d\,x)}{b}+1\right)^n\,(n-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b/cos(c + d*x))^n,x)
```

```
[Out] (cos(c + d*x)*(a + b/cos(c + d*x))^n*hypergeom([1 - n, -n], 2 - n, -(a*cos(
c + d*x))/b))/(d*((a*cos(c + d*x))/b + 1)^n*(n - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n\sin(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*sin(c + d*x), x)
```

3.271 $\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=115

$$\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a-b}\right)}{2d(n+1)(a-b)}-\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a+b}\right)}{2d(n+1)(a+b)}$$

```
[Out] 1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a-b))*(a+b*sec(d*x+c))^(1+n)
/(a-b)/d/(1+n)-1/2*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*se
c(d*x+c))^(1+n)/(a+b)/d/(1+n)
```

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.210, Rules used = {3874, 73, 712, 68}

$$\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a-b}\right)}{2d(n+1)(a-b)}-\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a+b}\right)}{2d(n+1)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*Se
c[c + d*x])^(1 + n))/(2*(a - b)*d*(1 + n)) - (Hypergeometric2F1[1, 1 + n, 2
 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(2*(a + b
)*d*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 73

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

Rule 712

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 3874

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[f^(-1), Subst[Int[((-1 + x)^((p - 1)/2)*(1 + x)^((p
- 1)/2)*(a + b*x)^m)/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e,
 f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx = -\frac{\text{Subst}\left(\int \frac{(a-bx)^n}{(-1+x)(1+x)}\,dx, x, -\sec(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{(a-bx)^n}{-1+x^2}\,dx, x, -\sec(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{(a-bx)^n}{2(1-x)} - \frac{(a-bx)^n}{2(1+x)}\right) dx, x, -\sec(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-bx)^n}{1-x}\,dx, x, -\sec(c+dx)\right)}{2d} + \frac{\text{Subst}\left(\int \frac{(a-bx)^n}{1+x}\,dx, x, -\sec(c+d\right.}{2d}$$
$$= \frac{{}_2F_1\left(1, 1+n; 2+n; \frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{2(a-b)d(1+n)} - \frac{{}_2F_1\left(1, 1+n; 2\right.}{}$$

Mathematica [A] time = 0.98, size = 132, normalized size = 1.15

$$\frac{(a+b\sec(c+dx))^n\left({}_2F_1\left(1,-n;1-n;\frac{(a+b)\cos(c+dx)}{b+a\cos(c+dx)}\right) - 2^n\left(\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b)}{b}\right)^{-n}{}_2F_1\left(-n,-n;1-n;\frac{(b-a)\cos\right.\right.}{2dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] ((Hypergeometric2F1[1, -n, 1 - n, ((a + b)*Cos[c + d*x])/(b + a*Cos[c + d*x
])] - (2^n*Hypergeometric2F1[-n, -n, 1 - n, ((-a + b)*Cos[c + d*x]*Sec[(c +
 d*x)/2]^2)/(2*b)])/(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/b)^n)*(a + b
*Sec[c + d*x])^n)/(2*d*n)
```

fricas [F] time = 0.62, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^n\csc(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

maple [F] time = 1.07, size = 0, normalized size = 0.00

$$\int \csc(dx+c)(a+b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)*(a+b*sec(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n \csc(dx+c)\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sin(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^n/sin(c + d*x),x)
[Out] int((a + b/cos(c + d*x))^n/sin(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n \csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))**n,x)
[Out] Integral((a + b*sec(c + d*x))**n*csc(c + d*x), x)
```

3.272 $\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=231

$$\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a-b}\right)}{4d(n+1)(a-b)}-\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a+b}\right)}{4d(n+1)(a+b)}+$$

```
[Out] 1/4*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a-b))*(a+b*sec(d*x+c))^(1+n)
/(a-b)/d/(1+n)-1/4*hypergeom([1, 1+n],[2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*se
c(d*x+c))^(1+n)/(a+b)/d/(1+n)+1/4*b*hypergeom([2, 1+n],[2+n],(a+b*sec(d*x+c
))/(a-b))*(a+b*sec(d*x+c))^(1+n)/(a-b)^2/d/(1+n)+1/4*b*hypergeom([2, 1+n],[
2+n],(a+b*sec(d*x+c))/(a+b))*(a+b*sec(d*x+c))^(1+n)/(a+b)^2/d/(1+n)
```

Rubi [A] time = 0.19, antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3874, 180, 68, 712}

$$\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a-b}\right)}{4d(n+1)(a-b)}-\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a+b}\right)}{4d(n+1)(a+b)}+$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*Se
c[c + d*x])^(1 + n))/(4*(a - b)*d*(1 + n)) - (Hypergeometric2F1[1, 1 + n, 2
 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(4*(a + b
)*d*(1 + n)) + (b*Hypergeometric2F1[2, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(
a - b)]*(a + b*Sec[c + d*x])^(1 + n))/(4*(a - b)^2*d*(1 + n)) + (b*Hypergeo
metric2F1[2, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x
])^(1 + n))/(4*(a + b)^2*d*(1 + n))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 712

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 3874

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[f^(-1), Subst[Int[((-1 + x)^((p - 1)/2)*(1 + x)^((p
- 1)/2)*(a + b*x)^m)/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e,
 f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx = -\frac{\text{Subst}\left(\int \frac{x^2(a-bx)^n}{(-1+x)^2(1+x)^2}\,dx, x, -\sec(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{(a-bx)^n}{4(-1+x)^2} + \frac{(a-bx)^n}{4(1+x)^2} + \frac{(a-bx)^n}{2(-1+x^2)}\right) dx, x, -\sec(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a-bx)^n}{(-1+x)^2}\,dx, x, -\sec(c+dx)\right)}{4d} - \frac{\text{Subst}\left(\int \frac{(a-bx)^n}{(1+x)^2}\,dx, x, -\text{sec}\right.}{4d}$$

$$= \frac{b\,{}_2F_1\left(2,1+n;2+n;\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)^2d(1+n)} + \frac{b\,{}_2F_1\left(2,1\right.}{}$$

$$= \frac{b\,{}_2F_1\left(2,1+n;2+n;\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)^2d(1+n)} + \frac{b\,{}_2F_1\left(2,1\right.}{}$$

$$= \frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)d(1+n)} - \frac{{}_2F_1\left(1,1+\right.}{}$$

Mathematica [B] time = 17.05, size = 710, normalized size = 3.07

$$\frac{\left(\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n \left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^{-2n} \left(1-\tan^4\left(\frac{1}{2}(c+dx)\right)\right)^n \left(\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)^n \left(\cos(c+d\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] ((Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(a + b*Sec[c + d*x])^n*((1 - Tan[(c +
d*x)/2]^2)^(-1))^n*(1 - Tan[(c + d*x)/2]^4)^n*(b + (a - a*Tan[(c + d*x)/2]^
2)/(1 + Tan[(c + d*x)/2]^2))^n*(2*(a + b + b*n)*Hypergeometric2F1[1, -n, 1
- n, ((a + b)*(-1 + Tan[(c + d*x)/2]^2))/(a*(-1 + Tan[(c + d*x)/2]^2) - b*(
1 + Tan[(c + d*x)/2]^2))]*(1 - Tan[(c + d*x)/2]^2)^n - (Cot[(c + d*x)/2]^2*
(2^(1 + n)*(a - b)*(1 + n)*(a + b + b*n)*Hypergeometric2F1[-n, -n, 1 - n, (
(a - b)*(-1 + Tan[(c + d*x)/2]^2))/(2*b)]*Tan[(c + d*x)/2]^2*(2 - 2*Tan[(c
+ d*x)/2]^2)^n + n*(1 - Tan[(c + d*x)/2]^2)^n*(a*(-1 + Tan[(c + d*x)/2]^2)
- b*(1 + Tan[(c + d*x)/2]^2))*(2^n*(a - b)*(1 + n)*(-1 + Tan[(c + d*x)/2]^2
)*((a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/b)^n - 2*a*Hyperge
ometric2F1[n, 1 + n, 2 + n, (a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)
/2]^2)/(2*b)]*Tan[(c + d*x)/2]^2*(-(((-1 + Tan[(c + d*x)/2]^2)*(-2*a*b*Tan[
(c + d*x)/2]^2 + a^2*(-1 + Tan[(c + d*x)/2]^2) + b^2*(1 + Tan[(c + d*x)/2]^
2)))/b^2))^n)))/(2^n*(a - b)*(1 + n)*((a + b - a*Tan[(c + d*x)/2]^2 + b*Tan
[(c + d*x)/2]^2)/b)^n)))/(8*(a + b)*d*n*(b + a*Cos[c + d*x])^n*(Cos[c + d*x
]*Sec[(c + d*x)/2]^4)^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(1 - Tan[(c + d
*x)/2]^2)^(2*n))
```

fricas [F] time = 0.98, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^n\csc(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n\csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

maple [F] time = 1.14, size = 0, normalized size = 0.00

$$\int \left(\csc^3(dx+c)\right)(a+b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n\csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sin(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^n/sin(c + d*x)^3,x)
```

```
[Out] int((a + b/cos(c + d*x))^n/sin(c + d*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

3.273 $\int (a+b\sec(c+dx))^n \sin^4(c+dx)\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\sin^4(c+dx)(a+b\sec(c+dx))^n,x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sec(c+dx))^n \sin^4(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^4,x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Sin[c + d*x]^4, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n \sin^4(c+dx)\,dx = \int (a+b\sec(c+dx))^n \sin^4(c+dx)\,dx$$

Mathematica [A] time = 14.44, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n \sin^4(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^4,x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^4, x]
```

fricas [A] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\cos(dx+c)^4 - 2\cos(dx+c)^2 + 1\right)(b\sec(dx+c)+a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="fricas")
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(b*sec(d*x + c) + a)^n, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n \sin(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

maple [A] time = 3.60, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^n \left(\sin^4(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^{n}\sin(dx+c)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \sin(c+d\,x)^4\left(a+\frac{b}{\cos(c+d\,x)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + b/cos(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)^4*(a + b/cos(c + d*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.274 $\int (a+b\sec(c+dx))^n \sin^2(c+dx)\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\sin^2(c+dx)(a+b\sec(c+dx))^n,x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sec(c+dx))^n \sin^2(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^2,x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Sin[c + d*x]^2, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n \sin^2(c+dx)\,dx = \int (a+b\sec(c+dx))^n \sin^2(c+dx)\,dx$$

Mathematica [A] time = 3.90, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n \sin^2(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^2,x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^2, x]
```

fricas [A] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos(dx+c)^2-1\right)(b\sec(dx+c)+a)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="fricas")
[Out] integral(-(cos(d*x + c)^2 - 1)*(b*sec(d*x + c) + a)^n, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n \sin(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

maple [A] time = 2.83, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^n \left(\sin^2(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x)
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n \sin(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \sin(c+d\,x)^2 \left(a+\frac{b}{\cos(c+d\,x)}\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + b/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^2*(a + b/cos(c + d*x))^n, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n \sin^2(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**2,x)
[Out] Integral((a + b*sec(c + d*x))**n*sin(c + d*x)**2, x)
```

3.275 $\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=136

$$\frac{\sqrt{2}\,bn\tan(c+dx)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},1-n;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d(a+b)\sqrt{\sec(c+dx)+1}}-\text{cot(}$$

```
[Out] -cot(d*x+c)*(a+b*sec(d*x+c))^n/d+b*n*AppellF1(1/2,1-n,1/2,3/2,b*(1-sec(d*x+
c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^n*2^(1/2)*tan(d*x+c)/(a+b)/d
/(((a+b*sec(d*x+c))/(a+b))^n)/(1+sec(d*x+c))^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3875, 3834, 139, 138}

$$\frac{\sqrt{2}\,bn\tan(c+dx)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},1-n;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d(a+b)\sqrt{\sec(c+dx)+1}}-\text{cot(}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] -((Cot[c + d*x]*(a + b*Sec[c + d*x])^n)/d) + (Sqrt[2]*b*n*AppellF1[1/2, 1/2
, 1 - n, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*
Sec[c + d*x])^n*Tan[c + d*x])/((a + b)*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec
[c + d*x])/(a + b))^n)
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] &&  !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) &&  !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
```

m}, x]

Rubi steps

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx = -\frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d} + (bn)\int \sec(c+dx)(a+b\sec(c+dx)$$

$$= -\frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d} - \frac{(bn\tan(c+dx))\,\text{Subst}\left(\int \frac{(a+bx)^{-1+n}}{\sqrt{1-x}\,\sqrt{1+x}}\right.}{d\sqrt{1-\sec(c+dx)}\,\sqrt{1+\text{se}}}$$

$$= -\frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d} - \frac{\left(bn(a+b\sec(c+dx))^n\left(-\frac{a+b\sec(c+dx}{-a-b}\right.\right.}{(a+b)d}$$

$$= -\frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d} + \frac{\sqrt{2}\,bnF_1\left(\frac{1}{2};\frac{1}{2},1-n;\frac{3}{2};\frac{1}{2}(1-\sec(c+\right.}{}$$

Mathematica [B] time = 18.46, size = 3614, normalized size = 26.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] ((b + a*Cos[c + d*x])^n*Cot[(c + d*x)/2]*Csc[c + d*x]^2*Sec[c + d*x]^n*(a +
 b*Sec[c + d*x])^n*(-((AppellF1[-1/2, n, -n, 1/2, Tan[(c + d*x)/2]^2, ((a -
 b)*Tan[(c + d*x)/2]^2)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n)/(((b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))^n) + (3*(a + b)*AppellF1[1/2
, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan
[(c + d*x)/2]^2)/(3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, (
(a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2*n*((-a + b)*AppellF1[3/2, n, 1 - n
, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*
AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]
^2)/(a + b)])*Tan[(c + d*x)/2]^2)))/(2*d*(-1/4*((b + a*Cos[c + d*x])^n*Csc[
(c + d*x)/2]^2*Sec[c + d*x]^n*(-((AppellF1[-1/2, n, -n, 1/2, Tan[(c + d*x)/
2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^n)/(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))^n) + (3*(a + b)*A
ppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(
a + b)]*Tan[(c + d*x)/2]^2)/(3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d
*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2*n*((-a + b)*AppellF1[3/
2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]
 + (a + b)*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(
c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2))) - (a*n*(b + a*Cos[c + d*x])^
(-1 + n)*Cot[(c + d*x)/2]*Sec[c + d*x]^n*Sin[c + d*x]*(-((AppellF1[-1/2, n,
 -n, 1/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*(Cos[c
+ d*x]*Sec[(c + d*x)/2]^2)^n)/(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a
 + b))^n) + (3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a -
b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2)/(3*(a + b)*AppellF1[1/2
, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2
*n*((-a + b)*AppellF1[3/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[
(c + d*x)/2]^2)/(a + b)] + (a + b)*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d
*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2)))/2 +
(n*(b + a*Cos[c + d*x])^n*Cot[(c + d*x)/2]*Sec[c + d*x]^(1 + n)*Sin[c + d*x
]*(-((AppellF1[-1/2, n, -n, 1/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)
/2]^2)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n)/(((b + a*Cos[c + d*x])
```

```
*Sec[(c + d*x)/2]^2)/(a + b))^n) + (3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan
[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2)/
(3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c +
d*x)/2]^2)/(a + b)] + 2*n*((-a + b)*AppellF1[3/2, n, 1 - n, 5/2, Tan[(c + d
*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*AppellF1[3/2, 1 +
 n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan
[(c + d*x)/2]^2)))/2 + ((b + a*Cos[c + d*x])^n*Cot[(c + d*x)/2]*Sec[c + d*x
]^n*(-(((Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(((a - b)*n*AppellF1[1/2, n, 1
- n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2])/(a + b) - n*AppellF1[1/2, 1 + n, -n, 3/2, Tan
[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2]))/(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))^n) -
(n*AppellF1[-1/2, n, -n, 1/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]
^2)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(-1 + n)*(-(Sec[(c + d*x)/2]
^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(((b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))^n + n*AppellF1[-1/2, n, -n,
1/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*(Cos[c + d*x
]*Sec[(c + d*x)/2]^2)^n*(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))
^(-1 - n)*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c +
 d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)) + (3*(a + b)*AppellF1[
1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*
Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3*(a + b)*AppellF1[1/2, n, -n, 3/2, T
an[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2*n*((-a + b)*Ap
pellF1[3/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)
/(a + b)] + (a + b)*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a -
 b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2) + (3*(a + b)*Tan[(c +
 d*x)/2]^2*(-1/3*((a - b)*n*AppellF1[3/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2
, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]
)/(a + b) + (n*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*T
an[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3))/(3*(a
+ b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2
]^2)/(a + b)] + 2*n*((-a + b)*AppellF1[3/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]
^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*AppellF1[3/2, 1 + n, -n
, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan[(c +
d*x)/2]^2) - (3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a -
 b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2*(2*n*((-a + b)*AppellF1
[3/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a +
b)] + (a + b)*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Ta
n[(c + d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 3*(a + b)
*(-1/3*((a - b)*n*AppellF1[3/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)
*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)
+ (n*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*
x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3) + 2*n*Tan[(c + d*
x)/2]^2*((-a + b)*((3*(a - b)*(1 - n)*AppellF1[5/2, n, 2 - n, 7/2, Tan[(c +
 d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2])/(5*(a + b)) + (3*n*AppellF1[5/2, 1 + n, 1 - n, 7/2, Tan[(c + d*
x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/5) + (a + b)*((-3*(a - b)*n*AppellF1[5/2, 1 + n, 1 - n, 7/2, Tan[(
c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan
[(c + d*x)/2])/(5*(a + b)) + (3*(1 + n)*AppellF1[5/2, 2 + n, -n, 7/2, Tan[(
c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan
[(c + d*x)/2])/5))))/(3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^
2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2*n*((-a + b)*AppellF1[3/2, n, 1
 - n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (a +
 b)*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x
)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2)^2))/2))
```

fricas [F] time = 0.76, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^n\csc(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(dx+c) + a)^n \csc(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

maple [F] time = 1.14, size = 0, normalized size = 0.00

$$\int \left(\csc^2(dx+c)\right)(a+b\sec(dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
[Out] int(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(dx+c) + a)^n \csc(dx+c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sin(c+d\,x)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^n/sin(c + d*x)^2,x)
[Out] int((a + b/cos(c + d*x))^n/sin(c + d*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sec(c+dx))^n \csc^2(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*sec(d*x+c))**n,x)
[Out] Integral((a + b*sec(c + d*x))**n*csc(c + d*x)**2, x)
```

3.276 $\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=424

$$-\frac{\cot^3(c+dx)(\sec(c+dx)+1)^{3/2}(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}F_1\left(-\frac{3}{2};\frac{5}{2},-n;-\frac{1}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-}{}\right.}{6\sqrt{2}d}$$

```
[Out] -1/12*AppellF1(-3/2,-n,5/2,-1/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*
cot(d*x+c)^3*(1+sec(d*x+c))^(3/2)*(a+b*sec(d*x+c))^n/d/(((a+b*sec(d*x+c))/(
a+b))^n)*2^(1/2)-3/4*AppellF1(-1/2,-n,5/2,1/2,b*(1-sec(d*x+c))/(a+b),1/2-1/
2*sec(d*x+c))*cot(d*x+c)*(a+b*sec(d*x+c))^n*(1+sec(d*x+c))^(1/2)/d/(((a+b*s
ec(d*x+c))/(a+b))^n)*2^(1/2)+1/2*AppellF1(1/2,-n,3/2,3/2,b*(1-sec(d*x+c))/(
a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^n*tan(d*x+c)/d/(((a+b*sec(d*x+c))
/(a+b))^n)*2^(1/2)/(1+sec(d*x+c))^(1/2)+1/4*AppellF1(1/2,-n,5/2,3/2,b*(1-se
c(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b*sec(d*x+c))^n*tan(d*x+c)/d/(((a+b*
sec(d*x+c))/(a+b))^n)*2^(1/2)/(1+sec(d*x+c))^(1/2)
```

Rubi [F] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^n,x]
[Out] Defer[Int][Csc[c + d*x]^4*(a + b*Sec[c + d*x])^n, x]
```

Rubi steps

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx=\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

Mathematica [B] time = 23.72, size = 6403, normalized size = 15.10

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^n,x]
[Out] Result too large to show
```

fricas [F] time = 1.27, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^n\csc(dx+c)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n\csc(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

maple [F] time = 1.26, size = 0, normalized size = 0.00

$$\int \left(\csc^{4}\left(dx+c\right)\right)\left(a+b\sec\left(dx+c\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(dx+c\right)+a\right)^{n}\csc\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{n}}{\sin\left(c+d\,x\right)^{4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^n/sin(c + d*x)^4,x)
```

```
[Out] int((a + b/cos(c + d*x))^n/sin(c + d*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

3.277 $\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\sin^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^n,x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^(3/2),x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Sin[c + d*x]^(3/2), x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx = \int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$$

Mathematica [A] time = 1.87, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^(3/2),x]
[Out] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^(3/2), x]
```

fricas [A] time = 0.87, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^n \sin(dx+c)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n \sin(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

maple [A] time = 0.90, size = 0, normalized size = 0.00

$$\int (a + b\sec(dx + c))^{n} \left(\sin^{\frac{3}{2}}(dx + c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx + c) + a)^{n} \sin(dx + c)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \sin(c + d\,x)^{3/2} \left(a + \frac{b}{\cos(c + d\,x)}\right)^{n} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^(3/2)*(a + b/cos(c + d*x))^n,x)
[Out] int(sin(c + d*x)^(3/2)*(a + b/cos(c + d*x))^n, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**(3/2),x)
[Out] Timed out
```

3.278 $\int (a+b\sec(c+dx))^n \sqrt{\sin(c+dx)}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\sqrt{\sin(c+dx)}\,(a+b\sec(c+dx))^n,x\right)$$

[Out] Unintegrable((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sec(c+dx))^n \sqrt{\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]],x]

[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]], x]

Rubi steps

$$\int (a+b\sec(c+dx))^n \sqrt{\sin(c+dx)}\,dx = \int (a+b\sec(c+dx))^n \sqrt{\sin(c+dx)}\,dx$$

Mathematica [A] time = 4.99, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n \sqrt{\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]],x]

[Out] Integrate[(a + b*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]], x]

fricas [A] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left((b\sec(dx+c)+a)^n\sqrt{\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((b*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n \sqrt{\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((b*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)

maple [A] time = 0.80, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^n \left(\sqrt{\sin}\,(dx+c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
```

```
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c)+a)^n\sqrt{\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \sqrt{\sin(c+d\,x)}\left(a+\frac{b}{\cos(c+d\,x)}\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^(1/2)*(a + b/cos(c + d*x))^n,x)
```

```
[Out] int(sin(c + d*x)^(1/2)*(a + b/cos(c + d*x))^n, x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*sqrt(sin(c + d*x)), x)
```

3.279 $\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]],x]
[Out] Defer[Int][(a + b*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]], x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx = \int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$$

Mathematica [A] time = 2.75, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]],x]
[Out] Integrate[(a + b*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]], x]
```

fricas [A] time = 0.87, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{(b\sec(dx+c)+a)^n}{\sqrt{\sin(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral((b*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^n}{\sqrt{\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate((b*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

maple [A] time = 0.82, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^n}{\sqrt{\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
[Out] int((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^n}{\sqrt{\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate((b*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sqrt{\sin(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))^n/sin(c + d*x)^(1/2),x)
[Out] int((a + b/cos(c + d*x))^n/sin(c + d*x)^(1/2), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n/sin(d*x+c)**(1/2),x)
[Out] Integral((a + b*sec(c + d*x))**n/sqrt(sin(c + d*x)), x)
```

3.280 $\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=26

$$\operatorname{Int}\left(\frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)},x\right)$$

```
[Out] Unintegrable((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n/Sin[c + d*x]^(3/2),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n/Sin[c + d*x]^(3/2), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

Mathematica [A] time = 2.91, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n/Sin[c + d*x]^(3/2),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^n/Sin[c + d*x]^(3/2), x]
```

fricas [A] time = 1.05, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(-\frac{(b\sec(dx+c)+a)^n\sqrt{\sin(dx+c)}}{\cos(dx+c)^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*sec(d*x + c) + a)^n*sqrt(sin(d*x + c))/(cos(d*x + c)^2 - 1), x
)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^n}{\sin(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="giac")
```

[Out] integrate((b*sec(d*x + c) + a)^n/sin(d*x + c)^(3/2), x)

maple [A] time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(dx+c))^n}{\sin(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)

[Out] int((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c)+a)^n}{\sin(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sec(d*x + c) + a)^n/sin(d*x + c)^(3/2), x)

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+\frac{b}{\cos(c+d\,x)}\right)^n}{\sin(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a + b/cos(c + d*x))^n/sin(c + d*x)^(3/2),x)

[Out] int((a + b/cos(c + d*x))^n/sin(c + d*x)^(3/2), x)

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**n/sin(d*x+c)**(3/2),x)

[Out] Integral((a + b*sec(c + d*x))**n/sin(c + d*x)**(3/2), x)

3.281 $\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=190

$$-\frac{2ae^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d}+\frac{ae^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\tan^{-1}\left(\sqrt{s\right.}}{d}$$

```
[Out] -2/3*a*e^2*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/d-2/3*a*e^2*csc(d*x+c)*(e*csc(d*
x+c))^(1/2)/d+a*e^2*arctan(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c
)^(1/2)/d+a*e^2*arctanh(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(
1/2)/d-2/3*a*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d
*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d
*x+c)^(1/2)/d
```

Rubi [A] time = 0.17, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3878, 3872, 2838, 2564, 325, 329, 212, 206, 203, 2636, 2641}

$$-\frac{2ae^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d}+\frac{ae^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\tan^{-1}\left(\sqrt{s\right.}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*a*e^2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(3*d) - (2*a*e^2*Csc[c + d*x]*
Sqrt[e*Csc[c + d*x]])/(3*d) + (a*e^2*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[
c + d*x]]*Sqrt[Sin[c + d*x]])/d + (a*e^2*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[e
*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d + (2*a*e^2*Sqrt[e*Csc[c + d*x]]*Ellipt
icF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{a+a\sec(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}dx$$

$$= -\left(\left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))\sec(c+$$

$$= \left(ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sin^{\frac{5}{2}}(c+dx)}dx + \left(ae^2\sqrt{e\csc$$

$$= -\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} + \frac{1}{3}\left(ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+$$

$$= -\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2ae^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}$$

$$= -\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2ae^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}$$

$$= -\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2ae^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}$$

$$= -\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2ae^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}$$

Mathematica [A] time = 1.52, size = 135, normalized size = 0.71

$$-\frac{a(e\csc(c+dx))^{5/2}\left(4\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{\csc(c+dx)}+3\log\left(1-\sqrt{\csc(c+dx)}\right)-3\log\left(\sqrt{\csc(c+dx)}+1\right)+}{6d\csc^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/6*(a*(e*Csc[c + d*x])^(5/2)*(6*ArcTan[Sqrt[Csc[c + d*x]]] + 4*Cot[(c + d
*x)/2]*Sqrt[Csc[c + d*x]] + 3*Log[1 - Sqrt[Csc[c + d*x]]] - 3*Log[1 + Sqrt[
Csc[c + d*x]]] + 4*Sqrt[Csc[c + d*x]]*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*S
qrt[Sin[c + d*x]]))/(d*Csc[c + d*x]^(5/2))
```

fricas [F] time = 0.65, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(ae^2\csc(dx+c)^2\sec(dx+c)+ae^2\csc(dx+c)^2\right)\sqrt{e\csc(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*e^2*csc(d*x + c)^2*sec(d*x + c) + a*e^2*csc(d*x + c)^2)*sqrt(e*
csc(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\csc(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a), x)
```

maple [C] time = 2.10, size = 679, normalized size = 3.57

$$\frac{a\left(-1+\cos\left(dx+c\right)\right)\left(4i\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}},\frac{\sqrt{2}}{2}\right)\sin\left(dx+c\right)\sqrt{\frac{-i\cos(dx+c)+\sin(dx+c)+i}{\sin(dx+c)}}\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/6*a/d*(-1+cos(d*x+c))*(4*I*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x
+c))^(1/2),1/2*2^(1/2))*sin(d*x+c)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c)
)^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/
sin(d*x+c))^(1/2)-3*I*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(
1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*
x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+
c))/sin(d*x+c))^(1/2)-3*I*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c
))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*((-I*cos(d*x+c)+sin(d*x+c)+I)/si
n(d*x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(-1+cos(
d*x+c))/sin(d*x+c))^(1/2)-3*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x
+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*((-I*cos(d*x+c)+sin(d*x+c)+I)/
sin(d*x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(-1+co
s(d*x+c))/sin(d*x+c))^(1/2)+3*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d
*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)*((-I*cos(d*x+c)+sin(d*x+c)+I
)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(-1+
cos(d*x+c))/sin(d*x+c))^(1/2)+2*2^(1/2))*(1+cos(d*x+c))^2*(e/sin(d*x+c))^(5
/2)/sin(d*x+c)*2^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{e}{\sin\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(5/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.282 $\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=169

$$-\frac{2ae\sqrt{e\csc(c+dx)}}{d}-\frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d}-\frac{ae\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d}+$$

```
[Out] -2*a*e*(e*csc(d*x+c))^(1/2)/d-2*a*e*cos(d*x+c)*(e*csc(d*x+c))^(1/2)/d-a*e*a
rctan(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+a*e*arctanh
(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+2*a*e*(sin(1/2*c
+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4
*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.16, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3878, 3872, 2838, 2564, 325, 329, 298, 203, 206, 2636, 2639}

$$-\frac{2ae\sqrt{e\csc(c+dx)}}{d}-\frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d}-\frac{ae\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*a*e*Sqrt[e*Csc[c + d*x]])/d - (2*a*e*Cos[c + d*x]*Sqrt[e*Csc[c + d*x]])
/d - (a*e*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]
])/d + (a*e*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d
*x]])/d - (2*a*e*Sqrt[e*Csc[c + d*x]]*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt
[Sin[c + d*x]])/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \left(e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{a+a\sec(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

$$= -\left(\left(e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))\sec(c+}{\sin^{\frac{3}{2}}(c+dx)}\right.$$

$$= \left(ae\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sin^{\frac{3}{2}}(c+dx)}\,dx + \left(ae\sqrt{e\csc(c\right.$$

$$= -\frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \left(ae\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right.$$

$$= -\frac{2ae\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\sqrt{e\csc}}{}$$

$$= -\frac{2ae\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\sqrt{e\csc}}{}$$

$$= -\frac{2ae\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\sqrt{e\csc}}{}$$

$$= -\frac{2ae\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{ae\tan^{-1}\left(\right.}{}$$

Mathematica [C] time = 1.28, size = 146, normalized size = 0.86

$$\frac{a(e\csc(c+dx))^{3/2}\left(\frac{2\sin(2(c+dx))\csc^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};\csc^2(c+dx)\right)}{\sqrt{-\cot^2(c+dx)}} - 4(\cos(c+dx)+1)\sqrt{\csc(c+dx)} - \log\left(1-\sqrt{\csc}\right.\right.}{2d\csc^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*(e*Csc[c + d*x])^(3/2)*(2*ArcTan[Sqrt[Csc[c + d*x]]] - 4*(1 + Cos[c + d*
x])*Sqrt[Csc[c + d*x]] - Log[1 - Sqrt[Csc[c + d*x]]] + Log[1 + Sqrt[Csc[c +
 d*x]]] + (2*Csc[c + d*x]^(3/2)*Hypergeometric2F1[-1/4, 1/2, 3/4, Csc[c + d
*x]^2]*Sin[2*(c + d*x)])/Sqrt[-Cot[c + d*x]^2]))/(2*d*Csc[c + d*x]^(3/2))
```

fricas [F] time = 1.52, size = 0, normalized size = 0.00

$$\text{integral}\left((ae\csc(dx+c)\sec(dx+c) + ae\csc(dx+c))\sqrt{e\csc(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*e*csc(d*x + c)*sec(d*x + c) + a*e*csc(d*x + c))*sqrt(e*csc(d*x
+ c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\csc(dx+c))^{\frac{3}{2}}\,(a\sec(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] integrate((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a), x)

maple [C] time = 1.66, size = 1522, normalized size = 9.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x)

[Out] 1/2*a/d*(I*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-I*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+4*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-2*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+I*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-I*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+4*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-2*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-4*2^(1/2))*(e/sin(d*x+c))^(3/2)*sin(d*x+c)*2^(1/2)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{e}{\sin\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(3/2)*(a+a*sec(d*x+c)),x)
[Out] Timed out
```

3.283 $\int \sqrt{e\csc(c+dx)}\,(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=121

$$\frac{a\sqrt{\sin(c+dx)}\,\sqrt{e\csc(c+dx)}\,\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{a\sqrt{\sin(c+dx)}\,\sqrt{e\csc(c+dx)}\,\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{2a\sqrt{}}{}$$

```
[Out] a*arctan(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+a*arctan
h(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d-2*a*(sin(1/2*c+
1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*
Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d
```

Rubi [A] time = 0.14, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3878, 3872, 2838, 2564, 329, 212, 206, 203, 2641}

$$\frac{a\sqrt{\sin(c+dx)}\,\sqrt{e\csc(c+dx)}\,\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{a\sqrt{\sin(c+dx)}\,\sqrt{e\csc(c+dx)}\,\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{2a\sqrt{}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d +
(a*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d +
 (2*a*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*
x]])/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{e\csc(c+dx)}\,(a+a\sec(c+dx))\,dx = \left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{a+a\sec(c+dx)}{\sqrt{\sin(c+dx)}}\,dx$$

$$= -\left(\left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))\sec(c+d$$

$$= \left(a\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx + \left(a\sqrt{e\csc(c+$$

$$= \frac{2a\sqrt{e\csc(c+dx)}\,F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d} + \frac{\left(a\sqrt{e\csc(c-$$

$$= \frac{2a\sqrt{e\csc(c+dx)}\,F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d} + \frac{\left(2a\sqrt{e\csc(c$$

$$= \frac{2a\sqrt{e\csc(c+dx)}\,F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d} + \frac{\left(a\sqrt{e\csc(c-$$

$$= \frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}{d} + \frac{a\tanh^{-1}\left(\sqrt{}\right.$$

Mathematica [A] time = 0.85, size = 111, normalized size = 0.92

$$-\frac{a\sqrt{e\csc(c+dx)}\left(\log\left(1-\sqrt{\csc(c+dx)}\right)-\log\left(\sqrt{\csc(c+dx)}+1\right)+2\tan^{-1}\left(\sqrt{\csc(c+dx)}\right)+4\sqrt{\sin(c+d}\right.}{2d\sqrt{\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/2*(a*Sqrt[e*Csc[c + d*x]]*(2*ArcTan[Sqrt[Csc[c + d*x]]] + Log[1 - Sqrt[C
sc[c + d*x]]] - Log[1 + Sqrt[Csc[c + d*x]]] + 4*Sqrt[Csc[c + d*x]]*Elliptic
F[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]]))/(d*Sqrt[Csc[c + d*x]])
```

fricas [F] time = 0.85, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{e\csc\left(dx+c\right)}\left(a\sec\left(dx+c\right)+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\csc\left(dx+c\right)}\left(a\sec\left(dx+c\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*csc(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a), x)
```

maple [C] time = 1.63, size = 291, normalized size = 2.40

$$-\frac{a\sqrt{2}\,\sqrt{\frac{e}{\sin(dx+c)}}\,\left(-1+\cos\left(dx+c\right)\right)\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}}\,\sqrt{-\frac{i\cos(dx+c)-i-\sin(dx+c)}{\sin(dx+c)}}\,\sqrt{-\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}}\,\left(i\,\mathrm{Elliptic}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*csc(d*x+c))^(1/2),x)
```

```
[Out] -1/2*a/d*2^(1/2)*(e/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*((I*cos(d*x+c)-I+sin(
d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*(
-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(I*EllipticPi(((I*cos(d*x+c)-I+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+I*EllipticPi(((I*cos(d*x+c)-I
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi(((I*cos(d*
x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-EllipticPi(((I*
cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2)))/sin(d*x+
c)^2*(1+cos(d*x+c))^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\csc\left(dx+c\right)}\left(a\sec\left(dx+c\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\sqrt{\frac{e}{\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))*(e/sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \sqrt{e\csc\left(c+dx\right)}\,dx+\int \sqrt{e\csc\left(c+dx\right)}\,\sec\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*csc(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(e*csc(c + d*x)), x) + Integral(sqrt(e*csc(c + d*x))*sec(c
+ d*x), x))
```

3.284 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx$

Optimal. Leaf size=122

$$-\frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{a\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[Out] -a*arctan(sin(d*x+c)^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+a*arcta
nh(sin(d*x+c)^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)-2*a*(sin(1/2*c
+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4
*Pi+1/2*d*x),2^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3878, 3872, 2838, 2564, 329, 298, 203, 206, 2639}

$$-\frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{a\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sqrt[e*Csc[c + d*x]],x]
```

```
[Out] -((a*ArcTan[Sqrt[Sin[c + d*x]]])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]
)) + (a*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d
*x]]) + (2*a*EllipticE[(c - Pi/2 + d*x)/2, 2])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt
[Sin[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+a\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx &= \frac{\int (a+a\sec(c+dx))\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}\\
&= -\frac{\int (-a-a\cos(c+dx))\sec(c+dx)\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}\\
&= \frac{a\int \sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}+\frac{a\int \sec(c+dx)\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}\\
&= \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}+\frac{a\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}\\
&= \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}+\frac{(2a)\,\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx,x,\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}\\
&= \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}+\frac{a\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}-\frac{a\,\text{Subst}\left(\int\right.}{d\sqrt{e\csc}}\\
&= -\frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}+\frac{a\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}+\frac{2aE\left(\frac{1}{2}\left(c-\right.\right.}{d\sqrt{e\csc(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 0.83, size = 130, normalized size = 1.07

$$\frac{a\left(\sqrt{-\cot^2(c+dx)}\,\sqrt{\csc(c+dx)}\,\left(-\log\left(1-\sqrt{\csc(c+dx)}\right)+\log\left(\sqrt{\csc(c+dx)}+1\right)+2\tan^{-1}\left(\sqrt{\csc(c+dx)}\right.\right.\right.}{2d\sqrt{-\cot^2(c+dx)}\,\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sqrt[e*Csc[c + d*x]],x]
```

```
[Out] (a*(-4*Cot[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Csc[c + d*x]^2] + Sqr
t[-Cot[c + d*x]^2]*Sqrt[Csc[c + d*x]]*(2*ArcTan[Sqrt[Csc[c + d*x]]] - Log[1
 - Sqrt[Csc[c + d*x]]] + Log[1 + Sqrt[Csc[c + d*x]]])))/(2*d*Sqrt[-Cot[c +
d*x]^2]*Sqrt[e*Csc[c + d*x]])
```

fricas [F] time = 0.95, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e \csc\left(dx+c\right)}\left(a \sec\left(dx+c\right)+a\right)}{e \csc\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)/(e*csc(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a \sec\left(dx+c\right)+a}{\sqrt{e \csc\left(dx+c\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(e*csc(d*x + c)), x)
```

maple [C] time = 1.44, size = 1535, normalized size = 12.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x)
```

```
[Out] 1/2*a/d*(I*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*c
os(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d
*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x
+c))^(1/2)-I*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I
*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d
*x+c))^(1/2)+cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I
*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d
*x+c))^(1/2)+cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I
*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d
*x+c))^(1/2)-4*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-
(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)-I+si
n(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1
/2)+2*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*
x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+I*((I
*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/s
in(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),
1/2+1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-I*((I*cos(d*x+
c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)
)^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I
```

```
,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+((I*cos(d*x+c)-I+sin(d*
x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*Ell
ipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2
))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+((I*cos(d*x+c)-I+sin(d*x+c))/sin(d
*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I
*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(-1+
cos(d*x+c))/sin(d*x+c))^(1/2)-4*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1
/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c
)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+
c))^(1/2)+2*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-
I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-2*cos(d*x+
c)*2^(1/2)+2*2^(1/2))/(e/sin(d*x+c))^(1/2)/sin(d*x+c)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec(dx+c)+a}{\sqrt{e\csc(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(e*csc(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{\sqrt{\frac{e}{\sin(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{\sqrt{e\csc(c+dx)}}\,dx+\int \frac{\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(e*csc(c + d*x)), x) + Integral(sec(c + d*x)/sqrt(e*csc(c
 + d*x)), x))
```

3.285 $\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=182

$$-\frac{2a}{de\sqrt{e\csc(c+dx)}}-\frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}}+\frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{a\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2a}{3de\sqrt{s}}$$

```
[Out] -2*a/d/e/(e*csc(d*x+c))^(1/2)-2/3*a*cos(d*x+c)/d/e/(e*csc(d*x+c))^(1/2)+a*a
rctan(sin(d*x+c)^(1/2))/d/e/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+a*arctanh
(sin(d*x+c)^(1/2))/d/e/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)-2/3*a*(sin(1/2
*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1
/4*Pi+1/2*d*x),2^(1/2))/d/e/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3878, 3872, 2838, 2564, 321, 329, 212, 206, 203, 2635, 2641}

$$-\frac{2a}{de\sqrt{e\csc(c+dx)}}-\frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}}+\frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{a\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2a}{3de\sqrt{s}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Csc[c + d*x])^(3/2),x]
```

```
[Out] (-2*a)/(d*e*Sqrt[e*Csc[c + d*x]]) - (2*a*Cos[c + d*x])/(3*d*e*Sqrt[e*Csc[c
+ d*x]]) + (a*ArcTan[Sqrt[Sin[c + d*x]]])/(d*e*Sqrt[e*Csc[c + d*x]]*Sqrt[Si
n[c + d*x]]) + (a*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*e*Sqrt[e*Csc[c + d*x]]*Sq
rt[Sin[c + d*x]]) + (2*a*EllipticF[(c - Pi/2 + d*x)/2, 2])/(3*d*e*Sqrt[e*Cs
c[c + d*x]]*Sqrt[Sin[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{3/2}}\,dx = \frac{\int (a+a\sec(c+dx))\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{\int (-a-a\cos(c+dx))\sec(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{a\int \sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\int \sec(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\,\text{Subst}\left(\int \frac{x^{3/2}}{1-x^2}\,dx, x, \sin(c+\right.}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+}$$

$$= -\frac{2a}{de\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\,\text{Subst}}{de\sqrt{}}$$

$$= -\frac{2a}{de\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{(2a)\,\text{Sul}}{de\sqrt{}}$$

$$= -\frac{2a}{de\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\,\text{Subst}}{de\sqrt{e}}$$

$$= -\frac{2a}{de\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\tanh}{de\sqrt{e\csc}}$$

Mathematica [A] time = 11.03, size = 135, normalized size = 0.74

$$-\frac{a\left(4\cos(c+dx)+3\sqrt{\csc(c+dx)}\log\left(1-\sqrt{\csc(c+dx)}\right)-3\sqrt{\csc(c+dx)}\log\left(\sqrt{\csc(c+dx)}+1\right)+6\sqrt{\csc(c+\right.}{6de\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Csc[c + d*x])^(3/2),x]
```

```
[Out] -1/6*(a*(12 + 4*Cos[c + d*x] + 6*ArcTan[Sqrt[Csc[c + d*x]]]*Sqrt[Csc[c + d*
x]] + 3*Sqrt[Csc[c + d*x]]*Log[1 - Sqrt[Csc[c + d*x]]] - 3*Sqrt[Csc[c + d*x
]]*Log[1 + Sqrt[Csc[c + d*x]]] + (4*EllipticF[(-2*c + Pi - 2*d*x)/4, 2])/Sq
rt[Sin[c + d*x]]))/(d*e*Sqrt[e*Csc[c + d*x]])
```

fricas [F] time = 1.08, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\csc(dx+c)}\,(a\sec(dx+c)+a)}{e^2\csc(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)/(e^2*csc(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec(dx+c)+a}{(e\csc(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/(e*csc(d*x + c))^(3/2), x)
```

maple [C] time = 1.32, size = 710, normalized size = 3.90

$$-\frac{a\left(3i\sqrt{-\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}}\,\sin(dx+c)\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}}\sqrt{-\frac{i\cos(dx+c)-i-\sin(dx+c)}{\sin(dx+c)}}\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*csc(d*x+c))^(3/2),x)
```

```
[Out] -1/6*a/d*(3*I*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*((I*cos(d*x+
c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c)
)^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I
,1/2*2^(1/2))-4*I*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*((I*cos(
d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*
x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^
(1/2))+3*I*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*((I*cos(d*x+c)-
I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(
1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/
2*2^(1/2))-3*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*((I*cos(d*x+c
)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))
^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,
1/2*2^(1/2))+3*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*((I*cos(d*x
+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c
))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*
I,1/2*2^(1/2))+2*cos(d*x+c)^2*2^(1/2)+4*cos(d*x+c)*2^(1/2)-6*2^(1/2))/(-1+c
os(d*x+c))/(e/sin(d*x+c))^(3/2)/sin(d*x+c)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec(dx+c)+a}{(e\csc(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/(e*csc(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{{\left(\frac{e}{\sin(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{1}{(e\csc(c+dx))^{\frac{3}{2}}}\,dx+\int \frac{\sec(c+dx)}{(e\csc(c+dx))^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))**(3/2),x)
```

```
[Out] a*(Integral((e*csc(c + d*x))**(-3/2), x) + Integral(sec(c + d*x)/(e*csc(c +
 d*x))**(3/2), x))
```

3.286 $\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{2a\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}}-\frac{2a\sin(c+dx)\cos(c+dx)}{5de^2\sqrt{e\csc(c+dx)}}-\frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{a\tanh^{-1}\left(\sqrt{\sin(c+d}\right.}{de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c}}$$

```
[Out] -2/3*a*sin(d*x+c)/d/e^2/(e*csc(d*x+c))^(1/2)-2/5*a*cos(d*x+c)*sin(d*x+c)/d/
e^2/(e*csc(d*x+c))^(1/2)-a*arctan(sin(d*x+c)^(1/2))/d/e^2/(e*csc(d*x+c))^(1
/2)/sin(d*x+c)^(1/2)+a*arctanh(sin(d*x+c)^(1/2))/d/e^2/(e*csc(d*x+c))^(1/2)
/sin(d*x+c)^(1/2)-6/5*a*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*P
i+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/d/e^2/(e*csc(d*x+c)
)^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3878, 3872, 2838, 2564, 321, 329, 298, 203, 206, 2635, 2639}

$$-\frac{2a\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}}-\frac{2a\sin(c+dx)\cos(c+dx)}{5de^2\sqrt{e\csc(c+dx)}}-\frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{a\tanh^{-1}\left(\sqrt{\sin(c+d}\right.}{de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Csc[c + d*x])^(5/2),x]
```

```
[Out] -((a*ArcTan[Sqrt[Sin[c + d*x]]])/(d*e^2*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d
*x]])) + (a*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*e^2*Sqrt[e*Csc[c + d*x]]*Sqrt[S
in[c + d*x]]) + (6*a*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*d*e^2*Sqrt[e*Csc[
c + d*x]]*Sqrt[Sin[c + d*x]]) - (2*a*Sin[c + d*x])/(3*d*e^2*Sqrt[e*Csc[c +
d*x]]) - (2*a*Cos[c + d*x]*Sin[c + d*x])/(5*d*e^2*Sqrt[e*Csc[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx = \frac{\int (a+a\sec(c+dx))\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= -\frac{\int (-a-a\cos(c+dx))\sec(c+dx)\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{a\int \sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\int \sec(c+dx)\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= -\frac{2a\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}} + \frac{(3a)\int \sqrt{\sin(c+dx)}\,dx}{5e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\,\mathrm{Subst}\left(\int \frac{x^{5/2}}{1-x^2}\,dx\right.}{de^2\sqrt{e\csc(c+dx}}$$
$$= \frac{6aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2a\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)\sin(c+dx}{5de^2\sqrt{e\csc(c+dx)}}$$
$$= \frac{6aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2a\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)\sin(c+dx}{5de^2\sqrt{e\csc(c+dx)}}$$
$$= \frac{6aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2a\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)\sin(c+dx}{5de^2\sqrt{e\csc(c+dx)}}$$
$$= -\frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{6aE\left(\frac{1}{2}\right.}{5de^2\sqrt{e\csc(}}$$

Mathematica [C] time = 1.51, size = 165, normalized size = 0.84

$$\frac{a\left(-72\cot(c+dx)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};\csc^2(c+dx)\right)-2\sqrt{-\cot^2(c+dx)}\left(20\sin(c+dx)+6\sin(2(c+dx))+15\sqrt{\csc}\right.\right.}{60de^2\sqrt{-\cot^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Csc[c + d*x])^(5/2),x]
```

```
[Out] (a*(-72*Cot[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Csc[c + d*x]^2] - 2*
Sqrt[-Cot[c + d*x]^2]*(-30*ArcTan[Sqrt[Csc[c + d*x]]]*Sqrt[Csc[c + d*x]] +
15*Sqrt[Csc[c + d*x]]*(Log[1 - Sqrt[Csc[c + d*x]]] - Log[1 + Sqrt[Csc[c + d
*x]]]) + 20*Sin[c + d*x] + 6*Sin[2*(c + d*x)])))/(60*d*e^2*Sqrt[-Cot[c + d*
x]^2]*Sqrt[e*Csc[c + d*x]])
```

fricas [F] time = 1.40, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{e\csc(dx+c)}\,(a\sec(dx+c)+a)}{e^3\csc(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)/(e^3*csc(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a\sec(dx+c)+a}{(e\csc(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/(e*csc(d*x + c))^(5/2), x)
```

maple [C] time = 1.32, size = 1529, normalized size = 7.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*csc(d*x+c))^(5/2),x)
```

```
[Out] 1/30*a/d*(15*I*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d
*x+c))/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*El
lipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/
2))-15*I*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)-I+
sin(d*x+c))/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/
2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*
2^(1/2))+18*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)
-I+sin(d*x+c))/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^
(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))
+15*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d
*x+c))/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*El
lipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/
2))+15*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)-I+si
n(d*x+c))/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)
*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^
(1/2))-36*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)-I
+sin(d*x+c))/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1
/2)*EllipticE(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+1
5*I*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d
*x+c))/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*El
lipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/
2))-15*I*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))
/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*Elliptic
Pi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+6*
2^(1/2)*cos(d*x+c)^3+18*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c
)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))
^(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2)
)+15*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin
(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi((
(I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+15*(-I
*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))
^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d
*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-36*(-I*(-1+cos
(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(
(-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)-I+s
in(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+10*cos(d*x+c)^2*2^(1/2)-24*cos(d*
x+c)*2^(1/2)+8*2^(1/2))/(e/sin(d*x+c))^(5/2)/sin(d*x+c)^3*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a \sec (dx + c) + a}{(e \csc (dx + c))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/(e*csc(d*x + c))^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+\frac{a}{\cos(c+d\,x)}}{\left(\frac{e}{\sin(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(5/2),x)
[Out] int((a + a/cos(c + d*x))/(e/sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))**(5/2),x)
[Out] Timed out
```

3.287 $\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=270

$$-\frac{4a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d}+\frac{5a^2e^2\tan(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{2a^2e^2\csc}{}$$

```
[Out] -2/3*a^2*e^2*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/d-4/3*a^2*e^2*csc(d*x+c)*(e*cs
c(d*x+c))^(1/2)/d-2/3*a^2*e^2*csc(d*x+c)*sec(d*x+c)*(e*csc(d*x+c))^(1/2)/d+
2*a^2*e^2*arctan(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+
2*a^2*e^2*arctanh(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d
-7/3*a^2*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*
EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c
)^(1/2)/d+5/3*a^2*e^2*(e*csc(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.33, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.520$, Rules used = {3878, 3872, 2873, 2636, 2641, 2564, 325, 329, 212, 206, 203, 2570, 2571}

$$-\frac{4a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d}+\frac{5a^2e^2\tan(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{2a^2e^2\csc}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a^2*e^2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(3*d) - (4*a^2*e^2*Csc[c + d
*x]*Sqrt[e*Csc[c + d*x]])/(3*d) - (2*a^2*e^2*Csc[c + d*x]*Sqrt[e*Csc[c + d*
x]]*Sec[c + d*x])/(3*d) + (2*a^2*e^2*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[
c + d*x]]*Sqrt[Sin[c + d*x]])/d + (2*a^2*e^2*ArcTanh[Sqrt[Sin[c + d*x]]]*Sq
rt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d + (7*a^2*e^2*Sqrt[e*Csc[c + d*x]]*
EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d) + (5*a^2*e^2*Sqr
t[e*Csc[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sin^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))^2\sec^2(c+d}{\sin^{\frac{5}{2}}(c+dx)}$$

$$= \left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\left(\frac{a^2}{\sin^{\frac{5}{2}}(c+dx)}+\frac{2a^2\sec(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\right.$$

$$= \left(a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sin^{\frac{5}{2}}(c+dx)}\,dx + \left(a^2e^2\sqrt{e\csc}\right.$$

$$= -\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}$$

$$= -\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{4a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}$$

$$= -\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{4a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}$$

$$= -\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{4a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}$$

$$= -\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{4a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}$$

Mathematica [C] time = 3.86, size = 195, normalized size = 0.72

$$-\frac{a^2e^2\cos^4\left(\frac{1}{2}(c+dx)\right)\tan(c+dx)\sqrt{e\csc(c+dx)}\sec^4\left(\frac{1}{2}\csc^{-1}(\csc(c+dx))\right)\left(7\sqrt{-\cot^2(c+dx)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\csc^2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/3*(a^2*e^2*Cos[(c + d*x)/2]^4*Sqrt[e*Csc[c + d*x]]*(-7 + 6*ArcTan[Sqrt[C
sc[c + d*x]]]*Sqrt[Cos[c + d*x]^2]*Sqrt[Csc[c + d*x]] - 6*ArcTanh[Sqrt[Csc[
c + d*x]]]*Sqrt[Cos[c + d*x]^2]*Sqrt[Csc[c + d*x]] + 4*Csc[c + d*x]^2 + 4*S
qrt[Cos[c + d*x]^2]*Csc[c + d*x]^2 + 7*Sqrt[-Cot[c + d*x]^2]*Hypergeometric
2F1[1/4, 1/2, 5/4, Csc[c + d*x]^2])*Sec[ArcCsc[Csc[c + d*x]]/2]^4*Tan[c + d
*x])/d
```

fricas [F] time = 0.79, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2e^2\csc(dx+c)^2\sec(dx+c)^2+2a^2e^2\csc(dx+c)^2\sec(dx+c)+a^2e^2\csc(dx+c)^2\right)\sqrt{e\csc(dx+c)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*e^2*csc(d*x + c)^2*sec(d*x + c)^2 + 2*a^2*e^2*csc(d*x + c)^2*
sec(d*x + c) + a^2*e^2*csc(d*x + c)^2)*sqrt(e*csc(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e\csc(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2, x)
```

maple [C] time = 1.81, size = 745, normalized size = 2.76

$$-\frac{a^2(-1+\cos(dx+c))\left(6i\cos(dx+c)\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}}\sqrt{-\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}}\sqrt{-\frac{i\cos(dx+c)-i-\sin(dx+c)}{\sin(dx+c)}}\sin(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/6*a^2/d*(-1+cos(d*x+c))*(6*I*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin
(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(
d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c)
)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*I*cos(d*x+c)*((I*cos(d*x+c)-I+
sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*co
s(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticF(((I*cos(d*x+c
)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+6*I*cos(d*x+c)*((I*cos(d*x+c
)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(
I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((I*cos(
d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*cos(d*x+c)*
((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*Ellipt
icPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+
6*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(
d*x+c)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,
1/2*2^(1/2))-7*cos(d*x+c)*2^(1/2)+3*2^(1/2))*(e/sin(d*x+c))^(5/2)*(1+cos(d*
x+c))^2/sin(d*x+c)/cos(d*x+c)*2^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\left(a+\frac{a}{\cos(c+d\,x)}\right)}^2\,{\left(\frac{e}{\sin(c+d\,x)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(5/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.288 $\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=240

$$-\frac{4a^2e\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\sec(c+dx)\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\sqrt{\sin(c+dx)}\,\sqrt{e}$$

```
[Out] -4*a^2*e*(e*csc(d*x+c))^(1/2)/d-2*a^2*e*cos(d*x+c)*(e*csc(d*x+c))^(1/2)/d-2
*a^2*e*sec(d*x+c)*(e*csc(d*x+c))^(1/2)/d-2*a^2*e*arctan(sin(d*x+c)^(1/2))*(
e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+2*a^2*e*arctanh(sin(d*x+c)^(1/2))*(e
*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+5*a^2*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2
))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+3*a^2*e*sin(d*x+c)*(e*csc(d*x+c)
)^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.33, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.520, Rules used = {3878, 3872, 2873, 2636, 2639, 2564, 325, 329, 298, 203, 206, 2570, 2571}

$$-\frac{4a^2e\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\sec(c+dx)\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\sqrt{\sin(c+dx)}\,\sqrt{e}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*a^2*e*Sqrt[e*Csc[c + d*x]])/d - (2*a^2*e*Cos[c + d*x]*Sqrt[e*Csc[c + d*
x]])/d - (2*a^2*e*Sqrt[e*Csc[c + d*x]]*Sec[c + d*x])/d - (2*a^2*e*ArcTan[Sq
rt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d + (2*a^2*e*Arc
Tanh[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d - (5*a^
2*e*Sqrt[e*Csc[c + d*x]]*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]
])/d + (3*a^2*e*Sqrt[e*Csc[c + d*x]]*Sin[c + d*x]*Tan[c + d*x])/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))^2\sec^2(c+$$

$$= \left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\left(\frac{a^2}{\sin^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sec(c+d}{\sin^{\frac{3}{2}}(c+dx}\right.$$

$$= \left(a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int\frac{1}{\sin^{\frac{3}{2}}(c+dx)}\,dx+\left(a^2e\sqrt{e\,cs\right.$$

$$= -\frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\sqrt{e\csc(c+dx)}\sec(c+d}{d}$$

$$= -\frac{4a^2e\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\sqrt{}}{}$$

$$= -\frac{4a^2e\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\sqrt{}}{}$$

$$= -\frac{4a^2e\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\sqrt{}}{}$$

$$= -\frac{4a^2e\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\sqrt{}}{}$$

Mathematica [C] time = 4.92, size = 195, normalized size = 0.81

$$\frac{2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(e\csc(c+dx))^{3/2}\sec^4\left(\frac{1}{2}\csc^{-1}(\csc(c+dx))\right)\left(5\sqrt{-\cot^2(c+dx)}\sqrt{\csc(c+dx)}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*Cos[(c + d*x)/2]^4*(e*Csc[c + d*x])^(3/2)*(3*ArcTan[Sqrt[Csc[c + d*x
]]]*Sqrt[Cos[c + d*x]^2] + 3*ArcTanh[Sqrt[Csc[c + d*x]]]*Sqrt[Cos[c + d*x]^
2] - 6*Sqrt[Csc[c + d*x]] - 6*Sqrt[Cos[c + d*x]^2]*Sqrt[Csc[c + d*x]] + 5*S
qrt[-Cot[c + d*x]^2]*Sqrt[Csc[c + d*x]]*Hypergeometric2F1[3/4, 3/2, 7/4, Cs
c[c + d*x]^2])*Sec[c + d*x]*Sec[ArcCsc[Csc[c + d*x]]/2]^4)/(3*d*Csc[c + d*x
]^(3/2))
```

fricas [F] time = 1.09, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(a^2e\csc(dx+c)\sec(dx+c)^2+2\,a^2e\csc(dx+c)\sec(dx+c)+a^2e\csc(dx+c)\right)\sqrt{e\csc(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*e*csc(d*x + c)*sec(d*x + c)^2 + 2*a^2*e*csc(d*x + c)*sec(d*x
+ c) + a^2*e*csc(d*x + c))*sqrt(e*csc(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (e \csc (dx+c))^{\frac{3}{2}} (a \sec (dx+c)+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2, x)
```

maple [C] time = 1.66, size = 1593, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/2*a^2/d*(2*I*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d
*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x
+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/
2*I,1/2*2^(1/2))-2*I*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I
*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/s
in(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),
1/2-1/2*I,1/2*2^(1/2))+2*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c
))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1
/2),1/2+1/2*I,1/2*2^(1/2))+2*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d
*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))-5*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-s
in(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x
+c))^(1/2),1/2*2^(1/2))+10*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/
2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x
+c))/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(
1/2),1/2*2^(1/2))+2*I*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(
1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x
+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+
c))/sin(d*x+c))^(1/2)-2*I*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c)
)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/
2),1/2-1/2*I,1/2*2^(1/2))+2*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x
+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*c
os(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(-1+co
s(d*x+c))/sin(d*x+c))^(1/2)+2*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d
*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I
*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(-1+
cos(d*x+c))/sin(d*x+c))^(1/2)-5*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin
(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((
I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c
))/sin(d*x+c))^(1/2)+10*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))
^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*
x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d
*x+c))^(1/2)-9*cos(d*x+c)*2^(1/2)+2^(1/2))*(e/sin(d*x+c))^(3/2)*sin(d*x+c)/
cos(d*x+c)*2^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^2\,{\left(\frac{e}{\sin\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(3/2),x)
[Out] int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(3/2)*(a+a*sec(d*x+c))**2,x)
[Out] Timed out
```

3.289 $\int \sqrt{e\csc(c+dx)}\,(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=154

$$\frac{a^2\tan(c+dx)\sqrt{e\csc(c+dx)}}{d}+\frac{2a^2\sqrt{\sin(c+dx)}\,\sqrt{e\csc(c+dx)}\,\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{2a^2\sqrt{\sin(c+dx)}\,\sqrt{e\csc(c}{}$$

```
[Out] 2*a^2*arctan(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+2*a^
2*arctanh(sin(d*x+c)^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d-3*a^2*(
sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(
1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/d+a^2*
(e*csc(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A] time = 0.26, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3878, 3872, 2873, 2641, 2564, 329, 212, 206, 203, 2571}

$$\frac{a^2\tan(c+dx)\sqrt{e\csc(c+dx)}}{d}+\frac{2a^2\sqrt{\sin(c+dx)}\,\sqrt{e\csc(c+dx)}\,\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{2a^2\sqrt{\sin(c+dx)}\,\sqrt{e\csc(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/
d + (2*a^2*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*
x]])/d + (3*a^2*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[
Sin[c + d*x]])/d + (a^2*Sqrt[e*Csc[c + d*x]]*Tan[c + d*x])/d
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{e\csc(c+dx)}\,(a+a\sec(c+dx))^2\,dx = \left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sin(c+dx)}}\,dx$$

$$= \left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))^2\sec^2(c+dx)}{\sqrt{\sin(c+dx)}}\,d$$

$$= \left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \left(\frac{a^2}{\sqrt{\sin(c+dx)}}+\frac{2a^2\sec(c+dx)}{\sqrt{\sin(c+dx)}}+\right.$$

$$= \left(a^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx + \left(a^2\sqrt{e\csc(c+\right.$$

$$= \frac{2a^2\sqrt{e\csc(c+dx)}\,F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d} + \frac{a^2\sqrt{e\csc(c+}}{}$$

$$= \frac{3a^2\sqrt{e\csc(c+dx)}\,F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d} + \frac{a^2\sqrt{e\csc(c+}}{}$$

$$= \frac{3a^2\sqrt{e\csc(c+dx)}\,F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d} + \frac{a^2\sqrt{e\csc(c+}}{}$$

$$= \frac{2a^2\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}{d} + \frac{2a^2\tanh^{-1}(}{}$$

Mathematica [C] time = 2.42, size = 168, normalized size = 1.09

$$-\frac{2a^2\sin\left(\frac{1}{2}(c+dx)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{e\csc(c+dx)}\,\sec^4\left(\frac{1}{2}\csc^{-1}(\csc(c+dx))\right)\left(3\sqrt{-\cot^2(c+dx)}\;{}_2\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a^2*Cos[(c + d*x)/2]^5*Sqrt[e*Csc[c + d*x]]*(-1 + 2*ArcTan[Sqrt[Csc[c +
 d*x]]]*Sqrt[Cos[c + d*x]^2]*Sqrt[Csc[c + d*x]] - 2*ArcTanh[Sqrt[Csc[c + d*
x]]]*Sqrt[Cos[c + d*x]^2]*Sqrt[Csc[c + d*x]] + 3*Sqrt[-Cot[c + d*x]^2]*Hype
rgeometric2F1[1/4, 1/2, 5/4, Csc[c + d*x]^2])*Sec[c + d*x]*Sec[ArcCsc[Csc[c
 + d*x]]/2]^4*Sin[(c + d*x)/2])/d
```

fricas [F] time = 0.73, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2\right)\sqrt{e\csc\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*sqrt(e*csc(d*x + c
)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{e\csc\left(dx+c\right)}\left(a\sec\left(dx+c\right)+a\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*csc(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)^2, x)
```

maple [C] time = 1.90, size = 744, normalized size = 4.83

$$\frac{a^2(-1+\cos(dx+c))\left(i\cos(dx+c)\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}}\sqrt{-\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}}\sqrt{-\frac{i\cos(dx+c)-i-\sin(dx+c)}{\sin(dx+c)}}\sin(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*csc(d*x+c))^(1/2),x)
```

```
[Out] 1/2*a^2/d*(-1+cos(d*x+c))*(I*cos(d*x+c)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*sin(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/
sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-s
in(d*x+c))/sin(d*x+c))^(1/2)-2*I*cos(d*x+c)*EllipticPi(((I*cos(d*x+c)-I+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*sin(d*x+c)*((I*cos(d*x+c)
-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I
*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)-2*I*cos(d*x+c)*EllipticPi(((I*c
os(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*sin(d*x+c)
*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)-2*cos(d*x+c)*((I
*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticP
i(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*c
os(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x
+c)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2
*2^(1/2))+cos(d*x+c)*2^(1/2)-2^(1/2))*(1+cos(d*x+c))^2*(e/sin(d*x+c))^(1/2)
/cos(d*x+c)/sin(d*x+c)^3*2^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^2\,\sqrt{\frac{e}{\sin\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\sqrt{e\csc(c+dx)}\,dx+\int 2\sqrt{e\csc(c+dx)}\sec(c+dx)\,dx+\int\sqrt{e\csc(c+dx)}\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*csc(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(sqrt(e*csc(c + d*x)), x) + Integral(2*sqrt(e*csc(c + d*x))*s
ec(c + d*x), x) + Integral(sqrt(e*csc(c + d*x))*sec(c + d*x)**2, x))
```

3.290 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx$

Optimal. Leaf size=153

$$\frac{a^2\tan(c+dx)}{d\sqrt{e\csc(c+dx)}}-\frac{2a^2\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2a^2\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{a^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[Out] -2*a^2*arctan(sin(d*x+c)^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+2*a
^2*arctanh(sin(d*x+c)^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)-a^2*(s
in(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1
/2*c+1/4*Pi+1/2*d*x),2^(1/2))/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+a^2*t
an(d*x+c)/d/(e*csc(d*x+c))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3878, 3872, 2873, 2639, 2564, 329, 298, 203, 206, 2571}

$$\frac{a^2\tan(c+dx)}{d\sqrt{e\csc(c+dx)}}-\frac{2a^2\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2a^2\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{a^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sqrt[e*Csc[c + d*x]],x]
```

```
[Out] (-2*a^2*ArcTan[Sqrt[Sin[c + d*x]]])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*
x]]) + (2*a^2*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin
[c + d*x]]) + (a^2*EllipticE[(c - Pi/2 + d*x)/2, 2])/(d*Sqrt[e*Csc[c + d*x]
]*Sqrt[Sin[c + d*x]]) + (a^2*Tan[c + d*x])/(d*Sqrt[e*Csc[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx = \frac{\int (a+a\sec(c+dx))^2\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{\int (-a-a\cos(c+dx))^2\sec^2(c+dx)\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{\int \left(a^2\sqrt{\sin(c+dx)}+2a^2\sec(c+dx)\sqrt{\sin(c+dx)}+a^2\sec^2(c+dx)\sqrt{\sin(c+dx)}\right) d}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{a^2\int \sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2\int \sec^2(c+dx)\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{\left(2a^2\right)\int \sec(c+}{\sqrt{e\csc(c+}$$
$$= \frac{2a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2\tan(c+dx)}{d\sqrt{e\csc(c+dx)}} - \frac{a^2\int \sqrt{\sin(c+dx)}\,dx}{2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} +$$
$$= \frac{a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2\tan(c+dx)}{d\sqrt{e\csc(c+dx)}} + \frac{\left(4a^2\right)\text{Subst}\left(\int \frac{x^2}{1-x^4}\,dx, x, \sqrt{\sin}\right.}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+}$$
$$= \frac{a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2\tan(c+dx)}{d\sqrt{e\csc(c+dx)}} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sqrt{\sin}\right.}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+}$$
$$= -\frac{2a^2\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2a^2\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}\right.\right.}{d\sqrt{e\csc(c+dx)}}$$

Mathematica [C] time = 8.52, size = 287, normalized size = 1.88

$$-\frac{\left(\cos\left(2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+1\right)^2\cos(c+dx)\left(\csc^2(c+dx)-1\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^2\left(\frac{\sqrt{1-\sin^2(c+dx)}\sqrt{\csc(c+dx)}}{\sqrt{1-\csc^2(}}\right.}{2d\sqrt{1-\sin^2(c+dx)}\csc^{\frac{3}{2}}(c+dx)\sqrt{e\csc(c+dx}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sqrt[e*Csc[c + d*x]],x]
```

```
[Out] -1/2*((1 + Cos[2*(c/2 + (d*x)/2)])^2*Cos[c + d*x]*(-1 + Csc[c + d*x]^2)*Sec
[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(-ArcTan[Sqrt[Csc[c + d*x]]] - Arc
Tanh[Sqrt[Csc[c + d*x]]] - (2*Sqrt[Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]^2]*H
ypergeometric2F1[3/4, 3/2, 7/4, Csc[c + d*x]^2])/(3*Sqrt[1 - Sin[c + d*x]^2
]) + (Sqrt[Csc[c + d*x]]*Hypergeometric2F1[-1/4, 3/2, 3/4, Csc[c + d*x]^2]*
Sqrt[1 - Sin[c + d*x]^2])/Sqrt[1 - Csc[c + d*x]^2]))/(d*(1 + Cos[2*(c/2 + (
-c + ArcCsc[Csc[c + d*x]])/2)])^2*Csc[c + d*x]^(3/2)*Sqrt[e*Csc[c + d*x]]*S
qrt[1 - Sin[c + d*x]^2])
```

fricas [F] time = 1.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2\right)\sqrt{e\csc\left(dx+c\right)}}{e\csc\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*sqrt(e*csc(d*x + c
))/(e*csc(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a \sec (dx+c)+a)^2}{\sqrt{e \csc (dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(e*csc(d*x + c)), x)
```

maple [C] time = 1.44, size = 1605, normalized size = 10.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x)
[Out] 1/2*a^2/d*(2*I*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d
*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x
+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/
2*I,1/2*2^(1/2))-2*I*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I
*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/s
in(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),
1/2-1/2*I,1/2*2^(1/2))+2*I*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+
c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*co
s(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(-1+cos
(d*x+c))/sin(d*x+c))^(1/2)-2*I*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d
*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))+2*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-s
in(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*
x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)
-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/si
n(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-2*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*
x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)-I+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2*2^(1/2))+cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-si
n(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+
c))^(1/2),1/2*2^(1/2))+2*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)
)^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(
d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d
*x+c))/sin(d*x+c))^(1/2)+2*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+
c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*co
s(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(-1+cos
(d*x+c))/sin(d*x+c))^(1/2)-2*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*
x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((I*c
os(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/
sin(d*x+c))^(1/2)+cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)
*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))
^(1/2)-2*cos(d*x+c)^2*2^(1/2)+cos(d*x+c)*2^(1/2)+2^(1/2))/sin(d*x+c)/cos(d*
x+c)/(e/sin(d*x+c))^(1/2)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a \sec (dx+c)+a)^2}{\sqrt{e \csc (dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(e*csc(d*x + c)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\sqrt{\frac{e}{\sin(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(1/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{1}{\sqrt{e\csc(c+dx)}}\,dx + \int \frac{2\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx + \int \frac{\sec^2(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*csc(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(1/sqrt(e*csc(c + d*x)), x) + Integral(2*sec(c + d*x)/sqrt(e*
csc(c + d*x)), x) + Integral(sec(c + d*x)**2/sqrt(e*csc(c + d*x)), x))
```

3.291 $\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=222

$$-\frac{4a^2}{de\sqrt{e\csc(c+dx)}}-\frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}}+\frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}}+\frac{2a^2\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2a^2\tanh^{-1}\left(\sqrt{\right.}}{de\sqrt{\sin(c+dx)}}$$

```
[Out] -4*a^2/d/e/(e*csc(d*x+c))^(1/2)-2/3*a^2*cos(d*x+c)/d/e/(e*csc(d*x+c))^(1/2)
+a^2*sec(d*x+c)/d/e/(e*csc(d*x+c))^(1/2)+2*a^2*arctan(sin(d*x+c)^(1/2))/d/e
/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+2*a^2*arctanh(sin(d*x+c)^(1/2))/d/e/
(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+1/3*a^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)
^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2
))/d/e/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3878, 3872, 2873, 2635, 2641, 2564, 321, 329, 212, 206, 203, 2566}

$$-\frac{4a^2}{de\sqrt{e\csc(c+dx)}}-\frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}}+\frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}}+\frac{2a^2\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2a^2\tanh^{-1}\left(\sqrt{\right.}}{de\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Csc[c + d*x])^(3/2),x]
```

```
[Out] (-4*a^2)/(d*e*Sqrt[e*Csc[c + d*x]]) - (2*a^2*Cos[c + d*x])/(3*d*e*Sqrt[e*Cs
c[c + d*x]]) + (a^2*Sec[c + d*x])/(d*e*Sqrt[e*Csc[c + d*x]]) + (2*a^2*ArcTa
n[Sqrt[Sin[c + d*x]]])/(d*e*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]) + (2*a
^2*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*e*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]
]) - (a^2*EllipticF[(c - Pi/2 + d*x)/2, 2])/(3*d*e*Sqrt[e*Csc[c + d*x]]*Sqr
t[Sin[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
```

```
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx = \frac{\int (a+a\sec(c+dx))^2\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{\int (-a-a\cos(c+dx))^2\sec^2(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{\int \left(a^2\sin^{\frac{3}{2}}(c+dx)+2a^2\sec(c+dx)\sin^{\frac{3}{2}}(c+dx)+a^2\sec^2(c+dx)\sin^{\frac{3}{2}}(c+dx)\right)}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{a^2\int \sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2\int \sec^2(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{(2a^2)\int \sec(}{e\sqrt{e\csc(}}$$
$$= -\frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}} + \frac{a^2\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{}{2e\sqrt{}}$$
$$= -\frac{4a^2}{de\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}} - \frac{a^2F\left(\frac{1}{2}\left(c-\frac{}{2}\right.\right.}{3de\sqrt{e\csc(c+dx}}$$
$$= -\frac{4a^2}{de\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}} - \frac{a^2F\left(\frac{1}{2}\left(c-\frac{}{2}\right.\right.}{3de\sqrt{e\csc(c+dx}}$$
$$= -\frac{4a^2}{de\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}} - \frac{a^2F\left(\frac{1}{2}\left(c-\frac{}{2}\right.\right.}{3de\sqrt{e\csc(c+dx}}$$
$$= -\frac{4a^2}{de\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}} + \frac{2a^2\tan^{-1}\left(\sqrt{\mathrm{si}}\right.}{de\sqrt{e\csc(c+dx)}}$$

Mathematica [C] time = 7.82, size = 164, normalized size = 0.74

$$\frac{2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\tan(c+dx)\sqrt{e\csc(c+dx)}\sec^4\left(\frac{1}{2}\csc^{-1}(\csc(c+dx))\right)\left(-6\sqrt{\cos^2(c+dx)}\,{}_2F_1\left(-\frac{1}{4},1;\frac{3}{4};\mathrm{cs}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Csc[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*Cos[(c + d*x)/2]^4*Sqrt[e*Csc[c + d*x]]*Sec[ArcCsc[Csc[c + d*x]]/2]^
4*(3 - 6*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[-1/4, 1, 3/4, Csc[c + d*x]^
2] + 3*Sqrt[-Cot[c + d*x]^2]*Hypergeometric2F1[1/4, 1/2, 5/4, Csc[c + d*x]^
2] + Sqrt[-Cot[c + d*x]^2]*Hypergeometric2F1[-3/4, 3/2, 1/4, Csc[c + d*x]^2
]*Sin[c + d*x]^2)*Tan[c + d*x])/(3*d*e^2)
```

fricas [F] time = 1.33, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(a^2\sec(dx+c)^2+2a^2\sec(dx+c)+a^2\right)\sqrt{e\csc(dx+c)}}{e^2\csc(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*sqrt(e*csc(d*x + c
))/(e^2*csc(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^2}{(e\csc(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/(e*csc(d*x + c))^(3/2), x)
```

maple [C] time = 1.33, size = 763, normalized size = 3.44

$$-\frac{a^2\left(6i\cos(dx+c)\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}}\sqrt{-\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}}\sqrt{-\frac{i\cos(dx+c)-i-\sin(dx+c)}{\sin(dx+c)}}\sin(dx+c)\,\mathrm{EllipticPi}\left(\sqrt{\frac{i\cos}{}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(3/2),x)
```

```
[Out] -1/6*a^2/d*(6*I*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(
-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+
c))^(1/2)*sin(d*x+c)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1
/2),1/2-1/2*I,1/2*2^(1/2))+6*I*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(
d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d
*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-13*I*cos(d*x+c)*EllipticF(((I*cos
(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*sin(d*x+c)*((I*cos(d*x
+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(
-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)-6*cos(d*x+c)*((I*cos(d*x+c)-
I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*
cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticPi(((I*cos(d*
x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+6*cos(d*x+c)*((
I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*Elliptic
Pi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+2*
2^(1/2)*cos(d*x+c)^3+10*cos(d*x+c)^2*2^(1/2)-15*cos(d*x+c)*2^(1/2)+3*2^(1/2
))/(-1+cos(d*x+c))/cos(d*x+c)/(e/sin(d*x+c))^(3/2)/sin(d*x+c)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^2}{(e\csc(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/(e*csc(d*x + c))^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(\frac{e}{\sin(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(3/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{1}{(e\csc(c+dx))^{\frac{3}{2}}}\,dx+\int \frac{2\sec(c+dx)}{(e\csc(c+dx))^{\frac{3}{2}}}\,dx+\int \frac{\sec^2(c+dx)}{(e\csc(c+dx))^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*csc(d*x+c))**(3/2),x)
```

```
[Out] a**2*(Integral((e*csc(c + d*x))**(-3/2), x) + Integral(2*sec(c + d*x)/(e*cs
c(c + d*x))**(3/2), x) + Integral(sec(c + d*x)**2/(e*csc(c + d*x))**(3/2),
x))
```

3.292 $\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=236

$$-\frac{4a^2\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}}+\frac{a^2\tan(c+dx)}{de^2\sqrt{e\csc(c+dx)}}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{5de^2\sqrt{e\csc(c+dx)}}-\frac{2a^2\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2a^2 \mathrm{t}}{de^2\sqrt{\mathrm{si}}}$$

```
[Out] -4/3*a^2*sin(d*x+c)/d/e^2/(e*csc(d*x+c))^(1/2)-2/5*a^2*cos(d*x+c)*sin(d*x+c
)/d/e^2/(e*csc(d*x+c))^(1/2)-2*a^2*arctan(sin(d*x+c)^(1/2))/d/e^2/(e*csc(d*
x+c))^(1/2)/sin(d*x+c)^(1/2)+2*a^2*arctanh(sin(d*x+c)^(1/2))/d/e^2/(e*csc(d
*x+c))^(1/2)/sin(d*x+c)^(1/2)+9/5*a^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/s
in(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/d/e^2
/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)+a^2*tan(d*x+c)/d/e^2/(e*csc(d*x+c))^
(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.480, Rules used = {3878, 3872, 2873, 2635, 2639, 2564, 321, 329, 298, 203, 206, 2566}

$$-\frac{4a^2\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}}+\frac{a^2\tan(c+dx)}{de^2\sqrt{e\csc(c+dx)}}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{5de^2\sqrt{e\csc(c+dx)}}-\frac{2a^2\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2a^2 \mathrm{t}}{de^2\sqrt{\mathrm{si}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Csc[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^2*ArcTan[Sqrt[Sin[c + d*x]]])/(d*e^2*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c
+ d*x]]) + (2*a^2*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*e^2*Sqrt[e*Csc[c + d*x]]*
Sqrt[Sin[c + d*x]]) - (9*a^2*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*d*e^2*Sqr
t[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]) - (4*a^2*Sin[c + d*x])/(3*d*e^2*Sqrt[
e*Csc[c + d*x]]) - (2*a^2*Cos[c + d*x]*Sin[c + d*x])/(5*d*e^2*Sqrt[e*Csc[c
+ d*x]]) + (a^2*Tan[c + d*x])/(d*e^2*Sqrt[e*Csc[c + d*x]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
```

```
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx = \frac{\int (a+a\sec(c+dx))^2 \sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{\int (-a-a\cos(c+dx))^2 \sec^2(c+dx)\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{\int \left(a^2\sin^{\frac{5}{2}}(c+dx)+2a^2\sec(c+dx)\sin^{\frac{5}{2}}(c+dx)+a^2\sec^2(c+dx)\sin^{\frac{5}{2}}(c+dx)\right) dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{a^2\int \sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2\int \sec^2(c+dx)\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{\left(2a^2\right)\int \sec(c}{e^2\sqrt{e\csc(c}$$
$$= -\frac{2a^2\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}} + \frac{a^2\tan(c+dx)}{de^2\sqrt{e\csc(c+dx)}} + \frac{\left(3a^2\right)\int \sqrt{\sin(c+dx)}\,dx}{5e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx}$$
$$= -\frac{9a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{4a^2\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)\sin(c+}{5de^2\sqrt{e\csc(c+dx)}}$$
$$= -\frac{9a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{4a^2\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)\sin(c+}{5de^2\sqrt{e\csc(c+dx)}}$$
$$= -\frac{9a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{4a^2\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)\sin(c+}{5de^2\sqrt{e\csc(c+dx)}}$$
$$= -\frac{2a^2\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2a^2\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{9a^2E\left(\frac{1}{2}\right.}{5de^2\sqrt{e\csc(c}$$

Mathematica [C] time = 11.31, size = 152, normalized size = 0.64

$$\frac{2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\tan(c+dx)\sec^4\left(\frac{1}{2}\csc^{-1}(\csc(c+dx))\right)\left(3\sqrt{-\cot^2(c+dx)}\left(\sin^2(c+dx)\,{}_2F_1\left(-\frac{5}{4},\frac{3}{2};-\frac{1}{4};\csc^2\right.\right.\right.}{15de^2\sqrt{e\csc(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Csc[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Cos[(c + d*x)/2]^4*Sec[ArcCsc[Csc[c + d*x]]/2]^4*(-10*Sqrt[Cos[c + d
*x]^2]*Hypergeometric2F1[-3/4, 1, 1/4, Csc[c + d*x]^2] + 3*Sqrt[-Cot[c + d*
x]^2]*(-10*Hypergeometric2F1[-1/4, 3/2, 3/4, Csc[c + d*x]^2] + Hypergeometr
ic2F1[-5/4, 3/2, -1/4, Csc[c + d*x]^2]*Sin[c + d*x]^2))*Tan[c + d*x])/(15*d
*e^2*Sqrt[e*Csc[c + d*x]])
```

fricas [F] time = 1.05, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(a^2\sec(dx+c)^2+2a^2\sec(dx+c)+a^2\right)\sqrt{e\csc(dx+c)}}{e^3\csc(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*sqrt(e*csc(d*x + c
))/(e^3*csc(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a\sec(dx+c)+a)^2}{(e\csc(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/(e*csc(d*x + c))^(5/2), x)
```

maple [C] time = 1.32, size = 1636, normalized size = 6.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(5/2),x)
```

```
[Out] -1/30*a^2/d*(-30*I*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2
)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)
-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(-1+cos(d*x+c))
/sin(d*x+c))^(1/2)-30*I*cos(d*x+c)^2*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c
))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos
(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*(-I*(-1+cos(
d*x+c))/sin(d*x+c))^(1/2)+30*I*cos(d*x+c)^2*((I*cos(d*x+c)-I+sin(d*x+c))/si
n(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(
((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*(-I*(
-1+cos(d*x+c))/sin(d*x+c))^(1/2)+30*I*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c
))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*Ellipt
icPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*
(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+27*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/s
in(d*x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*
x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2*2^(1/2))-54*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I
-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d
*x+c))^(1/2),1/2*2^(1/2))-30*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d
*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))-30*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-
sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d
*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*cos(d*x+c)^4*2^(1/2)+27*cos(d*x+c)*((
I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/
sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),
1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-54*cos(d*x+c)*((I*cos(d*
x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+
c))^(1/2)*EllipticE(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1
/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-30*cos(d*x+c)*((I*cos(d*x+c)-I+s
in(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2
)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2
^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-30*cos(d*x+c)*((I*cos(d*x+c)-
I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(
1/2)*EllipticPi(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2+1/2*I,1/
2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-20*2^(1/2)*cos(d*x+c)^3-6*
cos(d*x+c)^2*2^(1/2)+47*cos(d*x+c)*2^(1/2)-15*2^(1/2))/cos(d*x+c)/(e/sin(d*
x+c))^(5/2)/sin(d*x+c)^3*2^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(\frac{e}{\sin(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(5/2),x)
```

```
[Out] int((a + a/cos(c + d*x))^2/(e/sin(c + d*x))^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*csc(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

3.293 $\int \frac{(e\csc(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=155

$$-\frac{2e^2\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7ad}+\frac{2e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7ad}-\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{21ad}-$$

```
[Out] -4/21*e^2*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/a/d+2/7*e^2*cot(d*x+c)*csc(d*x+c)
^2*(e*csc(d*x+c))^(1/2)/a/d-2/7*e^2*csc(d*x+c)^3*(e*csc(d*x+c))^(1/2)/a/d-4
/21*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellip
ticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/
2)/a/d
```

Rubi [A] time = 0.22, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3878, 3872, 2839, 2564, 30, 2567, 2636, 2641}

$$-\frac{2e^2\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7ad}+\frac{2e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7ad}-\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{21ad}-$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*e^2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(21*a*d) + (2*e^2*Cot[c + d*x]*C
sc[c + d*x]^2*Sqrt[e*Csc[c + d*x]])/(7*a*d) - (2*e^2*Csc[c + d*x]^3*Sqrt[e*
Csc[c + d*x]])/(7*a*d) + (4*e^2*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 +
d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{(e\csc(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = \left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))\sin^{\frac{5}{2}}(c+dx)}\,dx$$

$$= -\left(\left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos(c+dx)}{(-a-a\cos(c+dx))\sin^{\frac{5}{2}}(c+dx)}\,dx\right)$$

$$= \frac{\left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}\,dx}{a} - \frac{\left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{c}{s}}{a}$$

$$= \frac{2e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7ad} + \frac{\left(2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{}{\sin^{\frac{5}{2}}(}}{7a}$$

$$= -\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{21ad} + \frac{2e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7ad} - \frac{2e^2\csc}{}$$

$$= -\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{21ad} + \frac{2e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7ad} - \frac{2e^2\csc}{}$$

Mathematica [A] time = 1.04, size = 131, normalized size = 0.85

$$-\frac{\sin^{\frac{5}{2}}(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)(e\csc(c+dx))^{5/2}\left(2\sqrt{\sin(c+dx)}\,(2\cos(c+dx)+\cos(2(c+dx))\right.}{168ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Csc[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/168*(Csc[(c + d*x)/2]^2*(e*Csc[c + d*x])^(5/2)*Sec[(c + d*x)/2]^4*((2 +
Cos[c + d*x] - 2*Cos[2*(c + d*x)] - Cos[3*(c + d*x)])*EllipticF[(-2*c + Pi
```

```
- 2*d*x)/4, 2] + 2*(4 + 2*Cos[c + d*x] + Cos[2*(c + d*x)])*Sqrt[Sin[c + d*x
]])*Sin[c + d*x]^(5/2))/(a*d)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\csc(dx+c)}\,e^2\csc(dx+c)^2}{a\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))*e^2*csc(d*x + c)^2/(a*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(e\csc(dx+c))^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*csc(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

maple [C] time = 1.33, size = 465, normalized size = 3.00

$$-\frac{(-1+\cos(dx+c))^3\left(2i\sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{-\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}}\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-i\cos(dx+c)+\sin(}{\sin(dx+c)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/21/a/d*(-1+cos(d*x+c))^3*(2*I*sin(d*x+c)*cos(d*x+c)^2*(-I*(-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*((-I*cos
(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+4*I*cos(d*x+c)*((-I*cos(d*x+c)+sin(d*x+c
)+I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(
-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticF(((I*cos(d*x+c)-I+sin(
d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*I*((-I*cos(d*x+c)+sin(d*x+c)+I)/si
n(d*x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(-1+cos(
d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))
/sin(d*x+c))^(1/2),1/2*2^(1/2))-2*cos(d*x+c)^2*2^(1/2)-2*cos(d*x+c)*2^(1/2)
-3*2^(1/2))*(1+cos(d*x+c))^2*(e/sin(d*x+c))^(5/2)/sin(d*x+c)^5*2^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\cos(c+d\,x)\,{\left(\frac{e}{\sin(c+d\,x)}\right)}^{5/2}}{a\,\left(\cos(c+d\,x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/sin(c + d*x))^(5/2)/(a + a/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)*(e/sin(c + d*x))^(5/2))/(a*(cos(c + d*x) + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.294 $\int \frac{(e \csc(c+dx))^{3/2}}{a+a\sec(c+dx)} dx$

Optimal. Leaf size=145

$$-\frac{2e\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{5ad}-\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{5ad}+\frac{2e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{5ad}-\frac{4e}{}$$

```
[Out] -4/5*e*cos(d*x+c)*(e*csc(d*x+c))^(1/2)/a/d+2/5*e*cot(d*x+c)*csc(d*x+c)*(e*c
sc(d*x+c))^(1/2)/a/d-2/5*e*csc(d*x+c)^2*(e*csc(d*x+c))^(1/2)/a/d+4/5*e*(sin
(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2
*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/a/d
```

Rubi [A] time = 0.23, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3878, 3872, 2839, 2564, 30, 2567, 2636, 2639}

$$-\frac{2e\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{5ad}-\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{5ad}+\frac{2e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{5ad}-\frac{4e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*e*Cos[c + d*x]*Sqrt[e*Csc[c + d*x]])/(5*a*d) + (2*e*Cot[c + d*x]*Csc[c
+ d*x]*Sqrt[e*Csc[c + d*x]])/(5*a*d) - (2*e*Csc[c + d*x]^2*Sqrt[e*Csc[c + d
*x]])/(5*a*d) - (4*e*Sqrt[e*Csc[c + d*x]]*EllipticE[(c - Pi/2 + d*x)/2, 2]*
Sqrt[Sin[c + d*x]])/(5*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{(e\csc(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx = \left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))\sin^{\frac{3}{2}}(c+dx)}\,dx$$

$$= -\left(\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos(c+dx)}{(-a-a\cos(c+dx))\sin^{\frac{3}{2}}(c+dx)}\,dx\right)$$

$$= \frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}\,dx}{a} - \frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos}{\sin^{\frac{7}{2}}}}{a}$$

$$= \frac{2e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{5ad} + \frac{\left(2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sin^{\frac{3}{2}}(c+d}}{5a}$$

$$= -\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{5ad} + \frac{2e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{5ad} - \frac{2e\csc^2(c}{}$$

$$= -\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{5ad} + \frac{2e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{5ad} - \frac{2e\csc^2(c}{}$$

Mathematica [C] time = 1.39, size = 230, normalized size = 1.59

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)(e\csc(c+dx))^{3/2}\left(-\frac{6\tan(c+dx)\left(\sec^2\left(\frac{1}{2}(c+dx)\right)+4\sec(c)\cos(dx)\right)}{d} + \frac{8\sqrt{2}e^{i(c-dx)}\sqrt{\frac{ie^{i(c+dx)}}{-1+e^{2i(c+dx)}}}\left(\left(1+e^{2ic}\right)e^{2idx}\sqrt{1-e^{2i(c+}}\right.}{\left(1+e^{2ic}\right)d\,\mathrm{cs}}\right.}{15a(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Csc[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(e*Csc[c + d*x])^(3/2)*((8*Sqrt[2]*E^(I*(c - d*x))*Sqrt
[(I*E^(I*(c + d*x)))/(-1 + E^((2*I)*(c + d*x)))]*(3 - 3*E^((2*I)*(c + d*x))
 + E^((2*I)*d*x)*(1 + E^((2*I)*c))*Sqrt[1 - E^((2*I)*(c + d*x))]*Hypergeome
tric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])*Sec[c + d*x])/(d*(1 + E^((2*I)
```

```
*c))*Csc[c + d*x]^(3/2)) - (6*(4*Cos[d*x]*Sec[c] + Sec[(c + d*x)/2]^2)*Tan[
c + d*x])/d))/(15*a*(1 + Sec[c + d*x]))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\csc\left(dx+c\right)}\,e\csc\left(dx+c\right)}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))*e*csc(d*x + c)/(a*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(e\csc\left(dx+c\right)\right)^{\frac{3}{2}}}{a\sec\left(dx+c\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*csc(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

maple [C] time = 1.29, size = 799, normalized size = 5.51

$$-\frac{\left(-1+\cos\left(dx+c\right)\right)\left(4\left(\cos^{2}\left(dx+c\right)\right)\sqrt{-\frac{i\left(-1+\cos\left(dx+c\right)\right)}{\sin\left(dx+c\right)}}\,\sqrt{\frac{i\cos\left(dx+c\right)-i+\sin\left(dx+c\right)}{\sin\left(dx+c\right)}}\,\sqrt{-\frac{i\cos\left(dx+c\right)-i-\sin\left(dx+c\right)}{\sin\left(dx+c\right)}}\ \text{Ellipt}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/5/a/d*(-1+cos(d*x+c))*(4*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*
x+c))/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^
(1/2),1/2*2^(1/2))-2*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I
*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/s
in(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1
/2*2^(1/2))+8*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(
I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)-I+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/
2)-4*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x
+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/
sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+4*((I*
cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/si
n(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/
2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-2*((I*cos(d*x+c)-I+sin(d*x
+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*Elli
pticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+c
os(d*x+c))/sin(d*x+c))^(1/2)-2*cos(d*x+c)*2^(1/2)-3*2^(1/2))*(e/sin(d*x+c))
^(3/2)/sin(d*x+c)*2^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)\,\left(\frac{e}{\sin(c+d\,x)}\right)^{3/2}}{a\,\left(\cos(c+d\,x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/sin(c + d*x))^(3/2)/(a + a/cos(c + d*x)),x)
[Out] int((cos(c + d*x)*(e/sin(c + d*x))^(3/2))/(a*(cos(c + d*x) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\csc(c+dx))^{\frac{3}{2}}}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
[Out] Integral((e*csc(c + d*x))**(3/2)/(sec(c + d*x) + 1), x)/a
```

3.295 $\int \frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=105

$$-\frac{2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3ad}+\frac{2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3ad}+\frac{4\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\csc(c+d}}{3ad}$$

```
[Out] 2/3*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/a/d-2/3*csc(d*x+c)*(e*csc(d*x+c))^(1/2)
/a/d-4/3*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Elli
pticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(1
/2)/a/d
```

Rubi [A] time = 0.20, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3878, 3872, 2839, 2564, 30, 2567, 2641}

$$-\frac{2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3ad}+\frac{2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3ad}+\frac{4\sqrt{\sin(c+dx)}\,F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\csc(c+d}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Csc[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(3*a*d) - (2*Csc[c + d*x]*Sqrt[e*Csc[
c + d*x]])/(3*a*d) + (4*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(3*a*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
```

```
e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx = \left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))\sqrt{\sin(c+dx)}}\,dx$$
$$= -\left(\left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos(c+dx)}{(-a-a\cos(c+dx))\sqrt{\sin(c+dx)}}\,dx\right)$$
$$= \frac{\left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx}{a} - \frac{\left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c}{\sin^{\frac{5}{2}}(c}}{a}$$
$$= \frac{2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3ad} + \frac{\left(2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3a} + \frac{\left(\sqrt{e\,c}\right.}{}$$
$$= \frac{2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3ad} - \frac{2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3ad} + \frac{4\sqrt{e\csc(c+dx)}\,F\left(\frac{1}{2}\left(c\right.\right.}{3}$$

Mathematica [A] time = 0.38, size = 60, normalized size = 0.57

$$\frac{2(e\csc(c+dx))^{3/2}\left(\cos(c+dx)-2\sin^{\frac{3}{2}}(c+dx)F\left(\frac{1}{4}(-2c-2dx+\pi)\middle|2\right)-1\right)}{3ade}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Csc[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*(e*Csc[c + d*x])^(3/2)*(-1 + Cos[c + d*x] - 2*EllipticF[(-2*c + Pi - 2*d
*x)/4, 2]*Sin[c + d*x]^(3/2)))/(3*a*d*e)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\csc(dx+c)}}{a\sec(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))/(a*sec(d*x + c) + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\csc(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] integrate(sqrt(e*csc(d*x + c))/(a*sec(d*x + c) + a), x)

maple [C] time = 1.34, size = 326, normalized size = 3.10

$$\frac{\sqrt{\frac{e}{\sin(dx+c)}}\,(1+\cos(dx+c))^2\,(-1+\cos(dx+c))^2\left(2i\cos(dx+c)\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}}\sqrt{-\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}}\sqrt{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x)

[Out] 1/3/a/d*(e/sin(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(2*I*cos(d*x+c)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*sin(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)+2*I*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+cos(d*x+c)*2^(1/2)-2^(1/2))/sin(d*x+c)^5*2^(1/2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\csc(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate(sqrt(e*csc(d*x + c))/(a*sec(d*x + c) + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)\,\sqrt{\frac{e}{\sin(c+d\,x)}}}{a\,(\cos(c+d\,x)+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e/sin(c + d*x))^(1/2)/(a + a/cos(c + d*x)),x)

[Out] int((cos(c + d*x)*(e/sin(c + d*x))^(1/2))/(a*(cos(c + d*x) + 1)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{e\csc(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*csc(d*x+c))**(1/2)/(a+a*sec(d*x+c)),x)

[Out] Integral(sqrt(e*csc(c + d*x))/(sec(c + d*x) + 1), x)/a

3.296 $\int \frac{1}{\sqrt{e\csc(c+dx)}\,(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=99

$$-\frac{2\csc(c+dx)}{ad\sqrt{e\csc(c+dx)}}+\frac{2\cot(c+dx)}{ad\sqrt{e\csc(c+dx)}}+\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{ad\sqrt{\sin(c+dx)}\,\sqrt{e\csc(c+dx)}}$$

```
[Out] 2*cot(d*x+c)/a/d/(e*csc(d*x+c))^(1/2)-2*csc(d*x+c)/a/d/(e*csc(d*x+c))^(1/2)
-4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticE(
cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2
)
```

Rubi [A] time = 0.21, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3878, 3872, 2839, 2564, 30, 2567, 2639}

$$-\frac{2\csc(c+dx)}{ad\sqrt{e\csc(c+dx)}}+\frac{2\cot(c+dx)}{ad\sqrt{e\csc(c+dx)}}+\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{ad\sqrt{\sin(c+dx)}\,\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] (2*Cot[c + d*x])/(a*d*Sqrt[e*Csc[c + d*x]]) - (2*Csc[c + d*x])/(a*d*Sqrt[e*
Csc[c + d*x]]) + (4*EllipticE[(c - Pi/2 + d*x)/2, 2])/(a*d*Sqrt[e*Csc[c + d
*x]]*Sqrt[Sin[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
```

```
e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\csc(c+dx)}\,(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\sqrt{\sin(c+dx)}}{a+a\sec(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= -\frac{\int \frac{\cos(c+dx)\sqrt{\sin(c+dx)}}{-a-a\cos(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\,dx}{a\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} - \frac{\int \frac{\cos^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\,dx}{a\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$

$$= \frac{2\cot(c+dx)}{ad\sqrt{e\csc(c+dx)}} + \frac{2\int \sqrt{\sin(c+dx)}\,dx}{a\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{x^{3/2}}\right.}{ad\sqrt{e\csc(c+}}$$

$$= \frac{2\cot(c+dx)}{ad\sqrt{e\csc(c+dx)}} - \frac{2\csc(c+dx)}{ad\sqrt{e\csc(c+dx)}} + \frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right.}{ad\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c-}}$$

Mathematica [C] time = 0.62, size = 95, normalized size = 0.96

$$\frac{6(\cot(c+dx)-\csc(c+dx)+2i)-4\sqrt{1-e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{2i(c+dx)}\right)(\cot(c+dx)+i)}{3ad\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] (6*(2*I + Cot[c + d*x] - Csc[c + d*x]) - 4*Sqrt[1 - E^((2*I)*(c + d*x))]*(I
 + Cot[c + d*x])*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])/(3*
a*d*Sqrt[e*Csc[c + d*x]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\csc(dx+c)}}{ae\csc(dx+c)\sec(dx+c)+ae\csc(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))/(a*e*csc(d*x + c)*sec(d*x + c) + a*e*csc(d*x
+ c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e \csc \left(dx + c\right)} \left(a \sec \left(dx + c\right) + a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)), x)
```

maple [C] time = 1.17, size = 536, normalized size = 5.41

$$-\frac{\left(4 \cos \left(dx + c\right) \sqrt{\frac{i \cos(dx+c) - i + \sin(dx+c)}{\sin(dx+c)}}\, \sqrt{-\frac{i \cos(dx+c) - i - \sin(dx+c)}{\sin(dx+c)}}\, \mathrm{EllipticE}\left(\sqrt{\frac{i \cos(dx+c) - i + \sin(dx+c)}{\sin(dx+c)}}, \frac{\sqrt{2}}{2}\right) \sqrt{-\frac{i(-1+\cos}{\sin(d}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x)
```

```
[Out] -1/a/d*(4*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*co
s(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)-I+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-2
*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-
I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(
d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+4*((I*cos(
d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*
x+c))^(1/2)*EllipticE(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^
(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-2*((I*cos(d*x+c)-I+sin(d*x+c))
/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*Elliptic
F(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d
*x+c))/sin(d*x+c))^(1/2)+cos(d*x+c)*2^(1/2)-2^(1/2))/(e/sin(d*x+c))^(1/2)/s
in(d*x+c)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e \csc \left(dx + c\right)} \left(a \sec \left(dx + c\right) + a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos \left(c + d\,x\right)}{a\,\sqrt{\frac{e}{\sin(c+d\,x)}}\,\left(\cos \left(c + d\,x\right) + 1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))*(e/sin(c + d*x))^(1/2)),x)
```

```
[Out] int(cos(c + d*x)/(a*(e/sin(c + d*x))^(1/2)*(cos(c + d*x) + 1)), x)
```

<u>**sympy**</u> **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{e\csc(c+dx)}\,\sec(c+dx)+\sqrt{e\csc(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))**(1/2),x)
[Out] Integral(1/(sqrt(e*csc(c + d*x))*sec(c + d*x) + sqrt(e*csc(c + d*x))), x)/a
```

3.297 $\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=106

$$\frac{2}{ade\sqrt{e\csc(c+dx)}}-\frac{2\cos(c+dx)}{3ade\sqrt{e\csc(c+dx)}}-\frac{4F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3ade\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[Out] 2/a/d/e/(e*csc(d*x+c))^(1/2)-2/3*cos(d*x+c)/a/d/e/(e*csc(d*x+c))^(1/2)+4/3*
(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos
(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a/d/e/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3878, 3872, 2839, 2564, 30, 2569, 2641}

$$\frac{2}{ade\sqrt{e\csc(c+dx)}}-\frac{2\cos(c+dx)}{3ade\sqrt{e\csc(c+dx)}}-\frac{4F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3ade\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] 2/(a*d*e*Sqrt[e*Csc[c + d*x]]) - (2*Cos[c + d*x])/(3*a*d*e*Sqrt[e*Csc[c + d
*x]]) - (4*EllipticF[(c - Pi/2 + d*x)/2, 2])/(3*a*d*e*Sqrt[e*Csc[c + d*x]]*
Sqrt[Sin[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\sin^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$
$$= -\frac{\int \frac{\cos(c+dx)\sin^{\frac{3}{2}}(c+dx)}{-a-a\cos(c+dx)}\,dx}{e\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$
$$= \frac{\int \frac{\cos(c+dx)}{\sqrt{\sin(c+dx)}}\,dx}{ae\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} - \frac{\int \frac{\cos^2(c+dx)}{\sqrt{\sin(c+dx)}}\,dx}{ae\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}}$$
$$= -\frac{2\cos(c+dx)}{3ade\sqrt{e\csc(c+dx)}} - \frac{2\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3ae\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}} + \frac{\text{Subs}}{ade\sqrt{e}}$$
$$= \frac{2}{ade\sqrt{e\csc(c+dx)}} - \frac{2\cos(c+dx)}{3ade\sqrt{e\csc(c+dx)}} - \frac{4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+\right.\right.}{3ade\sqrt{e\csc(c+dx)}\,\sqrt{}}$$

Mathematica [A] time = 0.40, size = 70, normalized size = 0.66

$$\frac{4F\left(\frac{1}{4}(-2c-2dx+\pi)\middle|2\right)-2\sqrt{\sin(c+dx)}\,(\cos(c+dx)-3)}{3ad\sin^{\frac{3}{2}}(c+dx)(e\csc(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (4*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] - 2*(-3 + Cos[c + d*x])*Sqrt[Sin[c +
 d*x]])/(3*a*d*(e*Csc[c + d*x])^(3/2)*Sin[c + d*x]^(3/2))
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\csc\left(dx+c\right)}}{ae^2\csc\left(dx+c\right)^2\sec\left(dx+c\right)+ae^2\csc\left(dx+c\right)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))/(a*e^2*csc(d*x + c)^2*sec(d*x + c) + a*e^2*cs
c(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\csc(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)), x)
```

maple [C] time = 1.15, size = 195, normalized size = 1.84

$$\frac{\left(2i\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}}, \frac{\sqrt{2}}{2}\right)\sin(dx+c)\sqrt{\frac{-i\cos(dx+c)+\sin(dx+c)+i}{\sin(dx+c)}}\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}}\sqrt{-\frac{i(-1+\cos(d}{\sin(dx+}}\right.}{3ad(-1+\cos(dx+c))\left(\frac{e}{\sin(dx+c)}\right)^{\frac{3}{2}}\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/3/a/d*(2*I*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)
-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(
d*x+c)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2)
)-cos(d*x+c)^2*2^(1/2)+4*cos(d*x+c)*2^(1/2)-3*2^(1/2))/(-1+cos(d*x+c))/(e/s
in(d*x+c))^(3/2)/sin(d*x+c)*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\csc(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)}{a\left(\frac{e}{\sin(c+d\,x)}\right)^{3/2}(\cos(c+d\,x)+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))*(e/sin(c + d*x))^(3/2)),x)
```

```
[Out] int(cos(c + d*x)/(a*(e/sin(c + d*x))^(3/2)*(cos(c + d*x) + 1)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{(e\csc(c+dx))^{\frac{3}{2}}\sec(c+dx)+(e\csc(c+dx))^{\frac{3}{2}}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(1/((e*csc(c + d*x))**(3/2)*sec(c + d*x) + (e*csc(c + d*x))**(3/2))
, x)/a
```

3.298 $\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=120

$$\frac{2\sin(c+dx)}{3ade^2\sqrt{e\csc(c+dx)}}-\frac{2\sin(c+dx)\cos(c+dx)}{5ade^2\sqrt{e\csc(c+dx)}}-\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{5ade^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[Out] 2/3*sin(d*x+c)/a/d/e^2/(e*csc(d*x+c))^(1/2)-2/5*cos(d*x+c)*sin(d*x+c)/a/d/e
^2/(e*csc(d*x+c))^(1/2)+4/5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1
/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a/d/e^2/(e*csc(
d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3878, 3872, 2839, 2564, 30, 2569, 2639}

$$\frac{2\sin(c+dx)}{3ade^2\sqrt{e\csc(c+dx)}}-\frac{2\sin(c+dx)\cos(c+dx)}{5ade^2\sqrt{e\csc(c+dx)}}-\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{5ade^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-4*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*a*d*e^2*Sqrt[e*Csc[c + d*x]]*Sqrt[
Sin[c + d*x]]) + (2*Sin[c + d*x])/(3*a*d*e^2*Sqrt[e*Csc[c + d*x]]) - (2*Cos
[c + d*x]*Sin[c + d*x])/(5*a*d*e^2*Sqrt[e*Csc[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
```

```
e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\sin^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{\int \frac{\cos(c+dx)\sin^{\frac{5}{2}}(c+dx)}{-a-a\cos(c+dx)}\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \cos(c+dx)\sqrt{\sin(c+dx)}\,dx}{ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{\int \cos^2(c+dx)\sqrt{\sin(c+dx)}\,dx}{ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{2\cos(c+dx)\sin(c+dx)}{5ade^2\sqrt{e\csc(c+dx)}} - \frac{2\int\sqrt{\sin(c+dx)}\,dx}{5ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{\text{Su}}{ad}$$

$$= -\frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5ade^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2\sin(c+dx)}{3ade^2\sqrt{e\csc(c+dx)}} - \frac{2\cos}{5ad}$$

Mathematica [C] time = 0.83, size = 100, normalized size = 0.83

$$\frac{8\sqrt{1-e^{2i(c+dx)}}\;{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{2i(c+dx)}\right)(\cot(c+dx)+i)+20\sin(c+dx)-6(\sin(2(c+dx))+4i)}{30ade^2\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (8*Sqrt[1 - E^((2*I)*(c + d*x))]*(I + Cot[c + d*x])*Hypergeometric2F1[1/2,
3/4, 7/4, E^((2*I)*(c + d*x))] + 20*Sin[c + d*x] - 6*(4*I + Sin[2*(c + d*x)
]))/(30*a*d*e^2*Sqrt[e*Csc[c + d*x]])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\csc(dx+c)}}{ae^3\csc(dx+c)^3\sec(dx+c)+ae^3\csc(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))/(a*e^3*csc(d*x + c)^3*sec(d*x + c) + a*e^3*cs
c(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\csc(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a)), x)
```

maple [C] time = 1.40, size = 551, normalized size = 4.59

$$\left(12\cos(dx+c)\sqrt{-\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}}\,\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}}\,\sqrt{\frac{-i\cos(dx+c)+\sin(dx+c)+i}{\sin(dx+c)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/15/a/d*(12*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c
)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))
^(1/2)*EllipticE(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2)
)-6*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d
*x+c))/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*El
lipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+3*2^(1/
2)*cos(d*x+c)^3+12*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)-I+s
in(d*x+c))/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2
)*EllipticE(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-6*(
-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c
))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(
d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-5*cos(d*x+c)^2*2^(1/2)+
3*cos(d*x+c)*2^(1/2)-2^(1/2))/(e/sin(d*x+c))^(5/2)/sin(d*x+c)^3*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\csc(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)}{a\left(\frac{e}{\sin(c+d\,x)}\right)^{5/2}(\cos(c+d\,x)+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))*(e/sin(c + d*x))^(5/2)),x)
```

```
[Out] int(cos(c + d*x)/(a*(e/sin(c + d*x))^(5/2)*(cos(c + d*x) + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.299 $\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=149

$$\frac{2\cos^3(c+dx)}{7ade^3\sqrt{e\csc(c+dx)}}-\frac{2\cos(c+dx)}{21ade^3\sqrt{e\csc(c+dx)}}+\frac{2\sin^2(c+dx)}{5ade^3\sqrt{e\csc(c+dx)}}-\frac{4F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{21ade^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[Out] -2/21*cos(d*x+c)/a/d/e^3/(e*csc(d*x+c))^(1/2)+2/7*cos(d*x+c)^3/a/d/e^3/(e*c
sc(d*x+c))^(1/2)+2/5*sin(d*x+c)^2/a/d/e^3/(e*csc(d*x+c))^(1/2)+4/21*(sin(1/
2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+
1/4*Pi+1/2*d*x),2^(1/2))/a/d/e^3/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3878, 3872, 2839, 2564, 30, 2568, 2569, 2641}

$$\frac{2\cos^3(c+dx)}{7ade^3\sqrt{e\csc(c+dx)}}-\frac{2\cos(c+dx)}{21ade^3\sqrt{e\csc(c+dx)}}+\frac{2\sin^2(c+dx)}{5ade^3\sqrt{e\csc(c+dx)}}-\frac{4F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{21ade^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Csc[c + d*x])^(7/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-2*Cos[c + d*x])/(21*a*d*e^3*Sqrt[e*Csc[c + d*x]]) + (2*Cos[c + d*x]^3)/(7
*a*d*e^3*Sqrt[e*Csc[c + d*x]]) - (4*EllipticF[(c - Pi/2 + d*x)/2, 2])/(21*a
*d*e^3*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]) + (2*Sin[c + d*x]^2)/(5*a*d
*e^3*Sqrt[e*Csc[c + d*x]])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx = \frac{\int \frac{\sin^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{\int \frac{\cos(c+dx)\sin^{\frac{7}{2}}(c+dx)}{-a-a\cos(c+dx)}\,dx}{e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \cos(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{ae^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{\int \cos^2(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{ae^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{2\cos^3(c+dx)}{7ade^3\sqrt{e\csc(c+dx)}} - \frac{\int \frac{\cos^2(c+dx)}{\sqrt{\sin(c+dx)}}\,dx}{7ae^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{\text{Subst}\Big(}{ade^3\sqrt{e\,\text{c}}}$$

$$= -\frac{2\cos(c+dx)}{21ade^3\sqrt{e\csc(c+dx)}} + \frac{2\cos^3(c+dx)}{7ade^3\sqrt{e\csc(c+dx)}} + \frac{2\sin^2(c+dx)}{5ade^3\sqrt{e\csc(c+d}}$$

$$= -\frac{2\cos(c+dx)}{21ade^3\sqrt{e\csc(c+dx)}} + \frac{2\cos^3(c+dx)}{7ade^3\sqrt{e\csc(c+dx)}} - \frac{4F\left(\frac{1}{2}\left(c-\right.\right.}{21ade^3\sqrt{e\csc(c+}}$$

Mathematica [A] time = 0.58, size = 91, normalized size = 0.61

$$\frac{\sqrt{e\csc(c+dx)}\left(126\sin(c+dx)+10\sin(2(c+dx))-42\sin(3(c+dx))+15\sin(4(c+dx))+80\sqrt{\sin(c+dx)}F\Big(\right.}{420ade^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Csc[c + d*x])^(7/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Sqrt[e*Csc[c + d*x]]*(80*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c +
d*x]] + 126*Sin[c + d*x] + 10*Sin[2*(c + d*x)] - 42*Sin[3*(c + d*x)] + 15*S
in[4*(c + d*x)]))/(420*a*d*e^4)
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{e\csc\left(dx+c\right)}}{ae^4\csc\left(dx+c\right)^4\sec\left(dx+c\right)+ae^4\csc\left(dx+c\right)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))/(a*e^4*csc(d*x + c)^4*sec(d*x + c) + a*e^4*cs
c(d*x + c)^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\csc\left(dx+c\right)\right)^{\frac{7}{2}}\left(a\sec\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*csc(d*x + c))^(7/2)*(a*sec(d*x + c) + a)), x)
```

maple [C] time = 1.23, size = 221, normalized size = 1.48

$$\frac{\left(10i\,\mathrm{EllipticF}\left(\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}}, \frac{\sqrt{2}}{2}\right)\sin\left(dx+c\right)\sqrt{\frac{-i\cos(dx+c)+\sin(dx+c)+i}{\sin(dx+c)}}\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}}\sqrt{-\frac{i(-1+}{\sin}}\right.}{105ad\left(-1+\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/105/a/d*(10*I*sin(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*
x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+
c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1
/2))+15*cos(d*x+c)^4*2^(1/2)-36*2^(1/2)*cos(d*x+c)^3+16*cos(d*x+c)^2*2^(1/2
)+26*cos(d*x+c)*2^(1/2)-21*2^(1/2))/(-1+cos(d*x+c))/(e/sin(d*x+c))^(7/2)/si
n(d*x+c)^3*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e\csc\left(dx+c\right)\right)^{\frac{7}{2}}\left(a\sec\left(dx+c\right)+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*csc(d*x + c))^(7/2)*(a*sec(d*x + c) + a)), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)}{a\,{\left(\frac{e}{\sin\left(c+d\,x\right)}\right)}^{7/2}\,\left(\cos\left(c+d\,x\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))*(e/sin(c + d*x))^(7/2)),x)
```

```
[Out] int(cos(c + d*x)/(a*(e/sin(c + d*x))^(7/2)*(cos(c + d*x) + 1)), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))**(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

3.300 $\int \frac{(e\csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=268

$$\frac{4e^2\csc^5(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d}-\frac{4e^2\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d}-\frac{2e^2\cot^3(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d}$$

```
[Out] -4/231*e^2*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/a^2/d+16/77*e^2*cot(d*x+c)*csc(d
*x+c)^2*(e*csc(d*x+c))^(1/2)/a^2/d-2/11*e^2*cot(d*x+c)^3*csc(d*x+c)^2*(e*cs
c(d*x+c))^(1/2)/a^2/d-4/7*e^2*csc(d*x+c)^3*(e*csc(d*x+c))^(1/2)/a^2/d-2/11*
e^2*cot(d*x+c)*csc(d*x+c)^4*(e*csc(d*x+c))^(1/2)/a^2/d+4/11*e^2*csc(d*x+c)^
5*(e*csc(d*x+c))^(1/2)/a^2/d-4/231*e^2*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/
sin(1/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*c
sc(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.50, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3878, 3872, 2875, 2873, 2567, 2636, 2641, 2564, 14}

$$\frac{4e^2\csc^5(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d}-\frac{4e^2\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d}-\frac{2e^2\cot^3(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*e^2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(231*a^2*d) + (16*e^2*Cot[c + d*
x]*Csc[c + d*x]^2*Sqrt[e*Csc[c + d*x]])/(77*a^2*d) - (2*e^2*Cot[c + d*x]^3*
Csc[c + d*x]^2*Sqrt[e*Csc[c + d*x]])/(11*a^2*d) - (4*e^2*Csc[c + d*x]^3*Sqr
t[e*Csc[c + d*x]])/(7*a^2*d) - (2*e^2*Cot[c + d*x]*Csc[c + d*x]^4*Sqrt[e*Cs
c[c + d*x]])/(11*a^2*d) + (4*e^2*Csc[c + d*x]^5*Sqrt[e*Csc[c + d*x]])/(11*a
^2*d) + (4*e^2*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(231*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
```

```
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(e\csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \left(e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))^2\sin^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \left(e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2\sin^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{\left(e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{13}{2}}(c+dx)}\,dx}{a^4}$$

$$= \frac{\left(e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{13}{2}}(c+dx)}-\frac{2a^2\cos^3(c+dx)}{\sin^{\frac{13}{2}}(c+dx)}+\frac{a^2\cos^4(c+dx)}{\sin^{\frac{13}{2}}(c+dx)}\right)dx}{a^4}$$

$$= \frac{\left(e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{\sin^{\frac{13}{2}}(c+dx)}\,dx}{a^2} + \frac{\left(e^2\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx}\right.}{a^2}$$

$$= -\frac{2e^2\cot^3(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d} - \frac{2e^2\cot(c+dx)\csc^4(c+dx)\sqrt{e\csc}}{11a^2d}$$

$$= \frac{16e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{77a^2d} - \frac{2e^2\cot^3(c+dx)\csc^2(c+dx)\sqrt{e\csc}}{11a^2d}$$

$$= -\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{231a^2d} + \frac{16e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{77a^2d} -$$

$$= -\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{231a^2d} + \frac{16e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{77a^2d} -$$

Mathematica [A] time = 1.20, size = 115, normalized size = 0.43

$$-\frac{e^3\csc^2\left(\frac{1}{2}(c+dx)\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\left(97\cos(c+dx)+4\cos(2(c+dx))+\cos(3(c+dx))+\sin^{\frac{11}{2}}(c+dx)\csc^4\right.}{3696a^2d\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Csc[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/3696*(e^3*Csc[(c + d*x)/2]^2*Sec[(c + d*x)/2]^6*(52 + 97*Cos[c + d*x] +
4*Cos[2*(c + d*x)] + Cos[3*(c + d*x)] + Csc[(c + d*x)/2]^4*EllipticF[(-2*c
+ Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(11/2)))/(a^2*d*Sqrt[e*Csc[c + d*x]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\csc(dx+c)}\,e^2\csc(dx+c)^2}{a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))*e^2*csc(d*x + c)^2/(a^2*sec(d*x + c)^2 + 2*a^
2*sec(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\csc(dx+c))^{\frac{5}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*csc(d*x + c))^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

maple [C] time = 1.40, size = 609, normalized size = 2.27

$$\frac{(-1+\cos(dx+c))^4 \left(2i\sin(dx+c)\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}}\sqrt{\frac{-i\cos(dx+c)+\sin(dx+c)+i}{\sin(dx+c)}}\ \mathrm{EllipticF}\left(\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/231/a^2/d*(-1+cos(d*x+c))^4*(2*I*sin(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/
sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF
(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^3*(
-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+6*I*sin(d*x+c)*((I*cos(d*x+c)-I+sin(d*
x+c))/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*Ell
ipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+
c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+6*I*EllipticF(((I*cos(d*x+c)-I+s
in(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)*((-I*cos(d*x+c)+sin(d*
x+c)+I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-
I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+2*I*((-I*cos(d*x+c)+sin(d*x+
c)+I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticF(((I*cos(d*x+c)-I+sin
(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-2*2^(1/2)*cos(d*x+c)^3-4*cos(d*x+c)
^2*2^(1/2)-47*cos(d*x+c)*2^(1/2)-24*2^(1/2))*(1+cos(d*x+c))^2*(e/sin(d*x+c)
)^(5/2)/sin(d*x+c)^7*2^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2\left(\frac{e}{\sin(c+d\,x)}\right)^{5/2}}{a^2\,(\cos(c+d\,x)+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/sin(c + d*x))^(5/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^2*(e/sin(c + d*x))^(5/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.301 $\int \frac{(e\csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=250

$$\frac{4e\csc^4(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d}-\frac{4e\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{5a^2d}-\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{15a^2d}-\frac{2e\cot^3(c+dx)}{}$$

```
[Out] -4/15*e*cos(d*x+c)*(e*csc(d*x+c))^(1/2)/a^2/d+16/45*e*cot(d*x+c)*csc(d*x+c)
*(e*csc(d*x+c))^(1/2)/a^2/d-2/9*e*cot(d*x+c)^3*csc(d*x+c)*(e*csc(d*x+c))^(1
/2)/a^2/d-4/5*e*csc(d*x+c)^2*(e*csc(d*x+c))^(1/2)/a^2/d-2/9*e*cot(d*x+c)*cs
c(d*x+c)^3*(e*csc(d*x+c))^(1/2)/a^2/d+4/9*e*csc(d*x+c)^4*(e*csc(d*x+c))^(1/
2)/a^2/d+4/15*e*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*
x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*
x+c)^(1/2)/a^2/d
```

Rubi [A] time = 0.49, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3878, 3872, 2875, 2873, 2567, 2636, 2639, 2564, 14}

$$\frac{4e\csc^4(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d}-\frac{4e\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{5a^2d}-\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{15a^2d}-\frac{2e\cot^3(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*e*Cos[c + d*x]*Sqrt[e*Csc[c + d*x]])/(15*a^2*d) + (16*e*Cot[c + d*x]*Cs
c[c + d*x]*Sqrt[e*Csc[c + d*x]])/(45*a^2*d) - (2*e*Cot[c + d*x]^3*Csc[c + d
*x]*Sqrt[e*Csc[c + d*x]])/(9*a^2*d) - (4*e*Csc[c + d*x]^2*Sqrt[e*Csc[c + d*
x]])/(5*a^2*d) - (2*e*Cot[c + d*x]*Csc[c + d*x]^3*Sqrt[e*Csc[c + d*x]])/(9*
a^2*d) + (4*e*Csc[c + d*x]^4*Sqrt[e*Csc[c + d*x]])/(9*a^2*d) - (4*e*Sqrt[e*
Csc[c + d*x]]*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(15*a^2*
d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
```

```
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(e\csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))^2\sin^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2\sin^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{11}{2}}(c+dx)}\,dx}{a^4}$$
$$= \frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{11}{2}}(c+dx)} - \frac{2a^2\cos^3(c+dx)}{\sin^{\frac{11}{2}}(c+dx)} + \frac{a^2\cos^4(c+dx)}{\sin^{\frac{11}{2}}(c+dx)}\right)dx}{a^4}$$
$$= \frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{\sin^{\frac{11}{2}}(c+dx)}\,dx}{a^2} + \frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right.}{a^2}$$
$$= -\frac{2e\cot^3(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d} - \frac{2e\cot(c+dx)\csc^3(c+dx)\sqrt{e\csc(c}}{9a^2d}$$
$$= \frac{16e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{45a^2d} - \frac{2e\cot^3(c+dx)\csc(c+dx)\sqrt{e\csc(c}}{9a^2d}$$
$$= -\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{15a^2d} + \frac{16e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{45a^2d} - 2e$$
$$= -\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{15a^2d} + \frac{16e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{45a^2d} - 2e$$

Mathematica [C] time = 1.83, size = 247, normalized size = 0.99

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(e\csc(c+dx))^{3/2}\left(-\frac{2\tan(c+dx)\left((13\cos(c+dx)+8)\sec^4\left(\frac{1}{2}(c+dx)\right)+24\sec(c)\cos(dx)\right)}{d} + \frac{16\sqrt{2}e^{i(c-dx)}}{\ }\right.}{45a^2(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Csc[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(e*Csc[c + d*x])^(3/2)*Sec[c + d*x]*((16*Sqrt[2]*E^(I*(
c - d*x))*Sqrt[(I*E^(I*(c + d*x)))/(-1 + E^((2*I)*(c + d*x)))]*(3 - 3*E^((2
*I)*(c + d*x)) + E^((2*I)*d*x)*(1 + E^((2*I)*c))*Sqrt[1 - E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])*Sec[c + d*x])/(d
*(1 + E^((2*I)*c))*Csc[c + d*x]^(3/2)) - (2*(24*Cos[d*x]*Sec[c] + (8 + 13*C
os[c + d*x])*Sec[(c + d*x)/2]^4)*Tan[c + d*x])/d))/(45*a^2*(1 + Sec[c + d*x
])^2)
```

fricas [F] time = 0.61, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\csc(dx+c)}\,e\csc(dx+c)}{a^2\sec(dx+c)^2+2a^2\sec(dx+c)+a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))*e*csc(d*x + c)/(a^2*sec(d*x + c)^2 + 2*a^2*se
c(d*x + c) + a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(e\csc(dx+c))^{\frac{3}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*csc(d*x + c))^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

maple [C] time = 1.34, size = 1044, normalized size = 4.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/45/a^2/d*(-1+cos(d*x+c))^2*(12*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(
1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+
c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^3*(-I*(-1+cos(d*
x+c))/sin(d*x+c))^(1/2)-6*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*((
-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+si
n(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^3*(-I*(-1+cos(d*x+c))/s
in(d*x+c))^(1/2)+36*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*((-I*cos
(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)-I+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x
+c))^(1/2)-18*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*((-I*cos(d*x+c
)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/si
n(d*x+c))^(1/2),1/2*2^(1/2))*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(
1/2)+36*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)-I+s
in(d*x+c))/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2
)*EllipticE(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-18*
cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c
))/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*Ellipt
icF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+12*(-I*(-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2
)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)-
I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))-6*(-I*(-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+si
n(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*
x+c))^(1/2),1/2*2^(1/2))-6*cos(d*x+c)^2*2^(1/2)-25*cos(d*x+c)*2^(1/2)-14*2^
(1/2))*(e/sin(d*x+c))^(3/2)/sin(d*x+c)^3*2^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2\,{\left(\frac{e}{\sin(c+d\,x)}\right)}^{3/2}}{a^2\,{\left(\cos(c+d\,x)+1\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e/sin(c + d*x))^(3/2)/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^2*(e/sin(c + d*x))^(3/2))/(a^2*(cos(c + d*x) + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(e\csc(c+dx))^{\frac{3}{2}}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral((e*csc(c + d*x))**(3/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)
/a**2
```

3.302 $\int \frac{\sqrt{e\csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=201

$$\frac{4\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d}-\frac{4\csc(c+dx)\sqrt{e\csc(c+dx)}}{3a^2d}-\frac{2\cot^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d}-\frac{2\cot(c+dx)\csc^2(c+}{7a^2}$$

```
[Out] 16/21*cot(d*x+c)*(e*csc(d*x+c))^(1/2)/a^2/d-2/7*cot(d*x+c)^3*(e*csc(d*x+c))
^(1/2)/a^2/d-4/3*csc(d*x+c)*(e*csc(d*x+c))^(1/2)/a^2/d-2/7*cot(d*x+c)*csc(d
*x+c)^2*(e*csc(d*x+c))^(1/2)/a^2/d+4/7*csc(d*x+c)^3*(e*csc(d*x+c))^(1/2)/a^
2/d-20/21*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ell
ipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))*(e*csc(d*x+c))^(1/2)*sin(d*x+c)^(
1/2)/a^2/d
```

Rubi [A] time = 0.45, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3878, 3872, 2875, 2873, 2567, 2636, 2641, 2564, 14}

$$\frac{4\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d}-\frac{4\csc(c+dx)\sqrt{e\csc(c+dx)}}{3a^2d}-\frac{2\cot^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d}-\frac{2\cot(c+dx)\csc^2(c+}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Csc[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (16*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(21*a^2*d) - (2*Cot[c + d*x]^3*Sqrt[
e*Csc[c + d*x]])/(7*a^2*d) - (4*Csc[c + d*x]*Sqrt[e*Csc[c + d*x]])/(3*a^2*d
) - (2*Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[e*Csc[c + d*x]])/(7*a^2*d) + (4*Csc
[c + d*x]^3*Sqrt[e*Csc[c + d*x]])/(7*a^2*d) + (20*Sqrt[e*Csc[c + d*x]]*Elli
pticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a^2*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{e\csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{\sin(c+dx)}}\,dx$$
$$= \left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2\sqrt{\sin(c+dx)}}\,dx$$
$$= \frac{\left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{9}{2}}(c+dx)}\,dx}{a^4}$$
$$= \frac{\left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{9}{2}}(c+dx)} - \frac{2a^2\cos^3(c+dx)}{\sin^{\frac{9}{2}}(c+dx)} + \frac{a^2\cos^4(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}\right)dx}{a^4}$$
$$= \frac{\left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}\,dx}{a^2} + \frac{\left(\sqrt{e\csc(c+dx)}\,\sqrt{\sin(c+dx)}\right)\int \frac{\cos}{\sin}}{a^2}$$
$$= -\frac{2\cot^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} - \frac{2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} - \frac{\left(2\sqrt{e\,c}\right.}{}$$
$$= \frac{16\cot(c+dx)\sqrt{e\csc(c+dx)}}{21a^2d} - \frac{2\cot^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} - \frac{2\cot(c+dx)\csc^2(c}{7a}$$
$$= \frac{16\cot(c+dx)\sqrt{e\csc(c+dx)}}{21a^2d} - \frac{2\cot^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} - \frac{4\csc(c+dx)\sqrt{e\csc(}}{3a^2d}$$

Mathematica [**A**] time = 0.75, size = 82, normalized size = 0.41

$$-\frac{4\csc^3(c+dx)\sqrt{e\csc(c+dx)}\left(2\sin^4\left(\frac{1}{2}(c+dx)\right)(11\cos(c+dx)+8)+5\sin^{\frac{7}{2}}(c+dx)F\left(\frac{1}{4}(-2c-2dx+\pi)\middle|2\right)\right)}{21a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Csc[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*Csc[c + d*x]^3*Sqrt[e*Csc[c + d*x]]*(2*(8 + 11*Cos[c + d*x])*Sin[(c + d
*x)/2]^4 + 5*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(7/2)))/(21*a
^2*d)
```

fricas [**F**] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\csc(dx+c)}}{a^2\sec(dx+c)^2 + 2a^2\sec(dx+c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^
2), x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{e\csc(dx+c)}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] integrate(sqrt(e*csc(d*x + c))/(a*sec(d*x + c) + a)^2, x)

maple [C] time = 1.56, size = 474, normalized size = 2.36

$$-\frac{\sqrt{\frac{e}{\sin(dx+c)}}\,(1+\cos(dx+c))^2\,(-1+\cos(dx+c))^3\left(10i\sin(dx+c)\sqrt{-\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}}\,\left(\cos^2(dx+c)\right)\sqrt{\frac{i\cos(d}{}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x)

[Out] -1/21/a^2/d*(e/sin(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^3*(10*I*sin(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+20*I*cos(d*x+c)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*sin(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)+10*I*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))+11*cos(d*x+c)^2*2^(1/2)-3*cos(d*x+c)*2^(1/2)-8*2^(1/2))/sin(d*x+c)^7*2^(1/2)

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Timed out

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c+d\,x)^2\,\sqrt{\frac{e}{\sin(c+d\,x)}}}{a^2\,(\cos(c+d\,x)+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e/sin(c + d*x))^(1/2)/(a + a/cos(c + d*x))^2,x)

[Out] int((cos(c + d*x)^2*(e/sin(c + d*x))^(1/2))/(a^2*(cos(c + d*x) + 1)^2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sqrt{e\csc(c+dx)}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e*csc(d*x+c))**(1/2)/(a+a*sec(d*x+c))**2,x)

[Out] Integral(sqrt(e*csc(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2

3.303 $\int \frac{1}{\sqrt{e \csc(c+dx)}\,(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=199

$$\frac{4\csc^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}-\frac{4\csc(c+dx)}{a^2d\sqrt{e\csc(c+dx)}}-\frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}-\frac{2\cot(c+dx)\csc^2(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}+\frac{16\cot(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}+$$

```
[Out] 16/5*cot(d*x+c)/a^2/d/(e*csc(d*x+c))^(1/2)-2/5*cot(d*x+c)^3/a^2/d/(e*csc(d*
x+c))^(1/2)-4*csc(d*x+c)/a^2/d/(e*csc(d*x+c))^(1/2)-2/5*cot(d*x+c)*csc(d*x+
c)^2/a^2/d/(e*csc(d*x+c))^(1/2)+4/5*csc(d*x+c)^3/a^2/d/(e*csc(d*x+c))^(1/2)
-28/5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ellipti
cE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a^2/d/(e*csc(d*x+c))^(1/2)/sin(d*x+c)
^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3878, 3872, 2875, 2873, 2567, 2636, 2639, 2564, 14}

$$\frac{4\csc^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}-\frac{4\csc(c+dx)}{a^2d\sqrt{e\csc(c+dx)}}-\frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}-\frac{2\cot(c+dx)\csc^2(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}+\frac{16\cot(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}+$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (16*Cot[c + d*x])/(5*a^2*d*Sqrt[e*Csc[c + d*x]]) - (2*Cot[c + d*x]^3)/(5*a^
2*d*Sqrt[e*Csc[c + d*x]]) - (4*Csc[c + d*x])/(a^2*d*Sqrt[e*Csc[c + d*x]]) -
 (2*Cot[c + d*x]*Csc[c + d*x]^2)/(5*a^2*d*Sqrt[e*Csc[c + d*x]]) + (4*Csc[c
+ d*x]^3)/(5*a^2*d*Sqrt[e*Csc[c + d*x]]) + (28*EllipticE[(c - Pi/2 + d*x)/2
, 2])/(5*a^2*d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2}dx = \frac{\int \frac{\sqrt{\sin(c+dx)}}{(a+a\sec(c+dx))^2}dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)\sqrt{\sin(c+dx)}}{(-a-a\cos(c+dx))^2}dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{7}{2}}(c+dx)}dx}{a^4\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{7}{2}}(c+dx)} - \frac{2a^2\cos^3(c+dx)}{\sin^{\frac{7}{2}}(c+dx)} + \frac{a^2\cos^4(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}\right)dx}{a^4\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}dx}{a^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{\int \frac{\cos^4(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}dx}{a^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{}{a}$$

$$= -\frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+dx)\csc^2(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{2\int \frac{}{\sin^{\frac{3}{2}}}}{5a^2\sqrt{e\csc(c+d}}$$

$$= \frac{16\cot(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+dx)\csc^2(c+d}{5a^2d\sqrt{e\csc(c+dx)}}$$

$$= \frac{16\cot(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{4\csc(c+dx)}{a^2d\sqrt{e\csc(c+dx)}} - \frac{2}{}$$

Mathematica [C] time = 1.60, size = 252, normalized size = 1.27

$$\frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\csc(c+dx)}\sec^2(c+dx)\left(-3\sqrt{\csc(c+dx)}\left((5\cos(2c)-23)\sec(c)\cos(dx)-2\left(5\sin(c)\sin(\right.\right.\right.}{15a^2d(\sec(c+dx)+1)^2\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (4*Cos[(c + d*x)/2]^4*Sqrt[Csc[c + d*x]]*Sec[c + d*x]^2*((-28*Sqrt[2]*E^(I*
(c - d*x))*Sqrt[(I*E^(I*(c + d*x)))/(-1 + E^((2*I)*(c + d*x)))]*(3 - 3*E^((
2*I)*(c + d*x)) + E^((2*I)*d*x)*(1 + E^((2*I)*c))*Sqrt[1 - E^((2*I)*(c + d*
x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))]))/(1 + E^((2*I)*
c)) - 3*Sqrt[Csc[c + d*x]]*((-23 + 5*Cos[2*c])*Cos[d*x]*Sec[c] - 2*(-10 + S
ec[(c + d*x)/2]^2 + 5*Sin[c]*Sin[d*x]))))/(15*a^2*d*Sqrt[e*Csc[c + d*x]]*(1
 + Sec[c + d*x])^2)
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\csc(dx+c)}}{a^2e\csc(dx+c)\sec(dx+c)^2 + 2a^2e\csc(dx+c)\sec(dx+c) + a^2e\csc(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))/(a^2*e*csc(d*x + c)*sec(d*x + c)^2 + 2*a^2*e*
csc(d*x + c)*sec(d*x + c) + a^2*e*csc(d*x + c)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\csc(dx+c)}\,(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)^2), x)
```

maple [C] time = 1.30, size = 811, normalized size = 4.08

$$\frac{(-1+\cos(dx+c))\left(28\left(\cos^2(dx+c)\right)\sqrt{-\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}}\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}}\sqrt{-\frac{i\cos(dx+c)-i-\sin(dx+c)}{\sin(dx+c)}}\ \text{Ellipti}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x)
```

```
[Out] 1/5/a^2/d*(-1+cos(d*x+c))*(28*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(
d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2*2^(1/2))-14*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c)
)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2
),1/2*2^(1/2))+56*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)
*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)-I
+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))
^(1/2)-28*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*co
s(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x
+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+2
8*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+
c))/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1
/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-14*((I*cos(d*x+c)-I+
sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/
2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-
I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+5*cos(d*x+c)^2*2^(1/2)+cos(d*x+c)*2^(1/
2)-6*2^(1/2))/(e/sin(d*x+c))^(1/2)/sin(d*x+c)^3*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{e\csc(dx+c)}\,(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2}{a^2\,\sqrt{\frac{e}{\sin(c+d\,x)}}\,\left(\cos(c+d\,x)+1\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(1/2)),x)
```

```
[Out] int(cos(c + d*x)^2/(a^2*(e/sin(c + d*x))^(1/2)*(cos(c + d*x) + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\sqrt{e\csc{(c+dx)}}\sec^2{(c+dx)}+2\sqrt{e\csc{(c+dx)}}\sec{(c+dx)}+\sqrt{e\csc{(c+dx)}}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*csc(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*csc(c + d*x))*sec(c + d*x)**2 + 2*sqrt(e*csc(c + d*x))*s
ec(c + d*x) + sqrt(e*csc(c + d*x))), x)/a**2
```

3.304 $\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=213

$$\frac{4\csc^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}}+\frac{4}{a^2de\sqrt{e\csc(c+dx)}}-\frac{4\cos(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}}-\frac{2\cot(c+dx)\csc(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}}-\frac{2\cos(c+dx)\cot}{3a^2de\sqrt{e\csc(c}}$$

```
[Out] 4/a^2/d/e/(e*csc(d*x+c))^(1/2)-4/3*cos(d*x+c)/a^2/d/e/(e*csc(d*x+c))^(1/2)-
2/3*cos(d*x+c)*cot(d*x+c)^2/a^2/d/e/(e*csc(d*x+c))^(1/2)-2/3*cot(d*x+c)*csc
(d*x+c)/a^2/d/e/(e*csc(d*x+c))^(1/2)+4/3*csc(d*x+c)^2/a^2/d/e/(e*csc(d*x+c)
)^(1/2)+4*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c+1/4*Pi+1/2*d*x)*Ell
ipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a^2/d/e/(e*csc(d*x+c))^(1/2)/sin(
d*x+c)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3878, 3872, 2875, 2873, 2567, 2641, 2564, 14, 2569}

$$\frac{4\csc^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}}+\frac{4}{a^2de\sqrt{e\csc(c+dx)}}-\frac{4\cos(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}}-\frac{2\cot(c+dx)\csc(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}}-\frac{2\cos(c+dx)\cot}{3a^2de\sqrt{e\csc(c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] 4/(a^2*d*e*Sqrt[e*Csc[c + d*x]]) - (4*Cos[c + d*x])/(3*a^2*d*e*Sqrt[e*Csc[c
 + d*x]]) - (2*Cos[c + d*x]*Cot[c + d*x]^2)/(3*a^2*d*e*Sqrt[e*Csc[c + d*x]]
) - (2*Cot[c + d*x]*Csc[c + d*x])/(3*a^2*d*e*Sqrt[e*Csc[c + d*x]]) + (4*Csc
[c + d*x]^2)/(3*a^2*d*e*Sqrt[e*Csc[c + d*x]]) - (4*EllipticF[(c - Pi/2 + d*
x)/2, 2])/(a^2*d*e*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
```

```
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sin^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)\sin^{\frac{3}{2}}(c+dx)}{(-a-a\cos(c+dx))^2}\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{5}{2}}(c+dx)}\,dx}{a^4e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{5}{2}}(c+dx)} - \frac{2a^2\cos^3(c+dx)}{\sin^{\frac{5}{2}}(c+dx)} + \frac{a^2\cos^4(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\right)dx}{a^4e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx}{a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{\int \frac{\cos^4(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx}{a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+d}$$

$$= -\frac{2\cos(c+dx)\cot^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+dx)\csc(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{}{3a^2e\sqrt{}}$$

$$= -\frac{4\cos(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\cos(c+dx)\cot^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+}{3a^2de\sqrt{}}$$

$$= \frac{4}{a^2de\sqrt{e\csc(c+dx)}} - \frac{4\cos(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\cos(c+dx)\cot^2}{3a^2de\sqrt{e\csc(c-}}$$

Mathematica [A] time = 0.56, size = 101, normalized size = 0.47

$$\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{\sin(c+dx)}\,(10\cos(c+dx)-\cos(2(c+dx))+15)+12(\cos(c+dx)+1)F\left(\frac{1}{4}(-2c-2dx+\right.\right.}{6a^2d\sin^{\frac{3}{2}}(c+dx)(e\csc(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (Sec[(c + d*x)/2]^2*(12*(1 + Cos[c + d*x])*EllipticF[(-2*c + Pi - 2*d*x)/4,
 2] + (15 + 10*Cos[c + d*x] - Cos[2*(c + d*x)])*Sqrt[Sin[c + d*x]]))/(6*a^2
*d*(e*Csc[c + d*x])^(3/2)*Sin[c + d*x]^(3/2))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\csc(dx+c)}}{a^2e^2\csc(dx+c)^2\sec(dx+c)^2 + 2\,a^2e^2\csc(dx+c)^2\sec(dx+c) + a^2e^2\csc(dx+c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))/(a^2*e^2*csc(d*x + c)^2*sec(d*x + c)^2 + 2*a^
2*e^2*csc(d*x + c)^2*sec(d*x + c) + a^2*e^2*csc(d*x + c)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\csc(dx+c))^{\frac{3}{2}}\,(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2), x)
```

maple [C] time = 1.23, size = 332, normalized size = 1.56

$$\frac{\left(-6 i \cos \left(d x+c\right) \sqrt{\frac{i \cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}}\, \sqrt{-\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}}\, \sqrt{-\frac{i \cos(dx+c)-i-\sin(dx+c)}{\sin(dx+c)}}\, \sin\left(dx+c\right) \mathrm{EllipticF}\left(\sqrt{\frac{i \cos(dx\cdots}{\cdots}}\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/3/a^2/d*(-6*I*cos(d*x+c)*sin(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+
c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((I*cos
(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/si
n(d*x+c))^(1/2)-6*I*sin(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/
2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)
-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c
))^(1/2)+2^(1/2)*cos(d*x+c)^3-6*cos(d*x+c)^2*2^(1/2)-3*cos(d*x+c)*2^(1/2)+8
*2^(1/2))/(e/sin(d*x+c))^(3/2)/sin(d*x+c)^3*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(e \csc\left(dx+c\right)\right)^{\frac{3}{2}} \left(a \sec\left(dx+c\right)+a\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2}{a^2\,{\left(\frac{e}{\sin\left(c+d\,x\right)}\right)}^{3/2}\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(3/2)),x)
```

```
[Out] int(cos(c + d*x)^2/(a^2*(e/sin(c + d*x))^(3/2)*(cos(c + d*x) + 1)^2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{\left(e \csc\left(c+dx\right)\right)^{\frac{3}{2}} \sec^{2}\left(c+dx\right)+2\left(e \csc\left(c+dx\right)\right)^{\frac{3}{2}} \sec\left(c+dx\right)+\left(e \csc\left(c+dx\right)\right)^{\frac{3}{2}}}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(1/((e*csc(c + d*x))**(3/2)*sec(c + d*x)**2 + 2*(e*csc(c + d*x))**(
3/2)*sec(c + d*x) + (e*csc(c + d*x))**(3/2)), x)/a**2
```

3.305 $\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=215

$$\frac{4\csc(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}}-\frac{2\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}}+\frac{4\sin(c+dx)}{3a^2de^2\sqrt{e\csc(c+dx)}}-\frac{2\cos^2(c+dx)\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}}-\frac{12\sin(c+dx}{5a^2de^2\sqrt{e\,c}}$$

```
[Out] -2*cot(d*x+c)/a^2/d/e^2/(e*csc(d*x+c))^(1/2)-2*cos(d*x+c)^2*cot(d*x+c)/a^2/
d/e^2/(e*csc(d*x+c))^(1/2)+4*csc(d*x+c)/a^2/d/e^2/(e*csc(d*x+c))^(1/2)+4/3*
sin(d*x+c)/a^2/d/e^2/(e*csc(d*x+c))^(1/2)-12/5*cos(d*x+c)*sin(d*x+c)/a^2/d/
e^2/(e*csc(d*x+c))^(1/2)+44/5*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1/2*c
+1/4*Pi+1/2*d*x)*EllipticE(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a^2/d/e^2/(e*
csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.47, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3878, 3872, 2875, 2873, 2567, 2639, 2564, 14, 2569}

$$\frac{4\csc(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}}-\frac{2\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}}+\frac{4\sin(c+dx)}{3a^2de^2\sqrt{e\csc(c+dx)}}-\frac{2\cos^2(c+dx)\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}}-\frac{12\sin(c+dx}{5a^2de^2\sqrt{e\,c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (-2*Cot[c + d*x])/(a^2*d*e^2*Sqrt[e*Csc[c + d*x]]) - (2*Cos[c + d*x]^2*Cot[
c + d*x])/(a^2*d*e^2*Sqrt[e*Csc[c + d*x]]) + (4*Csc[c + d*x])/(a^2*d*e^2*Sq
rt[e*Csc[c + d*x]]) - (44*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*a^2*d*e^2*Sq
rt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]) + (4*Sin[c + d*x])/(3*a^2*d*e^2*Sqrt
[e*Csc[c + d*x]]) - (12*Cos[c + d*x]*Sin[c + d*x])/(5*a^2*d*e^2*Sqrt[e*Csc[
c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
```

```
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}dx = \frac{\int \frac{\sin^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{\int \frac{\cos^2(c+dx)\sin^{\frac{5}{2}}(c+dx)}{(-a-a\cos(c+dx))^2}dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{3}{2}}(c+dx)}dx}{a^4e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{\int \left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)} - \frac{2a^2\cos^3(c+dx)}{\sin^{\frac{3}{2}}(c+dx)} + \frac{a^2\cos^4(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\right)dx}{a^4e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{\int \frac{\cos^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}dx}{a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{\int \frac{\cos^4(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}dx}{a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+}$$
$$= -\frac{2\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{2\cos^2(c+dx)\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{2\int}{a^2e^2\sqrt{e\csc}}$$
$$= -\frac{2\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{2\cos^2(c+dx)\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{4E\Big(}{a^2de^2\sqrt{e\,c}}$$
$$= -\frac{2\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{2\cos^2(c+dx)\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} + \frac{4\csc(}{a^2de^2\sqrt{e\,c}}$$

Mathematica [C] time = 2.14, size = 125, normalized size = 0.58

$$\frac{88\sqrt{1-e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};e^{2i(c+dx)}\right)(\cot(c+dx)+i) - 123\cot(c+dx) + \csc(c+dx)(-264i\sin(c+dx) - 20}{30a^2de^2\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (-123*Cot[c + d*x] + 88*Sqrt[1 - E^((2*I)*(c + d*x))]*(I + Cot[c + d*x])*Hy
pergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))] + Csc[c + d*x]*(140 - 2
0*Cos[2*(c + d*x)] + 3*Cos[3*(c + d*x)] - (264*I)*Sin[c + d*x]))/(30*a^2*d*
e^2*Sqrt[e*Csc[c + d*x]])
```

fricas [F] time = 0.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\csc(dx+c)}}{a^2e^3\csc(dx+c)^3\sec(dx+c)^2 + 2a^2e^3\csc(dx+c)^3\sec(dx+c) + a^2e^3\csc(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))/(a^2*e^3*csc(d*x + c)^3*sec(d*x + c)^2 + 2*a^
2*e^3*csc(d*x + c)^3*sec(d*x + c) + a^2*e^3*csc(d*x + c)^3), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\csc(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)^2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2), x)
```

maple [C] time = 1.30, size = 563, normalized size = 2.62

$$\frac{\left(-66\cos\left(dx+c\right)\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}}\,\sqrt{-\frac{i\cos(dx+c)-i-\sin(dx+c)}{\sin(dx+c)}}\ \mathrm{EllipticF}\left(\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}},\frac{\sqrt{2}}{2}\right)\sqrt{-\frac{i(-1+\cos}{\sin(}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/15/a^2/d*(-66*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(
-(I*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+s
in(d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(
1/2)+132*cos(d*x+c)*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos
(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)-I+sin(d*x+
c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+3*
2^(1/2)*cos(d*x+c)^3-66*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I
*cos(d*x+c)-I-sin(d*x+c))/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(
d*x+c))/sin(d*x+c))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2
)+132*((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(
d*x+c))/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c)
)^(1/2),1/2*2^(1/2))*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-10*cos(d*x+c)^2*
2^(1/2)+33*cos(d*x+c)*2^(1/2)-26*2^(1/2))/(e/sin(d*x+c))^(5/2)/sin(d*x+c)^3
*2^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2}{a^2\,{\left(\frac{e}{\sin\left(c+d\,x\right)}\right)}^{5/2}\,{\left(\cos\left(c+d\,x\right)+1\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(5/2)),x)
```

```
[Out] int(cos(c + d*x)^2/(a^2*(e/sin(c + d*x))^(5/2)*(cos(c + d*x) + 1)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

3.306 $\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=172

$$-\frac{4}{a^2de^3\sqrt{e\csc(c+dx)}}+\frac{2\cos^3(c+dx)}{7a^2de^3\sqrt{e\csc(c+dx)}}+\frac{26\cos(c+dx)}{21a^2de^3\sqrt{e\csc(c+dx)}}+\frac{4\sin^2(c+dx)}{5a^2de^3\sqrt{e\csc(c+dx)}}+\frac{52F\left(\frac{1}{2}\right.}{21a^2de^3\sqrt{\sin}}$$

```
[Out] -4/a^2/d/e^3/(e*csc(d*x+c))^(1/2)+26/21*cos(d*x+c)/a^2/d/e^3/(e*csc(d*x+c))
^(1/2)+2/7*cos(d*x+c)^3/a^2/d/e^3/(e*csc(d*x+c))^(1/2)+4/5*sin(d*x+c)^2/a^2
/d/e^3/(e*csc(d*x+c))^(1/2)-52/21*(sin(1/2*c+1/4*Pi+1/2*d*x)^2)^(1/2)/sin(1
/2*c+1/4*Pi+1/2*d*x)*EllipticF(cos(1/2*c+1/4*Pi+1/2*d*x),2^(1/2))/a^2/d/e^3
/(e*csc(d*x+c))^(1/2)/sin(d*x+c)^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3878, 3872, 2875, 2873, 2569, 2641, 2564, 14}

$$-\frac{4}{a^2de^3\sqrt{e\csc(c+dx)}}+\frac{2\cos^3(c+dx)}{7a^2de^3\sqrt{e\csc(c+dx)}}+\frac{26\cos(c+dx)}{21a^2de^3\sqrt{e\csc(c+dx)}}+\frac{4\sin^2(c+dx)}{5a^2de^3\sqrt{e\csc(c+dx)}}+\frac{52F\left(\frac{1}{2}\right.}{21a^2de^3\sqrt{\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Csc[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] -4/(a^2*d*e^3*Sqrt[e*Csc[c + d*x]]) + (26*Cos[c + d*x])/(21*a^2*d*e^3*Sqrt[
e*Csc[c + d*x]]) + (2*Cos[c + d*x]^3)/(7*a^2*d*e^3*Sqrt[e*Csc[c + d*x]]) +
(52*EllipticF[(c - Pi/2 + d*x)/2, 2])/(21*a^2*d*e^3*Sqrt[e*Csc[c + d*x]]*Sq
rt[Sin[c + d*x]]) + (4*Sin[c + d*x]^2)/(5*a^2*d*e^3*Sqrt[e*Csc[c + d*x]])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
```

```
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx &= \frac{\int \frac{\sin^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{\int \frac{\cos^2(c+dx)\sin^{\frac{7}{2}}(c+dx)}{(-a-a\cos(c+dx))^2}\,dx}{e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sqrt{\sin(c+dx)}}\,dx}{a^4e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{\int \left(\frac{a^2\cos^2(c+dx)}{\sqrt{\sin(c+dx)}}-\frac{2a^2\cos^3(c+dx)}{\sqrt{\sin(c+dx)}}+\frac{a^2\cos^4(c+dx)}{\sqrt{\sin(c+dx)}}\right)dx}{a^4e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{\int \frac{\cos^2(c+dx)}{\sqrt{\sin(c+dx)}}\,dx}{a^2e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{\int \frac{\cos^4(c+dx)}{\sqrt{\sin(c+dx)}}\,dx}{a^2e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx}}\\
&= \frac{2\cos(c+dx)}{3a^2de^3\sqrt{e\csc(c+dx)}}+\frac{2\cos^3(c+dx)}{7a^2de^3\sqrt{e\csc(c+dx)}}+\frac{2\int \frac{}{\sqrt{\sin(}}}{3a^2e^3\sqrt{e\csc(c+}}\\
&= \frac{26\cos(c+dx)}{21a^2de^3\sqrt{e\csc(c+dx)}}+\frac{2\cos^3(c+dx)}{7a^2de^3\sqrt{e\csc(c+dx)}}+\frac{4F\left(\frac{1}{2}\left(c-\right.\right.}{3a^2de^3\sqrt{e\csc(c}}\\
&= -\frac{4}{a^2de^3\sqrt{e\csc(c+dx)}}+\frac{26\cos(c+dx)}{21a^2de^3\sqrt{e\csc(c+dx)}}+\frac{2\cos^3(c+d}{7a^2de^3\sqrt{e\csc(c}}
\end{aligned}$$

Mathematica [A] time = 2.20, size = 94, normalized size = 0.55

$$\frac{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\sqrt{\sin(c+dx)}(305\cos(c+dx)-84\cos(2(c+dx))+15\cos(3(c+dx))-756)-520\right.}{210a^2de^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Csc[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (Sqrt[e*Csc[c + d*x]]*(-520*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + (-756 + 3
05*Cos[c + d*x] - 84*Cos[2*(c + d*x)] + 15*Cos[3*(c + d*x)])*Sqrt[Sin[c + d
*x]])*Sqrt[Sin[c + d*x]])/(210*a^2*d*e^4)
```

fricas [F] time = 0.74, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{e\csc(dx+c)}}{a^2e^4\csc(dx+c)^4\sec(dx+c)^2+2\,a^2e^4\csc(dx+c)^4\sec(dx+c)+a^2e^4\csc(dx+c)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))/(a^2*e^4*csc(d*x + c)^4*sec(d*x + c)^2 + 2*a^
2*e^4*csc(d*x + c)^4*sec(d*x + c) + a^2*e^4*csc(d*x + c)^4), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(e\csc(dx+c))^{\frac{7}{2}}(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*csc(d*x + c))^(7/2)*(a*sec(d*x + c) + a)^2), x)
```

maple [C] time = 1.24, size = 224, normalized size = 1.30

$$\frac{\left(-130i\sqrt{-\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}}\,\sin(dx+c)\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}}\sqrt{-\frac{i\cos(dx+c)-i-\sin(dx+c)}{\sin(dx+c)}}\,\text{EllipticF}\left(\sqrt{\frac{i\cos(dx+c)-i+\sin(dx+c)}{\sin(dx+c)}}\right.\right.}{105a^2d\,(-1+\cos(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/105/a^2/d*(-130*I*sin(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*co
s(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-I-sin(d*x+c))/sin(
d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)-I+sin(d*x+c))/sin(d*x+c))^(1/2),1/2*
2^(1/2))+15*cos(d*x+c)^4*2^(1/2)-57*2^(1/2)*cos(d*x+c)^3+107*cos(d*x+c)^2*2
^(1/2)-233*cos(d*x+c)*2^(1/2)+168*2^(1/2))/(-1+cos(d*x+c))/(e/sin(d*x+c))^(
7/2)/sin(d*x+c)^3*2^(1/2)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)^2}{a^2\left(\frac{e}{\sin(c+d\,x)}\right)^{7/2}(\cos(c+d\,x)+1)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a/cos(c + d*x))^2*(e/sin(c + d*x))^(7/2)),x)
```

```
[Out] int(cos(c + d*x)^2/(a^2*(e/sin(c + d*x))^(7/2)*(cos(c + d*x) + 1)^2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))**(7/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
   sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
   hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
   sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
   sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```